SET OF CRITERIA FOR EFFICIENCY OF THE PROCESS FORMING THE ANSWERS TO MULTIPLE-CHOICE TEST ITEMS

Alexander Aleksandrovich RYBANOV,
PhD in Technical Sciences, Associate Professor,
Informatics and programming techniques Department,
Volzhskii Polytechnic Institute,
Branch of the Volgograd State Technical University,
Volzhskii, RUSSIA

ABSTRACT

Is offered the set of criteria for assessing efficiency of the process forming the answers to multiple-choice test items. To increase accuracy of computer-assisted testing results, it is suggested to assess dynamics of the process of forming the final answer using the following factors: loss of time factor and correct choice factor. The model application results show the high efficiency of suggested set of criteria.

Keywords: **e-learning, distance learning, web-based e-learning system, computer-assisted testing, multiple-choice test items, computer-based assessment.**

INTRODUCTION

In web-oriented e-learning computer-assisted testing systems existing today, when assessing degree of conformity of the test item answer to the reference answer, final answer of the user is taken into account, but the answer formation process' dynamics (Rybanov, A.A., 2006a), by which it is possible to judge about (Bereby-Meyer, Y., Meyer, J., Flascher, O.M., 2002) choice by the user of the right answer at random, doubt of the user about correctness of his/her knowledge, use of prompts by the user, etc., is not taken into account.Analysis of the process of forming by the user the final answer is considered by example of multiple-choice test items of the type "several of several" (Figure: 1).

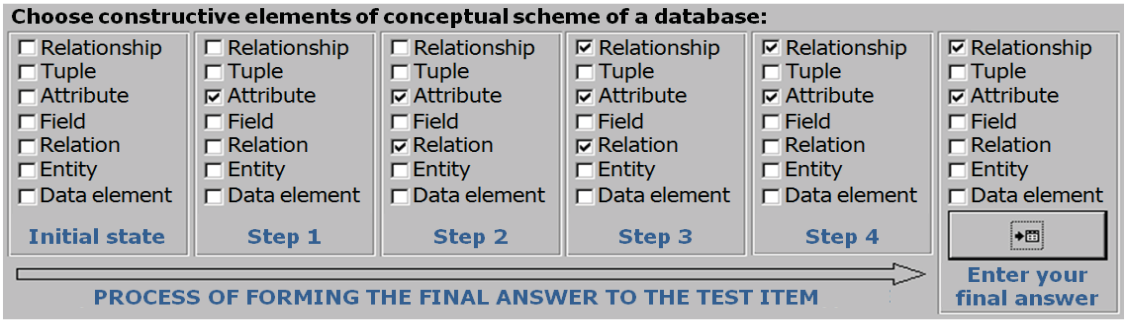


Figure: 1
Example of the process of forming the answer to multiple-choice test item

In web-oriented e-learning systems, the final assessment θ for the answer to the test item includes, as a rule, only comparison of the user's final answer (Rybanov, A.A., 2006b) (Figure: 1, step 4) with the reference answer (Figure: 2) and is calculated according to the formula:

$$\theta = \beta \cdot \delta, \quad (1)$$

where $\beta \in (0;1]$ - is the test item's difficulty measure; δ - is degree of conformity of the test item answer to the reference answer.

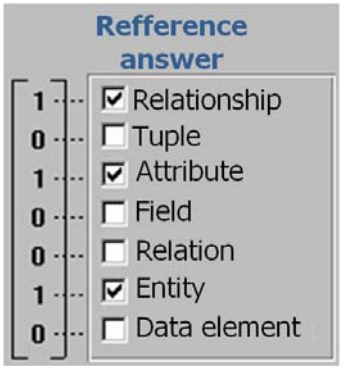


Figure: 2
Reference answer to the multiple-choice test item

The same final answer to the test item can be obtained by the user with various trajectories of the process of the final answer formation, therefore the formation process dynamics should be taken into account when making final assessment for the test item (Rybanov, A.A., 2008). Response time is an additional source of information necessary for assessing abilities of the tested user, and also for analysing the testing process (Van Der Linden, W.J., 2008).

To increase accuracy of computer-assisted testing results, it is suggested to assess dynamics of the process of forming the final answer using the following factors:

- **loss of time factor;**
- **η - correct choice factor.**

The factor μ takes into account loss of time, during forming final answer to test item, caused by choosing and subsequent cancelling the answer elements. For example, at *step 2* (Figure: 1), the user chooses the answer element "Relation", which is cancelled at *step 4* (Figure: 1).

