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Conceptualization, literature review, methodology, application, data analysis, review, translation, writing and editing

Abstract

This research aims to analyze the results of studies conducted in Turkey on reasoning skills in mathematics teaching and to reveal what kind of trend there is in this field. Within the scope of this study, databases were searched with the keywords "reasoning"(muhakeme, akıl yürütme) and "reasoning skill" (Muhakeme becerisi, akıl yürütme becerisi), and the results were examined in accordance with the inclusion criteria regarding mathematics teaching. One hundred sixty-three studies were included. Each of the studies included in the meta-synthesis study was analyzed descriptively according to type, year, method, sample type and size, data collection tools, statistical analysis, learning field, keywords, reasoning type and purpose. In addition, the studies' results were content analyzed and tabulated by coding the differences and similarities between them with a holistic approach. It has been determined that studies on the learning fields in the mathematics curriculum are mostly carried out in the field of learning numbers and operations in the secondary school mathematics curriculum. When evaluated in terms of reasoning types, almost half of the studies were conducted on mathematical reasoning. It has been observed that after mathematical reasoning, the most focus is on proportional reasoning. When the aims of the studies included in the research were examined, it was determined that the most focused ones were "examining the factors affecting reasoning skills", "measurement of reasoning skills" and "the effect of teaching practices on reasoning skills". In the studies examined, it was seen that there were 33 teaching practices whose effects on reasoning skills were examined.

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Research Article

Reasoning Skills in Mathematics Teaching: A Meta-Synthesis on Studies Conducted in Turkey *

Ali TUM [1]

Abstract

This research aims to analyze the results of studies conducted in Turkey on reasoning skills in mathematics teaching and to reveal what kind of trend there is in this field. Within the scope of this study, databases were searched with the keywords "reasoning"(muhakeme, akıl yürütme) and "reasoning skill" (Muhakeme becerisi, akıl yürütme becerisi), and the results were examined in accordance with the inclusion criteria regarding mathematics teaching. One hundred sixty-three studies were included. Each of the studies included in the meta-synthesis study was analyzed descriptively according to type, year, method, sample type and size, data collection tools, statistical analysis, learning field, keywords, reasoning type and purpose. In addition, the studies' results were content analyzed and tabulated by coding the differences and similarities between them with a holistic approach. It has been determined that studies on the learning fields in the mathematics curriculum are mostly carried out in the field of learning numbers and operations in the secondary school mathematics curriculum. When evaluated in terms of reasoning types, almost half of the studies were conducted on mathematical reasoning. It has been observed that after mathematical reasoning, the most focus is on proportional reasoning. When the aims of the studies included in the research were examined, it was determined that the most focused ones were "examining the factors affecting reasoning skills", "measurement of reasoning skills" and "the effect of teaching practices on reasoning skills". In the studies examined, it was seen that there were 33 teaching practices whose effects on reasoning skills were examined.

Keywords: Mathematics teaching, reasoning skill, meta-synthesis

1. INTRODUCTION

It is necessary to raise individuals who can adapt to the pace of development of technology in accordance with the needs of modern life, are questioning, productive, open to innovations, and have high-level thinking skills. As a matter of fact, constantly updated curricula due to developments in technology, changing needs of individuals and society, and changes in learning-teaching approaches indicate the need to raise individuals who can produce knowledge, use it in daily life, think critically by solving problems, etc (Ministry of National Education [MoNE], 2015; 2018, Salite et al., 2020). In this context, the individual is expected to analyze information and synthesize new information in the process of creating knowledge. While structuring information, in-depth thinking should occur, and mental processes should be activated on a fact or idea (Van De Walle, Karp & Bay –Williams, 2010). Reasoning is at the forefront of these mental processes. However, considering that reasoning not only creates new ideas but also undertakes the task of making the most accurate decision about an event, situation, or subject with existing information (Toulmin, et al., 1984), it can be seen that reasoning plays a role in structuring mathematical knowledge. In other words, it is the individual's reasoning ability that will decide the connection to be established between the information. Because this skill has the feature of examining the relevant situation in depth from many perspectives. Similarly, individuals

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with developed reasoning skills examine the current situation by asking some descriptive questions in order to discover the situations they are faced with (Ersoy, et al., 2017).

Reasoning skill, which is seen as the most important step of mathematical competence (Van de Walle et al., 2010), is seen as a behavior that shapes learning in learning mathematical knowledge (Harms, 2003). While mathematics, by its nature, deals with many topics in fields such as numbers, algebra, and geometry, it also includes discovering patterns, reasoning, making predictions, and justifying (Umay, 2003). Thus, learning mathematics includes not only the acquisition of basic concepts and skills but also the ability to think about mathematics, comprehend problem-solving strategies, and realize that mathematics has a very important place in real life (MoNE, 2018). Therefore, this skill is considered very important in terms of mathematics education and teaching (Erdem & Gürbüz, 2015).

A student's ability to discover the connections between subjects in mathematics and to use these connections to solve the problems he encounters will indicate that he is reasoning (Mandacı-Şahin, 2007). This student's ability to make predictions and assumptions will largely depend on his mathematical reasoning (Yavuz-Mumcu, 2011). As a matter of fact, what gives meaning and explains mathematics is seen as mathematical reasoning (Ev-Çimen, 2008). According to Umay (2003), in order for reasoning to occur, the thought formed through higher-level thinking must be based on knowledge and include approaches within the framework of logic. In other words, if the product obtained through the mental processes performed by the individual is not based on a knowledge base, cannot be justified, and does not contain a correct approach, mathematical reasoning cannot be considered (Gürbüz, et al., 2018). Reasoning teaches us to use discourses such as “if… then…” and “Because…” regarding drawing conclusions and making assumptions (Mason, 2001).

Mathematical reasoning aims to reach a reasonable conclusion by taking into account all aspects of a problem or situation through higher-level thinking (Gürbüz et al., 2018). In other words, this process directs mental processes to work with the data at hand, assigning meanings to the problem or situation from different perspectives, drawing conclusions from them, and ultimately making the right decisions from the results generated. In general terms, mathematical reasoning is seen as a culture that helps to give meaning to the events by examining the events in life from a mathematical perspective and making inquiries with questions such as "Why", "Why" and "How", and leads to make the right decisions as a result of this meaning (Erdem, 2015). Then, we can think that the culture of mathematical reasoning can be formed by changing depending on the mathematical knowledge, perspective, and experiences of the individual. As a matter of fact, Erdem (2015) attributed its formation primarily to the student's positive attitude towards mathematics. In this context, the student's positive attitude towards mathematics will activate the process that will reveal the individual's mathematical reasoning by providing cognitive and affective readiness (Tum, 2019). Therefore, it can be said that the student can take the first step in the process of making mathematical reasoning in this way. Because if an individual wants to achieve something, he must first want it. It can be said that taking the request step in the process of creating this culture of judgment will affect the other steps to be carried out easily. When this desire (positive attitude) that will motivate the individual is combined with sufficient knowledge of methods and mathematical concepts, a culture of mathematical reasoning will be formed. It is seen in studies that creating a culture for mathematical reasoning is possible through creating enriched learning environments that include various teaching methods and techniques of mathematics subjects to be taught at school (Erdem & Soylu, 2019; Kutluca, et al., 2020). Enriched learning environments will develop positive attitudes as they increase students' participation in class. At the same time, effective and permanent learning will occur, and the student will gain sufficient knowledge of methods and mathematical concepts as it encourages reasoning (Kutluca et al., 2020). Thus, the culture of reasoning will be firmly integrated into the mathematics taught at school.

Mathematics taught at school aims to provide the individual with basic mathematical skills to overcome problems that may be encountered in daily life (Fitriana, et al., 2018). In the primary and secondary school mathematics curriculum (MoNE, 2018); emphasis has been placed on raising students who can easily express their own ideas and reasoning in problem-solving processes, who can see the deficiencies in the reasoning of others, and who will logically explain and share mathematical thinking by using mathematical arguments correctly. Trends in International Mathematics and Science Study [TIMSS] sees reasoning as one of the most fundamental goals of mathematics education (Mullis, et al., 2020). The National Council of Teachers of Mathematics [NCTM] (2014); states that an effective mathematics teaching program should help students make sense of their mathematical ideas, involve them in the discussion process, and encourage them to use reasoning skills. On the other hand, Kilpatrick et al. (2001) emphasized that reasoning has a role in holding mathematics together. Therefore, developing students' mathematical reasoning and encouraging them to use it should be a goal in every mathematics course (Bragg, et al., 2018). Because confirming the accuracy of an assumption or event is done by observation in science and by reasoning in mathematics, the basis of mathematics is reasoning (Ross, 1998). Activities such as reasoning, making assumptions, and making logical inferences are the products of intelligence that give mathematics its identity (Baki, 2008). Reasoning appears as an important issue in understanding and learning mathematics (Herbert, et al., 2016). Therefore, we can compare mathematics and reasoning to intertwined wheels that turn each other. For a meaningful learning process to occur, these two wheels must be together. Because it is necessary to use reasoning when solving mathematical problems, and reasoning skills can be improved by learning mathematics (Niswah & Qohar, 2020).

In mathematics education and training, mathematical reasoning is an important part of mathematical ability (Sukirwan, et al., 2018). The way to fully learn mathematics and achieve success in this subject is through mathematical reasoning and mathematical thinking (Umay & Kaf, 2005). The duty of educators is to enable the development of mathematical reasoning, which is seen as one of the vital skills, and to raise awareness in individuals on this issue (Bal-İncebacak & Ersoy, 2016). It is known that reasoning skills are related to mathematics success and that individuals who use reasoning skills achieve success better by developing more effective solutions in problem solving (Kutluca & Tum, 2021). When students' reasoning skills are not developed, there is a perception that mathematics consists of calculations and drawings to which certain rules are applied and made without thinking or justifying (Ross, 1998). Therefore, while it has been demonstrated how important mathematical reasoning is for learning and that it is among the needed 21st-century skills, studies on this skill will have an important place.

In the literature review conducted by Ergün, et al. (2023), the theses on scientific reasoning conducted in Turkey were analyzed. In their meta-synthesis study, postgraduate theses that did not have any curriculum restrictions and dealt with the concept of Reasoning (Turkish version: "Muhakeme") were examined. Similarly, in the meta-synthesis study of Ersanlı, et al. (2018); Postgraduate theses on Reasoning skills (Turkish version: "akıl yürütme becerisi") in the field of educational sciences were examined according to the fields in Bloom's taxonomy, the disciplinary fields used (Cognitive, Affective and Psychomotor) and their main results. In his research, Mercan (2021) examined the type of reasoning skills used with a thematic approach in early childhood Reasoning Skills (Turkish version: akıl yürütme becerileri) in Turkey between 2010 and 2020, in relation to the year and method of the study. Similarly, Uçar (2022) studied 63 articles on the proportional reasoning skill, one of the reasoning types identified through the Web of Science database - without country restrictions - to determine the distributions according to publication year, method, country, sample groups, and sample sizes, themes, topics and results he did. Additionally, Akdoğan (2021), who examined postgraduate theses written about mathematical thinking and mathematical reasoning in Turkey, points out that theses on mathematical reasoning have increased in

recent years. Barak (2022) examined postgraduate theses written in Turkey regarding mathematical reasoning. Unlike the previous studies, our study will examine the articles and theses written in Turkey, which include the concept of reasoning in mathematics teaching and its Turkish synonym, " akıl yürütme ", and determine what kind of tendency the studies show. In addition, a meta-synthesis will be conducted to reveal the similarities and differences of the studies, create a general framework, and integrate the results. It was thought that more accurate results would be obtained if the study included both postgraduate articles and articles based on mathematics teaching and also addressed the synonyms of the concept. For this purpose, we tried to find answers to the following questions:

1. What is the distribution of research according to their types and the years they were conducted?
2. What is the distribution of the sample types and sizes used in the research?
3. What is the distribution of the methods, models, or approaches used in the research?
4. What are the data collection tools used?
5. What types of statistical analyzses were performed?
6. What is the distribution of research according to learning fields?
7. What is the distribution of keywords used in research?
8. What is the distribution of the types of reasoning used in research?
9. What aims were intended to be achieved in the research?
10. What are the results obtained in the research?

2. METHOD

2.1. Research Design

This study was conducted within the scope of qualitative research. The meta-synthesis (thematic content analysis) method, which is the qualitative version of the meta-analysis method, was used (Türkoğlu & Cihangir, 2017). These are studies that aim to evaluate and interpret the findings of research conducted in a specific discipline or subject by comparing their similarities and differences. Meta-synthesis studies involve in-depth synthesis (Polat & Ay, 2016). When the studies in which the meta-synthesis method is used (Aspfors & Fransson, 2015; Ergün et al., 2023; Ersanli et al., 2018; Polat & Ay, 2016) are examined, it is seen that the following process steps are generally followed:

i. Determining the subject,

ii. Determining keywords for the subject and scanning the literature,

iii. Selecting the studies to be evaluated by determining the inclusion criteria of the studies;

iv. Identifying and writing research problems;

v. Analysis and coding of findings for problems;

vi. Evaluating and interpreting the results obtained and interpreting them in the context of the literature

vii. Writing Conclusions and Suggestions.

2.2. Data Collecting

In order to access the studies to be included in the meta-synthesis, databases such as the Council of Higher Education (CoHER) national thesis center, Google Scholar, National Academic Network and Information Center (NANIC), Educational Resource Information Center (ERIC) were used. While scanning the literature, the keywords “reasoning” (Turkish Version: “muhakeme”, “akıl yürütme”) and

"reasoning skill" (Turkish Version: "muhakeme becerisi", "akıl yürütme becerisi") were used. As a result of the scans, a total of two hundred seventy-four studies, two hundred five of which were postgraduate theses and sixty-nine of which were articles, were found in Turkey. When these studies were examined, they were not included in the study because their content was done outside of mathematics teaching or some articles were produced from theses. Even if some articles were produced from a thesis, if one of the keywords specified in the title of the thesis did not appear, the article was included in the study instead of the thesis. No time period was determined when the studies were included in the meta-synthesis study in order to ensure that the concept of reasoning in Mathematics Teaching has a wide range and that its dimensions are examined in as wide a variety as possible and reflected in the research results. In meta-synthesis studies, after a certain amount of data was analyzed and findings started to be created, the relevant databases were scanned one last time on the day of the analysis, as the inclusion of new data would make the analysis and creation of findings difficult. The last scan was conducted on June 19, 2023. Therefore, ultimately, a total of one hundred sixty-three studies (See Appendix-1), including one hundred twenty postgraduate theses and forty-three articles, were included in the meta-synthesis.

