# Lower Secondary Students Using Mathematical Modeling for Managing Water Consumption: The Case of Asma

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**Abstract**: The main purpose of the current study is modifying students' daily drinking water and managing waste water through modelling activities as context and modelling cycle as tool. In this study, 12 students (13-14 years old) from the lower secondary school were participated. These students were participated in two sessions (60 minutes) each week for three months. Three modelling activities had been designed for this study. These modelling activities were related to the amount of water consumption. Classroom activities were video recorded and used as data in this study. In some cases, interviews conducted and used as complementary data. Tension which happened during the implementation of these two activities analyzed with activity system as a theoretical framework. The results of this study show that students were able to manage and modify their daily water consumption through doing modelling activities. Results of this study also show that power of mathematics could help students to solve their real-world problems and help them to have better life.

**Keywords:** Activity Theory, Modelling Cycle, Modeling Activity, Tension, Water Consumption.

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## Introduction

In the twenty-first century, climate changes have been happening in many parts of the world (Lowe, Lynch & Lowe 2014). These changes can make many problems including water shortage crises in many countries. For example, in Iran which have around 85 million populations, there is a crisis of water shortage (Madani Larijani, 2005). Therefore, the management of the water consumption is an essential issue for some countries. Modifying the consumption pattern is one of the water management strategies which appear in literature. Therefore, the modifying of water consumption pattern is an important issue in the real world that how doing it by using mathematical knowledge is very important. Hence, students need to have skills that by using them can to solve the problems of the real world. In this regard, the many countries including Germany and China, their main goals of mathematics education are based on skills development about the relationship between real-world and mathematics world. For example, in the curriculum of China this is referred explicitly the issue which students " *experience relationships between mathematics and real-world* " (Ludwig & Xu, 2010, p. 78). However, because of the difference of two contexts of the real world and mathematics world, the connection between them is not possible (Bonotto, 2007). The studies have attributed the main of the cause of disconnection between them to the gap between the two worlds that have made mathematical learning problems (Gravemeijer, 1999). This gap has been made that students could not use their school mathematics in the out-of-school situation. In this way, Lave (1988, cited in Boaler, 1998) stated that students are not capable use learned- mathematics of school in shopping situations. Therefore, the filling of the gap between the two worlds is an important goal that will follow in this study. A review of the research literature on the real world and the mathematical world shows that connection between the real world and the mathematical world is possible under certain conditions. Since mathematics is a human activity (Freudenthal, 1991), studies show that if human activities rely on everyday experience and mathematics, one can establish a connection between the real world and the mathematical world. For example, Civil (2002), in his study developed activities that relied on student experience and everyday mathematics. By using these activities and mathematizing the real situation, he was able to the connection between the real world and the mathematical world. Hence, the mathematization process called mathematical modelling in the mathematical education domain (Reusser & Stebler, 1997, Cited in Bonotto, 2004). Mathematical modelling can be used as a tool to connect the real world and the mathematical world (Freudenthal, 1991). In general, the mathematical modelling is a reciprocal process in the mathematics education that links the real world and the mathematical world (Stillman, 2010; Garcia, Gascón, Higueras, & Bosch, 2006).

However, it is necessary to look at the importance of mathematical modelling in mathematics education until to understand its role in the relationship between the real world and the mathematical world. Many researchers from different countries have mentioned in their studies that mathematical modelling is one of the main topics in the school mathematics curriculum, for example, Borromeo Ferri (2108) says that *"The teaching and learning of mathematical modelling has become a key competency within school curricula and educational standards in many countries of the world"* (p. ix). At this time, many countries in the world that considered modelling as one of the major topics in mathematics curriculum (e.g. US (Doerr & Lesh, 2011), Queensland and Singapore (Stillman, 2010), the South African Curriculum (Knott, 2014), China (Ludwig & Xu, 2010), Brazilian curriculum (Biembengut & Faria, 2011), Venezuelan curriculum (Ortiz & Santoz, 2011), New Zealand (Schaap, Vos & Goedhart, 2011), Japan (Matsuzaki, 2011), Germany (Bracke & Geiger, 2011) and *** National Curriculum (2011)). In addition to countries' curricula focusing on modelling, international assessments have also focused on mathematical modelling, for example, it can be pointed out the results of Pisa 2006 Cited. The results of Pisa 2006 (OECD, 2007) show that students all over the world have difficulty in modelling tasks. Another challenge about modelling is that teaching, according to Niss (2007, cited in Stillman & Galbraith, 2011), is one of the challenges of curriculum teaching of applications and modelling in school mathematics education.

