

## Developing Robust Forms of Pre-Service Teachers' Pedagogical Content Knowledge through Culturally Responsive Mathematics Teaching Analysis

Julia M. Aguirre  
*University of Washington Tacoma*

Maria del Rosario Zavala  
*San Francisco State University*

Tiffany Katanyoutanant  
*University of Washington Seattle*

This study documents and describes efforts to develop robust forms of pre-service teachers' pedagogical content knowledge through a culturally responsive mathematics teaching approach. Embedded in a university K-8 mathematics methods course emphasizing the connections among mathematics, children's mathematical thinking, and children's cultural/linguistic funds of knowledge, pre-service teachers (N=40) were given an assignment to analyse their own mathematics lessons utilizing a rubric tool with categories about children's mathematical thinking, academic language supports, cultural funds of knowledge, and critical math/social justice. Utilizing a mixed methods approach to analyse the pre-service teachers' (PST) work, we found the highest average self-ratings across the categories associated with children's mathematical thinking and high variability in the categories related to language, culture, and social justice. To understand the variation within the latter three categories we qualitatively analysed PST written reflections. We found strong PST receptivity to supporting academic language for second language learners and integrating cultural funds of knowledge into mathematics lessons, and mixed receptivity to integrating social justice into mathematics lessons. However, a more nuanced analysis of teacher resistance revealed challenges with pedagogy rather than ideology. Implications for mathematics teacher education and strengthening pedagogical content knowledge of pre-service teachers are discussed.

Calls continue to better prepare teachers to meet the mathematics education needs of increasingly culturally and linguistically diverse students (Grossman, Schoenfeld, & Lee, 2005; Sowder, 2007). What forms of knowledge and experiences do pre-service teachers need to develop into good mathematics teachers—teachers who meet the educational and aspirational demands of students? In this study we focus our attention on developing robust forms of pedagogical content knowledge (PCK) in pre-service teachers through a culturally responsive mathematics teaching approach. Grossman et al. (2005) describe pedagogical content knowledge as:

... the pedagogical understandings of the subject matter ... which include, among other things, the ability to anticipate and respond to typical student patterns of understanding and misunderstanding within a content area, and the ability to create multiple examples and representations of challenging topics that make the content accessible to a wide range of learners. (p. 201)

They further argue that teacher education must provide “intellectual tools” for teachers to continuously inquire, reflect, and develop productive strategies that help them further understand what it means to understand the subject matter in order to teach it to children.

This study describes the results of a university K-8 mathematics methods course in the United States that embraces this PCK definition and focuses pre-service teacher attention to four elements of teaching mathematics: children’s mathematical thinking, language, culture, and social justice. These elements are rooted in our definition of culturally responsive mathematics teaching (CRMT)—a set of specific pedagogical knowledge, dispositions, and practices that privilege mathematical thinking, cultural and linguistic funds of knowledge, and issues of power and social justice in mathematics education (Aguirre, 2009; Aguirre & Zavala, in press; Gutiérrez, 2009; Kitchen, 2005; Leonard, Brooks, Barnes-Johnson, & Berry III, 2010; Turner, Drake, Roth McDuffie, Aguirre, Bartell & Foote, 2012). Through the use of a culturally responsive mathematics teaching tool that embodies multiple elements of CRMT (Aguirre & Zavala, in press), pre-service teachers engaged in systematic self-assessment of a math lesson, rated the lesson based on specified criteria, and reflected on the quality of the lesson and areas for improvement. Our purpose was to understand how pre-service teachers (PSTs) made sense of their practice through lesson planning and how they envisioned altering or revising their lessons – thus creating reflection opportunities for deepening their pedagogical content knowledge and aiding their development as reflective practitioners (Philipp, 2007). The insights gained from this analysis provide important guideposts for mathematics teacher educators about potential sites to strengthen PST pedagogical content knowledge in more robust ways.

### Relevant Literature

We draw on two important literature bases for this analysis: Pedagogical Content Knowledge (PCK) and Culturally Responsive Pedagogy (CRP). We argue that both these literature bases have made great strides in shaping teacher education, particularly in the United States, Australia, and New Zealand. In bringing these literature bases together, we also acknowledge that while much literature on CRMT assumes that an integral part of teacher effectiveness is subject matter knowledge (for example, Averill et al., 2009; Gutstein, Lipman, Hernandez, Reyes, 1997), research that takes PCK of mathematics teachers as a starting point rarely expands into knowledge bases typically regarded as central to CRMT, namely community, cultural, and linguistic knowledge bases.

While the term PCK was coined by Lee Shulman (1986), Pamela Grossman’s work in teacher development (1990) delineated four specific components of this specialized form of teaching knowledge<sup>1</sup>:

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<sup>1</sup> Grossman’s work on PCK was developed through an in-depth study of secondary English/Language Arts teachers in the U.S. However, this concept is generalizable to other domains of teaching and teacher preparation (Grossman et al., 2005; Sowder, 2007).

- 1) An overarching knowledge and belief about teaching a subject at specific grade levels;
- 2) Knowledge of students' understandings, conceptions, and potential misunderstandings of particular topics of a subject;
- 3) Knowledge of curriculum and curricular materials, including horizontal and vertical directions within a subject;
- 4) Knowledge of the instructional strategies and representations for teaching particular topics.

In relation to mathematics teaching, Sowder (2007) argued that Grossman's delineation of PCK into four components "are helpful for those developing teacher education programs and professional development opportunities for mathematics teachers" (p. 164).

In contrast, culturally responsive educators have developed specific frameworks for instructional practices that promote excellence and equity for historically marginalized youth in the United States (Gay, 2000; Ladson-Billings, 1995) as well as Australia and New Zealand (Averill et al, 2009; Perry & Howard, 2008). According to Gay (2000),

Culturally responsive pedagogy simultaneously develops, along with academic achievement, social consciousness and critique, cultural affirmation, competence, and exchange; community-building and personal connections; individual self-worth and abilities; and an ethic of caring ... Culturally responsive teachers have unequivocal faith in the human dignity and intellectual capabilities of their students. They view learning as having intellectual, academic, personal, social, ethical, and political dimensions, all of which are developed in concert with one another (pp. 43-44).

CRP is a crucial component for teachers to have the knowledge and skills to work with youth from a variety of backgrounds, knowledge bases, and experiences. In the United States, however, a new teacher's introduction to CRP is usually reserved for multicultural education classes, not subject specific methods courses (Cochran-Smith, Davis, & Fries, 2004). Although new teachers may gain inspiration from this work, integration of these essential strands of CRP has been lacking in most mathematics classrooms (Nasir, Hand, & Taylor, 2008).