2.3. Data Analysis

In this research, the Microsoft Excel 2016 program was used to transfer the necessary information of all studies included in the meta-synthesis to the electronic environment. The way data is transferred to an electronic environment is shown in detail in Figure 1. As seen in Figure 1, the studies are transferred to the electronic environment respectively: author, publication year, publication type, keywords, the purpose of the research, learning field or development field, sample size, sample type, research method, data collection tools, data analysis methods and research results. It was conveyed as. Then, the data transferred to the electronic environment was analyzed with the NVIVO 12 Plus program, a qualitative data analysis software.

	Authors	Year	Publication Typ	Keywords	Aim	...rning or Development F	Sample Size	Sample Type	Methodology	Data Collection	Data Analysis
1	ÇOBAN, Halil	2019	Doctoral Dissertation	Differentiated Instruction, Mathematical Reasoning, Problem Solving	The effect of differentiated instructional	Numbers and Operations	93	Secondary School Stu	Quantitative experimental study/The control	Mathematical Reasoning	Descriptive Analysis, One-Way Anova, Scheffe Test, Kruskal Wallis,
2	ERTAMAN, Zeynep	2009	Master Thesis	Spatial reasoning, spatial visualization, spatial orientation	The effect of teaching activities on spatial	Geometry and Measurement	24	Secondary School Stu	Quantitative methods: The	Spatial Orientation Test	Wilcoxon signed rank test
3	BELİN, Mervenur	2016	Master Thesis	Unspecified	Pre-service mathematics teachers' quantitative reasoning on the development of decimal representation of real numbers and its effect on their comprehension of a related proof	Numbers and Algebra	19	Pre-service Mathema	mixed methods/Embedde d experimental study	Proof Comprehension T	Wilcoxon Signed Rank test
4	TUM, Ali	2019	Master Thesis	Styles, Enriched Learning, Mathe	Determine the effect of learning environments enriched using different teaching methods on mathematical reasoning skills and problem solving attitude within the context of learning styles and to provide the reflections of this process from the perspective of the participants	Numbers and Operations	24	Secondary School Stu	mixed methods/Embedde d experimental study	Learning Styles Scale, M	Wilcoxon Signed Rank test, K
5	AYDOĞDU, Gülbah	2023	Master Thesis	Proof, Errors of Reasoning, Colle	Examine the mathematical proof processes of secondary school students through argumentation and to determine the reasoning errors that occur	Numbers and Operations; Geometry and Measurement	64	Secondary School Stu	Qualitative methods/Case Study	Mathematical Proof Test	Content Analysis

Figure 1. How data is transferred to electronic media

The data uploaded to the Nvivo 12 Plus program was coded according to the research questions determined. While coding the sizes of the working groups, if a document review was carried out, the number of documents examined was added to the relevant group size category as a reference. Learning fields are coded according to the fields in the Ministry of Education's preschool, primary, secondary, and high school mathematics curriculum. Since there is a development field instead of a learning field

in preschool, it is coded according to the developmental field. If the learning field was not clearly stated in the studies examined, the learning field was determined by examining the subject examined in the study, the applications made, or the measurement tools applied in detail. If a clear learning field could not be identified after a detailed examination, it was categorized as unspecified. Similarly, the same method was followed when coding the types of reasoning that the studies focused on. While coding the aims of the research, not only the purpose sentence but also the problem and sub-problem sentences were examined. Studies examining postgraduate theses and articles were not included in the synthesis of the results of the studies included in the research, and these were evaluated in the discussion section.

The findings, along with the coding, were presented visually with tables, graphs, or word clouds. The coding was done by the researcher, and approximately 10 percent of the studies included in the study were sent to an academician in mathematics education and asked to code for the research questions. The coding made by the academician and the coding made by the researcher are compatible. The reliability formula of Miles and Huberman (1994) was used to calculate coding reliability. This formula is based on the ratio of coders' common code frequency to the sum code frequency. In this context, coding reliability was calculated as 96%.

3. FINDINGS

3.1. Findings Regarding the Publication Year and Type of Studies

The distribution of the studies included in the research according to publication year, and type is given in Chart 1.

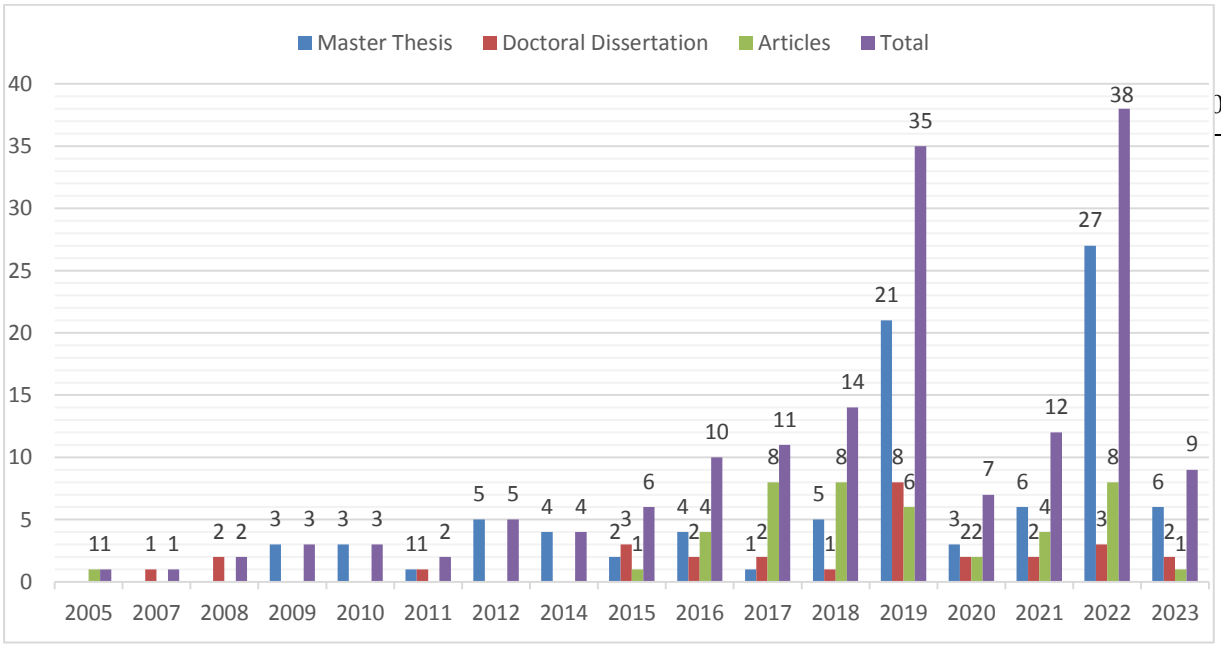

Chart 1. Distribution of publication types by years

When Chart 1 is examined, considering the publication dates of the studies, it is seen that the studies are distributed between the years 2005-2023. The studies included in the research were published mostly in 2022 and 2019. Of the studies examined, 91 are master's theses, 29 are doctoral theses and 43 are articles. Doctoral thesis studies on reasoning are rarely done. It is seen that the increase rate of studies was slow between 2005-2014, but it accelerated slightly in between 2015-2017, and although the number of studies decreased in 2020 and 2021, the number of studies increased after 2018. While the year in which the master's thesis was least published was 2011, the years in which the doctoral thesis was least published were 2007 and 2011. Additionally, the years in which

articles were published the least were 2005, 2015, and 2023. It is a striking finding that only one article on reasoning was published before 2015.

3.2. Findings Regarding the Study Group and Size Focused on in the Studies

Data regarding the study group are shown in Chart 2. The data shown in Chart 2 exceeds the number of studies due to the presence of more than one study group in the same study. The total frequency of the distribution according to the study group is 174. 54.29% of the studies were conducted with secondary school students, 15.52% with candidate teachers, 10.34% with teachers, 6.9% with secondary school students, 5.75% with documents, 3.45% with preschool students, 3.45% with primary school students and 0.57 of them were conducted with families.

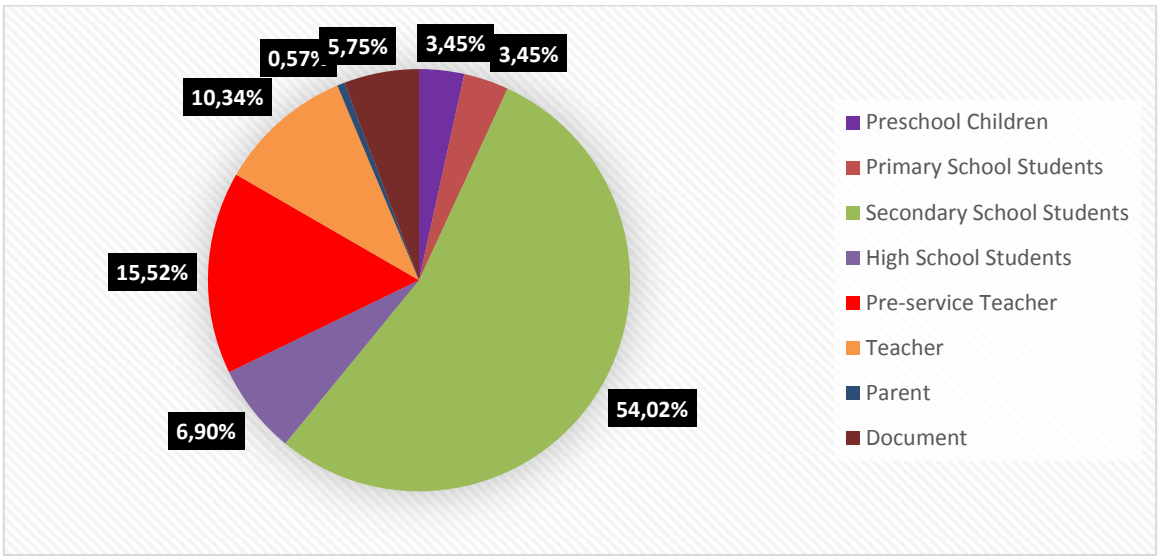

Chart 2. Distribution of working groups

In the studies examined, the size of the study group was also evaluated. The size of the study group was determined and coded within the ranges as given in Chart 3.

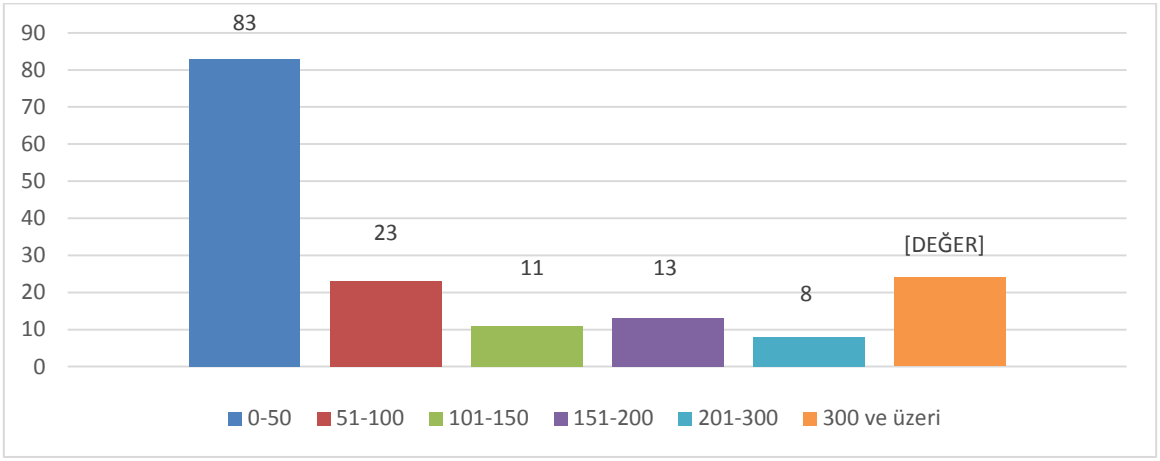

Chart 3. Distribution of the sizes of working groups

When Chart 3 is examined, it is seen that the most preferred working group size is between 0-50. We can say that these researchers prefer to work with small groups in studies on reasoning skills.

3.3. Findings Regarding the Research Methods Used in the Studies

The distribution of findings regarding the research methods used in the studies according to the methods and designs of the studies included in the research is given in Table 1.

Table 1. Distribution of the studies examined according to research methods and designs

Method	Design	f	f
Quantitative	Causal Comparative Research	3	55
	Causal Design	1	
	The Action Research	2	
	Descriptive Research Design	1	
	Experimental Study	13	
	Detailed Research	1	
	Developmental Research Method	1	
	Survey	13	
	Relational Survey	13	
	Descriptive Survey	5	
	Cross-Sectional Survey	2	
Qualitative	The Action Research	1	78
	Case Study	53	
	Ethnographic case Study	2	
	Phenomenology	4	
	Teaching Experiment	9	
	Design-Based Research	3	
	Grounded Theory	1	
	Document Analysis Method	5	
Mixed		26	26
Compilation Research	Bibliometric Analysis Research	1	5
	Document Analysis Method	4	
Evaluation tool development Study		1	1
	Total		165*

**Since the studies examined were used in more than one research design, the sum of the frequency for the research design is more than the number of studies. For example, "Relational Survey" and "Descriptive Survey".*

As seen in Table 1, it has been observed that quantitative, qualitative, mixed, compilation research and evaluation tool development studies were used in studies on reasoning skills in mathematics teaching. The most used method in the studies included in this research is the qualitative method (f = 78), while the mixed method (f = 26) is used relatively less than the qualitative and quantitative methods. In addition, compilation research (f = 5) and evaluation tool development studies (f = 1) were carried out the least. When the designs used in the methods are evaluated, the most used quantitative design is the Survey (f = 33), while the Experimental Study (f = 13) is used relatively less than the Survey. Causal Design, Descriptive Research Design, Descriptive Research Design, Detailed Research, and Developmental Research Method are the least used designs, with one study each. In studies where the qualitative method was adopted, the most used pattern was case study (f = 53), while the least used patterns were Action Research and Grounded Theory, with one study each. While Document analysis was the most used method in compilation research, it was observed that bibliometric analysis was the least used research method. Researchers prefer to study case study design, one of the qualitative research methods on reasoning skills in mathematics teaching.

