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Calculations and Expectations: How engineering students describe three-dimensional forces

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Calculations and Expectations: How engineering students describe three-dimensional forces

Abstract

The premise of student-centered teaching is to respond to the ways in which students engage with the context and content of their learning, and therefore the purpose of this study was to find out how students visualize three-dimensional statics problems from two-dimensional diagrams early in a first-year engineering course. Think-alouds were conducted where students were asked to describe magnitudes and directions of various forces acting in three-dimensional spaces. Three key themes emerged: students have more trouble visualizing points behind, or vectors pointing into, the plane of the page; students may not use contextual clues to aid in their visualization; and students rely on equations to answer problems even when not necessary or even possible to do so. These findings are important to instructors in disciplines where spatial visualization is important because as they are already "experts" in this skill, they may underestimate the difficulty students initially face in approaching these problems. The value of using think-alouds to reveal student thinking as they struggle with concepts is also discussed.

La prémisse de l'enseignement centré sur l'apprenant est de réagir à la manière dont les étudiants s'intéressent réellement au contexte et au contenu de leur apprentissage. En conséquence, le but de cette étude était de découvrir comment les étudiants visualisent les problèmes statiques tridimensionnels à partir de diagrammes bi-dimensionnels, dans un cours de génie de première année. Des exercices de réflexion à haute voix ont été effectués, au cours desquels on a demandé aux étudiants de décrire les magnitudes et les directions de diverses forces qui agissaient dans des espaces tridimensionnels. Trois thèmes clés sont apparus : les étudiants ont davantage de difficulté à visualiser les points qui se trouvent derrière le niveau de la page ou les vecteurs tournés dans la direction de la page; les étudiants n'utilisent pas toujours les indices contextuels dans leur visualisation; et enfin, les étudiants s'appuient sur les équations pour répondre aux problèmes, même quand ce n'est pas nécessaire ou quand c'est impossible à faire. Ces conclusions présentent un grand intérêt pour les enseignants de disciplines où la visualisation spatiale est importante car, puisqu'eux-mêmes sont déjà « experts » dans cette compétence, ils risquent de mésestimer la difficulté à laquelle les étudiants sont confrontés, au début, quand ils essaient de résoudre ces problèmes. L'article discute également de la valeur de l'utilisation d'exercices de réflexion à haute voix pour révéler ce que pensent les étudiants quand ils sont aux prises avec un problème.

Keywords

spatial visualization, statics, novice vs. expert, think-alouds

Cover Page Footnote

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Spatial visualization has been defined as "the ability to mentally manipulate, rotate, twist, or invert pictorially presented stimulus objects" (McGee, 1979, p. 893). Spatial visualization skills are important for students to accurately visualize structures in mechanics and are considered essential in engineering (Alias, Black, & Gray, 2002). Knowledge of spatial skills is directly linked to students' future success in their professional work (Hsi, Linn, & Bell, 1997). Visualization ability has even been related to problem solving skills in areas where the problem does not actually involve visualization (Bodner & Guay, 1997).

There is an overwhelming amount of literature about teaching spatial visualization skills for various ages and disciplines. At the post-secondary level, seemingly countless pre-post studies have indicated that spatial abilities can be improved through the use of spatial exercises, drafting instruction, physical models and computer models (e.g., Alias, Black, & Grey, 2002; Lord, 2006; Miller, 1990; Wiley, 1990). Sorby and Baartmans (1996) speculated that a diverse range of spatial activities may be the key to effective spatial ability. In fact, simply taking a physics course has been shown to improve students' visual-spatial abilities (Pallrand & Seeber, 1984). According to cognitive theories, a skill is the result of declarative knowledge (facts associated with the context) being integrated and transformed in a continuous process of refinement over time, as a result of deliberate practice on the part of the learner (Ericsson, Krampe, & Tesch-Romer, 1993). Therefore, the challenge for educators is not only to figure out better ways to teach this skill, but to figure out the challenges students have when learning to solve problems that require visualization skills, so that we can better meet their needs.

