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Does tracing worked examples enhance geometry learning?

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ABSTRACT

Cognitive load theory seeks to generate novel instructional designs through a focus on human cognitive architecture including a limited working memory; however, the potential for enhancing learning through non-visual or non-auditory working memory channels is yet to be evaluated. This exploratory experiment tested whether explicit instructions to trace out elements of geometry worked examples with the index finger would enhance learning, as measured by error rates and problems solved. Compared to a non-tracing condition, students in the tracing condition correctly solved more practice problems, and made fewer errors on a subsequent test. Recommendations for subsequent more sensitive experiments are made.

INTRODUCTION

Cognitive load theory (Sweller, 2011) is a theory of instructional design which foregrounds human cognitive architecture as a key consideration in the likely effectiveness of instruction. The theory stipulates an architecture including a working memory whose limitations in capacity and duration are substantially reduced when knowledge in schematic form is activated from long-term memory

The current version of the theory incorporates arguments from evolutionary theory (Geary, 2008; Sweller, 2011). When considering learning, a key distinction can be made between biologically *primary* knowledge that our genetic heritage supports through immersion in normal society (e.g., learning to speak), and biologically secondary knowledge which does not emerge naturally (e.g., mathematics). Secondary knowledge requires prolonged, effortful instruction, typically undertaken in purpose-designed institutions such as schools. Instructional redesigns generated by cognitive load theory apply to biologically secondary knowledge where the potential for working memory overload during learning is common. This distinction prompted Paas and Sweller (2012) to question whether some forms of biologically primary knowledge might support the development of biologically secondary knowledge.

A number of findings from both laboratory and educational settings argue for *pointing gestures* as biologically primary but supporting biologically secondary knowledge construction. Supporting this assertion, the literature on protodeclarative pointing has established pointing gestures as naturally

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emerging in the first year of life and playing a key role in subsequent vocabulary development (Liszkowski, Brown, Callaghan, Takada, & de Vos, 2012). Pointing gestures, and the related use of the index finger for tracing on a physical surface, are specific uses of the hands that act to guide attention, both for one's own benefit and in communicative settings.

For over a century, multisensory exercises involving tracing have been a staple of Montessori education, addressing biologically secondary knowledge such as writing (Montessori, 1914). For example, when teachers and students learn with "sandpaper letters", students listen to the sound made by the teacher, look simultaneously at its representation in the form of a letter cut out of sandpaper, and feel how it is written by tracing out the sandpaper letter. Experimental demonstrations of the efficacy of this pedagogy include studies on letter learning and phoneme identification (e.g., Bara, Gentaz, & Colé, 2007) and geometrical shape recognition by kindergarten children (e.g., Kalenine, Pinet, & Gentaz, 2011).

While such studies provide evidence that finger tracing can enhance recognition of visual stimuli, it remains to be established whether such activity enhances learning more advanced, multi-step problems such as those found later in the mathematics curriculum. Research on the modality effect (Mousavi, Low, & Sweller, 1995; for a review see Ginns, 2005) has found presenting to-be-learned information across multiple working memory channels (e.g., visual for diagrams and auditory for text) will reduce the load on each individual channel, and accordingly the effective capacity of working memory available for learning can be increased. Following the logic of the modality effect, by using multiple sensory modalities to undertake a learning task (i.e., visual, tactile and kinesthetic, rather than visual only), improved learning and test performance should result. In addition to generating additional cognitive resources available for use, the presence of a pointing finger could also serve as an attentional guide (Cosman & Vecera, 2010; Reed, Grubb, & Steele, 2006), effectively directing cognitive resources to the most important information for meaningful learning. Therefore, it is predicted that students instructed to trace out elements of worked examples in geometry are predicted to outperform students who simply study the materials.

METHOD

Participants.

Participants consisted of 56 Year 6 students, including 38 boys and 18 girls, from 2 independent schools in Sydney, Australia. All participants participated voluntarily, and were aged between 11 and 12 years old ($M = 11.20$, $SD = .44$). Students were largely novices with respect to the information presented in the instructional materials; they could identify parallel lines and angles but had not learned the angle relationships involving parallel lines at school. Participants were randomly assigned to the tracing or non-tracing conditions.

Materials and Procedure.

Students were tested individually, with each student being withdrawn from class for approximately 30 minutes. The experiment began with an initial instruction phase, identical for both groups. This phase was followed by an acquisition phase involving study with or without hand tracing two worked examples; each worked example was paired by a similar practice problem. The experiment concluded with a test phase of six questions.