The factor η characterizes correctness of choice by the user of the final answer, i.e. takes into account sequence of choosing correct and wrong answer elements taking into account their weight factors.

For example, trajectory of the process of the final answer formation presented on Figure: 1 (*step 1*: choosing "Attribute", *step 2*: choosing "Relation", *step 3*: choosing "Relationship", *step 4*: cancelling "Relation") leads to the final answer {"Attribute", " Relationship"} and should have higher value of the criterion η than the trajectory of obtaining the same final answer: *step 1*: choosing "Relation",

***step 2*: choosing "Attribute", *step 3*: choosing "Relationship", *step 4*: cancelling "Relation". I.e., the more quickly the user chooses correct answer elements with the biggest weight factors, the higher the correct choice factor η is.**

Thus, the final assessment θ for the answer to the test item should be determined not only by the difficulty measure β and the degree of conformity of the test item answer to the reference answer δ, but also by the factors μ and η.

MATHEMATICAL FORMULATION

The *multiple-choice test item model* of the type "several of several" can be presented as:

$$T = (Q, E, \Gamma, D, \beta), \quad (2)$$

where Q - is the question contents; $E = (e_i \mid i = \overline{1,n})$ - is a set of elements for forming the answer to the test item; $\Gamma = (\gamma_i \mid \gamma_i = f_1(e_i), \gamma_i \in (0;1), i = \overline{1,n})$ - is a set of weight factors, γ_i is weight factor of element e_i; $D = (d_i \mid d_i = f_2(e_i), d_i \in B = \{0,1\}, i = \overline{1,n})$ - is the reference answer to the test item, d_i - is descriptor of element e_i of the answer ($d_i = 1$ if e_i is element of the reference answer, otherwise $d_i = 0$); β - is the test item's difficulty measure.

For the sets D and Γ, the following conditions should be met:

$$1 < \sum_{i=1}^{n} d_i < \sum_{i=1}^{n} (1 - d_i), \quad \sum_{i=1}^{n} \gamma_i \cdot d_i = \sum_{i=1}^{n} \gamma_i \cdot (1 - d_i) = 1. \quad (3)$$

Example: 1. The multiple-choice test item on Figure: 1 can be formally presented as

$$T_1 = (Q, E, \Gamma, D, \beta),$$

where Q, E, Γ, D, β are shown on Figure: 3.

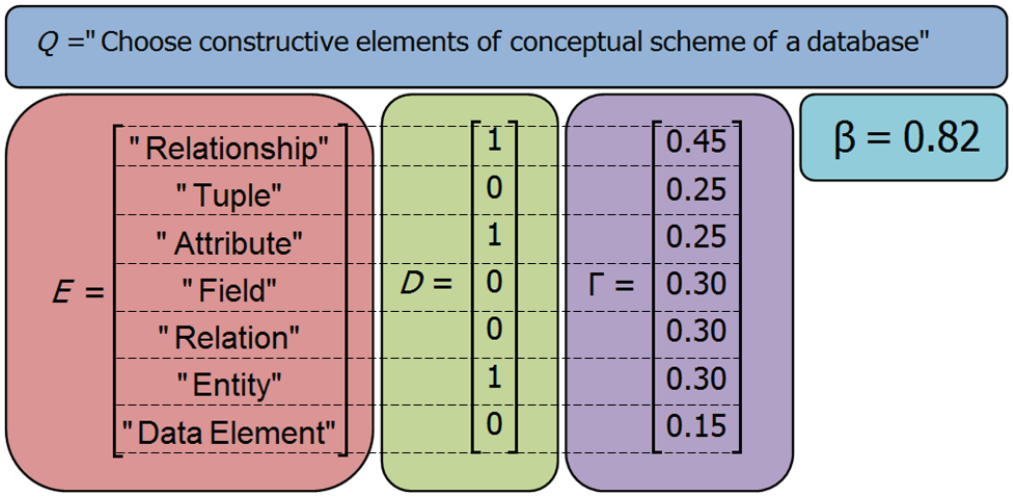


Figure: 3
The multiple-choice test item $T_1 = (Q, E, \Gamma, D, \beta)$

The user's final answer to the test item T can be presented as a set:

$$W = (w_i \mid w_i \in \mathrm{B} = \{0,1\}, i = \overline{1,n}). \quad \textbf{(4)}$$

Degree of conformity of the test item answer to the reference answer can be presented as:

$$\delta = f(w_1, w_2, ..., w_n) = \sum_{i=1}^{n} \gamma_i \cdot w_i \cdot (2d_i - 1), \quad \textbf{(5)}$$

where $w_i \in \mathrm{B}$ - is descriptor of i-th element of the final answer to the test item; $f(w_1, w_2, ..., w_n)$ - is a function for assessing conformity of the user's final answer to the reference answer.

