Online Stereo 3D Simulation in Studying the Spherical Pendulum in Conservative Force Field

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Abstract
The current paper aims at presenting a modern e-learning method and tool that is utilized in teaching physics in the universities. An online stereo 3D simulation is used for e-learning mechanics and specifically the teaching of spherical pendulum as part of the General Physics course for students in the universities. This approach was realized on bases of interactive simulations on a personal computer, a part of the free online e-learning system at http://ialms.net/sim/. This system was practically applied with students at Sofia University, Bulgaria, among others. The shown simulation demonstrates the capabilities of the Web for online representations and visualizations of simulated physics processes that are hard to observe in laboratory conditions with all the accompanying parameters, vectors, quantities and trajectories. The discussed simulation allows the study of spherical pendulum both in conservative and non-conservative fields. The conservative force field is created by the earth gravity force, whose magnitude may be varied in the simulation from positive to negative values, while its direction is always vertical. The simulation also supports various non-conservative forces that may be applied to the pendulum. The current article concentrates on the case of conservative forces acting.
Keywords: Simulation of spherical pendulum, 3D-stereo simulation, E-learning physics through stereo 3D visualization.

Introduction

This article describes the capabilities of the modern web technologies through online stereo 3D simulations of physics phenomena. Here we present a part of a free online virtual laboratory used for e-learning mechanics that is used for teaching the spherical pendulum. This simulation is implemented in the General Physics course for students at Sofia University, Bulgaria (see http://ialms.net/sim/). The student may observe the pendulum motion in conservative and non-conservative force fields. Earth gravity force (adjustable by value) creates the conservative force field. Non-conservative force fields are the dissipative force of friction and external periodical sinusoidal, and horizontal forces along the Ox and Oy axes, which may be superimposed in the simulation. The current material concentrates on the case of conservative force field.

The usage of computer-aided learning can be traced back to the early ages of the electronic computer. Modern technologies have brought this e-learning tool to unpredicted avenues (Ivanov & Neacsu, 2011). Further, the latest technological advances have enabled the realization of virtual laboratories that are accessed online on the Internet. These approaches, among others, include utilization of mobile devices (Vogt & Kuhn, 2013) and the application of stereo 3D vision in online physics education (Zabunov, 2012). The latter promises future astonishing implementations of distant e-learning systems. The usage of computer as a learning tool concerning the pendulum is well discussed in recent scientific papers (Lieto et al., 1991; Erkal, 2000; Aggarwal et al., 2005; Liang et al., 2008; Wadhwa, 2009; Gintautas & Hübler, 2009).

The spherical pendulum motion is a fruitful mechanical setting. While simple to visualize and comprehend it has been the subject of analysis and discussions from scientific perspective to educational purpose. The motion of a classical pendulum in a gravitational conservative force field has been comprehensively discussed in a number of articles (Butikov, 1999; 2012; 2008; Essén & Apazidis, 2009; Anicin et al., 1993; Lane, 1970). On the other hand, the publication (Czudková & Musilová, 2000) reveals the truth that spherical pendulum and even its special cases, such as conical pendulum, seem to be difficult to understand not only for secondary school students but even for students taking introductory university courses in physics. In (Butikov, 1999) the physical pendulum is analysed and further examined using computerized simulations.

When introducing a simulation on the spherical pendulum, it should be noted that its dynamics equations define an analytical solution in several cases (Ochs, 2011; Yang et al., 2010; Johannessen, 2011). On the other hand, dumped cases along with special external force cases (forced pendulums) require approximate solutions (Beléndez et al., 2010; Johannessen, 2010; Qing-Xin & Pei, 2010).

Description of the problem

Practical teaching of the spherical pendulum and its special cases is impeded by the impossibility to observe all vector and scalar quantities, inherent to its kinematics and dynamics during a laboratory exercise. There are studies of performing experiments with the spherical pendulum in order to help students better understand certain problems (Russeva et al. 2010; Lee & Wong, 2011). The Foucault pendulum has generated substantial interest among researcher as well (Stanovnik, 2006; Mattila, 1991).

The presented computer simulation creates the opportunity to watch a stereoscopic image of the simulated process along with its dynamical motion of the complete spherical pendulum dynamic model. The scalar quantities are printed with their momentary values, while all vector variables are rendered as arrows with different colours and also their component values are printed. In this way, a number of complex questions, related to the spherical pendulum, can be observed and clarified, and students may place the simulation in diverse situations and initial conditions targeting the explanation of certain phenomena.