The literature on expertise and teaching suggests that that "expertise can sometimes hurt teaching because many experts forget what is easy and what is difficult for students" (Bransford, Brown, & Cocking, 2000). Thus, the knowledge that underlies effective teaching, "pedagogical content knowledge" (Redish 1994; Shulman 1986; Shulman 1987), requires an awareness of the typical difficulties that students encounter when they are learning a new skill or topic. As an engineering instructor at Mount Royal University, I have taught our introductory statics course several times. In my experience, one of the difficulties students have in this course is learning to interpret and analyze three-dimensional (3D) static equilibrium problems. Three-dimensional structures and forces are typically presented in two-dimensional (2D) drawings from a textbook, and some students seem to be able to visualize the structures and forces quickly while others struggle.

In a similar vein, Meyer and Land (2005) propose that student-centered teaching has an element of responsiveness that is sensitive to variation in the manner in which students engage with the context and content of their learning. Thus, in order to be responsive, the variation in student learning must be externalised in a form that can be responded to. Similarly, Land, Cousin, Meyer, and Davies (2004) argue that "we can't second guess where students are coming from or what their uncertainties are" (p. 58). Therefore, the purpose of this study was to determine the difficulties students in a first-year engineering class experience in learning to visualize 3D statics problems from 2D drawings. The main data source was think-alouds using two visualization problems, supplemented by individual course work, both collected in the first two weeks after the topic had been introduced in class.

Context

This study took place in my first-year, first-semester statics (Mechanics I) course at Mount Royal University, a Canadian public undergraduate institution. Mechanics I is comprised of 40 engineering students. Similar to most engineering technical courses, Mechanics I is content heavy and there is minimal time to teach spatial visualization skills which are required early in the course. Fortunately, the course is typically taken in parallel with another course which includes technical drawing, where students learn isometric and orthographic projections, dimensioning and other engineering drawing and communication skills. As spatial visualization is a key baseline requirement for success in engineering programs, and students are expected to be competent at it after the first few weeks of classes, it is important for instructors to understand the difficulties students have when first presented with mechanics problems that require 3D visualization, in order to coach these skills better and to improve students' chance of success.

Methods

Participants: Recruitment and Sample

All students in my fall Engineering Mechanics I class were invited to participate in the study, which included allowing their coursework to be analyzed as data, with the option of also participating in a think-aloud interview. The participants were recruited at the beginning of the semester by an independent faculty member and consent forms were held until the final marks for the course were submitted, so that the instructor/researcher did not know who had agreed to participate in the study until after the course. This study was approved by the Human Research Ethics Board of Mount Royal University.

Ten students volunteered to participate in this study. Five of the ten participants attended think-aloud sessions, which took place outside of class time. Types of spatial visualization difficulties were identified from the think-alouds, and subsequently all ten participants' in-class quizzes were examined for evidence of these difficulties. Table 1 (below) shows that the five people who participated in the think-aloud sessions represented a range of students in terms of course performance. This is important to note because the think-aloud sessions were used to identify themes, and the think-aloud participants had difficulties associated with different themes, regardless of their grade level. Of the remaining five participants, two had final grades ranging from C to C+ and three others did not finish the course.

Table 1
Think-Aloud Participants' Course Performance

Participant	Midterm 1 mark (%)	Final grade
A	91	A-
B	86	A-
C	91	C
D	45	N/A
E	36	N/A

Data Collection

Think-aloud procedure. Think-alouds are a type of verbal protocol and are a mainstay in cognitive psychology. They are used to infer mental models by observing students while they are actually engaged in mental activities. Participants are asked to talk out loud about what they are thinking, while engaged in a task which could normally be carried out alone (Streveler, Litzinger, Miller, & Steif, 2008). Since it is assumed that the majority of students represent a very limited number of distinctively different ways of understanding, small sample sizes are usually considered appropriate (Marton, 2000).

