



## COMPARATIVE ANALYSIS OF THE MATHEMATICS PROBLEMS GIVEN AT INTERNATIONAL TESTS AND AT THE ROMANIAN NATIONAL TESTS

Iuliana Marchis

**Abstract.** The results of the Romanian pupils on international tests PISA and TIMSS in Mathematics are below the average. These poor results have many explanations. In this article we compare the Mathematics problems given on these international tests with those given on national tests in Romania.

**Zusammenfassung.** Die Ergebnisse der rumänischen Schüler auf den internationalen PISA- und TIMSS-Tests in der Mathematik sind weiterhin unter dem Durchschnitt. Diese schlechten Ergebnisse haben viele Erklärungen. In diesem Artikel werden wir präsentieren und vergleichen Mathematische-Aufgabe aus dieser internationalen Tests mit Aufgaben von nationalen Tests in Rumänien.

**Key words:** international test, Mathematics Education, Mathematics problem

### 1. Introduction

The Trends in International Mathematics and Science Study (TIMSS) is conducted under the auspices of the International Association for the Evaluation of Educational Achievement (IEA). The curriculum for TIMSS contains *content domains* and *Knowing facts and procedures*. The content domains covered by TIMSS 2003 are Number, Measurement, Geometry, Data and Algebra. The knowing facts and procedures are using concepts, solving routine problems, reasoning.

The Program for International Student Assessment (PISA) is conducted by the Organization for Economic Cooperation and Development (OECD). The main objective of PISA is to provide data on the competencies of 15-year-olds (students at an age that is near the end of compulsory schooling in most countries). PISA focuses on literacy - the ability to use and apply knowledge and skills to real-world situations encountered in adult life. The curriculum contains *contents* and *competency clusters*. The contents of the Mathematics test contain change and relationship, quantity, space and shape, uncertainty. As regarding competency clusters, those evaluated by PISA are reproduction, connections, and reflection. In this test the *Mathematical Literacy* of pupils is tested. Mathematical literacy is “the capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen” (OECD, 2003). It is related to wider, functional use of mathematics, engagement requires the ability to recognize and formulate mathematical problems in various situations.

Romanian pupils obtained poor results on these tests. For example In Table 1 we can compare the average of the Romanian pupils on TIMSS and PISA tests with the international average. Analyzing the Mathematics results at TIMSS and PISA test, Romania has the score lower than the international average, with one exception: TIMSS 2003. On this testing Romania obtained 475 points, as the average is 466, so the difference is not big. The highest difference between Romania’s score and international average was obtained at PISA 2006 test

**Table 1.** *The results of Romanian students at international tests in Mathematics*

TIMSS tests			PISA tests		
Year	TIMSS average	Romania's score	Year	OECD average	Romania's score
1995	519	474	2000	500	426
1999	521	472	2003	500	-
2003	466	475	2006	500	415
2007	500	461			

These poor results have more reasons. Analyzing the Romanian school curriculum, it stops at the analysis level of the cognitive domain taxonomy of Bloom. (Marchis, Ciascai, Ciomos, 2009). Reasoning, which is present at TIMSS test, is not covered. The real-life problems are missing.

The aim of this paper is to compare the problems given at PISA and TIMSS test with those given at Romanian national tests.

## 2. Analyzing the problems given at TIMSS test

Many of the problems given at TIMSS tests have base in the real-life. Problem 1 shows a real situation. Pupils need to analyse datas in order to make a decision. This problem can be set on the *analysis* level of the Bloom's taxonomy.

Betty, Frank, and Darlene have just moved to Zedland. They each need to get phone service. They received the following information from the telephone company about the two different phone plans it offers. They must pay a set fee each month and there are different rates for each minute they talk. These rates depend on the time of the day or night they use the phone, and on which payment plan they choose. Both plans include time for which phone calls are free. Details of the two plans are shown in the table below.

Plan	Monthly Fee	Rate per minute		Free minutes per month
		Day (8 am – 6 pm)	Night (6 pm – 8 am)	
Plan A	20 zeds	3 zeds	1 zed	180
Plan B	15 zeds	2 zeds	2 zeds	120

Betty talks for less than 2 hours per month. Which plan would be less expensive for her?

Less expensive plan \_\_\_\_\_

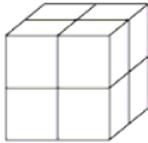
Explain your answer in terms of both the monthly fee and free minutes.