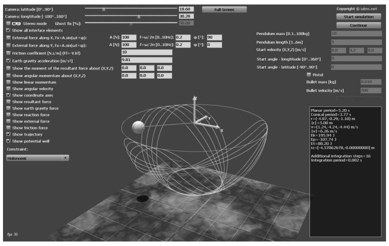

Figure 1. *Spherical pendulum in online simulation.*

A comparison with existing simulation systems

In recent years computers have become adequate to realizing sophisticated simulations of physical phenomena. 3D graphics are now capable of visualizing setups of laboratory settings and the realization of virtual laboratories is now possible (Gagnon, 2012; Lane, 2013; Wieman et al., 2010; Christian & Esquembre, 2007; Blanton, 2006).

Nevertheless, e-learning demands to reach further and requires more realistic visualizations and distant online access on the Internet. The author conducted research on the Internet and as a result an unassimilated resource for realizing 3D-computer graphical simulations of mechanical processes was revealed. Markedly distinct was the absence of serious simulations in the field of rigid body motion and the accompanying classical settings in teaching mechanics such as the spherical pendulum.

Some of the discovered online simulations that relate to pendulums are:

1. A planar pendulum simulation (http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=11). The viewer may observe pendulum momentum, period and other quantities.
2. Another planar pendulum simulation: http://www.myphysicslab.com/pendulum1.html. A plot of the angular acceleration depending on the angle is generated.

The presented simulations offer valuable visualization and interactive analysis of the studied settings, but they present only 2D graphical calculations and offer no visualization of all major vectors involved and other variables innate to the studied phenomenon. In contrast to the presented examples, the current simulation (Figure 1) offers an interactive process viewed in 3D stereoscopic view mode (Figure 2), which enhances dramatically the perception of the studied phenomenon and the understanding of a large number of relevant problems.

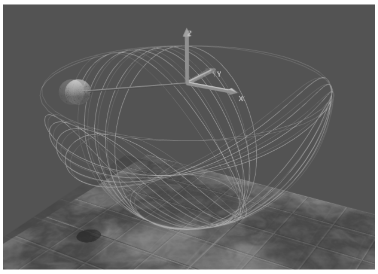

Figure 2. *The scene from Figure 1 shown in stereo 3D graphics mode (use red-cyan anaglyph glasses to observe).*

Description of the simulation interface and features

A user's manual may be accessed online on http://ialms.net/sim/3d-pendulum-simulation-tutorial. The described simulation realizes spherical pendulum motion under different conditions. The simplest case is when no external forces are acting. The motion is determined only by the constraint of pendulum massless bar (string). The second option is to introduce an external force (force of weight or gravity force), which is a conservative force and creates a conservative force field. Additionally, the observer may introduce non-conservative forces such as dissipative force of friction and external periodical sinusoidal forces. As mentioned earlier, the current paper gives an account of the conservative case. The latter is examined using gravity force acting along the vertical Oz axis.

We should mention that the pendulum is visualized in 3D graphics using textures and light shades to increase the ease of perception. All vectors are also presented in 3D view as arrows with different colours. Students may display the momentum of the pendulum as a vector. All acting forces can be shown with differently coloured vectors, along with angular momentum vector and moment of the resultant force vector. The two moments are calculated towards an origin point, which can be set with its three coordinates. The data panel presents values of the visualized vectors along with other scalar quantities related to the simulated process such as pendulum energies, the potential well and pendulum period (Gough, 1983; Turkyilmazoglu, 2010; Beléndez et al., 2011). The student may enter values for pendulum initial position and starting linear velocity. Learners may also select the pendulum length and mass in the corresponding fields. At any time the user may show or hide pendulum trajectory and its potential well by checking or unchecking the corresponding checkboxes.

Studying the spherical pendulum through stereoscopic 3D simulation

Studying the spherical pendulum in its special cases – planar and conical pendulum, is hampered because of the vector nature of its dynamical characteristics: force, momentum, moment of the resultant force (torque) and moment of momentum (angular momentum).

The simulation provides an opportunity to enhance the perception of vectors' mutual disposition in space. Vectors may be displayed consecutively, for example one could first display the forces acting, then the torque and then the angular momentum. Thus the following vector products are illustrated:

1. $\vec{M} = \vec{r} \times \vec{F}$. Here $\vec{F}$ is the resultant force. The moment of the resultant force is calculated towards different origins as mentioned above.
2. $\vec{L} = \vec{r} \times \vec{p}$. Again, the angular momentum is calculated towards different origins.