When taken together PCK and CRP make a powerful combination to develop culturally responsive mathematics teaching (see Figure 1). It is important to understand that one cannot engage in culturally responsive mathematics teaching without PCK. We argue that our field is at a crucial point where the research in CRMT now demands that we frame PCK in more robust ways, drawing on research in CRP, so that we can lead PSTs to develop effective mathematics teaching practices. In the following sections, we will articulate how culturally responsive mathematics teaching connects to and enhances Grossman's (1990) four components of PCK.

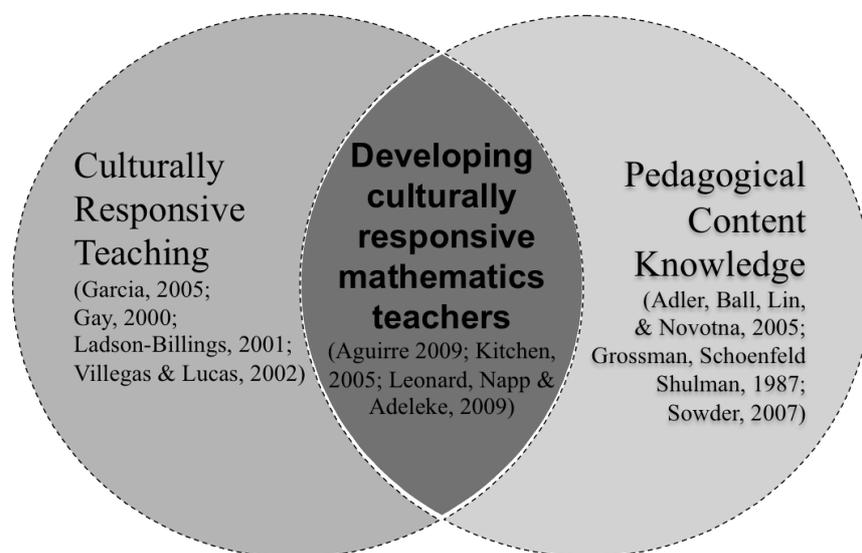


Figure 1. A framework for culturally responsive mathematics teaching.

### *Overarching Knowledge and Belief About the Purposes of Teaching Mathematics*

Teachers are well intentioned in their support of children to learn mathematics, but these intentions are mitigated by beliefs about the nature of mathematics, how children learn mathematics, and the teacher's role in this process. Beliefs act like "filters" that affect the development of instructional practice and thus the purpose of teaching (Borko, Mayfield, Marion, Flexer, & Cumbo, 1997; Philipp, 2007). Scholars also note, however, that beliefs about the purposes of teaching are impacted by beliefs about race, class, culture, and power (Diversity in Mathematics Education (DiME), 2007; Gutiérrez, 2010; Spielhagen, 2011; Sztajin, 2003). These beliefs are consequential in day-to-day interactions, as well as in school and district level policies aimed at improving mathematics learning and performance. Scholars have shown teacher beliefs to reify current status differences and racial stereotypes while resisting mathematics reform efforts to engage all students in cognitively demanding mathematical practices (Martin, 2007; Spielhagen, 2011; Sztajin, 2003). Other scholars document teacher beliefs that reframe the purpose of teaching mathematics to emphasize mathematical understanding, discourse, and reasoning; expand conceptions of mathematical competence (Boaler, 2008); and affirm student mathematical identities and social agency (Gutiérrez, 2010; Gutstein, 2006; Martin, 2007).

In culturally responsive teacher education, researchers argue that teachers must learn to see their teaching as a political activity rather than neutral activity, embrace multiple perspectives (rather than just their own), and develop an awareness of the role power plays in school policies and curriculum practices (Gay, 2000; Villegas & Lucas, 2002). Mathematics teachers must understand the

political role of mathematics in perpetuating and disrupting social inequities and specific actions that dismantle institutional structures and practices that mirror inequities as part of their teaching purpose (DiME, 2007; Gutiérrez, 2009, 2010).

### *Knowledge of Children's Mathematical Understandings and Misunderstandings*

Great strides have been made in research on this component of PCK (see Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Franke & Levi, 2003). This work on children's mathematical thinking spawned a critical parallel track for understanding teacher beliefs and the degree to which this knowledge of student thinking affects instructional decision-making (Fennema et al., 1996; Philipp, 2007; Vacc & Bright, 1999). As summarized by Philipp (2007), studies that investigated change in PSTs' beliefs related to children's mathematics thinking found that consistent exposure to children's strategies and sense-making in method courses and placement experiences, including conducting and reflecting upon clinical mathematics interviews with children, provided productive paths toward change (Ambrose, 2004; Vacc & Bright, 1999).

A key component of culturally responsive mathematics teacher preparation is to build on this work and provide examples of children's mathematical thinking in culturally and linguistically diverse student populations. Research similar to the Cognitively Guided Instruction (CGI) studies (conducted in predominantly white, affluent areas in the 1980s) have been replicated with groups of children from historically marginalized groups, such as working class Latino/a children and English language learners (ELLs). This recent research shows similarly consistent positive results for student learning (Turner & Celedón-Pattichis, 2011). Through data from such research, PSTs can see the benefits on mathematical learning through active attention to children's mathematical thinking. Examples emerging from this research also help to affirm the capabilities of students from non-dominant backgrounds to engage in complex mathematical problem-solving while still learning to master "the basics". Furthermore, these examples also help to counter any deficit-oriented notions a PST may have (consciously or unconsciously) that non-dominant students have "less adaptive or even maladaptive strategies" that must be overcome (Clements & Sarama, 2007, p. 32). Teacher preparation that integrates a culturally responsive mathematics teaching approach creates these experiences with the explicit intention to challenge deficit views by highlighting the power of mathematical thinking from children with diverse academic, cultural, linguistic, and socio-economic backgrounds (Turner et al., 2012).

### *Knowledge of Curriculum and Curricular Materials*

Integral knowledge of curricular materials (e.g., textbooks) is a core component in mathematics teaching. However, it is important to understand that the intended curriculum and the enacted curriculum are different (Philipp, 2007). Teachers' knowledge and beliefs of the nature of mathematics, purposes of

schooling, and how children learn mathematics affect how teachers use curriculum (Stein, Remillard, & Smith, 2007).