Example: 2. The user's final answer to the multiple-choice test item T_1:
$W = (1,0,1,0,1,1,0)$, $\delta = f(1,0,1,0,1,1,0) = 0.70$.

Let us name position in space $W(t_j) = (w_1(t_j), w_2(t_j), ..., w_n(t_j)) \in \mathrm{B}^n$ of the user's answer at fixed t_j as *image of the user's answer to the test item*, and let us name the time t_j as *point of fixing the answer image* $W(t_j)$.
When forming the final answer, the image of the user's answer to the test item changes, drawing in a phase plane B^n some curve, which can be named as *trajectory of process of forming the answer to the test item.*

Parameters of conformity of the image of the user's answer $W(t_j)$ to the reference answer D can be calculated according to the following formulas:

$$a(t_j) = \sum_{i=1}^{n} \gamma_i \cdot d_i \cdot w_i(t_j), \; h(t_j) = \sum_{i=1}^{n} \gamma_i \cdot (1 - d_i) \cdot w_i(t_j), \quad \textbf{(6)}$$

where $a(t_j)$ is the sum of weights of correct answer elements marked in the image $W(t_j)$ as correct elements; $h(t_j)$ is the sum of weights of wrong answer elements marked in the image $W(t_j)$ as correct elements.
***Starting point of trajectory of process of forming the answer to the test item* is the answer image $W(t_0)$, which meets the following conditions:**

$$W(t_0) = (w_i(t_0) \mid w_i(t_0) = 0, i = \overline{1,n}). \quad \textbf{(7)}$$

***Intermediate point of trajectory of process of forming the answer to the test item* is the answer image $W(t_k)$, where t_k is the time when the user sets (or unsets) the correct element mark (the action of "*setting/unsetting the mark*") in the process of forming the answer.**

***Final point of trajectory of process of forming the answer to the test item* is the answer image $W(t_m)$, where t_m is the time when the user enters the final answer.**

***Trajectory P of process of forming the answer to the multiple-choice test item* can be presented as ordered sequence of the answer images:**

$$P_r = (W(t_j) \mid t_j \in V_r, j = \overline{0,m}), \quad \textbf{(8)}$$

which meets the following conditions:

$$\sum_{i=1}^{n} | w_i(t_{k-1}) - w_i(t_k) | = 1, \; k = \overline{1, m-1}, \tag{9}$$

$$W(t_m) = W(t_{m-1}), \tag{10}$$

where $V_r = (t_j \mid j = \overline{0, m})$ **- is a set of points of fixing the answer image.**

The same answer to the multiple-choice test item can be obtained with various trajectories of the answer formation process. Example: 3. On Figure: 4 examples of trajectories of the process of forming the same final answer to the multiple-choice test item T_1 **are presented.**

$$P_0 = \begin{pmatrix} 0&1&1&1&1&1 \\ 0&0&0&0&0&0 \\ 0&0&1&1&0&1 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&1&1&1 \\ 0&0&0&0&0&0 \end{pmatrix} \quad P_1 = \begin{pmatrix} 0&1&1&1&1&1 \\ 0&0&0&0&0&0 \\ 0&0&1&1&1&1 \\ 0&0&0&0&0&0 \\ 0&0&0&0&1&1 \\ 0&0&0&1&1&1 \\ 0&0&0&0&0&0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0&1&1&1&1&1 \\ 0&0&0&0&0&0 \\ 0&0&0&0&1&1 \\ 0&0&0&0&0&0 \\ 0&0&0&1&1&1 \\ 0&0&1&1&1&1 \\ 0&0&0&0&0&0 \end{pmatrix}$$

$$V_0 = (0, 35, 50, 65, 80) \quad V_1 = (0, 35, 45, 55, 65, 80) \quad V_2 = (0, 35, 45, 55, 65, 80),$$

