Symposium: Big Ideas in School Mathematics

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The "Big Ideas in School Mathematics" (BISM) is a Research Project funded by the Ministry of Education, Singapore, and administered through the Office of Educational Research, National Institute of Education, Nanyang Technological University. The project began in 2020 and its aim is to investigate various areas in relation to teaching towards mathematical Big Ideas in Singapore schools. The study has currency in so far as "Big Ideas" were introduced in the latest Syllabus Revision by the Ministry of Education. There are three sub-studies in the project: the first is on the development of instruments to measure knowledge of BISM among primary- and secondary-level students and teachers; the second is on professional development work for secondary-level teachers on BISM; the third is similar to the second but for primary-level teachers. The papers in this symposium report information and findings on all these sub-studies.

Overview of the Symposium Papers and Presenters

Presenters: Associate Professor Leong Yew Hoong (Chair), Associate Professor Toh Tin Lam (Paper 1), Mr Mohamed Jahabar Jahangeer (Paper 2), Assistant Professor Choy Ban Heng (Paper 3), Professor Berinderjeet Kaur (Paper 4)

Paper 1: Overview of the research project on Big Ideas in School Mathematics

Authors: Toh Tin Lam, Tay Eng Guan, Berinderjeet Kaur, Leong Yew Hoong, Tong Cherng Luen

This paper provides a brief overview of the entire research project and the component sub-studies.

Paper 2: Assessment of Big Ideas in School Mathematics: Exploring an Aggregated Approach

Authors: Mohamed Jahabar Jahangeer, Toh Tin Lam, Tay Eng Guan, Tong Cherng Luen

This paper reports on developments under Sub-study 1. An item from the student BISM instrument will be discussed. It argues for the use of an "aggregated approach" in considering the scores of the student responses.

Paper 3: From Inert Knowledge to Usable Knowledge: Noticing Affordances in Tasks Used for Teaching Towards Big Ideas About Proportionality

Authors: Choy Ban Heng, Yeo Boon Wooi Joseph, Leong Yew Hoong

This paper reports on developments under Sub-study 2. Part of the professional development under this project involved teachers designing their own instructional materials to foreground a targeted Big Idea. Snippets of tasks in these instructional materials will be discussed.

Paper 4: Primary School Teachers Solving Mathematical Tasks Involving the Big Idea of Equivalence

Authors: Berinderjeet Kaur, Tong Cherng Luen, Mohamed Jahabar Jahangeer

This paper reports on developments under Sub-study 3. An item from the teacher BISM instrument will be discussed. Some data on teachers' responses to the item will be shared. There are thus implications to teacher professional development on the Big Idea of Equivalence.

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From Inert Knowledge to Usable Knowledge: Noticing Affordances in Tasks Used for Teaching Towards Big Ideas About Proportionality

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Teaching towards big ideas provide opportunities for teachers to think deeply about content and pedagogy in order to support their students to see connections in mathematics. However, teachers may not always activate or mobilise their knowledge in classroom situations. This paper looks into how a teacher, Peter, think about the tasks in his instructional materials he crafted to uncover what he may notice about the affordances of the tasks for teaching proportionality.

Teaching towards big ideas, a recent initiative included in the 2020 Singapore Mathematics Syllabus (Ministry of Education-Singapore, 2019), provides opportunities for teachers to think more deeply about what and how they teach in order to support their students to see connections in mathematics (Choy, 2019). Doing this requires teachers to pay attention to the mathematics embedded in the curriculum, discern the details of the big ideas, and perceive the affordances in tasks for bringing out these ideas (Choy, 2019). A key enabler is the mathematical knowledge for teaching (Ball et al., 2008) that teachers can activate during classroom instruction. This suggests a key distinction between inert knowledge (Renkl et al., 2010) and usable knowledge, or what they mobilise during teaching. Kersting et al. (2012) hypothesized that "teachers with more usable knowledge are able to apply that knowledge to the design and improvement of instruction in their classrooms" (p. 573). Furthermore, as Choy and Dindyal (2021) had pointed out, it is not trivial for teachers to notice the affordances of tasks and harness them to improve instruction. Here, we explore how teachers can be supported, through professional learning (PL) sessions, to transform their inert knowledge into usable knowledge through the discussion and design of instructional materials. This paper is guided by the following research question:

- How does a PL session that focuses on the design of instructional materials activate his inert knowledge of a big idea in mathematics?