In the United States, mathematics curriculum is highly controversial (Schoenfeld, 2004). It is important for new teachers to be able to critique and distinguish underlying theories of learning and purposes of teaching embedded in mathematics curricula (Grossman et al., 2005), while also understanding how attention to content and learning intersects with culture, language, and power. This latter point is directly connected to CRMT's approach to expand PCK knowledge related to curriculum and curricular material. This can be done in at least three ways. First, curricular materials, such as pre-packaged texts, have language demands that must be considered to help second language learners access the content. PSTs must learn techniques proven effective for engaging second language learners in mathematics as they adapt activities for their own classrooms: drawing on the students home language (L1) as a resource, activating prior knowledge, using realia, and other visual/tactile resources, as well as strategic grouping of students so that peers can be resources in mathematics discussions (Aguirre & Bunch, 2012; Aguirre et al., 2012; Barwell, 2009; Moschkovich, 2007, 2010).

Second, curriculum must be modified to make it more authentic to students' lives, leveraging funds of knowledge in students' home communities. Moll and Gonzalez (2004) define cultural funds of knowledge (CFoK) as "the knowledge base that underlies the productive and exchange activities of households" (p. 700). Scholars in mathematics education offer many examples of how family and community activities such as gardening, sewing, weaving, fishing, cooking, playing games, story-telling, and mapping land use are mathematical resources available to students and teachers to support children's mathematics learning (Averill et al., 2009; Civil, 2007; Perry & Howard, 2008; Turner et al., 2012; Wager, 2012).

Third, curriculum can be modified to engage students in mathematically-rich contexts that are meaningful to their lives and address social justice issues (Gutstein, 2006; Turner & Font Strawhun, 2007). This approach, which some call "teaching mathematics for social justice", requires that teachers make connections between the classical school mathematics that students must learn, the home communities of students, and how mathematics can be used as an analytical tool to critique and address injustice (Gutstein, 2006). Turner and Font Strawhun (2007) argue that the problem context must be authentic—arising from an actual situation that students find genuinely problematic—for students to engage meaningfully with the mathematics. Examples within the U.S. context include secondary students investigating the role of racism in mortgage lending practices and police actions (Gutstein, 2006); elementary students using multiplication, measurement, and data analysis to challenge district decisions to close a neighborhood school (Varley-Gutiérrez, 2011); utilizing measurement concepts to challenge societal messages about body image (Kitchen & Lear, 2000); and primary students engaging in data analysis and representation to address race and racism (Tenorio, 2004). With this expanded curricular knowledge that

includes attention to culture, language, and power, teachers can enhance their pool of curricular options to maximize student engagement and learning.

### *Knowledge of Instructional Strategies and Representations*

PSTs need to know specific instructional strategies to engage students in mathematics, such as steps to orchestrating productive mathematical discussions based on student work (Stein, Engle, Smith, & Hughes, 2008), as well as what talk moves are shown to be productive in facilitating mathematics discussions (Chapin & O'Connor, 2007). PSTs must develop skills in leading conversations and making mathematics discourse transparent for students. For second language learners, PSTs must also be knowledgeable about how to use discourse moves to position students as authors of mathematical ideas, with implications for their mathematics identities over time (Turner, Dominguez, Maldonado, & Empson, 2010).

One key role that representations play in teaching and learning mathematics is to be a focal point for collective thought and mathematical discourse (Aguirre & Bunch, 2012). Therefore PSTs need to know what representations are powerful accompaniments to which mathematical ideas. PSTs also need to know when and how to introduce representations to their students. For example, teachers would need to be able to find out what knowledge of arrays students had (whether school- or home-based), decide if the representation was an appropriate tool for the mathematics, decide how to introduce arrays to students, recognise when to move from closed to open arrays for advanced abstract modeling in the upper grades, as well as the common misconceptions or developmental stages that students progress through when learning to model multiplication using arrays. This requires coordination of many elements of teaching at once, and though arrays are introduced as a part of some elementary school mathematics curricula, PSTs would still need background knowledge to understand the intent of the curriculum.

CRMT would also require that teachers use representations as a tool for second language learners to access mathematics discussions and mathematical tasks. Representations are in themselves a language demand on students (Aguirre & Bunch, 2012). Furthermore, they are culturally situated and are not necessarily universal. For example, Perkins and Flores (2007) present several ways mathematics is represented differently in Latin American countries, including different measurement units, computational procedures and symbolic notation. For example, division can be noted by ":" in Latin American countries. A U.S. teacher might be alarmed if presented with an equation such as " $59 : 8 = 7 + 3 : 8$ ," which is another way to express the equivalent fractions  $\frac{59}{8} = 7\frac{3}{8}$ . PSTs need exposure to such possible mathematical representations that immigrant students and their parents might use as resources to support mathematical learning. Within enhanced PCK, bridges between school mathematical practices and home mathematical practices could be made.

In summary, CRMT embraces and expands the four components of PCK. CRMT forces teachers to rethink their relationships with students, mathematics,

curriculum, and the purpose of mathematics teaching—which we argue are all important elements of a robust framing of PCK.

## Methods

### *Setting*

Data were collected from a mathematics methods course at an urban university in the Pacific Northwest region of the United States taught by the first author. This mathematics methods course was part of a graduate-level one year K-8 teacher certification program with a master's degree option. It was a two-quarter sequenced course that met weekly for three hours. Data were collected from 40 PSTs during the second quarter of the methods course. PSTs represented a wide range of ages and experiences, including second career, home-schooling parents, and military veterans. The majority of PSTs were White and female with an average age of 28 years. At the time of the study, the PSTs' teaching responsibilities increased at their school placements, most with sizeable cultural/linguistic diversity evidenced by the range of 30-56 different languages spoken in collaborating urban school districts. At the time of data collection, PSTs were in their second quarter of their teacher certification program. Furthermore, PSTs' presence in their school placements increased along with their teaching responsibilities to plan and deliver daily lessons.

As part of a National Science Foundation funded TEACH MATH (Teachers Empowered to Advance Change in Mathematics) Project, this mathematics methods course utilised a theoretical framework and specifically-designed activity modules that promoted PST knowledge development connecting children's mathematical thinking and cultural/linguistic funds of knowledge, as well as supporting PSTs to teach mathematics in a manner that was responsive to their students and their particular learning contexts (Aguirre, 2009; Aguirre et al., in press; Turner et al., 2012). At this stage of their preparation, the PSTs were familiar with the goals of the methods course completing other assignments (e.g. mathematics inquiry, mathematics learning case study, community mathematics exploration<sup>2</sup>, language demand analysis) that required deep reflection about how those constructs fit together to help children learn mathematics.