Think-aloud sessions were conducted one to two weeks after the concepts of three-dimensional forces, unit vectors and direction cosines were introduced in the class. The sessions were conducted by a faculty member outside of engineering to ensure the students would feel comfortable during the session and to keep the participants' identity unknown to the instructor until the end of the course.

Participants were given standard instructions on paper to ensure consistency of presentation. First, they completed a simple warm-up activity to practice verbalizing their thinking process. When the participant had completed this warm-up and had no further questions, they were asked to answer two additional problems. For each question presented, they were instructed to read the question out loud and then to speak out loud what they were thinking as they worked through the questions; they were instructed not to try to explain or summarize until the end. They were free to write on the paper, which was kept for analysis by the researcher. Think-aloud protocols were audio recorded and transcribed by the researcher after the course was finished.

The first question for the think-aloud was the following:

a) Please describe the direction and magnitude of the forces in Figure 1 (whether they are into/out of the page, which one is larger, and which one has larger x, y and z components).
b) Explain how you visualized this problem and arrived at your answer.

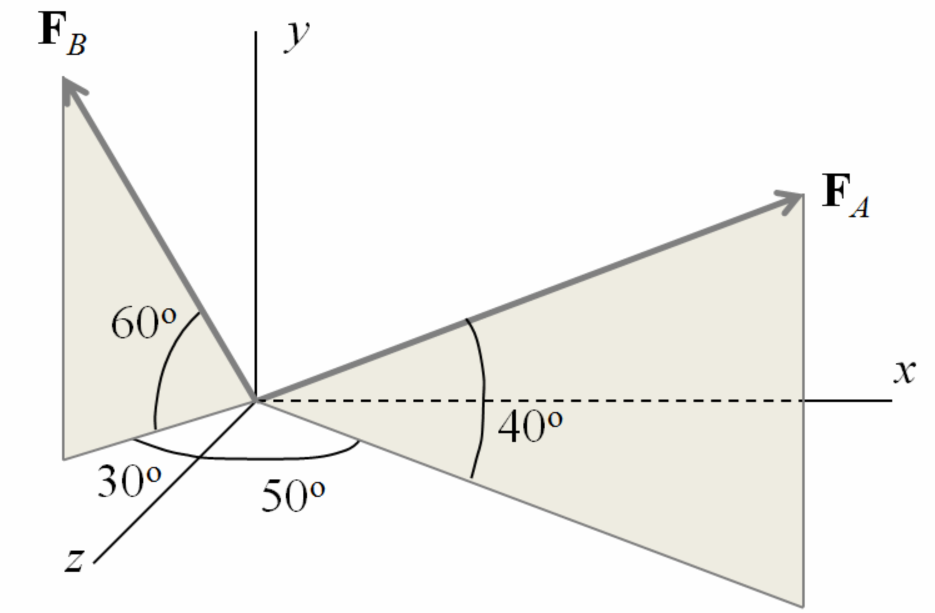

Figure 1. Diagram for question 1 from the think-aloud. From Bedford, A. M., & Fowler, W. (2008). *Engineering mechanics: Statics* (5th ed.) (p. 60). Upper Saddle River, NJ: Pearson Education. Reprinted with permission.

This problem was chosen because the associated diagram does not include any coordinates or magnitudes for the vectors shown. In order to answer the question, one must consider that each vector is pointing out of the page at a different angle from the x-y plane. Considering which vector has larger x, y and z components could help answer the question of which force is larger.

The second question was chosen because it involves the interpretation of forces acting on a physical structure:

a) Please describe the direction of the forces in Figure 2 (whether they go into/out of the page).
b) Write the 3-D co-ordinates of points A, B, C and D.
c) Explain how you visualized this problem and arrived at your answers.