Contexts and Methods

The six teacher participants in the study reported here is part of a larger project on "Big Ideas in School Mathematics", which focused on the notion of teaching towards big ideas in Singapore. These six teachers participated in a series of professional learning (PL) sessions to unpack big ideas about proportionality (Yeo, 2019) so that they can design instructional materials and lessons for teaching the topic of ratio and rates in Secondary One. In the first session, the second author discussed the idea of proportionality from a few perspectives: when one quantity is multiplied by *n*, the other quantity is also multiplied by *n* (which we will call proportional reasoning), the equality of two ratios (e.g. $\frac{y_2}{y_1} = \frac{x_2}{x_1}$ for direct proportion), the rate $\frac{y}{x}$ is constant for direct proportion, and the product xy is constant for inverse proportion. Two main approaches to solving problems involving proportionality were shared: proportional reasoning via the unitary method and using the constant rate $\frac{y}{x}$ directly. In the next two sessions, the second and third authors facilitated discussions on the use of these two approaches, as well as others (Weinberg, 2002), to solve problems involving constant rates and supported the teachers in thinking about the design of instructional materials to incorporate proportionality in questions involving ratio, percentage, currency exchange and speed. Of interest in this paper is the instructional material shared by Peter (pseudonym), one of the teachers, during the fourth PL session, which was facilitated by the first author. Data collected include video and voice recordings of the PL session, and the draft instructional material designed by Peter. For this paper, the findings were generated from Peter's sharing on his thinking behind the design of the instructional material used for teaching rate, as well as the interactions between him and the other teachers in the PL session. Analyses were guided by the following questions:

- What knowledge on proportionality did Peter utilise in his design?
- What inert knowledge on proportionality did Peter activate during the PL session?

Three Short Snippets of Peter's Thinking

In this section, we begin by describing three short snippets of Peter's thinking, juxtaposed with what the other teachers said in response to the questions or prompts by the first author (BH). We then unpack Peter's thinking behind his design or choice of tasks put into instructional material before we characterise his understanding of proportionality in terms of what he knew inertly (Renkl et al., 2010) and what he was able to access and use—usable knowledge (Kersting et al., 2012)—through his interactions during the PL session.

Snippet 1: Shampoo Investigation Task

Peter began by describing the investigation task he placed at the beginning of the instructional material (see Figure 1). He had wanted the students to rely on their intuition and explain how they solve the problem *before* teaching them about the concept of rate.

Investigation!

- Which bottle of shampoo is more **value for money** (cheaper for the same volume of shampoo)?
- How do you determine that?

Bottle A	Bottle B
	
400 mL: $5.00	500 mL: $6.00

Figure 1. Shampoo problem.

When asked about how students might respond to the task, Peter responded that "some of them might choose to ignore the idea of same volume and just superficially choose the cheapest" [38:26]. Teacher M then shared that "they would use the unitary method" to obtain the cost of shampoo for 100 mL and subsequently 1 mL [38:52]. With more prompting, Peter highlighted that students could "change the volume to 2 litres" [39: 47] and compare. Teacher N also offered a similar size-change strategy (Weinberg, 2002) by changing the price to $30. Building on this discussion, the first author highlighted that these different methods (without using rate explicitly) were all based on the overarching idea of proportionality.

Snippet 2: Fastest Typist Problem

After the investigative task, Peter defined rate as "a quantity per (one) unit of another quantity" and selected a series of tasks, meant for students to compute rates in his instructional material. One such task is given as follows: Jayden can type 720 words in 6 minutes, Ithiel can type 828 words in 18 minutes and Zhi Rui can type 798 words in 19 minutes. Who is the fastest typist?