In addition to mathematical modelling, which has an aspect of integrating, the researchers have pointed to the use of cultural artefacts in modelling activities. Among them are researchers such as Alsina (2007); Bonotto (2004) who in their study showed that the use of cultural artefacts is a prominent feature of modelling activities. Specifically, Bonotto (2007) points out in her study a variety of cultural artefacts such as bills of shopping, bottles, labels, etc. Because of these artefacts are as part of the everyday life experience of students, she considers them meaningful for students. According to Bonotto (2004) by using *"cultural artefacts" can "creating a new tension between school mathematics, and the students' everyday-life knowledge*." (p.315). Therefore, by using cultural constructs can create tension for students in the real world. So, these tension are the disagreements, dichotomies, problems, and concerns that arise concerning a subject for a person. The cultural artefacts used in this study are related to water consumption that creates tension for students. In fact, by using water consumption-based modelling activities can be creating new tension for students, which for analyze of them use the theory of Cultural-historical activity theory. In the activity theory, human activity is investigated on the base of the activity system that is consisted of six components. This activity system is shown in Figure 1.

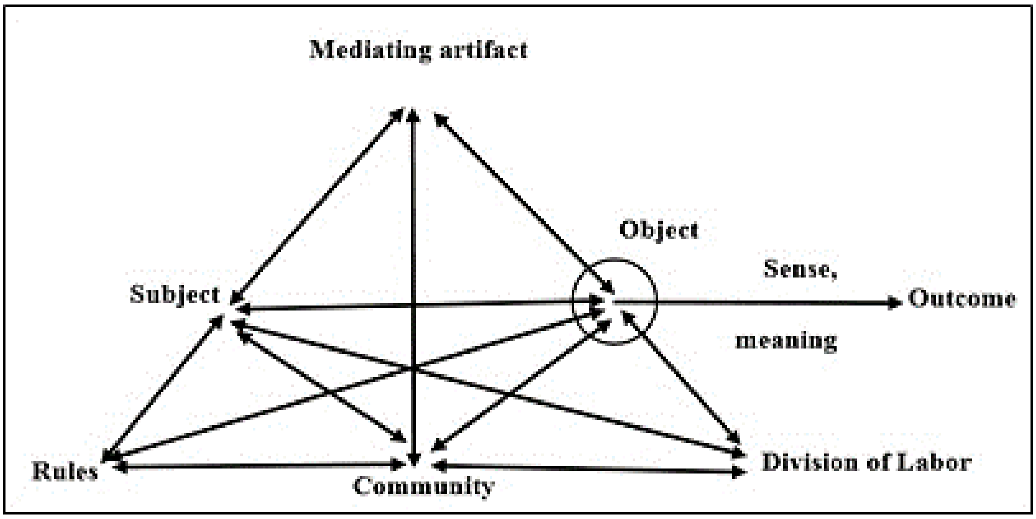


Figure 1. Structure of the Human Activity System (Engeström, 1987, p.6 cited in Engeström & Sannino, 2010)

> In Figure 1, *the subject refers to the individual or subgroup whose position and point of view are chosen as the perspective of the analysis. Object refers to the 'raw material' or 'problem space' at which the activity is directed. The object is turned into outcomes with the help of instruments, that is, tools and signs. The community comprises individuals and subgroups who share the same general object. Division of labor refers to the horizontal division of tasks and vertical division of power and status. Finally, rules refer to the explicit and implicit regulations, norms, conventions and standards that constrain actions within the activity system. The circle around the object in Figure 3 indicates the purpose of the system differs from the personal purpose. (Engeström and Sannino, 2010, p. 6).*