Initial Instruction phase. The materials used in this phase consisted of four pages of initial instruction about three angle relationships involving parallel lines. Students had five minutes to study the three angle relationships. For each angle relationship, there was a short text providing its definition, diagrams displaying the locations of the specific angles, and an example demonstrating how to use this angle relationship to solve a problem.

Acquisition phase. All students were then shown two worked examples applying the three angle relationships to find a missing angle. In the worked examples for the tracing group, every solution step was followed by instructions in brackets on hand tracing. Students in the tracing condition were given two minutes to read and try to understand the solution steps, while using their index finger of their writing hand to trace out the diagram following the instructions. For example, after reading, "When two lines cross, vertical angles are equal, so this angle is 50°. [Trace out the two vertical angles with your finger]", students had to find the designated vertical angles and trace them with their finger. Students in the non-tracing condition were instructed to read and try to understand the solution steps for two minutes, with their hands placed on their laps. Each worked example was paired with a similar practice problem, with a maximum of two minutes to solve the problem.

Test phase. The test phase consisted of two basic questions (with similar diagrams and solution steps to the worked examples but with different numbers), and four advanced questions which differed from the worked examples with moderate variations in diagrams and the sequence of solution steps. Students had up to two minutes for each question.

RESULTS

The variables under analysis were number of errors and number of correct solutions to practice problems in the Acquisition Phase, and number of errors and number of correct solutions to test questions in the Test Phase. Means and standard deviations are provided in Table 1; a Type 1 error rate of 0.05 was adopted.

Acquisition Phase. Due to substantial skewness of the data, a Mann-Whitney test was used to analyse number of errors and number of practice problems solved. For number of errors, the mean rank of the tracing condition (Mean rank = 27.57) was not statistically different to that of the non-tracing condition (Mean rank = 29.43), $U = 366.00$, $p = .551$, $d = -.16$; however, the tracing condition solved more practice problems (Mean rank = 31.52) than the non-tracing condition (Mean rank = 25.48), $U = 307.50$, $p = .045$, Cohen's $d = .63$.

Test Phase. Due to substantial skewness of the data, a Mann-Whitney test was used on the number of errors; the tracing condition (Mean rank = 24.64) made fewer errors than the non-tracing condition (Mean rank = 32.36), $U = 284.00$, $p = .047$, $d = -.54$. A ceiling effect was evident on test questions correctly answered (78.6 % students correctly solved all questions), preventing analysis of this variable.

Table 1: Means (and Standard Deviations in Parentheses) for Acquisition Phase Errors and Number of Correct Solutions, and Test Phase Errors and Number of Correct Solutions

	Non-tracing	Tracing
Acquisition Phase		
Number of errors	0.50 (1.04)	0.21 (.42)
Number of correct answers	1.71 (.53)	1.96 (.19)
Test Phase		
Number of errors	1.46 (1.90)	0.69 (1.25)
Number of correct answers	5.43 (1.17)	5.68 (0.86)

DISCUSSION

The results of this exploratory experiment provide preliminary evidence for the incorporation of tracing instructions into worked examples. Cognitive load theory argues well-designed sequences of worked examples and practice problems support schema construction and automation (Cooper & Sweller, 1987; Sweller, 2011). We hypothesized that pointing and tracing gestures are forms of biologically primary knowledge that will support learning from mathematics worked examples. Support is the result of the generation of additional (tactile and kinesthetic) working memory resources and focusing of visual attention on to-be-learned mathematical rules. During the acquisition phase, students who traced worked examples solved more practice problems than those who simply studied the materials, and during the test phase, students who traced made fewer errors than those who studied. Thus, tracing appears to assist initial learning, as well as reducing the number of errors during subsequent testing. The results of the present study are limited, however, by the ceiling effect obtained for test scores.

The results reported here speak to the importance in instructional design experiments of suitable alignment between the prior knowledge levels of participants and the complexity of the materials and test questions (see Cooper & Sweller, 1987). The high level of test performance achieved here informs two ways to improve the design of the next investigation of the tracing effect. First, since students in upper primary school may have sufficient prior knowledge to learn the properties of parallel lines without too much difficulty, recruiting younger students with less mathematics experience (e.g., Year 5 students) should reduce the likelihood of a ceiling effect on test scores. Second, if Year 6 students are to be used in future studies, increasing the difficulty level of the test should also act to reduce the chances of a ceiling effect. In this manner, the accuracy rate should be reduced and the error rate increased, creating more sensitive dependent variables. We will explore these options in subsequent replications and extensions of the present study.

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