**Problem 1.** *Example of problem given at TIMSS test (IEA, 2003)*

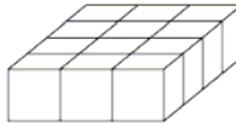
An example of a Geometry problem is Problem 2. This problem needs only counting of the cubes from the 3D shapes.

All the small blocks are the same size. Which stack of blocks has a different volume from the others?

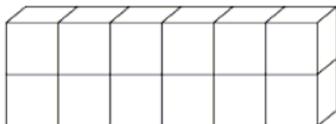
(A)



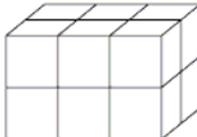
(B)



(C)



(D)



**Problem 2.** Example of problem given at TIMSS test (IEA, 2003)

### 3. Analyzing the problems given at PISA test

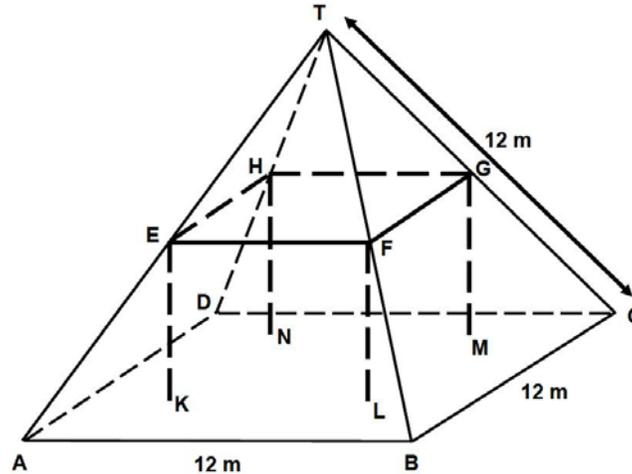
PISA test focuses on problem solving in real-world applications.

We could observe in case of Problem 3, that the geometrical problem is based on a situation from the everyday life. To answer question 1 pupils need to know how to calculate the area of a square when the edges are given. To calculate EF they have more possibility. If they know, that the length of a segment given by the middle point for two edges of a triangle is half of the length of the third edge, the answer is immediate. Otherwise they can calculate this length in triangle TEF, where they know that  $TE = TF = 6\text{cm}$ . This problem is a simple one; it needs just to apply some very well known formulas. The difficulty for pupils is in understanding the Mathematical text. This problem requires only *application* on some formulas or properties.

Here you see a photograph of a farmhouse with a roof in the shape of a pyramid.



Below is a student's mathematical model of the farmhouse **roof** with measurements added.



The attic floor, ABCD in the model, is a square. The beams that support the roof are the edges of a block (rectangular prism) EFGHKL MN. E is the middle of AT, F is the middle of BT, G is the middle of CT and H is the middle of DT. All the edges of the pyramid in the model have length 12 m.

**Question 1:** Calculate the area of the attic floor ABCD.

The area of the attic floor ABCD = \_\_\_\_\_ m<sup>2</sup>

**Question 2:** Calculate the length of EF, one of the horizontal edges of the block.

The length of EF = \_\_\_\_\_ m

**Problem 3.** Example of Geometry problem given at PISA test (OECD, 2006)

Question 1 of Problem 4 needs only counting for  $n=1, \dots, 4$ . But for  $n=5$  pupils have to draw the situation and then count. Another possibility is to find out a formula to calculating the number of apple and conifer trees. Actually, this formula is given in Question 2, so pupils only need to substitute the value of  $n$  in the right formula. Answering question 2 they have to solve an equation or to guess the number. These two questions are on the *application* level of the Bloom's taxonomy. Question 3 is more difficult; pupils have to find a way to compare the number of apple and the conifer trees. They have to create a method to do this. This question is on the *synthesis* cognitive level.

A farmer plants apple trees in a square pattern. In order to protect the apple trees against the wind he plants conifer trees all around the orchard.