Peter had intended the task to be used merely for computation. At this juncture, the first author highlighted the possibility of "looking more closely at the numbers used" and modify the numbers to bring out the idea of proportionality more explicitly. The first author suggested Peter to consider how the numbers can be changed to provide opportunities for students to exercise their proportional reasoning. In addition, he highlighted to Peter that the current set of numbers did not require students to do deliberate calculation using "proportional reasoning"; instead, students would just need to mentally estimate that Jayden has to be the fastest typist because he could type around 700 words within 6 minutes, as compared to what the other two could type in a much longer time (18 or 19 minutes). Of course, students could have multiplied 720 by 3 (proportional reasoning) to compare Jayden's typing speed against the other two. Through the discussion, Peter became aware of how the item could be used to emphasise different aspects of proportionality.

Snippet 3: Exchange Rate Problem

The rest of the instructional material focused on providing opportunities for students to calculate per (one) unit rates instead of looking out for opportunities to highlight the "power of proportionality" to make sense of comparisons between two quantities. For instance, Peter went through Example 2 (See Figure 2) as merely computational without noticing the alternative solution to part (b) of the question. When the first author prompted the teachers to look more closely at the answer to part (b), Peter realised that students could simply divide 1256 SGD (given in the stem of the question) by 10 using the idea of proportionality.

Example 2

On 13 June 2018, Cheryl exchanged 800 Euros (EUR) for 1256 Singapore dollars (SGD).

(a) Find the exchange rate, correct to 4 significant figures if it is not exact, between Euros and Singapore dollars in

(i) SGD/EUR, **(ii)** EUR/SGD.

(b) Cheryl spent 80 EUR on some gifts for her family. Find the price of the gifts in SGD.

Solution

(a) (i) 800 EUR = 1256 SGD $\div 800$

1 EUR = $\frac{1256}{800}$ SGD

= 1.570 SGD

∴ the exchange rate is 1.57 SGD/EUR.

(ii) 1256 SGD = 800 EUR $\div 1256$

1 SGD = $\frac{800}{1256}$ EUR

= 0.6369 EUR (correct to 4 s.f.)

∴ the exchange rate is 0.6369 EUR/SGD.

(b) Price of the gifts = 80 EUR

= 80 × 1.57 SGD

= 125.60 SGD

Figure 2. Exchange rate problem.

Discussion

Taken together, the three snippets detailed in this short paper suggest that while Peter and the other teachers were aware of the ideas of proportionality (as seen in Snippet 1), he might not always be able to *notice* these ideas and *harness the affordances* of the tasks embedded in the instructional materials he had designed (Choy & Dindyal, 2021). As seen from the three snippets, he was able to articulate the knowledge about teaching proportionality, especially the idea of providing opportunities for students to reason proportionally using different solution strategies (Weinberg, 2002). Yet, he did not always notice affordances of these tasks to bring out the idea of proportionality and instead focused on emphasising a fixed way of finding rate and solving missing value questions. In other words, it is not trivial for teachers to activate their inert knowledge about teaching proportionality to generate usable knowledge that can potentially enhance students' understanding of this big idea when designing instruction materials. What matters is not simply what the teachers know, but how they can learn to mobilise their knowledge in actual classroom situations (Ball et al., 2008; Kersting et al., 2012).

These snippets not only highlight the complex and perennial issue of knowledge activation in the act of teaching but also provide insights into how professional learning activities can be structured to support teachers to bridge the gap between their knowledge and classroom practice. First, such professional learning can be structured around discussion of lessons and more specifically, the design of instructional materials. Designing lesson materials provide an avenue for teachers to transform their knowledge into something usable, and hence enhance the possibility of them mobilising their inert knowledge. Second, we see the need for teachers to learn to notice affordances for using tasks and other instructional materials because doing this provides opportunities for teachers to generate new possibilities that can potentially change practices. Lastly, the role of a knowledge facilitator to support teachers to notice new possibilities in their design of instructional materials, in the context of professional learning sessions, should not be under-estimated. How such sessions could be facilitated remains under-studied and could be a fruitful area for future research.

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