The researchers consider the activity system of activity theory as the unit of analysis (Jonassen, 2000) which can be used to analyze the tension that occur in the activity system. However, the activity system that this study needs to use to define modelling activity systems is an activity system where elements are defined in the real world. In this regard, Jurdak (2006) uses Engstrom's activity system to present systems for problem-solving activities. He introduced the activity system of problem-solving in three different context which is included school context, simulated real world context and finally real-world context (see Figurre 2).

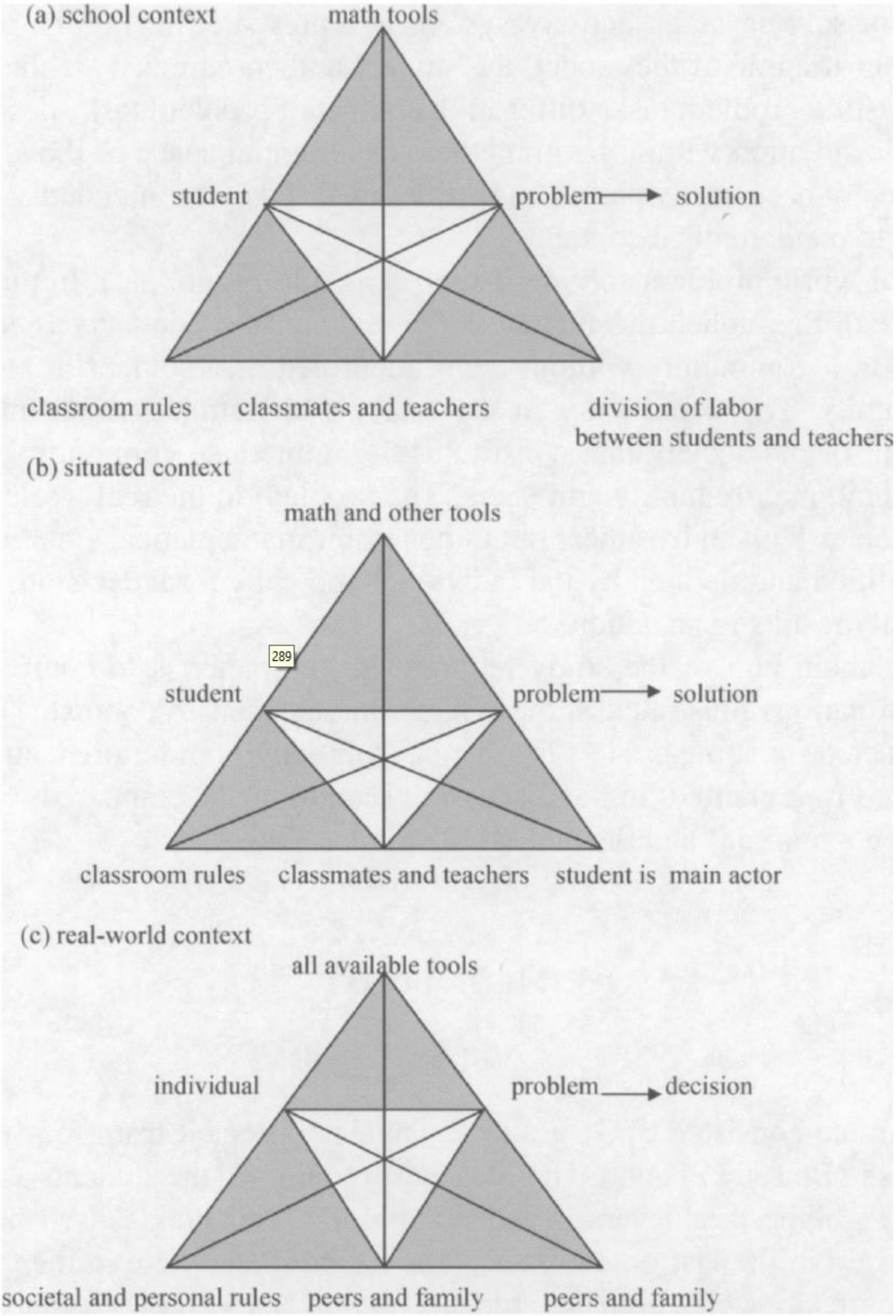


Figure 2. Problem Solving Activity System in (a) School Context, (b) Simulated Real-world Context and (c) Real-world Context

In the recent years, several studies have used activity theory for conducting research (e.g. David & Tomaz, 2012; David, Tomaz, & Ferreira, 2014; Engeström, 1996, 2015, 2016; Fleer, 2016; Hashim & Jones, 2007; Yamagata-

Lynch, 2010; Tomaz, & David 2015; Jonassen, 2000; Mwalongo, 2016; Otrel-Cass, Andreasen, & Bang, 2016; Ramanair, 2106; Russell, 2009), however in current study a composite model that integrates activity theory and modelling cycle (modelling activity system) was used to investigate students tension in solving real-world problem which related to students everyday life. It seems modelling activity system is a new idea for research in this domain. In general, this study, by using mathematical modelling, it creates a series of tension in toward changing student's water consumption with using activity theory for analysing data. Specifically, the following research question guided the present study.

- How students could manage, and modify water consumption patterns through modelling activities?

## Methodology

### Context of the Study

There is an important issue for many countries around the world for water consumption. Because the population of the world growing fast and we need more water resources and in the same time we have to a reduce and diversify our water use. Upon united Nation webpage (Water and Sanitation - United Nations Sustainable Development) water scarcity affects more than 40 present of the global population and is projected to rise. UN website define 6 target goals for sustainable development of water resources which some of them mostly related to water consumption and management for example 6.1 and 6.4 (see Table 1).

