

Investigating First Year Elementary Mathematics Teacher Education Students' Knowledge of Prism

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Abstract

The purpose of this study was to investigate first year elementary mathematics teacher education students' knowledge of prism. For this goal, the participants were asked to define the geometric concept of prism. The participants were 158 first year elementary mathematics teacher education students from a public university in Southern Turkey. The researchers analyzed the participants' definitions of prism. Additionally, 12 of the participants were selected for semi-structured interviews. The data were analyzed via content and frequency analysis techniques. Based on the content analysis, the themes that each participant used were determined. After this evaluation, the frequency of each of the themes was calculated. The findings indicated that the participants experienced difficulties in defining the concept of prism. It was also found out that the participants could not adequately use the mathematical language and define the concept.

Key Words

Geometry Teaching, Geometric Concepts, Prism, Concept Definition, Teacher Education.

Individuals understand their surrounding environment through sense making and concept mapping (Schoenfeld, 1992). A concept is an umbrella term that is used to portray main characteristics of an object or thought (Türk Dil Kurumu [TDK], 2005). It can be argued that concept is an element of understanding and knowledge (Öksüz, 2010). Teaching without conceptual understanding may just lead to memorizing, drill and practice, and computational learning (Sigler & Saam, 2006; Snowman & Biehler, 2003).

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There can be a wide range of individual differences in definitions of a single concept (Vinner, 1991). Hence, we can classify concept definitions in two large categories; formal and personal definitions. In mathematics, formal definitions are the ones which are formally accepted and respected by the large community of mathematicians (Tall & Vinner, 1981). On the other hand, personal definitions reflect individuals' thoughts and experiences or, more technically, their concept images (Tall, 1991). Definitions based on personal experiences make more sense than other definitions. If the concept image or the individuals' thoughts about the concept is inaccurate, their concept definitions may contain errors (Vinner); and the concept image draws boundaries of the concept (Keiser, Klee, & Fitch, 2003).

As a branch of mathematics, in geometry, it is essential to visualize geometric concepts in the mind for conceptual understanding (Hershkowitz, Parzys, & Dormolen, 1996). Higher level geometric thinking requires at least a basic understanding of concept definition (Linchevsky, Vinner, & Karsenty, 1992). Thus, understanding and accurate explanations of definitions is essential for geometric understanding (Hiebert & Lefevre, 1986).

In the literature, we see a number of studies that investigated K-12 and teacher education students' definitions of geometric concepts and their misconceptions (Clements, Sarama & Battista, 1998; Cunningham & Roberts, 2010; de Villers, 1998; Gutierrez & Jaime, 1999; Işık, Koç, & Osmanoglu, 2010; Koç & Bozkurt, 2011; Linchevsky et al., 1992; Matos, 1994; Tunç & Durmuş, 2012; Zembat, 2007). Such studies indicate that students experience difficulties in defining geometric concepts.

Teacher knowledge is important to form desired learning environments (Putnam, Heaton, Prawat, & Remillard, 1992). This is also valid in teaching geometry (Swafford, Jones, & Thornton, 1997). For this reason, teachers should understand the geometry they teach and also prepare the students for advanced levels. Teachers' content knowledge is generally shaped during their pre-service education and before. Yet, pre-service teachers do not always have adequate knowledge of the mathematics they would teach in the future. Pre-service teachers' limited knowledge of mathematics is one of the major obstacles in teacher education programs (Brown & Borko, 1992). Thus, it is essential to investigate teacher prospective teachers' level of conceptual knowledge in geometry.

Purpose

The purpose of this research is to study first year elementary mathematics teacher education students' knowledge of prism. To accomplish this purpose, the participants' definitions of the prism concept were investigated.

Method

Research Design

In this descriptive study, the data was qualitatively analyzed. It was collected via a data collection instrument which was developed as part of a large-scale research project.

Participants

The data was collected from 158 first year elementary mathematics teacher education students who were enrolled in a public university in Turkey during the 2010-2011 academic year. As requirements of the elementary and secondary school curricula (Milli Eğitim Bakanlığı [MEB], 2009, 2010), the participants were expected to have learned about the geometric concepts asked about in the data collection instrument.

Data Collection Instrument

In order to collect the data, the researcher designed and used an instrument on geometric concepts. The instrument was composed of open-ended questions where the participants were asked to define 17 major geometric concepts such as polygon, circle, prism, pyramid and cone. For the purpose of the present study, only responses regarding the definition of prism were analyzed. Prism was the focus of the study as it is a major three dimensional shape. Additionally, focusing on a single concept was essential to deeply investigate the participants' responses. In order to deeply understand the participants' thoughts, 12 participants were selected for a follow-up interview based on their responses. The ones with common responses were chosen for the interview where they were asked to talk about their written definition of the prism concept.