$$P_3 = \begin{pmatrix} 0&0&0&1&1&1 \\ 0&0&0&0&0&0 \\ 0&0&0&0&1&1 \\ 0&0&0&0&0&0 \\ 0&1&1&1&1&1 \\ 0&0&1&1&1&1 \\ 0&0&0&0&0&0 \end{pmatrix} \quad P_4 = \begin{pmatrix} 0&0&0&1&1&1 \\ 0&0&0&0&0&0 \\ 0&0&0&0&1&1 \\ 0&0&0&0&0&0 \\ 0&0&1&1&1&1 \\ 0&1&1&1&1&1 \\ 0&0&0&0&0&0 \end{pmatrix} \quad P_5 = \begin{pmatrix} 0&0&1&1&1&0&0&1&1&1 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&1&1&1&1 \\ 0&0&0&0&1&1&1&1&1&1 \\ 0&1&1&1&1&1&1&1&1&1 \\ 0&0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$V_3 = (0, 35, 45, 55, 65, 80) \quad V_4 = (0, 35, 45, 55, 65, 80) \quad V_5 = (0, 15, 25, 35, 45, 55, 60, 65, 70, 80)$$

Figure: 4
Trajectories of the process of forming the answer to multiple-choice test item
&
Figure: 5
Dynamic parameters $a(t)$ **and** $h(t)$ **for trajectory** P_2

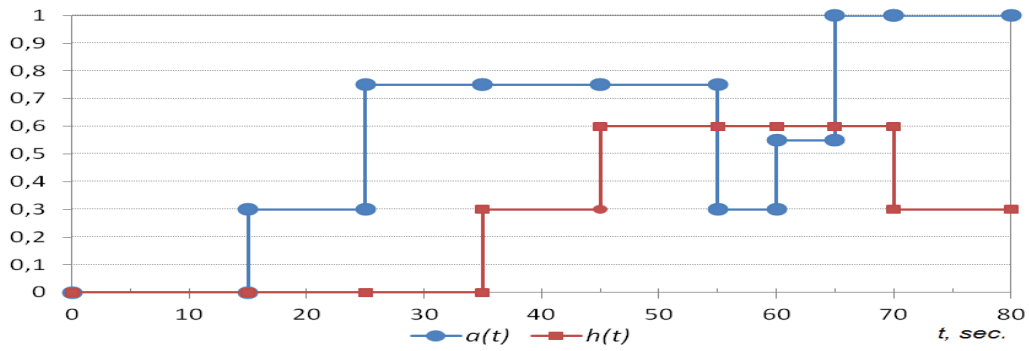


Figure: 6
Dynamic parameters $a(t)$ **and** $h(t)$ **for trajectory** P_5

On Figures. 5-6, dynamic characteristics $a(t)$ and $h(t)$ for trajectories P_2 and P_5 (Example: 3) of the process of forming the final answer to multiple-choice test item T_1 are presented.

Trajectory of the process of forming the answer reflects dynamics of choosing the answer to the test item. Taking into account that the user first chooses the answer elements, which he/she considers correct (factor η), and also quantity of steps to the final answer (factor μ), the *factor ξ of general effectiveness of the process of forming the final answer* can be used as a metric, which describes dynamics of the process of forming the answer.

$$\xi = \eta \cdot \mu. \quad (11)$$

Taking into account the factor ξ, the final assessment θ' for the tested user's answer to the test item will be calculated as follows:

$$\theta' = \beta \cdot \xi \cdot \delta. \quad (12)$$

The factor η of the correct choice in the process of forming the final answer to the test item can be presented as:

$$\eta = \frac{\sum_{i=0}^{m-1} g(t_i) \cdot (t_{i+1} - t_i)}{\sum_{i=0}^{m-1} (g(t_i) + b(t_i)) \cdot (t_{i+1} - t_i)}, \quad (13)$$

where $g(t)$, $b(t)$ are the functions of changing the sum of weights of all correct (wrong) answer elements chosen by the user in the process of forming the final answer to the test item.

The values $g(t)$, $b(t)$ show the sum of weights of all correct (wrong) answer elements at all points of fixing, starting from t_0 and including t_j.

For multiple-choice test items of the type "several of several":

$$g(t_j) = \sum_{k=0}^{j} a(t_k), \; b(t_j) = \sum_{k=0}^{j} h(t_k). \quad (14)$$

The loss of time factor μ can be presented as:

$$\mu = \frac{\sum_{i=1}^{m} I(t_i) \cdot (t_i - t_{i-1})}{\sum_{i=1}^{m} (I(t_i) + O(t_i)) \cdot (t_i - t_{i-1})},$$

where $I(t)$, $O(t)$ are the functions of changing the quantity of the actions

"***setting/unsetting the mark***" performed by the user in the process of forming the final answer to the test item.