3.4. Findings Regarding the Data Collection Tools Used in the Studies

The distribution of data collection tools used in studies focusing on reasoning skills in mathematics teaching is given in Chart 4. As can be seen from Chart 4, a wide variety of data collection tools were used. Among these tools, it is seen that open-ended questions problems or skill tests (f = 124) are used due to the nature of reasoning, and interview (f = 79) is used due to the nature of reasoning. The questions generally used in skill tests are open-ended. Additionally, "audio and video recording" is accompanied by interviews used as data collection tools.

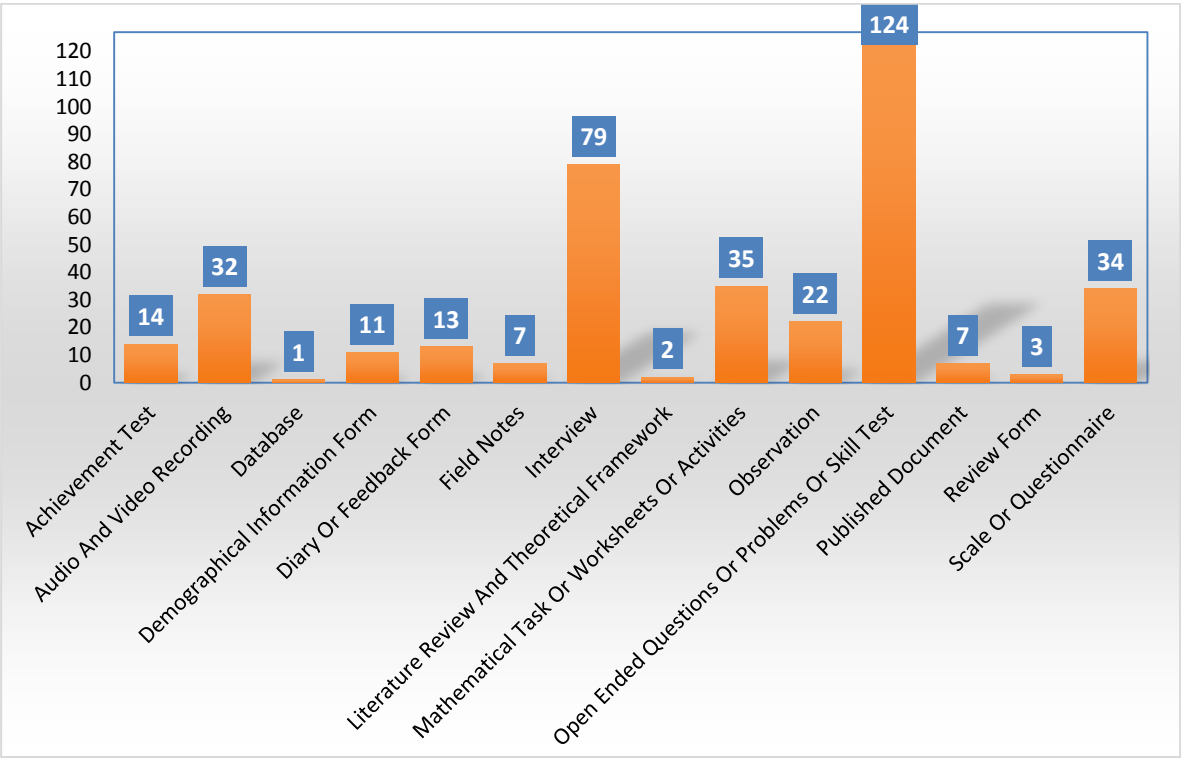

Chart 4. Distribution of data collection tools used in the studies examined

When Chart 4 is examined, it is seen that alternative data collection tools such as "demographic information form", "diary or feedback form", "field notes", and "mathematical task or worksheets or activities" are used.

3.5. Findings Regarding the Data Analysis Methods and Techniques Used in the Studies

The distribution of the findings regarding the data analysis methods and techniques used in the studies according to the data analysis methods and techniques used in the studies on reasoning skills in mathematics teaching in Turkey is given in Chart 5.

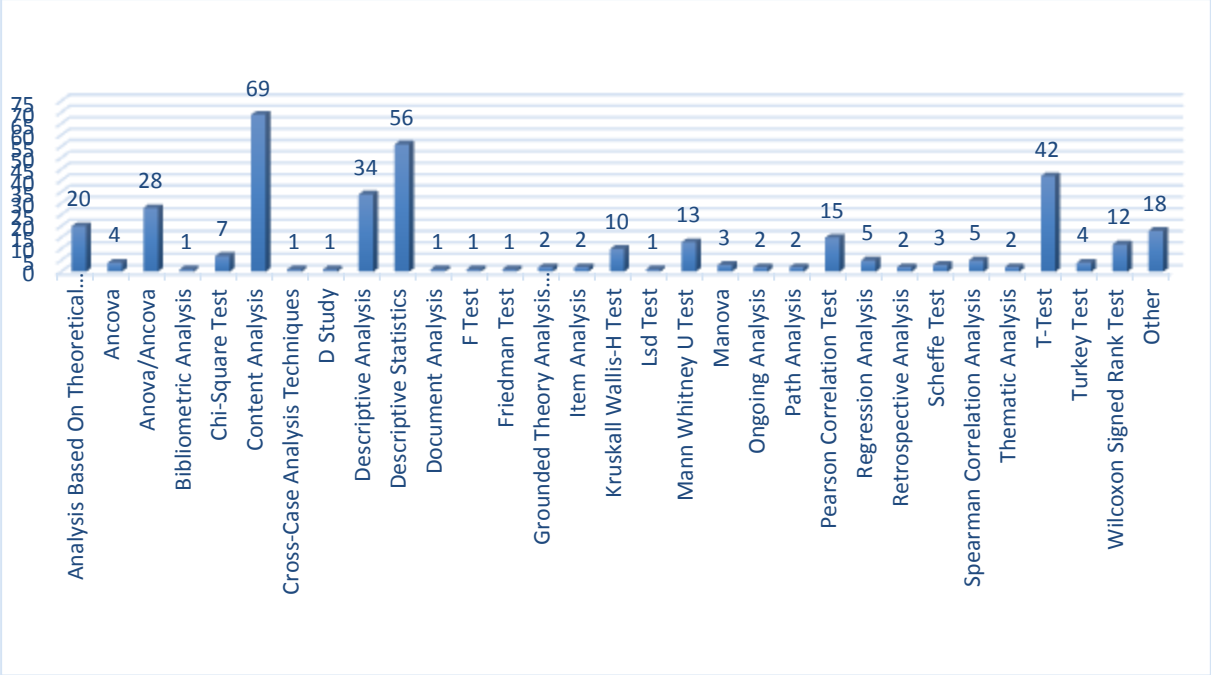

Chart 5. Distribution of methods and techniques used in data analysis in the studies examined

When Chart 5 is examined, it is seen that parametric tests are used more than non-parametric tests. Therefore, it can be said that in studies where a quantitative approach is adopted, the normality assumption required for the application of parametric tests is more met. Among the parametric tests, the t-Test (f=42), Anova/Ancova (f=27), and Pearson Correlation Test (f=15) were used most frequently. It is seen that among the non-parametric tests, Mann Whitney U Test (f = 13), Wilcoxon Signed Rank Test (f = 12), and Kruskall Wallis-H Test (f = 10) are used the most. It is seen that it is used in different analyses as well as parametric and non-parametric tests. The analysis based on the theoretical framework (f = 28) is one of the most prominent analyses. In studies where qualitative approaches were adopted, content analysis (f = 69) and descriptive analysis (f = 34) were most preferred. In addition, descriptive statistics were used in 56 of the 163 studies included in the research.

3.6. Findings Regarding Mathematics Course Learning Fields

The distribution of the learning field or development fields of the studies on reasoning skills in mathematics teaching in the mathematics curriculum published by the Ministry of Education is given in Table 2.

Table 2. Distribution of learning fields focused on in the studies examined

Curriculum	Learning Field or Development Field	f
Preschool Curriculum	Cognitive Development	6
Primary School Mathematics Curriculum	Numbers and Operations	4
	Geometry	2
	Measurement	1
	Data Processing	1
Secondary School Mathematics Curriculum	Numbers and Operations	65
	Algebra	25
	Geometry and Measurement	38
	Data Processing	14
	Probability	15
High School Mathematics Curriculum	Numbers and Algebra	11
	Geometry	10
	Data, Counting, and Probability	4
Unspecified		24
Total		220**

*** Since the studies examined were used about more than one learning field or development field, the sum of the learning field frequency is more than the number of studies.*

When Table 2 is examined, it is seen that studies focusing on reasoning skills in mathematics teaching mostly address the learning areas within the Secondary School mathematics curriculum. In this curriculum, the learning areas of numbers and operations (f=65), geometry and measurement f=38), and algebra (f=25) are covered the most, respectively, while the learning areas of Data Processing (f=14) and Probability (f=15) are covered the least. The least amount of work was carried out on the preschool curriculum. In the preschool curriculum, studies on reasoning skills were carried out only in the field of Cognitive Development (f = 6). When we look at the learning areas in the primary school mathematics curriculum, in the studies included in the research, the most learning areas were numbers and operations (f = 4) and geometry (f = 2), while the least was Measurement and Data Processing, with one study each. Similarly, studies have been conducted mostly on the learning areas of numbers and algebra (f=11) and geometry (f=10) for the secondary school mathematics curriculum. A minimum number of studies have been conducted on the data, counting and probability learning field in secondary school mathematics programs. Additionally, it needed to be specified which learning area the 24 studies focused on.

3.7. Findings Regarding the Keywords Used in Research

In line with the research, the frequency of use of the keywords used in the studies examined was analyzed and according to this result, the word cloud is given in Figure 2. The font size of the words in Figure 2 varies depending on the frequency of use of these words. In other words, as the frequency of use increases, the word size increases.

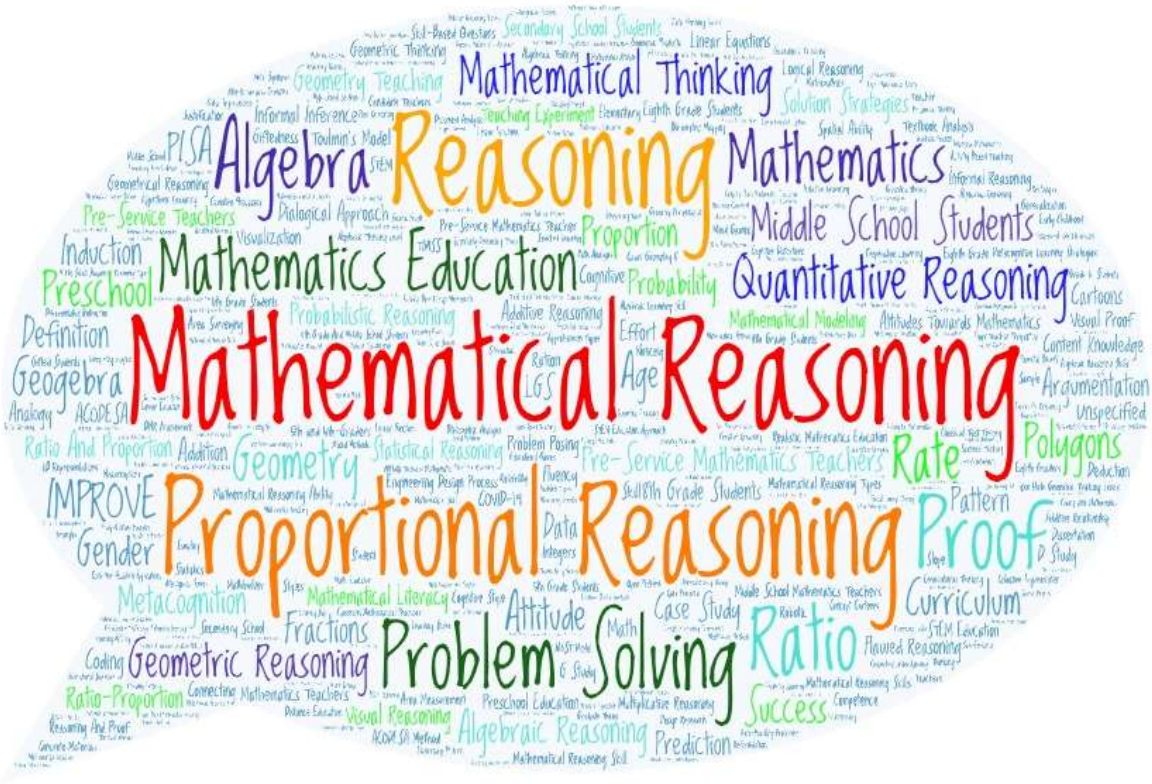

Figure 2. Word cloud of keywords in the studies examined

When Figure 2 is examined, it is understood that the words "Mathematical Reasoning" (f=45), "Proportional Reasoning" (f=31), and "Reasoning" (f=31) were most used as keywords in the studies. It can be said that the greater use of these keywords is due to the fact that the studies focus on the "Numbers and Operations" and "Numbers and Algebra" learning fields. Additionally, it was

determined that these keywords were followed by the words "Problem-Solving" (f=15), "Mathematics Education" (f=14), "Quantitative Reasoning" (f=9), "Mathematical Thinking (f=7) ", "Middle School Students" (f=7), "Algebra" (f=5), "Proof" (f=5) and "Ratio" (f=5).

3.8. Findings Regarding the Types of Reasoning That Research Focuses On

In line with the research, the types of reasoning that were the focal point of the studies examined were determined, and their frequencies were analyzed according to the number of studies. The word cloud in Figure 3 was created. The font size of the words or word groups in Figure 3 varies depending on the frequency of use of these words. In other words, as the frequency of use increases, the size of the words increases. In the studies examined, 21 types of reasoning were identified as focal points.