Thus the notions of moment towards origin and moment towards axis are clarified. The simulation demonstrates the relation between the two moments $\vec{M} = \frac{d\vec{L}}{dt}$, which are presented simultaneously (Arnold, 1989). Thus, their cause-effect dependency is manifested, i.e. the change of the angular momentum is due to moment of external force.

By the means of the simulation, a difficult to realize in practice effect is achieved – the altering of magnitude or introduction/elimination of external forces. For example, by the elimination of the gravity force, the pendulum continues to move along a circle solely under the action of the stretching (reaction) force of its bar (cord). The vector of the latter force points to the centre of the circular trajectory and is constant in its magnitude as is the magnitude of pendulum velocity, which is equal to the velocity magnitude in the moment of the gravity force elimination.

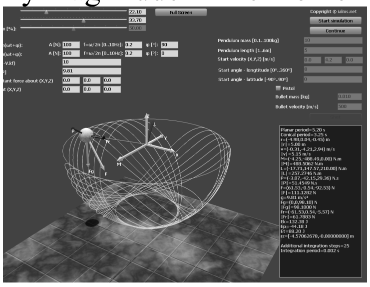

Figure 3. *Showing vectors along the simulation.*

Planar and conical pendulum

Both planar pendulum and conical pendulum proved to be a successful starting point to teaching mechanics in schools and universities (Tongaonkar & Khadse, 2011; Bender, 2007). On Figure 4 a planar pendulum is shown, which swings in the vertical plane Oxz (see the reference frame on Figure 4). Vector $\overrightarrow{Oy}$ is normal to the pendulum motion plane. Pendulum trajectory is a circle or a sector of a circle with radius r, equal to the pendulum bar length. Figure 5 demonstrates a conical pendulum.

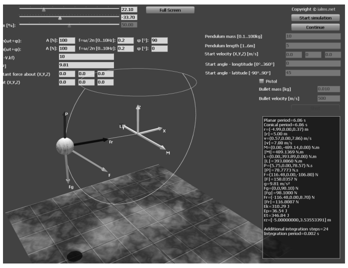

Figure 4. *Planar pendulum.*

These are, all, special cases of the spherical pendulum and hence they are examined first. The third special case, when the gravity force is not acting, is rather trivial, therefore it is not illustrated on a separate figure. The reader may place the simulation in such a condition and observe pendulum motion. It will be only mentioned that this case is also a planar case but with plane of movement not necessarily vertical as shown on Figure 4. It is important to remember that all examined cases are conservative and the force of air friction is excluded in the simulation. The only external force acting is the gravity force, while the bar force is defined as internal for the pendulum. The resultant force is, therefore, the sum of these two forces. The following dynamic variables are calculated and displayed:

1. The bar stretching force (reaction force) – blue vector.
2. Gravity force (constant) – green vector.
3. Resultant force – yellow vector.
4. Pendulum momentum proportional to its velocity – purple vector.
5. Moment of resultant force towards a given origin
6. Angular momentum towards a given origin

The simulation enables the student to compare motions of conical and planar pendulums. It is elucidated that pendulum trajectory shape is determined only by the initial conditions and not by the external acting forces, because these are the same in both cases. Even having the same initial pendulum orientation, the initial velocity may be chosen so that it would yield a planar or a conical pendulum. The latter two, being special cases of pendulum motion, are observed under specific initial conditions – the general case of the initial conditions yields spherical pendulum (see below). This is often a fact difficult to understand while studying mechanical motions. Assigning different initial conditions and observing the consequent trajectory shape is a beneficial task for students while they use the simulation.

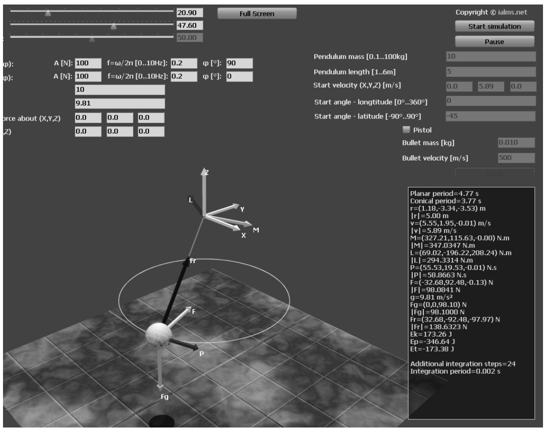

Figure 5. *Conical pendulum.*

Another point of study is the static equilibrium pendulum state (lowest position). It is frequently bound with dynamical equilibrium where there are equal forces acting in opposite directions and a zero resultant force. It is visualized through the simulation where in this lowest position the resultant force is always pointing upwards against the pivot point and is not zero. This fact results from the presence of normal acceleration proportional to the square of the pendulum velocity.