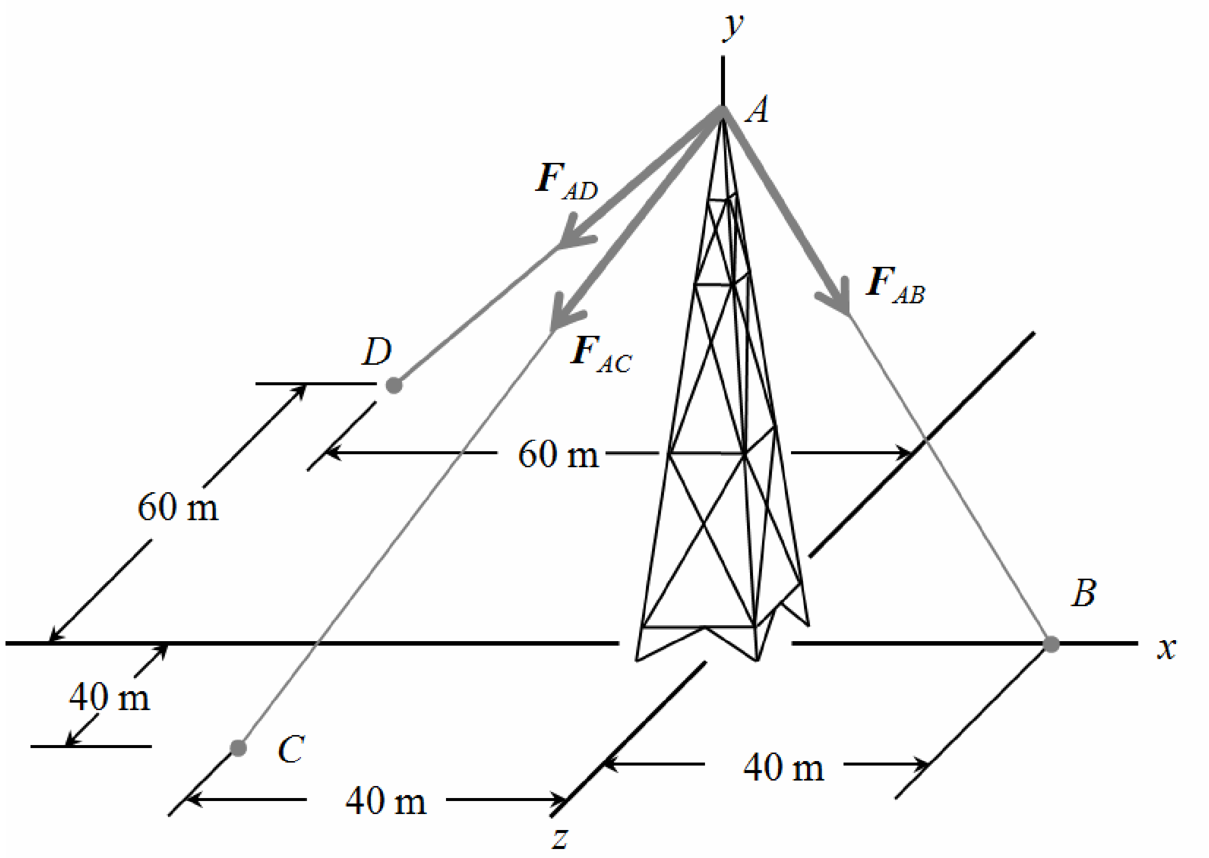

Figure 2. Diagram for question 2 from the think-aloud. Bedford, A. M., & Fowler, W. (2008). *Engineering mechanics: Statics* (5th ed.) (p. 61). Upper Saddle River, NJ: Pearson Education. Reprinted with permission.

Note that the forces are going in multiple directions which are defined in a different way than in question 1: not by angles, but by the locations of the endpoints of their line of action. It was expected that this would be a simpler problem to answer, since participants were not asked to interpret the magnitudes of the vectors but simply to describe directions and locate points.

The only intervention made by the interviewer during the think-aloud was to remind participants to think out loud if they appeared to be thinking but had stopped talking. After each question was completed the interviewer asked "Can you summarize how you arrived at your answer?" if the participant had not already done so.