Table 1. United Nation Target Goals for Water Consumption and Management

- 6.1 By 2030, achieve universal and equitable access to safe and affordable drinking water for all;
- 6.2 By 2030, achieve access to adequate and equitable sanitation and hygiene for all and end open defecation, paying special attention to the needs of women and girls and those in vulnerable situations
- 6.3 By 2030, improve water quality by reducing pollution, eliminating dumping and minimizing release of hazardous chemicals and materials, halving the proportion of untreated wastewater and substantially increasing recycling and safe reuse globally
- 6.4 By 2030, substantially increase water-use efficiency across all sectors and ensure sustainable withdrawals and supply of freshwater to address water scarcity and substantially reduce the number of people suffering from water scarcity
- 6.5 By 2030, implement integrated water resources management at all levels, including through transboundary cooperation as appropriate
- 6.6 By 2020, protect and restore water-related ecosystems, including mountains, forests, wetlands, rivers, aquifers and lakes
    - 6.A By 2030, expand international cooperation and capacity-building support to developing countries in water- and sanitation-related activities and programmes, including water harvesting, desalination, water efficiency, wastewater treatment, recycling and reuse technologies
    - 6.B Support and strengthen the participation of local communities in improving water and sanitation management

### Origin of the Study

This study is part of that larger study, "Developing Conceptual Understanding by Volume Measurement". In that large study, 12 modelling activities were designed during the study, which three activities were selected for the present study. The focus of them is on modifying the pattern of water consumption.

**Participants**

The participants in this study were the 12 students from the eighth-grade of a lower secondary school. These students were participated in two sessions (60 minutes) each week for three months. All the students are lived a village in south-east of Iran. The mean age of students was 13-14 years. Students had allowed to use handicraft, origami, and calculator. Students of the class were divided into several groups of three and two. They work on group at school time and then they continue their working at home individually. The task of each student was find a way for modifying their water consumption. Finally, all students present their idea through whole class discussions.