### *Data Sources*

Data were collected in March 2011 as part of a course assignment asking PSTs to select any mathematics lesson to analyse using the culturally responsive mathematics teaching (CRMT) tool (Aguirre & Zavala, in press). The CRMT tool is made up of eight dimensions that approximate the categories of mathematical thinking, language, culture, and social justice<sup>3</sup>: Intellectual Support (IS), Depth of Knowledge

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<sup>2</sup> The community math exploration (CME), also called the Community Math Walk, required students to visit several places in the local community, observe mathematical practices in the settings (by children, adults, consumers, employees, etc), and design a standards-based mathematics lesson informed by this activity. The CME and the math learning case study are specific TEACH MATH modules.

<sup>3</sup> See Aguirre & Zavala (in press) for a detailed discussion of the research that informs this CRMT tool.

and Student Understanding (DofK & SU), Mathematical Analysis (MA), Mathematics Discourse and Communication (MD & C), Student Engagement (SE), Academic Language Support for ELLs: Use of L1 (ALS:A) and Use of ESL Scaffolding Strategies (ALS:B), Funds of Knowledge/Culture/Community Support (CFoK), and Use of Critical Knowledge/ Power/Social Justice (CMSJ). Each category utilises a rubric scale of 1-5 with descriptions of what evidence constitutes a specific rating (see Appendix A for two sample categories: CFoK and CMSJ). PSTs were invited to select any mathematics lesson for this analysis. The PSTs self-assessed how their lesson attended to each category on a scale from 1 (little or no inclusion) to 5 (full inclusion). PSTs self-rated their lessons and, for each rubric category, elaborated on why those ratings were selected. PSTs then wrote a critical reflection about the strengths and limitations of the lesson based on the analysis, described a specific example of strengthening one area (in this lesson or in subsequent lessons), and presented general views of how this analysis might impact on future mathematics teaching. Data collected for this analysis included their written lesson plans, lesson self-assessment using the rubric tool, and critical reflection.

### *Data Analysis*

We employed a mixed methods approach to analyse the data (Creswell & Plano Clark, 2011). The quantitative analysis was based on data from a small (n=40), conveniently sampled population. For our purposes, the quantitative data in the form of student self-ratings enabled broader analysis to assess the areas where PSTs rated the quality of their lessons across the eight dimensions. Qualitative data from students' responses and reflections informed students' rationales for their self-ratings, and provided insights into their perceptions and opinions of culturally responsive mathematics teaching. Utilizing mixed methods enabled a more comprehensive analysis of PST ratings and responses, as well as enabled each strand of data to help explain results in the other strand.

As the qualitative data are entrenched within the quantitative rubric, an embedded design was employed, which enabled study of teachers' understanding of the rubric and how it applied to their own lessons (Creswell & Plano Clark, 2011). In this case, qualitative data were collected alongside scale ratings to illuminate explanations underlying the quantitative data. An embedded design also facilitated addressing multiple research questions simultaneously; in this case, trends and variability in PST self-ratings were examined, as well as PST understandings of language, culture, and social justice as part of lessons, which led to further analysis of teacher receptivity and resistance to critical examination of their teaching practice.

*Quantitative analysis.* The purpose of the quantitative analysis was to understand how well PSTs felt they could attend to each of the dimensions within a particular mathematics lesson, and to test a hypothesis that could account for variability among the last four dimensions. PST ratings were tallied to assess frequency. Initial descriptive statistics were generated to examine the distribution of PSTs' ratings. In cases where PSTs gave themselves in-between

scores (e.g. “between 3 and 4” or “3.5”), scores were rounded down for the sake of assessing frequency of ratings. Patterns in the variability of latter categories prompted further analysis.

We soon noticed that many PSTs selected to analyse the lesson from their community math exploration (CME) ( $n=17$ ), whereas others selected lessons that were linked to the mathematics curriculum currently being used at their placement or lessons developed for their mathematics inquiry assignment. For analysis purposes we called these types of lessons non-CME lessons ( $n=23$ ). With two distinct groups, we hypothesized that the PSTs who used their CME lesson would have a significant difference in mean score for the Funds of Knowledge category, given the explicit goal of the CME lesson assignment was to identify community funds of knowledge and design a lesson based on that information. Given the smaller sized group and how the rating scale used interval parameters (from 1-5) and since PSTs could rate themselves anywhere in between whole digit scores (and some did), a two-tailed t-test was appropriate to compare mean differences between PSTs who used their CME lesson and PSTs who did not.

*Qualitative analysis.* Data analysis began with first level descriptive codes using the rubric categories (e.g., mathematics discourse, student engagement, language, CFoK) to capture the general content of PST explanations and reflections. Analytic induction (Bogdan & Biklen, 1992) guided analysis and subsequent coding of PSTs’ rating explanations and reflections, focusing on how PSTs interpreted academic language, community funds of knowledge, and social justice elements, and their implications for instructional practice. PSTs’ rubric responses were analysed to assess rationale and understanding of the rubric; rating scores were then compared to lesson plans to gauge the fidelity of self-ratings, and how PST ratings reflected the rubric. We read rubric responses and critical reflections, then coded them for rationale related to each category. Instances in the reflections where PSTs discussed the strengths and limitations of lessons, the utility of the CRMT rubric, and possible improvements were highlighted and marked for analysis.

We created analytical codes to qualify the nature of different views PSTs seemed to express towards incorporating components of the academic language, funds of knowledge, and critical knowledge/social justice categories into their lesson plans (see Appendix B). These codes initially emerged from the data and were further framed and refined by linking to the mathematics/science teacher resistance framework offered by Rodriguez (2005). He described different forms of ideological and pedagogical resistance to teaching science for understanding and diversity, which we adapted for mathematics teaching. Ideological resistance was characterised by raising doubts about the purposes of incorporating categories into mathematics teaching. Pedagogical resistance was characterised by expressions of uncertainty due to lack of confidence or a lack of knowledge/skills to move away from traditional forms of mathematics instruction; finding reasons to downplay these dimensions of mathematics teaching; or explaining why they would not work in their classroom contexts. We created complementary “receptivity” codes to capture indications of openness

and willingness of PSTs to alter their practice. Ideological receptivity was characterised by agreement with the inclusion of academic language, funds of knowledge, and critical mathematics considerations into lesson plans, while pedagogical receptivity was indicated through agreement, willingness, and/or providing examples of how to incorporate these categories into practice.