Course work. Participants' individual work, which was completed over the same two-week period of time as the think-alouds, was also collected for this study. Their responses to any question that involved the interpretation or calculation of three-dimensional vectors were photocopied and kept for analysis. This amounted to one quiz which took place the week before the think-alouds began, as other coursework such as out-of-class assignments could not be expected to be reliable evidence of an individual student's understanding. The quiz question required students to redraw the diagram (Figure 3), showing the direction of the tension force in the cable BG, and to calculate the Cartesian components of the tension force acting at B, given a magnitude of 450 N.

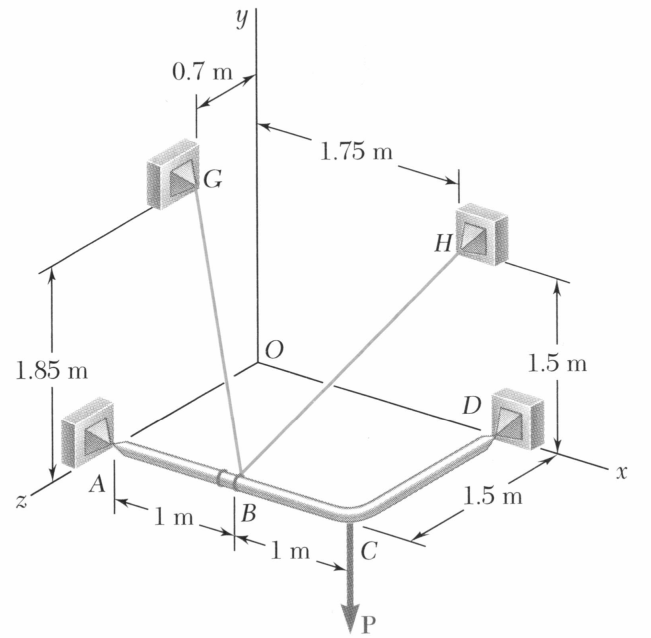

Figure 3. Diagram for quiz question. Beer, F., Johnston, E. R., & Eisenberg, E. (2007). *Vector mechanics for engineerings: Statics* (8^{th} ed.) (p. 56). New York: McGraw Hill. Reprinted with permission.

Data analysis. During transcription of the think-aloud recordings, some obvious themes immediately emerged; however, the transcripts were read several more times to obtain an overall feeling for them. Statements that pertained directly to difficulty interpreting the diagrams were identified and used to formulate additional themes. Subsequently, the participants' course work was examined to look for evidence of difficulties that were associated with these themes. Evidence included drawing forces the wrong way on a diagram and getting the sign (+/-) or value of a component of a force wrong according to a diagram, but mistakes such as algebra and notation errors were ignored.

Results and Discussion

Three major themes emerged from the think-aloud analysis. First, points behind the plane of the page or vectors going into the plane of the page gave students the most difficulty. Second, students did not use the real-life context of the problem in order to help interpret the diagrams. Third, students relied on using equations in their responses when this was neither necessary nor sufficient to answer the questions.

Theme 1: Visualizing Points or Vectors Behind the Plane of the Page is Difficult

Results indicate that visualizing points behind the plane of the page is a basic skills that needs to be mastered, as 5 out of 10 students struggled with this skill, as evidenced in both their think-aloud transcripts and written work. The students who performed better in the course (A, B) showed no signs of having this difficulty in the data analyzed. For example, in think-aloud question 2, three students (C, D, E) had trouble visualizing vector F_{AD} and/or identifying point D. When arriving at the question of whether F_{AD} goes into or out of the page, Subject C paused a very long time before he slowly said, "FofD……. you can't really tell which quadrant it goes in…… it looks like it stops over in quadrant 4… so FofD is going into the page." Subject D stated, "it looks like point D is floating up here…" but eventually wrote the co-ordinates of point D correctly and stated that F_{AD} was going into the page. Similarly, Subject E gave the co-ordinates of point D to be (-60, y, -60), which seems to indicate that he also saw the point as floating at some unknown height above the ground.