Data Analysis

For analyzing the data, the content analysis approach developed by Pilkington (2001) was used. The participants' responses were used to determine 10 coding themes (Patton, 2002, pp. 452-54) which were formed to accurately represent the participants' thoughts what a prism is. The reliability study of the coding process was completed through a number steps (Green & Gilhooly, 1996).

After determining all coding themes, participants' responses were coded. Then, the responses were classified by the number of themes they contain. To be more specific, the number of responses with one theme, two themes, and more were determined. Additionally, the frequencies of each theme were determined. Excerpts from the interview transcripts were also used to exemplify how the participants' define prism.

Findings

The findings indicate that while some of the themes were used by only five or six participants, some others were used by 27 or more participants. More specifically, 48 participants defined prism as a three dimensional shape, 27 of them defined it as a geometric shape and 19 of them indicated that prism is a solid. Additionally, 18 of the definitions were not found to be meaningful; so, no coding theme was assigned to such definitions. Also, only six participants defined prism as something that occupies a space and five of them indicated that it is the union of the top and bottom surfaces.

A big chunk of the participants, 51 (32%), did not write a definition of prism. Besides, 44 (28%) of them used only one theme, 49 (31%) used two themes and only 7 (4%) used three themes in their definitions of prism. Among the ones who used only one theme, 21 of them defined prism as a three dimensional shape, 7 others defined the prism as a three dimensional solid and 4 of them defined it via providing names of prisms such as rectangular prism and cube.

Among the ones who used two different themes, 6 of them defined it as a geometric shape which has the same top and bottom surfaces, 4 others indicated that prism is a geometric shape which is formed by different planes, and finally 4 participants defined it as a three dimensional shape which is formed by different planes. Unlike others, the ones who used three themes in their definitions used various combinations of all the themes. In all the seven responses, it was found out that all the participants who used three themes in their definitions chose to define prism as a geometric shape. Additionally, six of them indicated that the top and bottom surfaces of the prism are polygons. Furthermore, three participants defined prism as a geometric shape with bottom and top surfaces that are polygons, one participant defined that prism is a geometric shape with congruent top and bottom surfaces, and a height and finally one other participant indicated that prism is a geometric shape with top and bottom surfaces that are polygons.

Discussion

As seen from the findings, a majority of the participants (32%) did not provide a definition of prism. Additionally, 4% of them did not write a meaningful definition of the geometric concept. Furthermore, the participants who used one theme or characteristic in their definitions could not adequately define prism.

For example, some participants who used one theme indicated that prism is a three dimensional shape; but, this is a too general definition for prism as some other three dimensional solids also fall into this category such as pyramid and sphere. Similarly, many responses with two or three themes were not better than the ones with one theme in adequately defining prism.

The participants were admitted into the teacher education program via a nation-wide university entrance exam. Thus, they are expected to answer higher level geometry question; yet, most of them did not either define prism or could not provide an acceptable definition of the concept. It can be claimed that there can be a considerable gap be-

tween the participants' concept image and concept definition of prism. It is possible that they own a deep level of concept image (Tall & Vinner, 1981); but, their skill in defining prism does not seem to be at the same level.

It was also found out that the participants' skills in using the language of mathematics and defining a geometric concept were not adequate enough. It can be argued that not having a conceptual understanding of the concept may prevent them from defining prism (Linchevski et al., 1992). Another interesting finding is that some participants defined prism as a shape or geometric shape rather than three dimensional solid or a shape occupying a space. Also, the participants might have used geometric shape in place of geometric solid; but, formally in mathematics three dimensional shapes such as prism and pyramid are defined as three dimensional solids in Turkish school mathematics (MEB, 2009, 2010).

Recommendations

Regarding the findings, it is seen that conceptual learning should be paid an extra attention. Although the study just investigated concept definitions, they are essentially important for conceptual learning because conceptual learning involves the knowledge of the concept and facts, and the relation between them (Schneider & Stein, 2005).

Based on the findings and above discussions, it is essential that learners should work with physical models of geometric shapes and use them to solve geometric problems for a better conceptual understanding and high level definitions (Olkun, 2001). Moreover, it is suggested that to enhance learners' conceptual understanding of geometry a wide range of instructional materials should be developed and used in teaching geometry (İnan, 2006). In teacher education programs, the students should be introduced to the definitions of geometric concepts. Also, they should be given opportunities to see the connections among different concepts and communicate about them. Concept maps can be a useful tool to understand and explain the connections among geometric concepts.

While the study yields interesting and useful findings and make a significant contribution to the discipline, it has some limitations. First, the data was collected from a single teacher education institution. This can be a limitation for generalizing the findings to larger populations. It is suggested that future studies should be conducted with participants from institutions with different characteristics. Additionally, in the present study, only first

year mathematics teacher education students were the focus; so, for stronger conclusions, students who are in their second, third and fourth year should be studied to understand how individuals' definition of prism change across years. Having said that, similar studies should be conducted; but, different geometric concepts are needed to be investigated to improve our understanding.

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