Here you see a diagram of this situation where you can see the pattern of apple trees and conifer trees for any number ( $n$ ) of rows of apple trees:

n = 1

X X X  
X ● X  
X X X

n = 2

X X X X X  
X ● | ● X  
X X X X X

n = 3

X X X X X X X  
X ● ● ● X  
X ● ● ● X  
X X X X X X X

n = 4

X X X X X X X X X  
X ● ● ● ● X  
X ● ● ● ● X  
X ● ● ● ● X  
X ● ● ● ● X  
X ● ● ● ● X  
X X X X X X X X X

X = conifer tree  
● = apple tree

**Question 1:** Complete the table:

n	Number of apple trees	Number of conifer trees
1	1	8
2	4	
3		
4		
5		

**Question 2:** There are two formulae you can use to calculate the number of apple trees and the number of conifer trees for the pattern described above:

Number of apple trees =  $n^2$

Number of conifer trees =  $8n$

where  $n$  is the number of rows of apple trees.

There is a value of  $n$  for which the number of apple trees equals the number of conifer trees. Find the value of  $n$  and show your method of calculating this.

**Question 3:** Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifer trees? Explain how you found your answer.

**Problem 4.** Example of problem given at PISA test (OECD, 2006)

A problem type very frequent on PISA test is reading and interpreting data in tables and graphs. In many problems pupils need to explain, to judge. Thus the *evaluation* level is covered, too.

### 3. Analyzing the problems given at national Mathematics test for 8<sup>th</sup> grade in Romania

In Romania there are national tests two times a year for 7<sup>th</sup> and 8<sup>th</sup> grade pupils. Mathematics is present between the tested subjects. In the following, we analyze the 8<sup>th</sup> grade problems.

On this test the first 4 problems, with 3 questions each, are very simple, the last 3 are a bit more complicated.

In the first semester of the 2007/2008 school year the first three questions require calculations with real numbers (see Problem 5), the fourth problem some simple Geometrical calculations (see Problem 6). So these problems cover the *knowledge, understanding and applications* levels of Bloom's

taxonomy. The last three problems of this test usually require deeper mathematical knowledge and more calculations. But still remains on the *knowledge, understanding and application* levels.

The result of the calculation  $5,76 + 3,24$  is ....

The result of the calculation  $(\sqrt{5} + \sqrt{5})^2$  is ....

The result of the calculation  $|-8|-8$  is ....

**Problem 5.** Example of Algebra items given at Romanian national test for 8<sup>th</sup> grade (CNC, 2007)

The edges of a cube are 12 cm.

a) The sum of the length of all edges of the cube is ... cm.

b) The length of the segment DB is ... cm.

c) The perimeter of the quadrilateral BDD'B' is ... cm.

**Problem 6.** Example Geometry of items given at Romanian national test for 8<sup>th</sup> grade (CNC, 2007)

In the second semester of the 2007/2008 school year the first three questions the first 3 problems are mostly related with equations and functions (see examples in Problem 7). These problems require only application of some algorithms, so again only *knowledge, understanding and application* cognitive levels are covered. In the second part of the test there is a word-problem, which can be solved using an equation: "The sum of two numbers is 77. Dividing one of the numbers with the other one, the quotient is 4 and the rest is 2. Find the numbers." (MNT, 2008) This problem reaches the *analysis* level. Comparing with the international tests' problems, the text of this one is "mathematical", as those given on PISA and TIMSS tests are based on a problem from everyday life.

The real solution of the equation  $2x-5=3$  is ...

The function  $f : R \rightarrow R, f(x)=3+x$  is given.

The value of  $f$  for  $x=2$  is equal with ...

The intersection of the graph of  $f$  with the  $Ox$  axes is the point  $A(\dots, \dots)$ .

**Problem 7.** Example Algebra of items given at Romanian national test for 8<sup>th</sup> grade (CNC, 2008)

There is no problem which needs reading and interpreting data in tables and graphs.

### 3. Conclusion

Problems given at international tests (PISA, TIMSS) are related with everyday life. Most of these problems have a long description of the situation, and pupils have to discover the data needed for solving the problem in this text. These problems cover all the levels of Bloom's cognitive levels.

Most of the problems given on national Mathematics tests in Romania cover only the *knowledge, understanding and application* cognitive levels, to solve them it is required to apply formulas or algorithms. These problems have a mathematical formulation, they don't have any connection with real life. This has more consequences:

- pupils don't like Mathematics, as they don't see why they need to learn it;
- in their everyday life they meet problems, where Mathematics is present, and they don't know how to solve it, as they don't know how to transfer Mathematics to practical problems.

## Literature

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## Authors

**Iuliana Marchis**, Babes-Bolyai University, Cluj-Napoca, Romania, e-mail: [marchis\\_julianna@yahoo.com](mailto:marchis_julianna@yahoo.com)

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