Several of the participants' answers to quiz 1, where the tension force was going into the plane of the page, also indicated that they may have had difficulty determining the direction of the force going into the page. Participant E drew the force in the wrong direction but then also wrote the sign of the x and y components of the force incorrectly according to his diagram so that ultimately, he got the sign of the z-component backwards. Two other participants, who did not complete the think-alouds, also had trouble visualizing the direction of the force in the quiz, as evidenced by their errors in determining both the x- and z-components of the force. Note that for this question, both the x- and the z- direction are out of the plane of the page, which further supports the argument that beginning students have trouble visualizing forces going into the plane of the page. The fact that this difficulty was in evidence in the students' quiz responses is also important because it seems to indicate that it was not a difficulty that could be explained by nervousness or anxiety during the think-aloud protocol.

These findings are important for physics, drawing and mathematics instructors because they make visible the difficulty students have in interpreting three-dimensional drawings soon after this skill is introduced. The think-aloud results were particularly illuminating because normally instructors don't see the process students go through in solving a problem, just the final results that are produced as a solution. Participants C and D struggled with the second think-aloud question but ultimately got the correct answer for F_{AD}, so in a traditional type of assessment, their struggles would not have been visible. Being aware that these types of problems can initially be a struggle for some students will allow an instructor to present such a problem in class in a more effective way, for example by spending more time modelling how the instructor would interpret such diagrams, and by allowing more time for students to wrestle with such a question in class. Since practice is important for mastering visualization skills, if an instructor were to present the solution to this seemingly straightforward problem too quickly, the struggling student would not have the time to get the practice needed to develop his own visualization skills.

Theme 2: Not Using Context in Interpreting a Diagram

Question 2 in the think-aloud was chosen because it incorporated a diagram of an actual structure, which could help students in interpreting the directions of the forces in the diagram. In order for the tower to be stable, at least one guywire must be supporting it behind the plane of the page. Also, since guywires are attached the ground, the height of point D should be zero (assuming the ground is flat). However, students' not being able to locate point D, as described above, indicates that they were not using this context or physical understanding to interpret the diagram.

As mentioned previously, three students had difficulty with question 2 of the think-aloud, while the stronger students (A, B) again showed no signs of having difficulty with this question. Subject E was unable to pinpoint the location of point D. In his answer to this question, he also wrote the coordinates of points A, B, and C as having unknown *y* locations. He did not seem to use the fact that the drawing was of a structure with guywires attaching it to the ground to aid in his visualization of the locations of endpoints B, C, and D. Similarly, as reported above, subjects C and D had trouble determining the location of point D.

The fact that several participants drew the force in the wrong direction or got at least some of the components of the force wrong in quiz 1, also indicated that these participants were not considering context when solving the problem. In the case where the force was drawn in the wrong direction, as Subject E did, the answer was physically impossible. The two other participants who made errors in the direction of the force, as described above, did not actually draw the force on their diagram and due to the other errors they made, it is difficult to determine in which direction they thought the force was going. However, one of them gave the x-component of the force to be 0, which again from the context of the problem, is not physically possible. In summary, there is evidence that two of the participants who were unable to correctly determine the components of the force in the quiz question, did not use the context of the problem in formulating their answer.

These findings are aligned with the learning theory that a novice learning a new skill actually processes problems in a different way than an expert. Experts often recognize familiar features and meaningful patterns in a problem that are not noticed by novices (Bransford, Brown, & Cocking, 2000). Of course, this is an important consideration for an instructor trying to facilitate the learning of such a skill. It can be difficult as an expert in a subject to reverse engineer, or de-compile the knowledge that is used in thinking about a problem because the knowledge feels innate. This makes it hard for an instructor/expert to understand the students' difficulties when learning to solve the same problem. As instructors looking at an isometric projection of a three-dimensional force, we would use gestalt processing, where an image is transformed and organized as a whole (Smith, 1964; Spearman & Jones, 1950). In other words, we would look a drawing and instantly just "see" the direction and magnitude of lines and forces. We might automatically consider the contextual cues of the problem and expect students would do the same. However, some of the students used analytic processing to answer problem 2 of the think-aloud, in that they broke the problem down into individual parts and calculations, rather than solving it as a whole. The cognitive effort required in interpreting the diagram may have precluded them from using the contextual cues of the problem to help formulate their answers.