Coding was completed by all three authors, who then compared codes across PSTs to assess inter-rater reliability and strengthen internal validity (Merriam, 2009). Each rater gauged the accuracy of PST ratings and how their rationales and explanations compared to the rubric's guidelines. Analysis was completed separately by each rater, then collectively compared and revised to ensure agreement on constructs. While we coded student work from 40 PSTs, some were self-disqualified because they did not provide evidence that we could code for teacher receptivity in their critical reflections or narrative of rubric evidence. Thus the analytic coding resulted in a range of 31-33 PSTs lessons depending on the specific codes. Thematic patterns were documented through analytic memos.

## Findings

### *Quantitative Analysis*

From the frequency distribution, (see Figure 2) we saw overall high ratings in the first five categories, which accounted for categories that attend to children's mathematical thinking and align with traditional notions of PCK (such as student engagement, and the conceptual focus of mathematical discourse). There was more variability and lower self-ratings in the later categories, which accounted for language, culture, and critical mathematics. Though variability and more low scores were hypothesized in the last two categories (Cultural Funds of Knowledge and Critical Mathematics Knowledge/Social Justice) based on the CRMT literature that outlines tensions for pre-service teachers (Aguirre, 2009; Gutierrez, 2009), the variability in the language categories was not anticipated as PSTs in this teacher education program were learning the importance of attending to the needs of English Learner (EL) students.

The descriptive statistics (see Table 1) confirm the dramatic variation in the later categories while also ratifying the relatively high self-ratings for categories related to children's mathematical thinking. The children's mathematical thinking categories were not only on average higher than the language, culture, and social justice categories, they also had less variability.

One potential explanation for the variability in the last two categories of community/cultural funds of knowledge (CFoK) and critical math knowledge/social justice (CMSJ) is that two distinct types of lessons were used by the PSTs for this assignment: Community Math Exploration (CME) lessons ( $n=17$ ) and traditional (non-CME) lessons ( $n=23$ ). The  $t$ -test confirmed that there were significant differences in these two categories. In the CFoK category, PSTs who used the CME lesson had a high average self-rating ( $n=17$ ,  $M=4.06$ ,  $SD=1.09$ ) than PSTs who did not ( $n=23$ ,  $M=2.61$ ,  $SD=1.34$ ),  $t(40) = -3.77$ ,  $p < .01$ ,

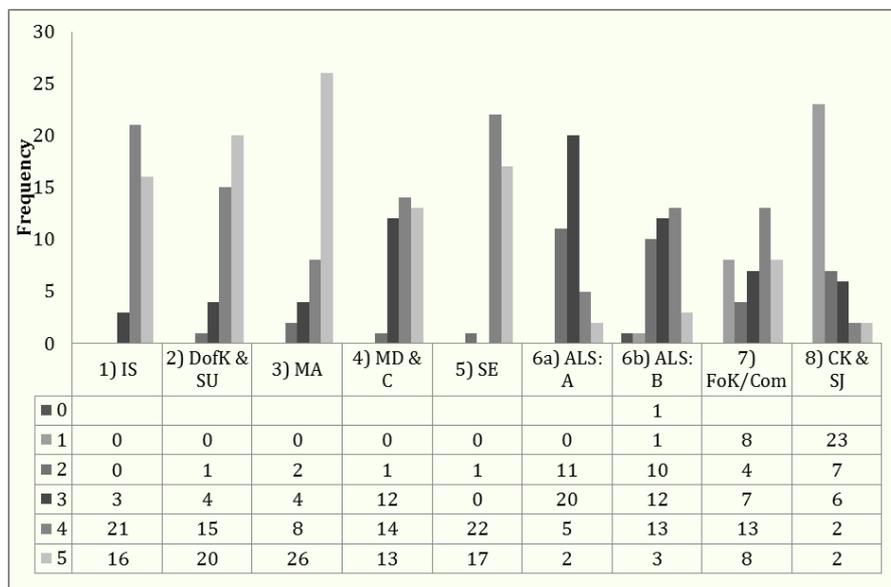


Figure 2. Rating frequency across all categories for all PSTs

$d=1.19$ . In the CMSJ category, PSTs who used the CME lesson had a higher average self-rating ( $n=17$ ,  $M=2.41$ ,  $SD=1.34$ ) than PSTs who did not ( $n=23$ ,  $M=1.39$ ,  $SD=.84$ ),  $t(40)=-2.79$ ,  $p<.01$ ,  $d=0.91$ .

Though both mean differences are significant, the major impact seemed to be mostly around the CFoK category, in which the group mean difference is from a 2.61 to a 4.06, suggesting that PSTs were able to rate themselves on the high end in how they attended to students' cultural funds of knowledge using a lesson from an assignment that explicitly required attention to children's funds of knowledge. Though significant, the mean differences between groups for the CMSJ category did not indicate a high rating from the CME lesson group, just a higher rating of 2.41 versus a mean rating of 1.39 in the traditional lesson group. Both mean ratings are low in the CMSJ category, suggesting that both groups of PSTs felt the evidence in their lessons was limited.

The high consistency of the self-ratings related to children's mathematical thinking categories suggest the PSTs self-ratings can be interpreted as over-all higher confidence in dimensions of lesson planning associated with traditional PCK, with few exceptions. However, this analysis only quantifies variability associated with the latter dimensions of language, culture, and social justice (i.e. non-traditional elements of PCK), but did not explain the nature of the variability. Hence we turned to the findings of the qualitative analysis.