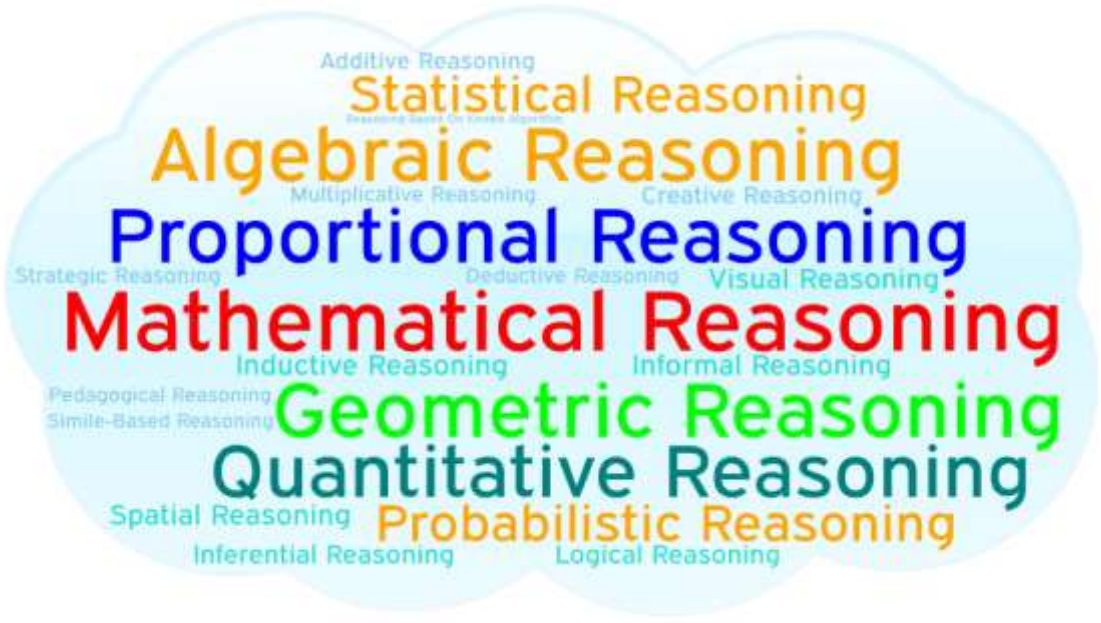

Figure 3. Word cloud of the type of reasoning that the studies examined focus on

When Figure 3 is examined, it can be seen that the studies were mostly focused on mathematical reasoning (f = 72). It was observed that the most focused reasoning after mathematical reasoning was proportional reasoning (f = 37). Geometric reasoning (f=13), quantitative reasoning (f=10), Algebraic reasoning (f=8), and probabilistic reasoning (f=6) are the types of reasoning discussed in more studies than other types of reasoning. Additive Reasoning, Creative Reasoning, Deductive Reasoning, Pedagogical Reasoning, and Strategic Reasoning are the least studied reasoning, with one study each. It has also been found that some of the studies focus on more than one reason.

3.9. Findings Regarding the Aims of the Studies

Findings regarding the aims of the studies included in the research are given in Table 3. Nine themes were created by examining the aims of the studies. These themes are document review, determining the relationships between different reasoning skills, examining the relationship between reasoning skills and other variables, examining the factors that affect reasoning skills in studies, measuring reasoning skills, reasoning process, determining opinions and field knowledge regarding reasoning, scale development, evaluation process and the effect of the applications on reasoning skills.

When Table 3 is examined, it is seen that the studies focus especially on examining the factors affecting reasoning skills (f = 62), measuring reasoning skills (f = 58), and the effect of teaching practices on reasoning skills (f = 54). It was investigated whether reasoning skills differ according to

variables such as gender, grade level, academic or mathematics achievement, age, and parental education level. In addition, under the umbrella of reasoning skills, it aims to measure different types of reasoning skills, such as mathematical reasoning, proportional reasoning, geometric reasoning, probabilistic reasoning, and quantitative reasoning. In addition, studies have been conducted to determine the effect of the use of different educational and instructional applications, such as methods, techniques, models, or activities on reasoning skills, originating from experimental approaches. In the studies examined, the effects of applications such as technology-supported learning, enriched learning environments, STEM, IMPROVE Model, intelligence games, hypothetical learning processes, mathematical modeling activities, multiple representation-based learning, and argumentation-based learning on different types of reasoning skills were examined.

Table 3. Distribution of the studies examined according to their purposes

Theme	Sub-theme	f	f
Document examination	Mathematical reasoning in postgraduate theses or articles	3	10
	Proportional reasoning in skill-based questions	1	
	Mathematical reasoning in early childhood	1	
	Proportional reasoning in articles	1	
	Logical reasoning in mathematics textbooks	1	
	Mathematical reasoning in mathematics textbooks	2	
	Statistical reasoning in the mathematics curriculum	1	
Determining the relationship between different reasoning skills	Algebraic Reasoning-probabilistic Reasoning	1	5
	Algebraic reasoning-spatial reasoning	1	
	Mathematical reasoning-Probabilistic Reasoning	1	
	Proportional reasoning-probabilistic reasoning	1	
	Spatial reasoning-probabilistic reasoning	1	
Examining the relationship between reasoning skills and other variables	Mathematics supported by Mind and Intelligence Games	1	29
	Metacognitive learning strategies	1	
	Algebraic thinking	1	
	Functional thinking	1	
	Visual estimation skills	1	
	Covariational thinking	1	
	High school entrance score	1	
	Mathematical achievement	2	
	Attitude towards mathematics	4	
	Coping with Math	1	
	Mathematical thinking profile	1	
	Reading comprehension	1	
	Learning styles	1	
	Patterning skills	1	
	Perception of self-efficacy	3	
	Problem solving success	2	
	Reflective thinking towards problem solving	1	
	Problem posing skills	1	
	Table and Chart Interpretation Performance	1	
	Spatial ability	1	
	Geometric thinking	2	
Examining the factors affecting reasoning skills	Socio-economic status	2	62
	Academic and mathematics achievement	7	
	Gender	19	
	COVID-19	1	
	Parents' education level	4	
	Duration of training	2	
	Number of siblings	1	

	Reading	1	
	Type of math teacher	1	
	Mathematical competence	1	
	School	1	
	School location	2	
	Problem type	2	
	Grade level	14	
	Age	4	
Examining and measuring reasoning skills	Algebraic reasoning	3	58
	Geometric reasoning	7	
	Visual reasoning	3	
	Statistical reasoning	3	
	Mathematical reasoning	16	
	Quantitative reasoning	4	
	Probabilistic reasoning	5	
	Proportional reasoning	12	
	Pedagogical reasoning	1	
	Strategic reasoning	1	
	Inductive reasoning	2	
	Spatial reasoning	1	
Reasoning process	Arguments and lesson plans used to elicit reasoning	6	24
	The effect of algebraic reasoning applications on misconceptions	1	
	Global argumentation structures	1	
	Reasoning error	5	
	Problem posing skills	2	
	Problem-solving strategies and reasoning types	9	
Determining opinion and field knowledge for reasoning	Awareness	3	7
	Statistical reasoning	1	
	Flawed reasoning	1	
	Mathematical reasoning	2	
Scale development and evaluation process	Development of the mathematical reasoning self-efficacy beliefs scale.	1	3
	Validity and reliability of the reasoning assessment rubric.	1	
	Reliability examination of data collection tool according to theories	1	
The effect of teaching practices on reasoning skills	Argumentation based learning	1	54
	Cognitively challenging activities	1	
	Individual learning	1	
	Mathematics teaching based on discovery learning approach	1	
	Knowledge framework for teaching algebra	1	
	Learning environment that supports algebraic reasoning	1	
	Multiple representations-based instruction	2	
	Educational robotic applications	1	
	Differentiated instruction	1	
	Realistic mathematics education	1	
	IMPROVE model	3	
	Teaching with concept cartoon	1	
	Coding education	1	
	In-service teacher training and practices in the field of mathematical literacy	1	
	Mathematics history activities	1	
	Mathematical modelling activities	2	
	Mathematics activities supported by Montessori materials	1	
	Reasoning and proof activities	1	
	Quantitative reasoning teaching approach	1	
	Focus group studies	1	
	Teaching module based on proportional reasoning	1	
	Origami based teaching	1	

	Game-based mathematics teaching	2
	Pattern-based mathematics education program	1
	Virtual manipulative team	1
	Scenario-based learning	1
	STEM	5
	Improve prediction skills activities	1
	Technology-enhanced learning	7
	Spatial visualization activities	1
	Hypothetical learning process	3
	Mind games	3
	Enriched learning environments	3
	Total	253*

*Since some of the studies examined included more than one purpose, the total frequency is higher than the number of studies included in the research.

In the studies, it is seen that after examining the effects of different teaching practices on reasoning skills, the relationship of this skill with other variables is discussed. When Table 3 is examined, it tries to reveal the relationship with variables such as mathematics attitude, self-efficacy perception, mathematics success, mathematical problem-solving success, geometric thinking, visual estimation, and algebraic thinking. In addition, although there are few studies, the relationship between reasoning types has also been examined. As can be seen in Table 3, the relationship between algebraic reasoning - probabilistic reasoning, algebraic reasoning - spatial reasoning, mathematical reasoning - probabilistic reasoning, proportional reasoning - probabilistic reasoning, and spatial reasoning - probabilistic reasoning was examined.

In the studies included in the research, the reasoning process was examined due to its structure. Especially since reasoning skills are demonstrated in problem-solving in mathematics, the strategies and types of reasoning used in this process have been tried to be examined. In addition, studies were carried out to identify the arguments and lesson plans used to reveal reasoning skills. Errors made while using reasoning skills were also tried to be identified. Reasoning skills are used not only when solving problems, but also in problem-posing studies. This aspect has been addressed in studies conducted.

When Table 3 is examined, studies have been conducted examining various documents according to different types of reasoning. These studies aim to examine mathematics textbooks, graduate theses, or articles with the main theme of mathematical reasoning, both published articles and skill-based questions published by MONE in terms of proportional reasoning, mathematics textbooks with a focus on logical reasoning, and mathematics curriculum in the context of statistical reasoning.

When we look at the studies examined, studies have been carried out to determine opinions and field knowledge regarding reasoning. It seems that the aim is to determine awareness of the skill in question and to determine opinions and field knowledge about mathematical reasoning, flawed reasoning, and statistical reasoning.

In the studies examined, a small number of scale development and evaluation studies have been carried out. It seems that the aim is to develop a mathematical reasoning self-efficacy scale. In addition, while it was aimed to conduct validity and reliability studies of the table prepared for the evaluation of reasoning skills, another study aimed to compare the reliability of a data collection tool for reasoning skills according to different theories.

3.10. Findings Regarding the Results Obtained in the Studies Examined

Findings regarding the results of the studies included in the research are given under nine themes. The results under the theme of examining documents in terms of reasoning are given in Table 4.

Table 4. Examination of documents in terms of reasoning

SUBJECT- THEME*	Result
• Proportional reasoning in skill-based questions • Mathematical reasoning and logical reasoning in mathematics textbooks • Statistical reasoning in the mathematics course curriculum	• Approximately one in every four questions published by MoNE involves proportional reasoning. The most were published on data analysis, the least on inequality and geometric concepts. Skills-based questions published are not evenly distributed across proportional reasoning problem types. • Reasoning and proof are not included enough in the mathematics textbook and are not sufficient for students to develop their own reasoning skills and strategies. • In current mathematics curriculum, there are deficiencies in some achievements in terms of collecting, defining, organizing, representing data and analyzing and interpreting data, and they are insufficient in terms of developing statistical reasoning.

**Systematic review studies examining theses or articles on reasoning skills are not included in the findings, and their results will be discussed in the discussion section.*

When Table 4 is examined, it can be seen that there are skill-based questions, mathematics textbooks, and mathematics curriculum published by MoNE for reasoning skills. Approximately 25% of published skill-based questions involve proportional reasoning. In addition, among skill-based questions involving proportional reasoning, most questions were asked about data analysis and the least about inequality and geometric concepts. Reasoning and proof are not included sufficiently in mathematics textbooks at different grade levels. Textbooks were deemed inadequate in terms of allowing students to develop their own reasoning skills. Mathematics teaching programs have been found inadequate in terms of statistical reasoning.

Findings regarding the theme of determining the relationship between different types of reasoning skills are given in Table 5.

Table 5. Determination of the relationship between different types of reasoning skills

Subject	Result
Algebraic Reasoning-Probabilistic Reasoning	• There is a weak positive significant relationship.
Algebraic Reasoning-Spatial Reasoning	• There is a positive, significant, and medium-level relationship.
Mathematical Reasoning-Probabilistic Reasoning	• There is a high level of positive correlation.
Proportional Reasoning-Probabilistic Reasoning	• There is a high level of positive correlation.
Spatial Reasoning-Probabilistic Reasoning	• There is a weak positive significant relationship.

Studies have found a positive relationship between algebraic reasoning and probabilistic reasoning, and between spatial reasoning and probabilistic reasoning, but this relationship is weak. If we look at the relationship between algebraic reasoning and spatial reasoning, it turns out that there is a moderate positive relationship. Mathematical reasoning and proportional reasoning separately have a high positive relationship with probabilistic reasoning.

Findings regarding the theme of examining the relationship between reasoning skills and other variables are given in Table 6.

Table 6. Examining the relationship between reasoning skills and other variables

Variable	Result
• Mathematics Teaching Supported by Mind and Intelligence Games	• There is a positive relationship.
• Metacognitive Learning Strategies	• There is a moderate relationship in the positive direction.
• High School Entrance Score	• There is a significant positive relationship.
• Coping with Mathematics	• There is a significant positive relationship.
• Reading Comprehension	• There is a significant positive relationship.
• Learning Styles	• There is a significant positive relationship.
• Skill of Pattering	• There is a significant positive relationship.
• Reflective Thinking Skills Towards Problem-Solving	• There is a significant positive relationship.
• Table Interpretation and Graphic Interpretation Performances	• There is a significant positive relationship.
• Spatial Ability	• There is a significant positive relationship.
• Algebraic Thinking	• There is a positive relationship.
• Functional Thinking	• There is a positive relationship.
• Covariational Thinking	• There is a positive relationship.
• Visual Estimation Skills	• There is a moderate relationship in the positive direction.
• Mathematical Thinking Profile	• There is a moderate relationship in the positive direction.
• Problem-Solving Success	• There is a moderate relationship in the positive direction.
• Problem Posing Skill	• There is a significant positive relationship.
• Mathematics Achievement	• There is a significant positive relationship.
• Mathematics Attitude	• There is a significant positive relationship.
• Perception of Self-Efficacy	• There is a significant positive relationship. (Moderate and weak level)
• Geometric Thinking	• There is a significant positive relationship.