Spherical pendulum – the general case

As previously mentioned, the general case of a pendulum is a motion whose trajectory does not develop on a plane, but rather on the surface of a sphere with radius equal to pendulum bar length. This general case is called spherical pendulum and may be observed in the simulation if initial values of velocity and orientation do not satisfy the planar and conical pendulum conditions (Figures 1, 2, 3, 7 and 8).

Sample problems, solved with the help of the described simulation

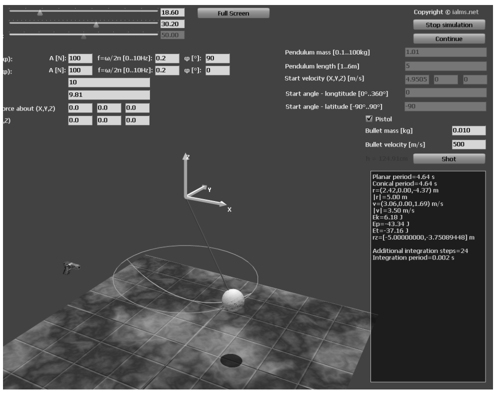

Figure 6. *Shooting the pendulum. Total momentum conserved.*

Sample problem 1. The momentum conservation law may be demonstrated with this problem. Let the pendulum be in lowest position and the gravity force acting. The pendulum will be at rest. Let us shoot the pendulum with a bullet having mass of 10 grams and initial horizontal velocity of 500 m/s (Figure 6).

If the pendulum has mass of 1 kg, to what height after the shot will it climb before falling back again?

Solution

The momentum of the system of two bodies – pendulum and bullet, is conserved. Pendulum initial momentum is zero, while after the impact, which is fully inelastic one, the system of two bodies forms one compound body with mass 1,01 kg and the same momentum as the momentum of the bullet, i.e.:

$$0.01v_1 = 1.01v_2 \Rightarrow v_2 = 4.9505\text{m/s}$$

Hence, the compound system kinetic energy after the impact would be:

$$\frac{mv_2^2}{2} = 12.3762\text{J}$$

At the highest point, pendulum kinetic energy would have been fully transformed into potential energy:

$$\frac{mv_2^2}{2} = mgh \Rightarrow h = \frac{v_2^2}{2g} = 1.2491\text{m}$$

(see Figure 6 and http://ialms.net/sim/). ■

Sample problem 2. On Figure 7 it is depicted a simulation of a conical pendulum. The angular momentum and moment of resultant force are shown. The origin point of both moments is chosen so that torque is zero and angular momentum is therefore constant.

a. What are the coordinates of this origin point?
b. Why is angular velocity vertical?

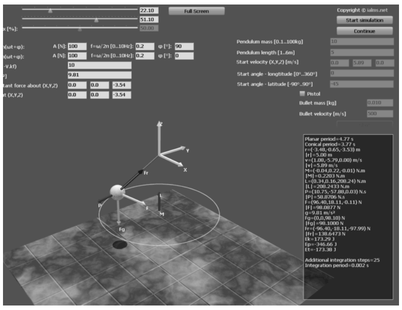

Figure 7. *Conical pendulum. Reference point for the moment of resultant force and angular momentum moved along the vertical axes to the plane of motion.*

Solution:

a. The torque is equal to the cross product of radius vector and resultant force: $\vec{M} = \vec{r} \times \vec{F}$. The latter is not zero and is pointing to the centre of the trajectory circle (Figure 6). The only possible variant that this vector product to be zero is when the radius vector is parallel to the resultant force, for which defines the origin point to coincide with the trajectory circle centre: (0,0,-3.54).

b. Angular velocity is equal to the cross product of radius vector and momentum: $\vec{L} = \vec{r} \times \vec{p}$. Momentum is horizontal. To satisfy sub-clause a. we chose a horizontal radius vector, orthogonal to momentum. This condition yields a vertical cross product. It should be mentioned that, due to orthogonality of momentum and radius vector, angular momentum magnitude towards origin point (0,0,-3.54) is equal to angular momentum towards Oz axis. If we choose another origin point along the Oz axis the z-component of the respective angular momentum will be always the same and equal to the magnitude of angular momentum with origin point (0,0,-3.54).