Table 1  
*Descriptive Statistics for all PST, and Differences Between Lesson Type*

Category	All PST			Group 1: CME lesson			Group 2: Traditional lesson		
	<i>M</i>	<i>SD</i>	<i>N</i>	<i>M</i>	<i>SD</i>	<i>n</i>	<i>M</i>	<i>SD</i>	<i>n</i>
1) Intellectual Support	4.33	0.62	40	4.35	0.50	17	4.30	0.70	23
2) Depth of Student Knowledge	4.38	0.74	40	4.59	0.51	17	4.17	0.89	23
3) Mathematical Analysis	4.45	0.88	40	4.41	0.87	17	4.48	0.90	23
4) Mathematics Discourse and Communication	3.98	0.86	40	4.06	0.97	17	3.91	0.79	23
5) Student Engagement	4.38	0.63	40	4.35	0.79	17	4.39	0.50	23
6a) Academic Language Support: Use of L1	2.92	0.85	38	3.12	0.78	17	2.76	0.89	21
6b) Academic Language Support: Use of ESL Scaffolding Strategies	3.10	1.11	40	3.47	0.94	17	2.83	1.15	23
7) Funds of Knowledge*	3.23	1.42	40	4.06	1.09	17	2.61	1.34	23
8) Critical Math and Social Justice*	1.83	1.17	40	2.41	1.33	17	1.39	0.84	23

\* statistically significant group difference of means,  $p < .01$

### *Tracking' Receptivity To Language, Culture and Social Justice*

The variability of the self-rated categories related to language, cultural funds of knowledge, and critical math/social justice suggest that PSTs found these dimensions more challenging to incorporate into mathematics lessons. A detailed examination of the PSTs' lesson analyses and critical reflections suggested strong receptivity to connecting cultural funds of knowledge into mathematics lessons and integrating academic language supports for second language learners (see Table 2). In contrast, there was mixed receptivity for connecting lessons to critical math/social justice issues, with the bulk of the resistance focused more on pedagogy (e.g., "I'm not sure how to do this with arrays") rather than ideology (e.g., "Maybe for older students, I'm not sure social justice issues work in the

primary grades"). One important finding is that PSTs who utilized their CME lessons were more receptive to identifying ways to connect to social justice issues than those who analysed non-CME lessons.

Table 2  
*Receptivity and Resistance among PSTs*

Code	Ideological and Pedagogical Receptivity	Ideological Receptivity, Pedagogical Resistance	Ideological Resistance, Pedagogical Receptivity	Ideological and Pedagogical Resistance
Language Categories ( <i>n</i> =31)	28 (90%)	2 (7%)	0	1 (3%)
Cultural Funds of Knowledge ( <i>n</i> =31)	28 (90%)	3 (10%)	0	0
Critical Math/Social Justice ( <i>n</i> =33)	15 (45%)	12 (36%)	3 (9.5%)	3 (9.5%)

*Strong receptivity to integrating academic language supports.* Pre-service teachers (*N*=31) expressed strong receptivity toward the use of L1 and/or ESL scaffolding to support academic language development of EL students. A full 90% (28/31) of the lessons with evidence related to academic language supports were coded as receptive ideologically and receptive pedagogically. For the first language category, Use of L1, the majority of PST comments reflected tolerance for allowing EL students to utilise their L1 (e.g., "native language") as a resource for students to express their ideas and access the mathematical tasks, as shown in these quotes:

Students with strong background in math and their native languages are allowed to share their math learning with classmates in L1. [PST 191]

We allowed students to plan their party in their native language. [PST 189]

Many PSTs expressed receptivity toward getting better at using L1 intentionally in the classroom (such as having students or parents help them translate mathematical terms for word walls or activity sheets), or to get more training themselves in a particular language so they could better communicate with emerging bilingual students. Many of these PSTs had clear strategies in their evidence or in their reflections of what they could do as teachers to support the use of L1. For example, PST 189 described in her reflection how she would use her students' "linguistic funds of knowledge" in subsequent lessons, writing, "I

will encourage the use of native languages and embrace the sharing and allow students to elaborate in their native language.”

Some PSTs also conveyed a need to get better at explicitly incorporating ESL scaffolding strategies. After rating both language categories a 3 and a 2, respectively, PST 156 reflected on how to change his lesson planning practice through explicit attention to L1 and ESL scaffolding:

My lesson plan indicated that ELL students would be grouped with bilingual students, but no mention was made of encouraging the use of L1 or of welcoming emerging language strategies, such as gestures. And while, over time, this may become automatic in my teaching practice, as a beginning teacher I find it most effective to include in my lesson plans in order to decrease the likelihood of forgetting or glossing over the practice. Simply including some examples, such as allowing use of L1, gestures, pictures and other means of communication, in the lesson plan would inform my thinking and be a mental reminder to provide necessary scaffolding for language.

There was one PST, however, who stood out as resistant ideologically and pedagogically to using students' L1 in the classroom. PST 187 was placed in a kindergarten class and expressed a lack of receptivity towards children's early language skills:

At kindergarten age, my two EL students have not reached proficiency in either language. Especially for academic language, since they do not have the words in L1, they would not be of use in teaching L2. This, combined with the fact that no adults in the classroom speak either of the L1s of my EL students, make English vocabulary teaching the focus.

This PST's characterization of kindergarten students as having “not reached proficiency in their language” may be accurate, but may also indicate that she may require support to develop her own sense of how the emerging linguistic resources of kindergarteners could be utilized to learn mathematics.

*Strong receptivity to cultural funds of knowledge integration.* PSTs' receptivity to integrating cultural funds of knowledge into mathematics lessons was overwhelmingly positive. All PST reflections addressing this category ( $N=31$ ) expressed ideological receptivity to connecting math lessons with cultural funds of knowledge. Moreover, 90% (28/31) expressed being both receptive pedagogically and ideologically. The remaining 10% expressed uncertainty as to how to connect cultural funds of knowledge to a specific grade level (e.g., kindergarten) or expressed hesitancy to modify the given sequence of lessons. For example, one PST commented about her reluctance to change her traditional lesson on the “doubling strategy” or its sequence in the curriculum because of her curricular knowledge that included later lessons with “relatable” contexts. While the sequence remained unaltered, she considered framing the lesson in a more relevant context to encourage student engagement and provide a “sneak peek” at a future lesson that would make those connections explicit. This PST's comments reflect an ideological receptivity, but her hesitancy to change this lesson coupled with a reliance on the curriculum with few specifics related to what might be

more “relevant” contexts for her students suggests a pedagogical resistance.

As the quantitative results reveal, lesson type plays a role in self-rating for this category. The PSTs who explicitly selected their CME lesson rated their lesson consistently high in this dimension. Of those who chose a more traditional lesson, almost all of the PSTs expressed willingness to update their selected lesson or make the connection explicit in future lessons. For example, PST 163 rated her first grade lesson on fact families a 2 and was clear she wanted to improve connections to CFoK:

Another area of weakness I came across in my lesson is a connection to the community. The only cultural connection I made in this lesson was to students’ families. This was a very superficial correlation because though it provided an opportunity for students to share about their different family structures, the rest of the lesson had nothing to do with their families. Instead the lesson went on to show how numbers are related to each other. The recent Community Math Walk (CME) assignment gave me valuable experience in practicing bringing the community into the classroom. In addition to helping my students become aware of how present math is in their everyday lives, I would like to incorporate relevant issues from the community into my lessons. By doing so I hope to promote critical thinking from my students as well as support their development and understanding of mathematical concepts through interesting and authentic connections from their lives.