**Data Collection**

Study data were collected during the 3 months from April 2022 to June 2022 through modelling activities, students and teacher notes, interview and observation of the class film. These data were from three modelling activities that students face "real challenge" during a modelling process (Alsina, 2007). "Real challenge", in this study, is the consumption water problem. These three activities were divided into two parts, the first part being water consumption modelling activities $A_0$ and the second part being waste modelling activities $A_1$. Drinking-Water modelling activities consist of three interconnected activities, that respectively the modelling activity of daily drinking Water (see Figure 3 for activity $A_{0,1}$).

Figure 4 clearly states the student's idea: *" I filed a bottle from water in the previous activity ($A_{0,1}$) and it put on the refrigerator, I drank to water 450 ml till night. What is the doctor's opinion about my drinking water?".*

The teacher uses from student' idea " you see everybody in our city that a bottle of water in with themselves " and designs the activity A0.,3. In this activity, based on software information that indicates daily drinking water for exercise and non-exercise days, from students are asked to calculate their drinking water based on their weight and pour it into a bottle and always carry the bottle with themselves that the activity described Figure 6. In this activity, the students made the origami boxes using the instructions in the activity. Next, by busing made- boxes, students calculate their drinking water at home and report it to the classroom, and then the teacher checked the students' responses. The students had a problem in doing the modelling activity, their problem was solved. In finally, again students did the activity until they can right report their drinking water. Some students took this activity for two weeks and others for three weeks to confirm their initial report on the amount of drinking water consumed by the teacher. In this activity, 28 interviews were conducted with students, with an average of 15 minutes per interview. After the activity and the next activity in the classroom, students were asked to perform the above activity at home to determine the exact amount of drinking water. After that, the students will present their report to the classroom. If the students do not do their calculations correctly, the students will do the activity again to confirm the answer by the teacher. After the teacher accepts the answer, the amount of drinking water by the student is presented to the doctor in a report. the teacher's interview with the doctor is recorded for half an hour, and it plays in the classroom the next day. And then, teacher interviews with each student in the classroom.