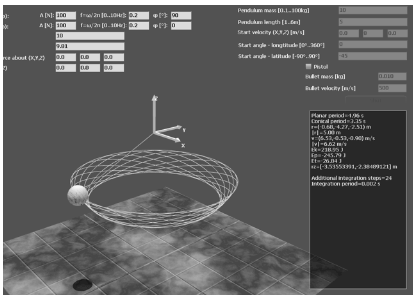

Figure 8. *Spherical pendulum with initial velocity close but not equal to the conical case. Potential well is formed.*

Sample problem 3. Figure 5 shows the simulation of a pendulum under initial conditions satisfying a conical case. In this example the bar length is $r = 5$m and its starting angle is 45^{o} towards vector $(-\overrightarrow{Oz})$.

What starting horizontal velocity should the pendulum have in order to start moving in a conical case?

Solution:

The condition defining a conical pendulum is:

$$v^2 = -g\frac{r^2 - r_z^2}{r_z}$$

Where r_z is the z-component of the initial position of the pendulum? The 45^{o} starting angle defines the relation $r_z = -\frac{r}{\sqrt{2}}$. Thus for the velocity it is calculated:

$$v^2 = \frac{gr}{\sqrt{2}} = \frac{9.81 \times 5}{\sqrt{2}} = 34.68 \Rightarrow |v| = 5.89\,\mathrm{m/s}$$

As the pendulum is initially oriented in the Oxz plane the starting velocity is towards the Oy axis (Figure 5) and the vector of the initial velocity is $(0, \pm 5.89, 0)$. After setting these values in the simulation, pendulum starts to move conically and the vertical component of its velocity is always zero, the vertical component of its position is constant (the momentary values of these vectors are printed in the data panel on Figure 5 in the lower-right corner of the interface).

What would happen if the above condition for initial velocity is not obeyed? If the initial velocity has the same direction (along the Oy axis) but is greater in magnitude, a spherical pendulum is observed and the pendulum will start to raise after the initial moment. On the contrary, if the velocity magnitude is lower, pendulum will start to fall after initial moment. Students may observe these two cases by setting the initial velocity along Oy axis to 9 and 3 meters per second respectively.

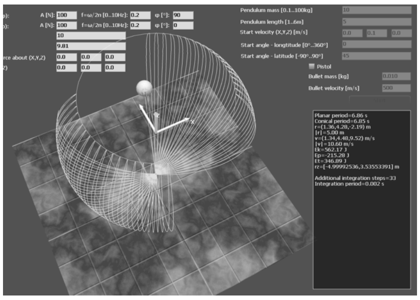

Figure 9. *Spherical pendulum with low initial velocity. A deep potential well is formed, but point (0,0,-l) is never reached.*

Sample problem 4. Each spherical pendulum has its potential well – limiting maximum and minimum heights (z-components of its position) beyond which it cannot travel (Beukers & Cushman, 2001). This potential well is defined by its initial conditions and the gravity force (Figures 8 and 9).

Calculate the potential well of the pendulum if its initial conditions are,

1. $g = 9.81\,\mathrm{m/s^2}$
2. $r = 5\mathrm{m}$
3. $\vec{r}_0 = \left(\frac{5\sqrt{2}}{2}, 0, -\frac{5\sqrt{2}}{2}\right)\mathrm{m}$
4. $\vec{v}_0 = (0, 8, 0)\,\mathrm{m/s}$

Solution

The potential well of the pendulum is defined by the cubic inequality:

$$gr_z^3 - \frac{E}{m} r_z^2 - gr^2 r_z + r^2 \frac{E}{m} - \frac{L_z^2}{2m^2} = ar_z^3 + br_z^2 + cr_z + d \geq 0$$

Let us first calculate the constants:

1. $\frac{E}{m} = \frac{v_0^2}{2} + gr_{0z} = 32 - 9.81 \frac{5\sqrt{2}}{2} = -2.6836$

2. $\frac{L_z}{m} = r_{0x}v_{0y} - r_{0y}v_{0x} = \frac{5\sqrt{2}}{2}8 = 20\sqrt{2} \Rightarrow \frac{L_z^2}{m^2} = 800$ ($L_z > 0$ - pendulum is rotating to the left)

Further, let us calculate the four quotients of the cubic inequality:

$a = 9.81$
$b = 2.6836$
$c = -245.25$
$d = -467.09$

We solve the cubic inequality and get $r_{z1} \in [-3.5355, -2.3849]\text{m}$.