In the studies examined, the relationship of 21 variables with reasoning skills was examined. It is seen that there is a positive relationship between all the variables given in Table 6 and reasoning skills. There is no variable in the studies for which no significant relationship was found.

Findings regarding the theme of factors affecting reasoning skills are given in Table 7.

Table 7. Factors affecting reasoning skills

Factor	Result
Socio-Economic Level	• As the socio-economic level increases, judgment becomes better.
Academic or Math Achievement	• Generally, as the level of success increases, the level of reasoning skills increases.

Gender	• In terms of mathematical reasoning, both girls perform better than boys and boys perform better than girls. However, most studies show that gender is not effective. • Although female students have higher level skills in probabilistic reasoning than male students, there are results showing that gender has no effect. • Although proportional reasoning shows that girls are more successful than boys, there are also results showing that gender is not effective. • It does not affect spatial reasoning, geometric reasoning, and algebraic reasoning.
Covid-19	• It negatively affected mathematical reasoning skills.
Educational Status of Parents	• It affects reasoning skills. As parents' education level increases, children use higher-level reasoning skills.
Duration of Training	• Whether children's duration of preschool education affects their mathematical reasoning varies from sample to sample.
Number of siblings	• It does not affect mathematical reasoning skills.
Reading	• It does not affect mathematical reasoning skills.
Mathematics Teaching Type	• It does not affect statistical reasoning skills.
Mathematical Proficiency	• It affects proportional and probabilistic reasoning. As mathematical proficiency increases, success in using reasoning skills will increase.
School Type	• It affects mathematical reasoning.
School Location	• It affects mathematical reasoning.
Problem Type	• It affects proportional judgment.
Grade Level	• It generally affects proportional reasoning. Proportional reasoning skills do not increase consistently as grade level increases. • It is not effective in inductive reasoning skills. • Although it affects mathematical reasoning skills in some samples, it exists in cases where it does not. • Although it affects geometric reasoning skills in some samples, there are cases where it does not. • It is effective in probabilistic reasoning skills.
Age	• It affects mathematical reasoning. As age increases, mathematical reasoning improves.

When Table 7 is examined, socio-economic level, academic and mathematics achievement, parents' education level, mathematical competence, school type, school location, problem type, and age factors affect reasoning skills. It also appears that the Covid-19 factor negatively affects reasoning skills. Type of mathematics teacher, reading books, and number of siblings are factors that do not affect your reasoning skills. Apart from this, when we look at the factors of gender, duration of education, and grade level, we see that there are different results indicating that they affect or do not affect reasoning skills. When we look at the gender factor, results show that it affects mathematical reasoning, probabilistic reasoning, and proportional reasoning. However, it seems that gender has no effect on spatial reasoning, geometric reasoning, and algebraic reasoning. Similarly, in the grade level factor, it can be seen that there are different results in proportional reasoning, inductive reasoning, geometric reasoning, mathematical reasoning, and probabilistic reasoning.

Findings regarding the theme of examining and measuring reasoning skills are given in Table 8. When Table 8 is examined, 12 types of reasoning are examined, including algebraic reasoning, geometric reasoning, visual reasoning, statistical reasoning, mathematical reasoning, quantitative reasoning, probabilistic reasoning, proportional reasoning, pedagogical reasoning, strategic reasoning, inductive reasoning, and spatial reasoning.

Algebraic reasoning skills have been measured, and it is seen that there are different levels of results in the studies. In studies conducted on geometric reasoning skills, it was found to be at an intermediate level. In addition, the most used components in geometric reasoning have been identified. In studies on visual reasoning, the stages used when using the skill have been identified. Statistical

reasoning skills were measured at the temporal level and quantitative level, and it was determined that there were difficulties in explaining ideas, making predictions, and making inferences.

Table 8. Examination and measurement of reasoning skills

Reasoning Type	Result
Algebraic Reasoning	• Reasoning Skill level was measured (Low, Medium, and Good).
Geometric Reasoning	• Measured as medium level. • The most common components are putting forward ideas and making inferences. They also make more flawed or faulty judgments.
Visual Reasoning	• When using visual reasoning, stages such as reading the question, explaining, making a plan, examining geometric shapes, explaining relationships, showing given mathematical expressions on a visual, expressing conceptual knowledge, adapting information to a visual, showing with a figure or model, proving, and evaluating are followed.
Statistical Reasoning	• Measured at Level 2 (temporary) and Level 3 (quantitative). • There is difficulty in expressing ideas, making predictions, and making inferences in questions that require reasoning.
Mathematical Reasoning	• Generally measured at medium and low levels. Expected strategies are not used sufficiently. • It is seen that they show a wide range of performance in all reasoning processes, such as analysis, generalization, and reasoning in mathematical problems. • A mathematical relationship is established between what is wanted and what is given, and the right and appropriate decision is made to solve the problem. • They exhibited superficial thinking structures and showed that they primarily preferred mathematical reasoning types based on analogy. • It has been observed that there are difficulties in the sub-steps of reasoning, such as "solving non-routine problems, developing logical arguments for the solution, making generalizations, determining and using appropriate reasoning".
Quantitative Reasoning	• He/she has weak and insufficient quantitative reasoning skills. The quantitative reasoning levels of those who resort to memorized knowledge and formulas that they do not understand, instead of reasoning by establishing relationships between quantities in the problems they encounter in daily life and at school, remain at a low level.
Probabilistic Reasoning	• Probabilistic reasoning skills were measured at poor, moderate, and good levels.
Proportional Reasoning	• It seems that they are not sufficient in terms of proportional reasoning skills.
Pedagogical Reasoning	• Teachers make pedagogical judgments by considering how the activities to be chosen for mathematics teaching will affect students' learning processes and how the teacher will organize his/her own teaching process.
Strategic Reasoning	• Different solutions are made in the process of making sense of the context in open-ended problems.
Inductive Reasoning	• Measured as good or moderate.
Spatial Reasoning	• Measured at a good level.

Mathematical reasoning skills were measured at medium and low levels, and it was determined that the expected strategies were not used sufficiently. It was determined that they exhibited very variable performances in the reasoning process steps and had difficulties in some sub-steps. When we look at the quantitative reasoning skill measurements, an inadequate and not strong performance was detected. It is seen that the use of rote knowledge and formulas that do not make sense in problem-solving decreases quantitative reasoning performance.

Probabilistic reasoning skills showed different levels of performance. In studies examining proportional reasoning, it is seen that the study group is not at a sufficient level. How teachers use their pedagogical reasoning in planning their lessons and selecting activities for this purpose was examined. In open-ended problems, different solution strategies are used, and convincing explanations are given. However, it is seen that there are problems in identifying connections and

relationships. Regarding spatial reasoning skills, good measurements were obtained in the studies, while different levels of measurements were obtained in the inductive reasoning skill.

Findings on the reasoning process theme are given in Table 9. When Table 9 is examined, the reasoning process is discussed over six topics. Arguments and lesson plans used to reveal reasoning, the effect of algebraic reasoning practices on misconceptions, global argumentation structure, reasoning errors, problem-posing skills and strategies used in problems, and reasoning types were included.

Teachers use arguments such as asking for answers, explanations, and evaluations to emphasize students' reasoning skills in the teaching and learning process. In the case of algebraic reasoning, lesson plans were developed, and learning environments were designed based on the indicators of this skill. In addition, it was determined that learning based on peer collaboration with computer-aided applications and the teacher's behavior to support this would have a positive effect on reasoning skills. It was determined that mathematics teachers act according to the reasoning framework in the literature to encourage the use of reasoning skills in their geometry lessons. Despite all these, it was also found that teachers provided limited opportunities to encourage students to use their mathematical reasoning.

Table 9. The reasoning process

Theme	Result
Arguments and lesson plans used to elicit reasoning	• Actions such as asking for an answer, confirming the correct answer, clarifying, asking for evaluation, giving conceptual information and making an explanation are performed. • The lesson was planned according to algebraic reasoning indicators, and the learning environment was developed within this framework. • Students are provided with limited opportunities to use their mathematical reasoning. • Prioritizing peer collaboration through computer-aided applications and providing teacher support reflects positively on the use of reasoning skills. • Mathematics teachers act according to Duval's reasoning framework in order to use geometric reasoning skills in geometry courses.
The Effect of Algebraic Reasoning Applications on Misconceptions	• Although algebraic reasoning applications are effective in eliminating misconceptions, it is not right to generalize this for all students.
Global argumentation structure	• It was revealed that mathematical reasoning skills were weak, and therefore, simple global argumentation structures were mostly used.
Errors in judgment	• Treating special cases as proof and starting with wrong assumptions are errors in reasoning. Missing mathematical definitions or including unnecessary steps and expressions in the proof creates a gap in reasoning. • In problems requiring reasoning, mistakes are made in additive relationships, neglecting data, using numbers and lack of context, responding emotionally, and not being able to determine non-proportional situations. • In proportional problems, additive or absolute is considered. • Students not completing the reasoning process or experiencing conceptual deficiencies and turning to formulaic solutions are used to create reasoning errors.
Problem posing skill	• The problems posed are routine problems that require judgment, contain quantitative data rather than being of a qualitative character, are far from original and uncreative, and can be solved directly with algorithms. • There are difficulties in posing problems based on semi-structured reasoning.
Strategies and types of reasoning used in problems	• The strategies used vary depending on grade level. For example, while there is no clear strategy at the 6th-grade level of secondary school, 7th-graders mostly use the inside-outside strategy and unit ratio, and 8th-graders use the unit ratio and equivalent fraction strategy. • Teachers mostly use curriculum-dependent and routine problems and avoid problems that require superior reasoning. • Students mostly use algorithm-based mathematical reasoning.

Although algebraic reasoning practices seem to be effective in eliminating misconceptions, this effect does not seem to be valid for every student or student group. It is seen that the argumentation structure used by individuals with weak mathematical reasoning skills in the reasoning process remains at a simple level.

When we look at the issue of judgment error, it is seen that treating special situations as proof, starting with wrong assumptions, including unnecessary expressions, additive relationships, neglecting data, using numbers and lack of content, responding emotionally, and not being able to determine non-proportional situations allow judgment errors to occur. In addition, students' use of formulaic solutions in their minds causes errors in reasoning.

In problem-posing studies, it has been determined that problems are posed that are quantitative rather than qualitative, that do not involve reasoning, that are far from originality and creativity, and that can be solved directly with algorithms. It has also been revealed that there are difficulties when posing problems that require semi-structured reasoning.

In the strategies used in problem-solving at the secondary school level, it is seen that there is no clear strategy in the 6th-grade, while the inside-outside strategy and unit ratio are used in the 7th-grade, and the equivalent fraction strategy with unit ratio is used in the 8th-grade. In addition, it is seen that teachers do not include problems that require reasoning and high-level reasoning in their lessons but rather routine problems related to the curriculum and related strategies. Students mostly use algorithm-based mathematical reasoning in problem-solving.

Findings regarding the theme of determining opinion and field knowledge regarding reasoning are given in Table 10. Under this theme, it can be seen that there are results regarding awareness of reasoning, statistical reasoning, flawed reasoning, and mathematical reasoning. When Table 10 is examined, it can be seen that students need to be made aware of how important reasoning is for their mathematical development and that they need to have sufficient experience in reasoning. It has also been determined that secondary school mathematics teachers do not have sufficient knowledge about mathematical reasoning. Despite this, it has been determined that teachers and teacher candidates are sufficient in terms of paying attention to the solutions made by students, interpreting their meanings in mathematics based on the solutions, and giving feedback to students regarding their mistakes. It has been determined that there are deficiencies in teachers' pedagogical content knowledge regarding statistical reasoning, and their level of knowledge about flawed reasoning is insufficient.

Table 10. Determination of opinion and field knowledge regarding reasoning

Theme	Result
Reasoning Awareness	• Students are not aware of the importance of reasoning in terms of mathematical development. Additionally, they do not have sufficient experience in terms of reasoning. • Secondary school mathematics teachers do not have comprehensive and sufficient knowledge and views on mathematical reasoning. • In general, it seems that teachers and teacher candidates are competent in paying attention to student solutions, interpreting students' understanding of mathematics from student solutions, and deciding how to give feedback on student mistakes.
Statistical Reasoning	• There are deficiencies in teachers' pedagogical content knowledge regarding statistical reasoning.
Flawed Reasoning	• Teachers' level of knowledge about flawed reasoning is insufficient.
Mathematical Reasoning	• Pre-service mathematics teachers gave various answers about what mathematical reasoning is. For example, relating the subject to with daily life, realizing the relationship between operations and concepts. • Pre-service mathematics teachers with high levels of Mathematical Thinking have more comprehensive views on reasoning.

It is seen that prospective mathematics teachers try to make sense of mathematical reasoning with expressions such as associating mathematical reasoning with daily life and determining the relationship between operations and concepts. In addition, in studies conducted with mathematics teachers, it was determined that teachers with a higher level of mathematical thinking had more comprehensive and qualified views on reasoning.

Findings regarding the theme of scale development and evaluation process are given in Table 11. When Table 11 is examined, it can be seen that the validity and reliability of the mathematical reasoning self-efficacy beliefs scale and reasoning evaluation table were developed. In addition, the reliability of the existing scale measuring mathematical reasoning performances of 7th-grade secondary school students was compared according to classical and generalizability theories, and it was determined that reliability was achieved according to both theories. Of these two theories, it is seen that the generalizability theory gives more detailed results.