The values $I(t_j)$, $O(t_j)$ show the quantity of the actions "***setting/unsetting the mark***" the user performed at all points of fixing, including t_j, from the beginning t_0 of the process of forming the answer.
Initial condition of the process of forming the final answer to the multiple-choice test item: $I(t_0) = O(t_0) = 0$.

For multiple-choice test items:

$$I(t_j) = \sum_{k=1}^{j} \sum_{i=1}^{n} (1 - w_i(t_{j-1})) \cdot w_i(t_j), \quad (16)$$

$$O(t_j) = \sum_{k=1}^{j} \sum_{i=1}^{n} w_i(t_{j-1}) \cdot (1 - w_i(t_j)). \quad (17)$$

The factor ξ of general effectiveness of the process of forming the final answer changes within the limits $[0;1]$.

RESULTS AND DISCUSSION

Let us consider trajectories P_1, P_2, P_3, P_4 (Example: 3) of forming by the user the same final answers to the test item T_1. Let us order the trajectories P_r ($r = \overline{1,4}$) by the time t_{P_r} of performing by the user the action "***setting the mark***" for the answer element e_5 with descriptor $d_5 = 0$: $t_{P_3} = 35 < t_{P_4} = 45 < t_{P_2} = 55 < t_{P_1} = 65$. Correct choice factors η_r for trajectories P_r form the similar sequence by index r: $\eta_3 = 0.551 < \eta_4 = 0.720 < \eta_2 = 0.870 < \eta_1 = 0.950$ that proves influence of sequence of user's choice of correct and wrong answer elements on factor η.
Let us consider trajectory P_5 (Example: 3), which describes process of forming the final answer during which the user performs the actions "***setting the mark***" for the answer elements e_1 (at $t = 25$) and e_4 (at $t = 45$) and then performs the actions "***unsetting the mark***" for e_1 (at $t = 55$) and e_4 (at $t = 70$). On Figures: 7-8, dynamic characteristics $g(t)$, $b(t)$ and $I(t)$, $O(t)$ for trajectory P_5 of the process of forming the final answer are presented.