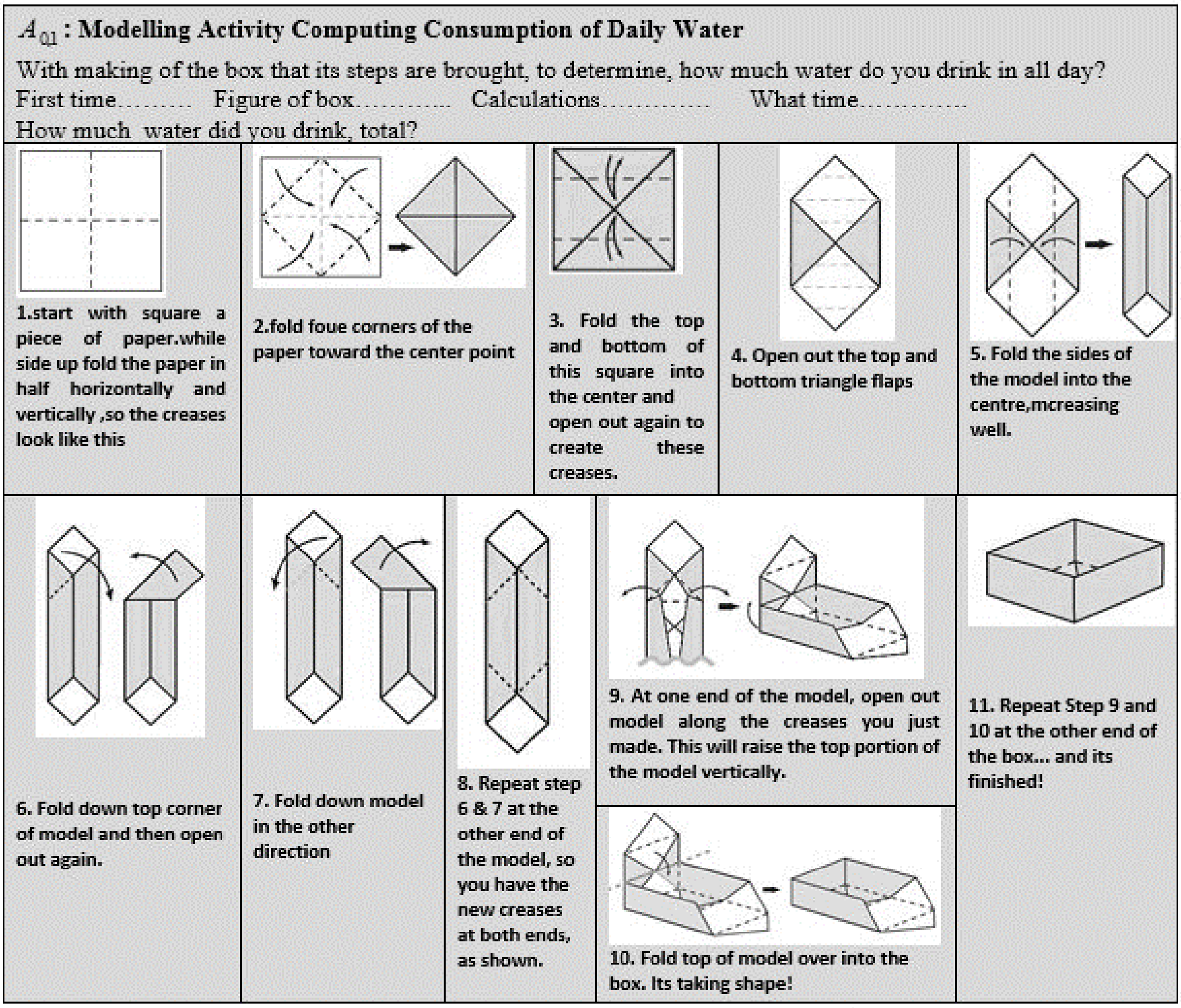


Figure 3. The Modelling Activity of Daily Drinking Water ($A_{0,1}$) (cited in site: www.origami-fun.com)

### Data Analysis

As has been shown, activities related to water consumption pattern modification have developed over a while of the study, which is characteristic of the historical theory of activity referred to by Engeström (1996). These interconnected activities with each other form an activity system that uses the activity theory to analyze it. According to Engstrom, the activity theory is used to analyze systems of activity in which human activity is defined as the unit of analysis. Activity theory is used as a method and a theoretical framework applied to qualitative methodology topics for the analysis of qualitative data related to situations in which have cultural or social contexts (Yamagata-Lynch, 2010; Mwalongo, 2016; Hashim & Jones, 2007). The data of this study have cultural or social aspects because they are modelling activities related to water consumption. On the other hand, research-based on the theory of activity should examine human activities in contexts of real-life activities (Ramanair, 2106). Therefore, the study examines the modelling activities that come from students' real life. Many studies of cultural-historical activity theory are conducted through the ethnographic methodology and case studies (Russell, 2009, p. 41), where data collection requires participant observation, interviews, and discussion in real-life contexts (Scriber 1985, Vygotosky, 1980 cited in Li, 2016, p 109). Therefore, this study was conducted by adopting a theoretical framework of activity theory using ethnography.

In this section, to emphasize the analysis of tension is that by the creation of them in modelling activity subsystems were caused modifying of water consumption pattern. These tension are as a source of change in students' water consumption patterns whose analysis of them is very important in the study. Hence, their analysis is carried out by Modelling Activity System.