Table 11. Scale development and evaluation process

Theme	Result
Development of the mathematical reasoning self-efficacy beliefs scale.	• It was developed for prospective mathematics teachers, and its validity and reliability have been ensured.
Validity and reliability of the reasoning assessment rubric.	• It was developed based on the mathematical reasoning criteria in the PISA 2021 Mathematics Framework and is sufficiently valid and reliable.
Reliability comparison of the data collection tool according to theories	• The reliability of the scale measuring mathematical reasoning performance for seventh graders was calculated according to classical test theory and generalizability theory. According to both theories, scale reliability is at a sufficient level, and generalizability theory provides more detailed results.

The findings regarding the theme of the effect of the applications on reasoning skills are given in Table 12. When the table is examined, it is seen that there were 33 teaching practices whose effects on reasoning skills were examined in the studies examined. It is seen that each of these practices has a positive effect on reasoning skills and improves reasoning skills.

Table 12. Effect of applications on reasoning skills

Application	Result
• Argumentation Based Learning • Cognitively challenging activities • Individual Teaching • Mathematics teaching is based on the discovery learning approach. • Knowledge framework for teaching algebra • Learning environment that supports algebraic reasoning • Multiple Representation-Based Teaching • Educational Robotic Applications • Differentiated Instruction • Realistic Mathematics Education • IMPROVE model • Teaching with concept cartoons • Coding Training • In-service teacher training and practices in the field of mathematical literacy • History of Mathematics Activities • Mathematical modeling activities • Mathematics activities supported by Montessori materials • Reasoning and Proof activities • Quantitative reasoning teaching approach • Focus Group Studies • Teaching module based on proportional reasoning • Origami Based Teaching • Game-Based Mathematics Teaching • Pattern-Based Mathematics Education Program • Virtual Manipulative Team • Scenario-based learning • STEM • Activities that improve forecasting skills • Technology supported learning • Spatial visualization activities • Hypothetical Learning process • Intelligence games • Enriched Learning environments	• It affects positively.

4. DISCUSSION and CONCLUSION

Reasoning skill is one of the most basic indicators of mathematical competence. This skill is considered extremely important in teaching, learning, and making sense of mathematics. While it is mentioned in the literature that reasoning skills positively affect mathematics achievement, it is necessary to develop reasoning skills in terms of mathematics education and training or to allow teachers to use this skill by giving students enough opportunities. Therefore, it is thought that conducting a metasynthesis study by holistically evaluating the studies on reasoning in mathematics teaching in Turkey will contribute to both the literature and researchers who want to work on reasoning skills in mathematics teaching. A total of 163 studies from theses and articles were included in the scope of the research. These studies were analyzed descriptively according to type, year, method, sample type and size, data collection tools, statistical analysis, learning field, keywords, reasoning type, and purpose. In addition, by combining the results of the studies included in the research with a holistic approach, the differences, and similarities between them were coded and tabulated.

Of the 163 studies within the scope of the research, 91 are master's theses, 29 are doctoral theses, and 43 are articles. In line with expectations, most master's theses were published. The number of theses at the doctoral level is quite low. It is recommended that you write a more qualified and comprehensive doctoral thesis. It is seen that there has been an increasing trend over the years in studies on reasoning skills in mathematics teaching. It is seen that there is a gradual increase in the number of studies after 2005-2014, 2015-2017, and 2018. It is thought that this gradual increase may be due to the updating of the 2013, 2015, and 2018 mathematics curriculum, which started in 2005, and the reasoning skill, one of the 20th-century skills, finding a place in these programs. In addition, despite the increasing trend, the reason for the decrease in studies in some years (2020 and 2021) is the negative effects of the COVID-19 epidemic, which has affected the whole world. The results regarding the publication year and type are parallel to the studies of Akdoğan (2021) and Barak (2022), who examined postgraduate theses on mathematical reasoning.

In terms of sample, most studies were conducted with secondary school students. After secondary school students, most studies were conducted on teacher candidates and teachers. The least studied sample type was parents. These results are parallel to the studies of Akdoğan (2021) and Barak (2022), who examined theses written on mathematical reasoning. Similar results were obtained with meta-synthesis studies on proportional reasoning (Uçar, 2022) and scientific reasoning in science education (Ergün, et al., 2023). According to Piaget's theory of cognitive development, the period in which the transition from the concrete operations period to the abstract operations period will be achieved corresponds to the secondary school level. With the transition to the abstract operations period, children will begin to use mathematical thinking skills. Therefore, it can be said that the use of reasoning skills will be more prominent and active in this period. However, a conclusion can be drawn more easily about the students' reasoning levels. For these reasons, studies on reasoning skills may have been carried out mostly with secondary school students. Additionally, there may be only one explanation for the fact that most studies are conducted on candidate teachers and teachers after secondary school students. It will be the teacher himself who will encourage these students to use reasoning skills and create suitable learning environments to develop these skills. Therefore, it is thought that the teacher or teacher candidate who will do these should have sufficient knowledge of reasoning skills and be able to use reasoning skills at a certain level.

When evaluated in terms of the methods and designs used in the studies, it was concluded that more studies using qualitative methods were conducted. While this result is parallel to the results of Akdoğan's (2021) and Barak's (2022) studies, it does not agree with Uçar's (2022) study. Fewer studies have been conducted with mixed methods compared to quantitative and qualitative methods, and while this result supports Barak's (2022) study, it does not agree with Akdoğan's (2021) study. It is thought that the reason for this is that more studies can be accessed due to the inclusion of articles other than the thesis in our study. When looking at the research designs, the most used quantitative ones were Survey and Experimental studies, while the most used qualitative ones were case studies. It is a striking result that, in general, case studies are the most preferred among the designs, regardless of whether they are quantitative or qualitative. It can be said that conducting more surveys and experimental studies in quantitative research may be prompting researchers to use these patterns, as it is emphasized in the literature how important the reasoning skill is in teaching mathematics, the current status of this skill should be determined, and learning environments should be designed to develop this skill with contemporary approaches. In addition, it is thought that the reason why case study is used more than other designs is due to the fact that it gives the opportunity to examine in depth how this skill is used with the questions "why?", "how?", rather than determining the current situation due to the structure of the reasoning skill. While this result overlaps with some studies in the literature (Akdoğan, 2021; Barak, 2022), it is also possible to come across a study that does not overlap (Ergün et al., 2023).

When the data collection tools used in the research are evaluated, it is concluded that skill tests and interviews are used more. The fact that reasoning is an abstract concept due to its nature and structure encourages researchers to use skill tests and interviews as data collection tools. In addition, it seems that skill tests consist mostly of open-ended questions. It is thought that open-ended problems, which cannot be easily solved, require the use of more than one strategy, and require students to think at a higher level, will better demonstrate reasoning skills. It is emphasized in the literature that open-ended problems are important in terms of reasoning skills in mathematics teaching (Erdem, 2015; Tum, 2019). In addition, it is thought that using interviews as a data collection tool is extremely important in terms of fully concretizing and in-depth examination of reasoning, which is a cognitive skill. In this sense, it can be said that conducting interviews will support the results of the skill tests. It can be said that the results obtained according to the data collection tools are parallel to the studies in the literature (Akdoğan, 2021; Barak, 2022; Uçar, 2022; Ergün et al., 2023).

When the methods and techniques used in the analysis of data are examined, it is seen that parametric tests (t-Test, ANOVA/ANCOVA, and Pearson Correlation Test) are used more than non-parametric tests (Mann Whitney U Test, Wilcoxon Signed Rank Test, and Kruskall Wallis-H Test) in studies where quantitative approaches are adopted. This indicates that the normality assumptions are more met in the studies. Content and descriptive analyses are seen more frequently in studies where qualitative approaches are adopted. It is seen as a striking result that content analysis and descriptive analysis are used more than other methods and techniques. The results regarding the methods and techniques used in data analysis coincide with the study of Ergün et al. (2023).

Studies on the learning fields in the mathematics curriculum are mostly carried out in the field of learning numbers and operations in the secondary school mathematics curriculum. This result is parallel to the studies of Akdoğan (2021) and Barak (2022). After the field of learning numbers and operations, most studies have been carried out in the fields of learning geometry and measurement, algebra, probability, and data analysis, which are also included in the secondary school mathematics curriculum. It is a remarkable result that among all learning fields, there are the least number of studies in the measurement and data analysis learning fields in the primary school mathematics curriculum compared to the others.

When the keywords used in the research are examined, it is seen that the words "Mathematical Reasoning", "Proportional Reasoning" and "Reasoning" are most used. It can be said that the use of these keywords is due to the fact that more studies are conducted in the field of numbers and operations in the secondary school mathematics curriculum and in the field of learning numbers and algebra in the secondary school mathematics curriculum, at each curriculum level, compared to the others. It can be seen that the keywords mentioned above are followed by the words "Problem-Solving", "Mathematics Education", "Quantitative Reasoning", "Mathematical Thinking", "Middle School Students", "Algebra", "Proof" and "Ratio", respectively. Çoban and Tezci (2022), who examined 545 publications published in different journals on mathematical reasoning with bibliometric analysis, revealed that the keywords "mathematical reasoning", "problem-solving", and "mathematical thinking" were used most. This result coincides with our result for keywords.

When evaluated in terms of reasoning types, which are the focus of the studies, almost half of the studies were conducted on mathematical reasoning. After mathematical reasoning, the focus was on proportional reasoning, followed by geometric reasoning, quantitative reasoning, algebraic reasoning, and probabilistic reasoning. Additive Reasoning, Creative Reasoning, Deductive Reasoning, Pedagogical Reasoning, and Strategic Reasoning are the types of reasoning that are least focused on with one study each. Another result is that some of the studies focus on more than one type of reasoning. In general, the results obtained in terms of the most and least focused reasoning type are parallel to Barak's (2022) study. Barak (2022) concluded that there was no study on deductive reasoning in the theses he examined in his study. It can be seen that our study focuses on the deductive

reasoning type. In his study, Mercan (2021) concluded that the theses and articles he examined regarding reasoning skills in early childhood in Turkey focused mostly on general reasoning, followed by mathematical reasoning and auditory/verbal reasoning types, respectively. In the same study, it was determined that the least focused type of reasoning was visual/spatial reasoning. Ersanlı et al. (2018), who examined the theses on reasoning skills in educational sciences, revealed that studies focused on the subject-based sub-dimensions of mathematical reasoning skills, mostly on algebraic reasoning and proportional reasoning. Studies have been carried out at least in terms of geometric reasoning and statistical reasoning. When we look at all these results, there are similarities with our study.

When the aims of the studies included in the research are examined, the most focused ones are examining the factors affecting reasoning skills, measuring reasoning skills, and the effects of teaching practices on reasoning skills. Reasoning skills have been mostly examined according to gender and grade level factors. Reasoning skills were measured by examining twelve types. Among these skills, mathematical reasoning and proportional reasoning skills were measured the most, respectively, compared to the others. Therefore, examination and skill measurement studies on the remaining ten types of reasoning are insufficient. It may be recommended to conduct studies on other types of reasoning on this subject. In terms of the effect of teaching practices on reasoning skills, Technology-supported learning, STEM, enriched learning environments, intelligence games, hypothetical learning proceses, and Improve model were discussed most, respectively. In studies, the relationship between mathematical reasoning skills or other types of reasoning skills and twenty-one variables has been examined. Among these variables, mathematics attitude, self-efficacy perception, mathematics success, problem-solving success and 'the relationship of geometric thinking variables with the focused reasoning skill' were investigated the most. In addition, in this process where the reasoning process was examined, lesson plans were prepared with the arguments used by teachers to reveal reasoning skills, as well as the strategies used in the problems, and studies were carried out to detect reasoning errors. When we look at the studies included in the scope of the research, it can be seen that at least studies on the scale development and evaluation process, studies on determining the relationship between reasoning types, and studies on determining opinions and field knowledge regarding reasoning skills have been carried out, respectively. It is a remarkable result that there are very few scale development studies on a skill that is extremely important for mathematics. The reason for this result is that the researchers created a scale within the study and presented it in the data collection tools section without directly aiming it at the purpose of the study. In addition, it is thought that due to the nature of reasoning, it is due to the greater use of one or more open-ended problems that can reveal the reasoning process. Similar results were obtained with the studies of Ergün et al. (2023), who conducted a meta-synthesis of theses on scientific reasoning, Barak (2022), who examined theses on mathematical reasoning, and Ersanlı et al. (2018), who examined theses in the field of educational sciences.

The results of the studies included in the research were examined under nine themes. When we look at the theme of examining the documents in terms of reasoning, it is seen that approximately one-quarter of the skill-based questions published by the Ministry of Education contain proportional reasoning, most topics are data analysis, and the least topics are about inequality. It seems that mathematics textbooks do not include enough room for reasoning and proof and are insufficient to allow students to develop their own reasoning skills and strategies. Similarly, it has been revealed that learning fields such as data processing and data analysis in the current mathematics curriculum are insufficient in terms of the development of statistical reasoning.

When we look at the results regarding the theme of determining the relationship between different types of reasoning skills, it turns out that mathematical reasoning and proportional reasoning have a highly positive relationship with probabilistic reasoning. There is a moderate positive relationship between algebraic reasoning and spatial reasoning. It has been determined that there is a

weak positive relationship between algebraic reasoning, spatial reasoning, and probabilistic reasoning. If we look at the theme of the relationship between reasoning skills and other variables, the relationship between twenty-one variables such as mathematics achievement, mathematics attitude, self-efficacy perception, geometric thinking, mathematical thinking profile, visual estimation skill, problem-solving success, algebraic thinking, spatial ability, learning styles, reading comprehension, and the focused reasoning skill was examined. It was revealed that reasoning skills had a positive and significant relationship with each of these variables at different levels. When we look at these variables, it can be said that all of them are directly or indirectly related to mathematics teaching. Therefore, nothing could be more natural than that reasoning skills have an important place in understanding and learning mathematics and that their relationship with reasoning skills has a positive relationship with the variables examined (Herbert et al., 2016).

The results of the studies included in the research were examined under nine themes. When we look at the theme of examining documents in terms of reasoning, it is seen that approximately one-quarter of the skill-based questions published by the Ministry of Education contain proportional reasoning and are published mostly on data analysis and least on inequality. It seems that mathematics textbooks do not include enough room for reasoning and proof and are insufficient to allow students to develop their own reasoning skills and strategies. Similarly, it has been revealed that learning fields such as data processing and data analysis in current mathematics curricula are inadequate in terms of the development of statistical reasoning.