This PST expressed her full receptivity, both ideologically and pedagogically, to integrating cultural funds of knowledge in a manner consistent with the kind of robust PCK we hope to support. Furthermore, while this PST selected a traditional lesson to analyse, the CME experience had a positive impact to guide her future mathematics lesson construction. For this PST, there is an emphasis of developing mathematical understanding through authentic connections to her students’ experiences.

*Mixed receptivity about connecting lessons to critical mathematics/social justice.* Pre-service teachers voiced mixed views about connecting critical mathematics/social justice to their mathematics lessons. The degree of receptivity varied among the critical reflections ( $N=33$ ). Forty-five percent ( $15/33$ ) were ideologically receptive and pedagogically resistant and 36% ( $12/33$ ) were both ideologically and pedagogically receptive. However, three PSTs were resistant ideologically while receptive pedagogically and three PSTs were fully resistant both ideologically and pedagogically.

Twelve PSTs reflected an ideological and pedagogical receptivity to integrating social justice into their math lessons. In these cases, PSTs argued this was an important strength of their lesson, or if not explicitly addressed, could imagine ways to modify and/or create new lessons with this explicit focus. For example, PST 174 critiqued her CME lesson and expressed strategies that would create dialogue about economic realities and inequities with her students:

This lesson could have done a better job at paying attention to social justices. Coming from the typical white middle class family I often forget many of the social injustices that occur. I think that one strategy I could include in my lesson

would be to bring attention to these issues. I do think that it is important that kids realize that there are different realities in life. If they had to work within a budget that would bring these issues to life. Yes we would all love to have the big elaborate party but can we afford that? What can we do to change our budget? I could continue to ask kids questions such as are you going to make or buy your dessert? Questions like these will get kids thinking about economic inequalities. When it does come up I need to embrace them instead of ignore it.

This PST is willing to “embrace” issues of social justice and generates pedagogical ideas to make this connection more explicit in future lessons. In addition, 10 out of the 12 PSTs reflecting full receptivity analysed their CME lesson, suggesting that the CME activity supported PSTs in reflecting on meaningful engagement with social justice in mathematics through children’s funds of knowledge.

Overall, 15 PSTs reflected a stance that was ideologically receptive but pedagogically resistant to making this connection. For these PSTs, it was clear that social justice was an important component they recognized was missing from their lesson, but they voiced a variety of concerns related to teaching, such as not knowing how to generate good examples of authentic problems for this specific math topic (e.g., multiplication with arrays) or this grade level (e.g., kindergarten). For example, PST 188 rated her multiplication lesson using arrays as a 1 in this category. While receptive to incorporating social justice issues, she expressed pedagogical uncertainty, stating, “I’m not quite sure how I can promote social justice in a 3rd grade math lesson that focuses on arrays.” In addition, other PSTs voiced their pedagogical resistance to social justice in relation to curricular sequence. For example, PST 154 made the case that in her first grade lesson on place value social justice connections are important but cannot precede number concepts:

As for the critical knowledge piece, the lesson would require more depth to meet this standard. Maybe placing the numbers into a situational problem may help address the critical knowledge component, but as an introductory lesson exploring ones and tens, it might be that simpler is better.

This PST positioned social justice contexts as an application rather than authentic situation that might introduce a mathematical idea, thus demonstrating a pedagogical resistance to making a math lesson (at least the first lesson of a math unit) connected to social justice.

The rest of the PST examples (n=6) exhibit resistance to incorporating social justice from an ideological standpoint. While three PSTs completely resisted connecting social justice issues in math lessons, three other PSTs voiced possible teaching strategies that might be considered to make this connection. In these cases, the PSTs struggled with what was “psychologically” appropriate for students in relation to studying issues of power with mathematics, yet offered some solid teaching strategies for math lessons.

I was aware before doing this analysis that I was not incorporating the use of critical knowledge/power/social justice into my lessons, and I have been

struggling with how to do that in a Kindergarten class. In my CRMT analysis, I noted that I am working on how to design some math lessons around issues such as fairness and equal shares to address this at an appropriate mathematical (and also psychological) level for 6-year-olds. I think I can make it meaningful by relating the math to the students' interactions with each other as well as student/teacher interactions in the classroom, but the challenge so far has been finding something mathematically simple enough to work. [PST 176 reflection]

This PST struggled with the appropriateness of social justice math lessons for primary students, but possible pedagogical leverage points exist that drives this teacher to think about possible lesson topics that meet the mathematical needs of the kindergarteners and address social justice issues.

## Conclusions

From this study, we learned that there are productive and challenging dimensions for PSTs to develop more robust forms of PCK. Overall, PSTs felt very confident that they could or did address important dimensions of children's mathematical thinking within their lesson, including an emphasis on analysis, discourse, and student engagement. Since a subset of PCK clearly links to beliefs about children's mathematical thinking, knowing the curriculum, and instructional strategies, these responses reflect a positive disposition toward these traditional elements of PCK and provide evidence of integrating these elements into their developing practice. However, the variability and responses related to the categories associated with language, cultural funds of knowledge and critical mathematics/social justice revealed a range of teacher receptivity and resistance that are consequential to improving their lessons from a culturally responsive standpoint. Within a robust framing of PCK, the findings suggest that PSTs will need additional support to attend to and integrate these constructs into their practice.

As mathematics teacher educators, we can build on the importance of children's mathematical thinking, strengthen PST understanding of the role language plays in mathematics learning and teaching for second language learners, and increase ways PSTs can integrate children's cultural funds of knowledge into their mathematics lessons. These elements are part of expanding PSTs' knowledge about the purposes of education, leveraging children's mathematical thinking, and expanding curricular knowledge to promote mathematical engagement and learning. Connecting mathematics lessons to social justice contexts is more challenging, yet many PSTs were receptive. The pedagogical tensions related to curricular knowledge and instructional strategies expressed by many PSTs related to social justice are consistent with other research (Aguirre, 2009; Christiansen, 2008; Gutierrez, 2009; Gutstein, 2006; Leonard et al, 2010). But, as evidenced by those selecting their CME lessons, PSTs made productive moves to strengthen this component of their teaching. Moreover, the CME enabled some PSTs to imagine how they could modify their traditional math lessons to make explicit connections to funds of knowledge in

children's families and communities. Multiple opportunities to develop that expanded view of the curriculum outside the school provides PSTs with enhanced opportunities to develop a culturally responsive mathematics teaching practice (Aguirre et al, in press).