In the quiz results, the students' method of processing was not made visible, but nevertheless, there was evidence that at least some of them had not considered context in answering the problem. Thus, it would be worthwhile for instructors to consider coaching students to use contextual clues in formulating a solution to even the most apparently "simple" problems.

Theme 3: Writing Equations Rather than Answers

The most surprising result of the think-aloud was that none of the five students answered the questions as anticipated. This was equally true of participants A and B, the stronger students who did not show difficulties associated with the other two themes. In fact, all participants did not actually answer parts of the questions at all. In short, all students/students A and B tried to calculate the directions and magnitudes of the vectors rather than describe them, even when there was not enough information to calculate an answer.

Some were overly concerned with remembering formulas. For example, participant D was unable to answer question 1 of the think-aloud. He was very pre-occupied with not being able to remember equations, repeatedly saying things like, "I'm not quite sure what the formula is…", "I'm trying to remember the formula…", and "I can't exactly remember how to do [it], but anyways that's how I'd go through it and do that one.", but did not actually outline a process for answering the question and did not give any written answer.

Participant E was also focussed on writing equations. He said things like, "For F_A, like the magnitude of it, you can just write the equation for it" when analyzing question 1 of the think-aloud. As an answer to which force had larger x, y and z components, he simply wrote for F_A, "F_h=$F_A$cos40, F_x=$F_h$cos40, F_y=$F_h$sin40", and for F_B, "F_{By}=$F_B$sin60, F_{Bx}=$F_h$sin30, F_{Bz}=$F_h$cos30". Of course, these equations would answer the question if the magnitudes of the two forces had been given, but they were not. After finishing his answer, he reread the question and said, "so which one's larger… ok so those are the magnitudes of each of the forces", seemingly satisfied that his response had fully answered the question. This participant also did not specifically say whether the forces went into or out of the page, but instead wrote the response, "direction of F_A = positive x, y, negative z. direction of F_B = negative x, but positive y, z" which again does not directly answer the question. (He originally wrote "positive z" for F_A, which is of course correct, but later crossed it out and changed it, indicating that he had difficulty seeing whether the force went into or out of the page.)

Participant A, the strongest student in terms of his performance in the course, also answered question 1 of the think-aloud using equations that did not fully answer the question. This participant promptly set about writing equations, and did not pause for any amount of time when interpreting the diagram. His answer, both orally and written, to question 1 was, "$F_x = F_B\cos60\ \cos30$, $\theta_x = \cos^{-1}(F_y/F_B)$, $F_y = F_B\ \sin60$, $\theta_y = \cos^{-1}(F_x/F_B) > 90^o$, $F_z = F_B\ \cos60\ \sin60$, $\theta_z = \cos^{-1}(F_z/F_B)$." He indicated that he felt that he had answered the question sufficiently. He also treated the think-aloud questions as if they were assignment questions and wrote out things that are typically asked for on an assignment (e.g. summarizing the given information and what the question asks for), which seems to indicate that the course assessments influenced how he responded in the think-aloud.

The other students all gave similar answers to question 1 in terms of writing equations but not answering the questions, although some struggled more than others with the fact that all the necessary information to calculate an answer was not given, and some students took longer to visualize/calculate than others.

Participants relied on equations to answer question 2 of the think-aloud as well. Participant E's answer to whether the forces go into/out of the page was similar to his response to question 1. He correctly indicated whether the components of each force were positive or negative, but did not directly answer the question. Two others wrote the forces in Cartesian vector format that indicated they could correctly "see" the direction of the forces, but they made mathematical errors in representing the vectors. Therefore, although they did not have trouble visualizing the points and directions for this problem, their reliance on using mathematical formulations and equations to answer this simple question prevented them from answering it correctly.