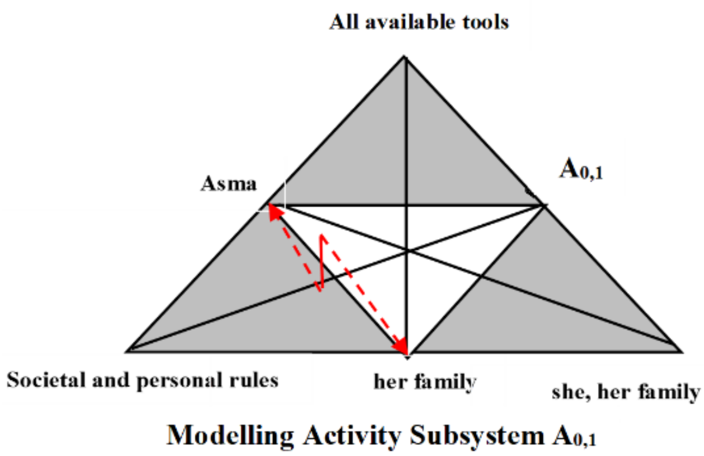


Figure 4. Tension ($T_1$) of Consuming Culture of Daily Water in the Modelling Activity Subsystem $A_{0,1}$

| | |
|---|---|
| **Interview Sample of Tension** | 48 Student (Asma): I would pour my amount of drinking water in the bottle and to have with myself that I drink to based on my weight. My mother said to me, you can't drink to water with and poured it in bottle every day.<br>49 Teacher: all of students should be to do Asma's work<br>50 Student: we can't to do this work became we have work<br>51 Student: I forget that to calculate amount of drinking water for each time<br>52 Student: I went to the party. What to do?<br>53 Student: people tell to us that you're crazy who drinking water pour in the box and to calculate it. |

Figure 5. Interview of the Tension of the Modelling Activity Subsystem $A_{0,1}$

As can be seen in Figure 4, for Asma, who is one of the subjects of the consumption of drinking water modelling activity subsystem ($A_{0,1}$). In the activity, the tension was created for her by her family as a member of the community. This tension ($T_1$) is related to consuming culture of daily water in the families that it can be found in interview No. 48 (see Figure 5) for Asma at home. This is clear that her mother is created the tension for her by saying the word *"You cannot drink this water and pour your water into the bottle every day"*. This interview reveals that the factor of tension, between Asma and her mother, is the artefact of a bottle that Asma's mother points it out in the interview. The same tension is created for another student who is members of the community. They say, *"We can't work on Thursday, we can't count on water," and "We went to a party, what to do then?"* In interviews 50, 51 and 52 illustrate the above tension. The intensity of this tension comes in Interview the *People tell us you are crazy*." However, the focus of the analysis of study tension in this section is on the tension created for the Asma's.

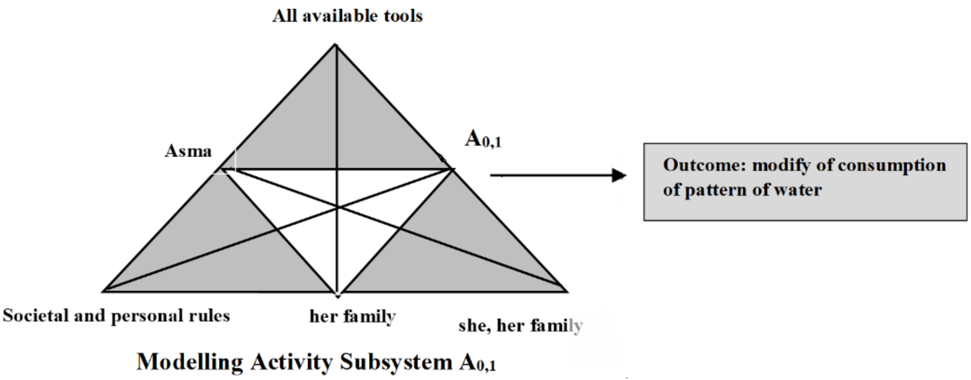


Figure 6. The Modifying of Tension ($T_1$) of Consuming Culture of Daily Water in the Modelling Activity Subsystem $A_{0,1}$

| | |
|---|---|
| **Interview Sample of** | 54 Student (Asma): I said to my mother I want to measure my drinking of water and then filled the bottle. Then I poured the water bottle into the box of activity $A_{0,1}$. The length, wide and height were 10 cm. when I measured the volume of the water bottle that was, 1000 ml. so, I would drink one or two glasses of water in each day. when I did this action, I understood that I can to do this work.<br>55 Teacher: how many days did you do it?<br>56 Student (Asma): I did it one week and the action is a habit for me. Now, I have a bottle in my room until night I drank water. My mother saw to me that I can do this work. |