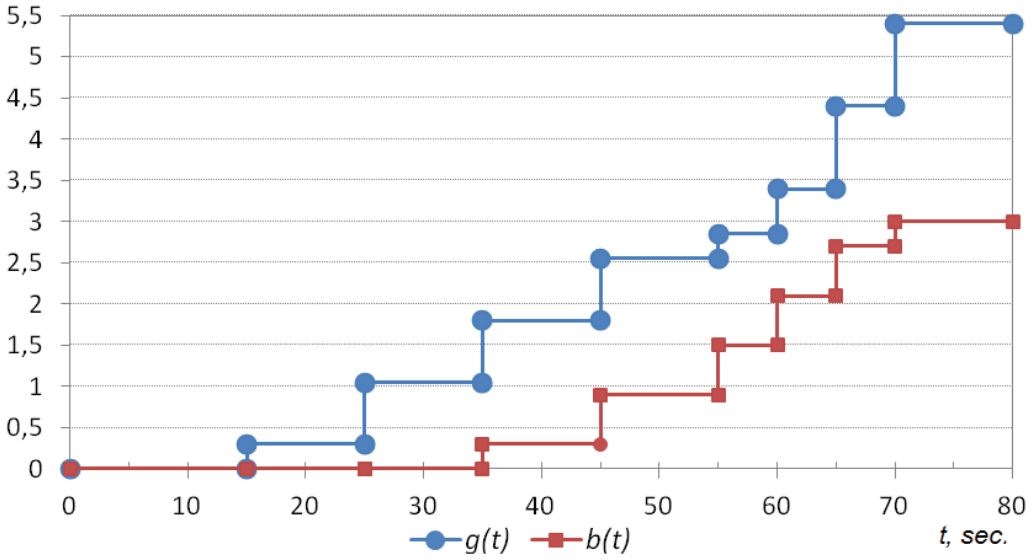


Figure: 7
Dynamic parameters $g(t)$ and $b(t)$ for trajectory P_5

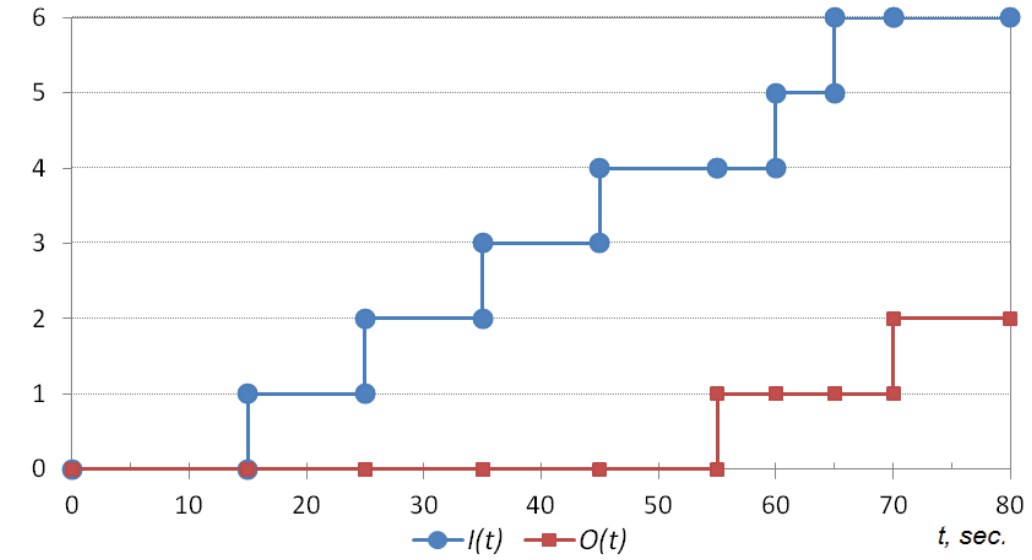


Figure: 8
Dynamic parameters $I(t)$ and $O(t)$ for trajectory P_5

Value of the loss of time factor μ_5 for trajectory P_5 is less than value μ_1 for trajectory P_1, during whose forming the user does not perform the action "*unsetting the mark*". The result $\mu_5 = 0.870 < \mu_1 = 1$ proves dependence between the factor μ and the action "*unsetting the mark*". In Table: 1, the criteria, which describe dynamics of the process of forming by the user the answer to the test item for the Example: 3 trajectories are specified.

Table: 1

Taking into account dynamics of the process of forming the answer to the test item

Trajectory of the process of forming the answer to the test item	δ	β	η	μ	ξ = η·μ	Final assessment θ = β·δ	Final assessment θ′ = β·ξ·δ
P_0	**1.000**	**.820**	**1.000**	**1.000**	**1.000**	**.820**	**.820**
P_1	**.700**	**.820**	**.950**	**1.000**	**.950**	**.820**	**.545**
P_2	**.700**	**.820**	**.870**	**1.000**	**.870**	**.820**	**.499**
P_3	**.700**	**.820**	**.551**	**1.000**	**.551**	**.820**	**.316**
P_4	**.700**	**.820**	**.720**	**1.000**	**.720**	**.820**	**.413**
P_5	**.700**	**.820**	**.691**	**.870**	**.601**	**.820**	**.345**

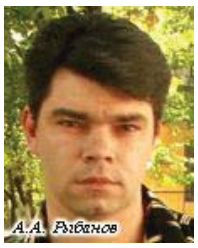
It follows from the Table: 1 that without taking into account dynamics of the process of forming the answer by the user, the final assessment θ for the same answer to the test item is identical for various trajectories of the answer formation process.

Taking into account dynamics of the process of forming the answer to the test item allows to make more objective assessment θ′ = β·ξ·δ.

CONCLUSION

The suggested set of criteria *(11-17)*: η, μ, ξ, δ, θ′ which takes into account dynamics of the process of forming by user the answers to multiple-choice test items of the type "several of several", is introduced in LMS Moodle that ensures correct assessment of knowledge in distance education testing.

BIODATA and CONTACT ADDRESSES of AUTHOR



Alexander Aleksandrovich Rybanov, PhD in Technical Sciences, Associate Professor. *He's head* **of the Informatics and Programming Techniques Department, Volzhskii Polytechnic Institute, Branch of the Volgograd State Technical University. He has more than 15 years teaching and administration experience in education. His research interests are technological pedagogical content knowledge, pedagogical measurements, web-based distance education, e-learning, communication and information technologies. He's a member of the editorial board of the Vestnik Magistratury Journal published by the Scientific Publishing Centre "Colloquium" (Russia) and *member of the Russian Academy of Natural History.***

Alexander Alexandrovich RYBANOV
PhD in Technical Sciences, Associate Professor.
Informatics and programming techniques Department,
Volzhskii Polytechnic Institute, Branch of the Volgograd State Technical University,
404121, 42a Engelsa Street, Volzhskii, Volgograd region, RUSSIA
Phone: +7 (88443) 41-22-62
Email: vit@volpi.ru
URL: www.volpi.ru

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