When we look at the results regarding the theme of determining the relationship between different types of reasoning skills, it turns out that mathematical reasoning and proportional reasoning have a highly positive relationship with probabilistic reasoning. There is a moderate positive relationship between algebraic reasoning and spatial reasoning. It has been determined that there is a weak positive relationship between algebraic reasoning, spatial reasoning, and probabilistic reasoning. If we look at the relationship between reasoning skills and other variables, The relationship of twenty-one variables such as mathematics achievement, mathematics attitude, self-efficacy perception, geometric thinking, mathematical thinking profile, visual estimation skill, problem-solving success, algebraic thinking, spatial ability, learning styles, reading comprehension, and the focused reasoning skill was examined. It was revealed that reasoning skills had a positive and significant relationship with each of these variables at different levels. When we look at these variables, it can be said that all of them are directly or indirectly related to mathematics teaching. Therefore, it can be said that reasoning skills have an important place in understanding and learning mathematics (Herbert et al., 2016), and nothing could be more natural than the positive relationship between reasoning skills and the variables examined.

When we look at the results regarding the theme of factors affecting reasoning skills, the number of siblings, book reading, school type, and school location factors do not affect mathematical reasoning skills, and the type of mathematics teaching does not affect statistical reasoning skills. It can be said that as the socio-economic level increases, it will be better to make judgments. In terms of academic and mathematical success, it has been revealed that generally, as the level of success increases, the level of reasoning skills also increases. Although most of the studies examining the gender factor in terms of mathematical reasoning showed that gender had no effect, in some of the studies in which its effect was detected, it was observed that girls performed more successfully than boys, and in others, boys performed more successfully than girls. Similarly, the majority of studies show that gender is not effective in probabilistic reasoning and proportional reasoning. Studies indicating that it has an effect show that girls perform better than boys. It has also been shown that gender has no effect on spatial reasoning, geometric reasoning, and algebraic reasoning. As with the gender factor, although the duration of preschool education does not affect mathematical reasoning skills in general, there are a few studies indicating that it does. In the studies examined, a study was

found examining the effect of the COVID-19 factor on mathematical reasoning skills, and this study revealed that COVID-19 had a negative effect on this skill. As it is known, in this global epidemic, education and training were interrupted, especially in Turkey, and education activities were continued with distance education for a long time. It can be said that distance education activities are completely inadequate in eliminating students' learning losses in mathematics lessons. Therefore, we can easily state that a negative impact on the development of students' mathematical skills is an expected result. When the education level of the parents is examined, it is concluded that as the education level increases, their children use higher-level reasoning skills. The mathematical proficiency factor was examined in terms of proportional reasoning and probabilistic reasoning, and it was revealed that it affects both types of reasoning. It has been observed that those with higher mathematical proficiency have better reasoning performances. It has been revealed that the grade level factor affects proportional reasoning and probabilistic reasoning skills but does not affect inductive reasoning skills. It is seen that there are studies on both results in terms of whether the grade level factor affects mathematical reasoning and geometric reasoning skills or not. It has been concluded that the age factor affects mathematical reasoning and that mathematical reasoning develops as age increases. When we look at the results for all factors, we see that only a small number of reasoning types were examined. Therefore, it is recommended to conduct studies on other types of reasoning to fill the gap in the literature in terms of reasoning skills.

In the studies included in the research, 12 types of reasoning were examined, including algebraic reasoning, geometric reasoning, visual reasoning, statistical reasoning, mathematical reasoning, quantitative reasoning, probabilistic reasoning, proportional reasoning, pedagogical reasoning, strategic reasoning, inductive reasoning, and spatial reasoning. In studies conducted in terms of algebraic reasoning and probabilistic reasoning skills, skill levels vary (low, medium, and good). Geometric reasoning skills were determined to be at an intermediate level. The components of putting forward ideas and making inferences are more frequently encountered in geometric reasoning skills. It turns out that flawed and erroneous reasoning is quite common when using geometric reasoning. While using visual reasoning skills, it is seen that stages such as reading the question, making a plan, examining geometric shapes, explaining relationships, showing given mathematical expressions on a visual, expressing conceptual knowledge, adapting information to a visual, showing it with a figure or model, making a proof, and evaluating are followed. In studies examining statistical reasoning, two levels of skills were measured: temporal and quantitative. It is concluded that there are difficulties in explaining ideas, making predictions, and making inferences in questions that require statistical reasoning. In studies examining mathematical reasoning skills, it has been concluded that medium and low-level skills are generally used, and the expected strategies are not used at a sufficient level. It has been revealed that different performances are shown in reasoning processes such as analysis, generalization, and justification in mathematical problems. It has been revealed that students can establish a mathematical relationship between what is given and what is required in the problems and make appropriate and correct decisions to solve the problem. However, in solving non-routine problems, there are difficulties in the sub-steps of reasoning, such as developing arguments about the solution, making generalizations, and determining and using appropriate reasoning. When the studies examining quantitative reasoning are examined, it is stated that there are insufficient reasoning skills and a lack of strong reasoning skills. It has been stated that the reason for this is that students use memorized information and formulas that need to be clarified instead of establishing relationships between quantities in the problems they encounter in daily life and at school. Studies on proportional reasoning skills show that groups are inadequate in using this skill. In studies examining teachers' pedagogical reasoning, it has been determined that they reason by considering how the activities to be chosen for mathematics teaching will affect the learning processes of students and how the teacher will organize his own teaching process. It has been revealed that in strategic reasoning skills, different

solution strategies are used in open-ended problems, and although there is difficulty in determining the quantities and the relationships between them in the context of the problem, convincing explanations are made on algebraic grounds. In studies conducted in terms of inductive reasoning, the level of use of this skill varies (medium and good). In studies examining spatial reasoning skills, it was determined that the reasoning levels of the groups were at a good level.

When looking at the results for the reasoning process theme, results were found on six topics: arguments and lesson plans used to reveal reasoning, the effect of algebraic reasoning applications on misconceptions, global argumentation structure, reasoning errors, problem-posing skills and strategies, and reasoning types used in problems. In order to reveal reasoning, teachers take actions such as asking students for answers, confirming the correct answers, asking for evaluation, and making explanations by giving conceptual information. However, another striking result is that students are given limited opportunities to use their mathematical reasoning. The ideal learning environment for revealing and developing reasoning skills should be such that students interact with each other and can express their thoughts easily (Yankelewitz, et al., 2010). In addition, students will have a more detailed report card for their own learning as their mistakes are analyzed, and feedback is given by their teachers (Kramarski & Zoldan, 2008). The teacher should be a guide in this regard. During this guidance process, effective feedback should be provided that will reveal the quality of learning behaviors, reveal the aspects of the learner's deficiencies, and allow errors or deficiencies to be corrected (Çevikbaş & Argün, 2016). It is known that mathematics teachers do not provide sufficient feedback to make the student realize the mistake he made and correct it during the learning and problem-solving process (Smith & Ragan, 2005). If the lesson should be planned by taking algebraic reasoning indicators into consideration and the learning environment is developed within this framework, it is seen that the reasoning process will be created in a healthy way. It is also stated that prioritizing peer collaboration through computer-aided applications and providing teacher support will have a positive impact on reasoning skills. It was concluded that in geometry courses, mathematics teachers act in a structure that overlaps with Duval's reasoning framework in terms of geometric reasoning. It has been stated that algebraic reasoning applications are effective for some students in eliminating misconceptions. In the study conducted on the global argumentation structure, it was concluded that prospective mathematics teachers had weak mathematical reasoning skills, and therefore used simple global argumentation. Treating special cases as proof and starting with wrong assumptions are errors in reasoning. In addition, in problems that require reasoning, mistakes such as additive relations, neglect of data, using numbers and lack of content, responding emotionally and not being able to determine non-proportional situations are made. It is stated that failure to complete the reasoning process or experiencing conceptual deficiencies and turning to routine solutions to which they are accustomed causes errors in judgment. In terms of problem-posing skills, it is concluded that the problems are not of a qualitative character that requires reasoning but are posed with a quantitative content, original and uncreative, and routine that can be solved with algorithms. When an evaluation is made in terms of the strategies and reasoning types used in the problems, it can be said that the strategies used vary according to the grade level. For example, It has been revealed that while there is no clear strategy in the 6th-grades at the secondary school level, the inside-outside strategy and unit ratio are mostly used in the 7th-grades, and the unit ratio and equivalent fraction strategy are used in the 8th-grades. It is seen that teachers mostly use routine problems depending on the curriculum, and their students mostly use algorithm-based mathematical reasoning. We can say that the education system's dependence on the curriculum and exam-oriented teaching prevents the use of non-routine problems. Therefore, the student will focus more on problems that can be solved with algorithms.

Under the theme of determining opinion and field knowledge regarding reasoning, there are results on four topics: awareness, statistical reasoning, flawed reasoning, and mathematical reasoning. When students' awareness of reasoning skills was evaluated, it was stated that students were not aware

of the importance of reasoning for mathematical development and that students did not have sufficient experience in reasoning. In addition, it stands out that mathematics teachers do not have sufficient knowledge and opinions about mathematical reasoning. In this regard, it can be said that it will be extremely important for teachers to develop correct reasoning and be aware of reasoning in order to provide students with correct reasoning skills (Tum, 2019). In general, teachers and teacher candidates are not competent in interpreting students' understanding of mathematics and deciding how to give feedback to student mistakes, even if student solutions are not taken into consideration. It was stated that they were not sufficient in their pedagogical content knowledge regarding statistical reasoning. Studies examining teachers' level of knowledge about flawed reasoning have concluded that they are at an inadequate level. When the knowledge of pre-service mathematics teachers in terms of mathematical reasoning is examined, it is seen that they give various answers about what mathematical reasoning is, such as associating the subject with daily life and realizing the relationship between operations and concepts. In addition, it was determined that teacher candidates with high levels of mathematical thinking had more comprehensive views.

In the studies conducted on the scale development and evaluation process, results were obtained in three sub-themes: "development of the mathematical reasoning self-efficacy beliefs scale", "validity and reliability of the reasoning evaluation table", and "reliability comparison of the data collection tool according to theories". The mathematical reasoning self-efficacy beliefs scale was developed for prospective mathematics teachers, and its validity and reliability have been ensured. Criteria were determined for the evaluation of reasoning, taking into account the PISA 2021 mathematics framework, and the evaluation table was developed using these criteria, and its validity and reliability were ensured. Reliability calculations of the scale measuring mathematical reasoning performances of seventh-grade secondary school students were made according to two different reliability theories, and it was seen that the generalizability theory gave more detailed results than the classical theory.

When the results regarding the theme of the effect of the applications on reasoning skills were examined, it was seen that 33 teaching applications (enriched learning environments, intelligence games, technology-supported learning, game-based mathematics teaching, mathematical modeling activities, reasoning, and proof activities, teaching with concept cartoons, differentiated teaching, realistic mathematics teaching, etc.) were made. It is seen that each of these practices has a positive effect on reasoning skills and improves reasoning skills. It is stated that in order for reasoning to develop, environments are needed where a constructivist approach is adopted, learner-centered, evaluation is multidimensional, the teacher guides the student, and activities, technological tools, and daily life situations are used to develop reasoning (Garfield & Ben-Zvi, 2009). Creating learning environments that allow students to internalize mathematical activities and provide the opportunity for collaborative work and express their ideas about mathematical activities is considered important in terms of developing reasoning skills (Francisco & Maher, 2005). It is stated that, especially in learning environments, the use of problem situations that will make the student active in the learning process, include technology, and enable the student to make different types of reasoning that will increase their motivation for the course will play an important role in the development of reasoning skills (NCTM, 1989). It is also emphasized that the use of concrete materials in learning environments will allow students to put forward their mathematical ideas, discuss them, and use their reasoning skills (Pham, 2015). In the literature, it is stated that teaching in a way that relates to daily life (Fitriana et al., 2018), examining and discussing solution-related strategies in problem-solving (Naksutthi & Chidmongkol, 2017), and using open-ended problems whose answers cannot be found immediately (Kasmer & Kim, 2011) have an effect on the development of reasoning skills.

In this study, studies on reasoning skills in mathematics teaching in Turkey are discussed and it is suggested that future studies can be conducted on different types of reasoning. In addition, in

terms of the development of reasoning skills, teachers can use such sample applications in classroom environments appropriate to student readiness and the nature of the subject to be taught. As stated before, since there are few studies on some types of reasoning, it is recommended that researchers design studies based on these types of reasoning to close the gaps in the literature.