Figure 7. Interview of the Modifying of the Tension of Modelling Activity Subsystem $A_{0,1}$

But after the creation of tension ($T_1$), the students overcome this tension by doing the modelling activity $A_{0,1}$ and modify $T_1$. For example, one can refer to Asma's modifying of tension that is done in Figure 6 and Figure 7. When she confronts with this tension, actions according to the teacher's telling and succeed to change her water consumption into a daily habit and modified this tension. The modelling process this student has successfully done is as follows: *"I said to my mother I want to measure my drinking of water "*(Real World Problem) *and then filled the bottle. Then I poured the bottled water into the box of activity $A_{0,1}$. The length, wide and height were10 cm* (Mathematical Model). *When I measured the volume of the water bottle that was 1,000 ml* (Mathematical Solution), after this student states that *I would drink one or two glasses of water in each day* (Real World Meaning Solution), this student's answer It doesn't make sense because she hasn't consumed all the water in the bottle. So this answer doesn't make sense in the real world, so this student has to go through the modelling cycle again. In student interview 56, she states that *"I did this for one week and the action is a habit for me* ", indicating that the student has gone through the modelling cycle several times and after one week has been able to consume bottled water overnight (Accept Solution). Finally, this student has been reached to a meaningful answer to the real world, drinking daily water (Modify Consumption Water).

## Discussion & Conclusion

This study has added to the existing literature by 1) Exploring students' tension in the process of conceptual development of volume for modifying their daily drinking water. 2) The sample was selected from a region where previous studies in relation to activity theory and modelling have not been undertaken. 3) It seems that this study is a first study of its kind that have used modelling activity theory/system (a composite model that integrates activity theory and modelling cycle) for studying student's conceptual development during solving a real-world problem. 4) Finally this study shows that how out of school experiences could integrate with school mathematics for fostering students conceptual understanding. In David and Tomaz (2015) study, some activities developed and used which related to students a real-life, but in the current study all modelling activity ($A_{0,1}$), In addition to being taken from students' everyday life situations, students were able to solve one the real situated problem related to water consumption. In $A_{0,1}$ activity, students measure their water consumption for 24 hours using an origami artefact. Cultural artefacts had crucial rules in this study. Students make a box with a sheet of paper (origami) and calculate the volume of water consumption. They also used a water bottle for their estimation. These artefacts help students to make a connection between the real world and the math world. Different type of artefact was used in previous researches such as water bill in Tomaz and David (2015), supermarket bill and food label in Bonotto (2004 and 2007).

Finding the current study shows, that participated students were able to use the experimental process during modelling cycle. Indeed, they examine their math solution in the real world context and if necessary, they apply a new modelling cycle for refining their real-world solutions. In this way, students were able to decrease their water consumption and manage their water consumption pattern during this study. So, the results of this study reveal the importance of modelling cycle for changing math classes toward an experimental environment. According to Carreira and Baioa (2011) with using experimental modelling activities, we can provide a new approach in school mathematics, which students can encounter with a real-world problem with a practical perspective.

Finally, the results of this show the power of mathematics, which help students to solve their real-world problems and help them to change the situation in hard living conditions. Students, which participated in this study, help themselves and improve their health through managing their water consumption. Indeed, the output of this study caused improvement in the quality of student life and preserve water for the next generation in a dry environment. In this way, we reach sustainable development in water consumption, which is an important issue for all people all around the world.

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