Sample problem 5. Under what initial conditions would the pendulum form a 3-leaved daisy-like closed trajectory?

Solution:

Only an outline of the solution will be given. The pendulum conservative case has exact analytical solution and its motion properties can be formulated in terms of elliptic integrals and elliptic functions. Its motion is periodic while its trajectory may be a closed curve in finite time or not, depending on the initial conditions.

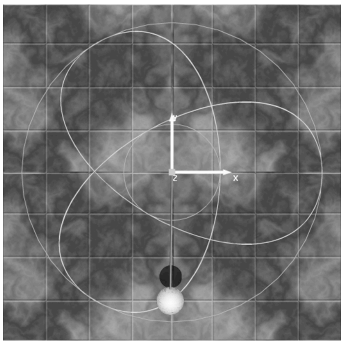

Figure 10. *Spherical pendulum with closed trajectory having three petals.*

Let us present the pendulum motion in cylindrical coordinates such that $x = \rho\cos\varphi$, $y = \rho\sin\varphi$ and $z = z$ (ρ is the horizontal projection of the radius vector and φ is the angle between ρ and Ox axis). Then the relation between $\frac{d\varphi}{dz}$ and z is easily expressed. Integrating both sides of this equation for a full period $z_{\max} \to z_{\min} \to z_{\max}$, or two times the half period $z_{\min} \to z_{\max}$, leads to an equation presenting period angle φ_T in terms of initial conditions, involving elliptic integrals. Finding the reverse function (involves elliptic functions) yields a presentation of the initial conditions in terms of the period angle φ_T. From the condition $\pi < \varphi_T < 2\pi$ it follows that a closed trajectory is observed when $m\varphi_T = n2\pi$ (m and n are natural numbers and m should be greater than n). The trajectory will be a closed curve with $\frac{mn}{2}$ petals. For three petals we find $m = 3$, $n = 2$ and $\varphi_T = \frac{4\pi}{3}$.

Figure 10 presents the simulation of the pendulum, whose trajectory is a closed curve with three petals. The initial conditions for a horizontal pendulum are horizontal tangential velocity of 3.995m/s.

Students' response

The simulation was used in lectures with students taking the General Physics course at Sofia University (Figure 11).

Figure 11. *The author on a lecture with students at Sofia University.*

Lecturers and professors at the Faculty of Physics were among the many parties interested in this new approach and its efficiency. As a result a research questionnaire was developed and students were asked to answer the questions. Here an excerpt of the questionnaire is presented in order to show students' attitude towards this novel visualization learning approach. Table 1. presents a small excerpt of the results from the study conducted after the simulation was presented to students as a method of blended tutoring. It was found that the simulation helps in two directions mainly: forming concepts such as vector variables and other quantities related to the simulated process (force, acceleration, trajectories of motion, etc.); improving the comprehension of concepts and relations between those. The students were informed that they could use the simulation free without limitations from the Internet at web address http://ialms.net/sim.

After using the simulation, did you acquire a clearer notion of quantities and laws in mechanics?

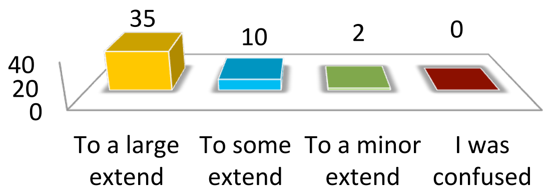

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Table 1. *An extract of students' responses to questions regarding the effects of teaching spherical pendulum using the 3D stereo online simulation*

The reported observations lead to a conclusion that the implementation of 3D-stereo online simulations promotes the comprehension and intensifies the motivation of students in learning the taught material.

Conclusion

This article focuses on the use of online e-learning simulation that visualizes through stereo 3D graphics the spherical pendulum in conservative force field. The author presents the benefits from using the simulation and how new technologies may be utilized to create virtual online laboratories with efficient and effective 3D stereo presentation approach.

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