The way participants responded (providing equations but generally not answering the questions) supports other studies' results regarding how expectation schemas can have a big impact in how students respond (Redish & Smith 2008), as well as concerns that, in the absence of deep learning, students resort to using surface routines and language (Davies, 2003) and that students' ability to solve problems does not necessarily demonstrate that they have a conceptual understanding (Davies & Mangan, 2007; Halloun & Hestenes, 1985; Montfort, 2007). Since the students in the current study were primarily tested in class on their calculations and were typically (but not always) given problems for which a numerical answer could be calculated, in hindsight it is not surprising that their expectations of what they were going to be asked to do may have influenced the way they answered the question. It is not possible to tell from the students' responses in this study whether they didn't fully answer question 1 because of their expectation of what was going to be asked, or because of their lack of realization that the problem could be answered without doing numerical calculations. However, the fact remains that participants felt that they had answered the question fully, or as fully as possible, with the information given, when they had not.

The results of this study support similar findings in physics education which document that students often have attitudes that memorizing equations is important and expectations that any numbers a student needs will always be given to them (Redish, Saul, & Steinberg, 1998) and that beginners typically rely on equations rather than major principles in problem solving (Larkin, 1981, 1983). It has also been shown that they often get stuck using a limited group of skills or reasoning and fail to notice that a different set of tools could quickly and easily solve their problem (Bing & Redish, 2009). One might be tempted to blame these students' instructors, but even when instructors stress the value of understanding and reasoning in their lectures, students may not understand what this means.

In the context of coaching the visualization of three-dimensional structures, it is important for an instructor to be aware that students may be stuck in a "plug and chug" mode when trying to interpret a diagram rather than simply "seeing" it. This is because, as Tuminaro and Redish (2007) suggest, if a question only asks for an equation or numerical answer, and not the "intuitive sense-making the instructor expects to go on behind it, we can be misled as to what the students are doing and the students can misinterpret what we are trying to teach" (p. 18). Certainly, being able to perform calculations does not necessarily indicate that students have a deep understanding of the underlying concepts and principles. Therefore, to scaffold students' deeper learning as they begin to work with these diagrams, an instructor might consider presenting more open-ended problems which require students to explain their reasoning, and as mentioned previously, spending more time modelling how the instructor would interpret such problems.

Conclusions and Future Research

The importance of spatial visualization ability for success in engineering is well-recognized. This study identified three main difficulties which students in a first year engineering course experience when learning to interpret isometric projections and to analyze three-dimensional forces and structures: (a) students have more trouble visualizing points behind, and vectors pointing into, the plane of the page; (b) students may not use contextual clues to aid in their visualization; (c) students rely on equations to answer problems even when not necessary or even possible to do so. Stronger students, as identified by their academic grade in the course, did not demonstrate evidence of difficulty visualizing, however relied on equations just as much as the other students. These findings are important because while other studies have found similar results in other contexts, it may be surprising to some who are already "experts" in this skill, that students experienced these difficulties with seemingly simple problems. The think-aloud exercises, which were not standard practice in this course before the study was designed, gave the instructor new insights into the types of visualization problems students initially struggle with. While the evidence may have existed before in students regular coursework, such as the quizzes analyzed in this study, the problems were not visible to the instructor until the think-alouds illuminated them. As Calder (2006) says, "listening to my students think out loud as they tried to make sense of [documents] is the single most eye-opening experience I have had in my years as a teacher" (p. 1368).

Suggestions for instructors to consider when teaching this topic include spending time modelling how the instructor would interpret such diagrams, allowing more time for students to wrestle with interpreting diagrams in class, engaging students in discussions during class, coaching students to use contextual clues, and presenting more open-ended problems which require students to explain their reasoning.

This study looked at a small population in a particular course, however the findings are aligned with other studies' results and may resonate with other instructors of courses where spatial visualization is a learning goal. The next steps in this line of research should include an examination of the effect of some of the teaching strategies suggested here. Think-aloud sessions could be very effective in assessing changes in spatial visualization skills over a period of time.

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