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APPENDIX-1

Studies Examined in Order of Transfer to Electronic Media

1. Çoban, H. (2019). *The effect of differentiated instructional design on students' mathematical reasoning skills, levels of using metacognitive learning strategies and problem solving skills* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 580371)
2. Eryaman, Z. (2009). *A study on sixth grade students' spatial reasoning regarding 2D representations of 3D objects* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 250710)
3. Belin, M. (2016). *Prospective mathematics teachers' quantitative reasoning on the development of decimal representation of real numbers and its effect on their comprehension of a related Proof* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 459452)
4. Tum, A. (2019). *The effect of enriched learning environments on mathematical reasoning and problem solving attitude in the context of learning styles* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 583517)
5. Aydoğdu, G. (2023). *Investigation of proff processes of secondary school students through argumentation and determination of their reasoning errors* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 782200)
6. Turan-Kurudirek, P. (2023). *Examination of middle school students' quantitative reasoning skills in STEM activities: A teaching experiment* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 794228)
7. Uyguç, A. (2023). *Middle school students' quantitative reasoning in pictorial, symbolic and iconographic problems* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 778157)
8. Alphayta, B. (2022). *Examining the relationship between geometric reasoning skills and self-efficacy perceptions of 8th grade students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 715667)
9. Akdemir, M. (2022).*Investigation of secondary school students' geometrical reasoning on the polygons* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 753433)
10. Baki, Ü. (2022). *Reflection of game-based mathematics teaching on 6th grade students' reasoning skills* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 743558)
11. Özaydın, Z. (2022). *Evaluation of mathematical reasoning competence in the process of in-service teacher training and practices of mathematical literacy given via distance Education* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 738691)
12. Özkırış, M. (2022). *Reasoning skills of fifth grade students in scenario based learning environment* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 743519)
13. Öztürk, V. (2022). *Prediction of the effect game-based teaching lessons on the mathematical reasoning process of 5th grade students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 739343)
14. Çınargil, T. (2022). *Investigation of mathematical reasoning skills of gifted students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 749287)
15. Arpacı, Y. (2022). *Mathematical problems of mind and intelligence games examining the realitionship to reasoning with teacher's view* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 710217)
16. Kaya, C. (2022). *Examination of the relationship between seventh grade students' mathematical reasoning and self-efficacy for mathematical reasoning perception* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 764765)
17. Adıgüzel-Doğan, F. (2021). *Analyzing ninth-grade students' algebraic thinking in geometry: An application in triangles sub-learning domain* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 673596)
18. Akdoğan, E. (2021). *Investigation of postgraduate theses on mathematical thinking and mathematical reasoning in Turkey* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 684521)

19. Kızıltoprak, A. (2020). *Development of middle school students' geometric reasoning in the context of quadrilaterals* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 647265)
20. Erol, G. (2019). *Investigation of inductive reasoning skills of secondary school students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 553046)
21. Üstün, A. (2019). *Investigating 5th grade students' reasoning skills on the topic of fractions* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 555971)
22. Koçyiğit, Ş. (2019). *An analysis of mathematical reasoning, attitudes towards mathematics and self-efficacy of students in STEM-oriented teaching processes* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 594419)
23. Tutan, S. (2019). *The examination of middle school matematics teachers' geometry-based courses in the context of geometric reasoning processes* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 567525)
24. Mutluoğlu, A. (2019). *The effects of virtual manipulatives, developed for 6th grade mathematics lesson in geometry and meausurement learning area, on students' academic achievement, attitudes towards geometry, and geometrical reasoning processes* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 559767)
25. Kara-Çalışkan, A. L. (2019). *Examination of mathematical reasoning skills of 7 th and 8 th grade students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 584017)
26. Özdemir, F. (2019). *Investigation of reasoning and metacognitive development of high school students on the subject of limit and continuity with model of Improve process* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 580945)
27. Güvendiren, G. N. (2019). *Investigation of the algebraic thinking of sixth grade students with three parameters: The quantitave reasoning, the covariational thinking and the functional thinking* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 579098)
28. Çokyaşa, M. Ç. (2019). *Investigation of the pre-service mathematics teachers' processes of mathematical reasoning and proving in the context of reasoning-proving framework and metacognition* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 608906)
29. Yöndemli, E. N. (2018). *The effect of intelligence games (Strategy and geometry) on secondary school students' mathematical reasoning skills and effort shown in mathematic lesson* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 532119)
30. Demir, E. (2017). *Investigation of pre-service secondary mathematics teachers' reasoning errors reasoning deficiences and reasoning gaps within the context of Proof* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 472004)
31. Selçuk, A. S. (2016). *The effects of technology supported measures of central tendency and variability activities on 9th grade students' informal inferential reasoning* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 435378)
32. Akın, A. (2016). *An analysis of supporting middle school students' mathematical literacy through strengthening their quantitative reasoning* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 449976)
33. Erdem, E. (2015). *The effect of enriched learning environment on mathematical reasoning and attitude* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 381651)
34. Bağcı, V. (2015). *The comparison of different designs in generalizability theory with Classical Test Theory in the measurement of mathematical reasoning* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 429476)
35. Kaya, D. (2015). *A study on the effects of multiple representations-based instruction on students' algebraic reasoning skills, algebraic thinking levels and attitudes towards mathematics* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 395240)
36. Erdem, E. (2011). *An investigation of the seventh grade students? mathematical and probabilistic reasoning skills* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 301094)

37. Bike-Kalkan, D. (2014). *Eighth grade students' ways of conceptual understanding and algebraic reasoning* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 361721)
38. Çoban, H. (2010). *Investigating the relationship between the level of students? using mathematical reasoning skills and using metacognitive learning strategies* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 258052)
39. Albayrak-Bahtiyari, Ö. (2010). *The awareness in proof and reasoning concepts and their importances in mathematics education of the eight class* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 266577)
40. Pilten, P. (2008). *The effect of metacognitive instruction on mathematical reasoning of fifth grade primary school students* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 214521)
41. Arslan, Ç. (2007). *The development of elementary school students on their reasoning and proof ideas* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 210145)
42. Uçar, R. (2022). *A systematic reviewing study on proportional reasoning* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 772882)
43. Demir, A. (2022). *Examining a mathematics teacher's moves for supporting students' proportional reasoning skills* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 754387)
44. Adıyaman, D. (2019). *Reflections from learning environment supporting algebraic reasoning skills of eighth grade students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 584629)
45. Aksu, N. (2022). *Investigation of seventh grade students' mathematical reasoning on polygons through Geogebra enhanced Acodesa method* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 766292)
46. Aladağ, A. (2009). *The examination of secondary school students? mathematical word problem solving abilities that depend on proportional reasoning and that require actual answer* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 241511)
47. Altaylı, D. (2012). *The effect of realistic mathematics education on teaching the subject of ratio and proportion and development of proportional reasoning skills* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 325796)
48. Angay, M. (2022). *The effect of the mathematics course taught with logical reasoning methods on the success of students in skill-based questions and student opinions on teaching* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 742432)
49. Arıcı, S. (2012). *The effect of origami-based instruction on spatial visualization, geometry achievement and geometric reasoning of tenth-grade students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 301704)
50. Atay, H. (2022). *Examination of 7th grade coursebooks in mathematics teaching in terms of logical reasoning* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 771597)
51. Ayan-Civak, R. (2020). *The evolution of mathematical practices in a seventh-grade classroom: Analyzing students' development of proportional reasoning* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 625104)
52. Bostancı, Ü. (2019). *The investigation of the relationship between eighth-grade students'geometryself-efficacy perceptions and geometric reasoning skills* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 538851)
53. Boyacı, H. S. (2019). *The proportional reasoning ability of preservice mathematics teachers: A mixed method study* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 582283)
54. Bursalı, G. G. (2019). *Investigation of Probabilistic Reasoning Levels of Secondary School Students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 570871)
55. Somuncu, B. (2021). *The effect of coding training on the mathematical reasoning skills of preschool children* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 657623)
56. Çetiner, S. (2022). *The effect of concept cartoons the proportional reasoning skills of 7th grade students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 736424)

57. Çatak, M. (2011). *An elementary mathematics teacher's pedagogical reasoning in selecting learning activities* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 286177)
58. Çelik, A. (2010). *The relationship between elementary school students? proportional reasoning skills and problem posing skills* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 265321)
59. Çetin, Ş.G. (2022). *Investigating the relationship between eighth grade students'proportional reasoning and Van Hiele geometric thinking levels* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 770515)
60. Çiftci, Z. (2015). *Investigation of the mathematical reasoning skills of pre-service mathematics teachers* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 418254)
61. Çilingir-Altıner, E. (2018). *Examination of 4th grade students' visual estimation - spatial reasoning skills and problem-solving performances based on mathematical thinking profiles* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 534185)
62. Çırakoğlu, T. (2020). *The effect of algebraic reasoning applications on misconceptions and mistakes in addition and subtraction* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 627887)
63. Çomruk, B. (2018). *Analysis of proportional reasoning strateiges of rural middle school students in material-assisted problem solving process* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 514476)
64. Dayan, M. (2021). *Analysis of data processing in mathematics lesson curriculums in terms of statistical reasoning* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 671897)
65. Demirtaş, T. M. (2022). *Examination of inductive reasoning process of secondary mathematics teacher candidates* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 713522)
66. Dinçer, B. (2019). *Development of algebraic reasoning in linear equations of 7th grade students with their educational robotic implementations: A teaching experiment* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 569512)
67. Duran, V. (2019). *Investigations of reasoning styles, cognitive distortions and critical thinking tendencies of teacher candidates (Ondokuz Mayıs University Faculty of Education sample)* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 537005)
68. Öztürk, E. (2023). *Investigation of geometry problems posed by middle school students for situations requiring proportional reasoning* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 787413)
69. Öz, E. (2020). *Investigation of high school students' proportional reasoning ability* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 646636)
70. Öztürk, F. (2023). *Noticing of matematics teachers and preservice teachers to the proportional reasoning strategies used by students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 790368)
71. Geçici, M. E. (2022). *Investigation of middle school students' visual reasoning skills in geometry problems* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 760519)
72. Gergin, G. (2022). *Investigating 8th grade students' strategic reasoning in algebra* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 711972)
73. Gök-Çolak, F. (2016). *The effect of pattern based mathematic education programme on 61-72 months old children's reasoning skills* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 429558)
74. Gökce, R. (2019). *Examining middle school mathematics teachers' content and pedagogical content knowledge of statistical reasoning* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 566542)
75. Göncü, H. B. (2020). *The effect of improving prediction skills on reasoning and intuitional mathematics abilities of children from 60 to 72 months* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 607937)

76. Gülay, O. (2020). *Examining the types of reasoning of different types of problems about measuring lengths of 4th grade student in primary school* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 621188)
77. Güler-Baran, H. (2023). *Investigation of mathematical reasoning processes of middle school students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 779335)
78. Gülşen, İ. (2012). *Investigation of the visual reasoning process of the pre-service mathematics teachers* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 317054)
79. Gündoğdu, N. S. (2021). *Examining the development of the proportional reasoning skills of 7th grade students with FeTeMM activities* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 685773)
80. Güneş, M. (2022). *Preservice middle school mathematics teachers' proportional reasoning knowledge resource using skills in measuring the areas of rectangles* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 768359)
81. Gürler, A. (2019). *İnvestigation of the middle school students development of the proportional reasoning in the hypothetical learning trajectory* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 546138)
82. Hançer, D. N. (2021). *An investigation of middle school students' tendency for additive and multiplicative reasoning according to different variables* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 685874)
83. İlhan, A. (2019). *Investigation of the relationship of elementry mathematics teacher candidates' between visual mathematics literacy perception level, reasoning skill on geometry and performance levels* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 548400)
84. Kanbur-Tekerek, B. (2023). *Investigation of reasoning skills of gifted primary students through geometry problems* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 785487)
85. Karacaköylü, A. (2021). *4, 5, 6, 7 and 8th grade students' understanding in the solution of proportional and non-proportional reasoning problems* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 730337)
86. Karaduman, B. (2018). *Investigation of proportional reasoning skills and attitudes towards mathematics lecture of middle school 6th, 7th, and 8th graduate students on some variables: Gender and class level perspective* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 534992)
87. Karatoprak, R. (2014). *Assessing preservice mathematics teachers' statistical reasoning* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 371783)
88. Karslı, N. (2016). *The effects of discovery - based mathematics education on 8th grade students' reasoning and connecting skills* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 414633)
89. Kazez, H. (2015). *The effects of the use of lego moretomath instructional tool on the 2nd graders' math problem solving, fluency, understanding and reasoning skills: The case study* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 423422)
90. Kocaman, M. (2017). *Investigation of mathematical thinking and the reasoning skills of the 11th grade students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 456169)
91. Küpçü, A. R. (2008*The effect of activity based teaching approach over the proportional reasoning related problem solving achievement* (Doctoral Dissertation).Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number:226373)
92. Öztürk, L. (2019). *Investigation of informal inferential reasoning processes of 9th grade students about the data groups* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 560871)
93. Öztürk, L. (2022). *An examination of secondary school students' mathematical reasoning about triangles in the GeoGebra supported learning environment designed with the ACODESA method* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 791159)
94. Nacar, S. (2022). *Investigation of proportional reasoning changes of teacher candidates as a result of application of the developed STEM education modüle*(Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 750936)

95. Nemutlu-İnanır, Ş. (2019). *Examining of highly gifted student's proportional reasoning skill* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 582917)
96. Ön-Hallumoğlu, K. (2019). *The effect of montessori materials supported individual and cooperative math activities on early mathematical reasoning skills* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 548998)
97. Özbay, S. (2012).*Students' thinking processes and reasoning about sample during informal inferential reasoning* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 325765)
98. Özen-Yılmaz, G. (2019). *Investigating middle school students' achievement and strategies in proportional reasoning problems* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 590716)
99. Öztürk-Zora, L. (2019). *The investigation of the statistical reasoning levels of 8th grade students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 557444)
100. Pakmak, G. S. (2014). *An İnvestigation about the items on required quantitive and qualitative proportional reasoning skills on understanding of 6th grade students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 375696)
101. Pay, G. (2018). *The investigation of mathematical reasoning skills of preschool children* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 525734)
102. Pelen, M. S. (2014). *The examination of 6th grade students' proportional reasoning with respect to problem classification and problem number structures* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 370848)
103. Pişkin-Tunç, M. (2016). *Pre-service middle school mathematics teachers' proportional reasoning before and after a practice-based instructional modüle* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 439216)
104. Sarıbaş, H. M. (2019). *Investigating 6th, 7th and 8th grade students' probabilistic reasoning levels in terms of gender, grade level and mathematics achievement* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 547859)
105. Sırtmaç, G. (2018). *An investigation of geometric reasoning skills of eighth grade students* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 512962)
106. Sivri, Y. E. (2022).*The effect of the application process of a mathematics–based STEM module on the development of seventh grade students' proportional reasoning skills* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 769961)
107. Somuncu, B. (2022). *Investigating the effects of reasoning games on middle school students' learning levels and mathematical reasoning abilities about linear equations* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 753139)
108. Şahin, Y. (2012). *An investigation on geometric reasoning of pre-service elementary mathematics teachers in terms of some variables* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 317786)
109. Şahinkaya, T. (2022). *Investigation of the efficiency of metacognitive IMPROVE technique in teaching ratio-proportion and developing proportional reasoning skills* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 738320)
110. Tokgöz, M. (2022). *Enhancing 7th grade students' mathematical reasoning throught cognitively demanding tasks* (Master's Thesis). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 711815)
111. Öz, T. (2017). *The examination of mathematical reasoning processes of the 7 th grade students* (Doctoral Dissertation). Retrieved from the Thesis Center of the Higher Education Institution. (Thesis Number: 475981)
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