Furthermore, as a teacher education community, we need "intellectual tools" to support and extend PST development of pedagogical content knowledge in rich and rigorous ways (Grossman et al., 2005). The CRMT rubric tool used by the PSTs in this study combines the traditional elements of PCK with key components of culturally responsive mathematics teaching (Aguirre & Zavala, in press). Through opportunities to reflect on their practice from a culturally responsive mathematics teaching perspective, PSTs can develop robust forms of PCK to help them become effective mathematics teachers.

As mathematics teacher educators we can also learn from documenting the points of PST receptivity and resistance as long as we are willing to engage in critical reflection ourselves to address the needs of PSTs in our teacher education programs, and use these points as guideposts for our course designs. Embracing a more robust definition of PCK will aid mathematics teacher educators in their quest to make teacher education relevant, and prepare our future teachers for the multifaceted humanity of their students.

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### Authors

Julia Aguirre, University of Washington, Tacoma, USA.

Email: <[jaguirre@uw.edu](mailto:jaguirre@uw.edu)>

Maria del Rosario Zavala, San Francisco State University, San Francisco, USA.

Email: [mza@sfsu.edu](mailto:mza@sfsu.edu)

Tiffany Katanyoutanant, University of Washington, Seattle, USA.

Email: <[tkatan@uw.edu](mailto:tkatan@uw.edu)>

### Appendix A

#### Sample Rubric Categories: FoK and CMSJ

Rating	1	2	3	4	5
Category	<i>Guiding Question: How does my lesson help students connect mathematics with relevant/authentic situations in their lives?</i>				
<b>7) Funds of Knowledge/Culture/Community Support</b>	No evidence of connecting to students' cultural funds of knowledge (parental/community knowledge, student interest). Lesson incorporates culturally neutral contexts that "all students" will be interested in.	There is at least one instance in connecting math lesson to community/cultural knowledge and experience. Lesson draws on student knowledge and experience. Focus is with one student or a small group of students.	There is at least one sustained episode of sharing and developing collective understanding about mathematics that involves connecting to community/cultural knowledge.  Or, brief episodes of sharing and developing collective understandings occur sporadically throughout the lesson.	There are many sustained episodes of sharing and developing collective understandings about mathematics that involves connecting to cultural/community knowledge (e.g. student experiences are mathematized, student/parent connections with math work; math examples are embedded in local community/cultural contexts and activities – i.e. games).	The creation and maintenance of collective understandings about mathematics that involves intricate connections to community/cultural knowledge and permeates the entire lesson. This would include hook/intro, main activities, assessment, closure and homework. Students are asked to analyze the mathematics within the community context and how the mathematics helps them understand that context.
	<i>Guiding Question: How does my lesson support students' use of mathematics to understand, critique, and change an important equity or social justice issue in their lives?</i>				
<b>8) Use of critical knowledge/social justice</b>	No evidence of connection to critical knowledge (socio-political contexts, issues that concern students)	Opportunity to critically mathematize a situation went unacknowledged or unaddressed when present.	There is at least one instance of connecting mathematics to analyze a sociopolitical/cultural context.	There is at least one major activity in which students collectively engage in mathematical analysis within a sociopolitical/authentic or problem-posing context. Mathematical arguments are provided to solve the problems. Pathways to change/transform the situation are briefly addressed.	Deliberate and continuous used of mathematics as an analytical tool to understand an issue/context, formulate mathematically-based arguments to address the issues and provide substantive pathways to change/transform the issue.

## Appendix B

## Coding Scheme for Receptive/Resistant Ideological and Pedagogical Markers with Examples from PST Lesson Analyses

Code Category	<i>Both Ideologically and Pedagogically Receptive to improving in this area</i>	<i>Ideologically Receptive but Pedagogically Resistant to improving in this area</i>	<i>Ideologically Resistant but Pedagogically Receptive to improving in this area</i>	<i>Both Ideologically and Pedagogically Resistant to improving in this area</i>
<b>Use of L1</b>	“I will encourage the use of native languages and embrace the sharing and allow students to elaborate in their native language. In addition, I will encourage students to embrace and share ideas and thoughts in their first language. While opportunities are given for students to use their native language, support will be planned in the actual lesson”(189)	“There are a variety of languages spoken in this classroom, though out of all the ELL students not one speaks the same L1 as another. This makes the use of L1 particularly difficult to incorporate, especially since neither I nor the cooperating teacher speaks any of the native languages of our ELL students. As a result however, more concerted effort should be put towards scaffolding ESL students” (154)	N/A	“My ELL kindergarten students are still gaining language skills generally, so there is no use of L1 to tolerate, as 2 indicates.... At kindergarten age, my two ELL students have not reached proficiency in either language. Especially for academic language, since they do not have the words in L1, they would not be of use in teaching L2. This, combined with the fact that no adults in the classroom speak either of the L1s of my ELL students, make English vocabulary teaching the focus” (187)
<b>Use of ESL Scaffolding Strategies</b>	“Another way to support understanding in ELL students is through careful use of cognates and checks for understanding throughout a lesson” (185)	N/A	N/A	
<b>Funds of Knowledge</b>	“This can be achieved by getting to know the students and their community and incorporating that into the lesson. This way, learning can become a more meaningful experience for the students” (173)	“I want my students to play games and explore probability concepts to make the math topic fun before I lead them to investigating more complex probability math tasks which relate to their real-life events or scenarios in their communities.” (191)	N/A	“I must first examine my own students and their academic, cultural, and linguistic needs before I can create an appropriately accommodating lesson and apply this rubric” (190)
<b>Critical Knowledge /Social Justice</b>	“This lesson is a good way of showing the importance of building on one’s math skills, but it is very important to also point out the power of mathematics. Since this lesson didn’t address that, I will be sure to address this in another lesson” (178)	“Because we are still learning our basic math facts, we have not discussed any social issues or ways in which we see these equations in our community. I know this is coming up in later lessons, but as far as this lesson is concerned we are simply creating a 10’s Fact Family Book.” (177)	“I think I can make it meaningful by relating the math to the students’ interactions with each other as well as student/ teacher interactions in the classroom, but the challenge so far has been finding something mathematically simple enough to work” (176)	“The use of critical knowledge/power/social justice is lacking in this lesson because it did not seem appropriate or necessary in the development of place value for a first grade classroom” (168)