Analysis of the Correlation between the Use of Written Algorithms and Success in Mental Calculation

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Abstract: This paper explores the correlation between mental calculation performance and the frequency of using written algorithms in mental calculation tasks. Mental calculation is a mathematical tool used in everyday life situations during and after our formal education. After presenting an overview of the professional literature on this topic, the paper will present calculation methods and show how represented they are in the Curriculum. For the empirical part of the research, a total of 233 Croatian students aged 10 to 22 years were tested and interviewed. The previously mentioned correlation was then analyzed. An overview of the interview results will be presented as well. It was found that school mathematics does not always contribute to the development and flexibility in using mental calculation strategies because of the student preference for acquired written algorithms. Definitely, recommendation is shifting the focus from written calculation and procedures to the mental, discussing the associated strategies and different concepts of number. In this way, formal education could contribute to what students really need later on, in both private and professional situations in which they may find themselves on a daily basis.

Keywords: Mental Calculation, Mental Calculation Strategies, School Mathematics, Mathematics Curriculum, Written Calculation

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Introduction

Cambridge's Dictionary (Dictionary, 2018) defines *calculation* as the process of using information and operations of addition, subtraction, multiplication and division to estimate the number or quantity of something, while according to Collins' Dictionary (Forsyth & Mangan, 2014), *calculation* is something we think about and do mathematically; what we think about very carefully and based on which we come to conclusions taking into account all relevant factors. Calculation is involved in solving real problems that we encounter on a daily basis.

As for calculation methods, numerical calculations can be performed using three methods: mental calculation, written calculation or written algorithms (using paper and pencil) or using calculators (McIntosh et. al., 1995).

In this paper, we will give a theoretical overview of written and mental calculation and investigate the correlation between mental calculation performance and the use of written calculation.

Written Calculation

As previously mentioned, one of calculation methods is written calculation, i.e. using paper and pencil. Teachers usually use the term algorithm to describe written calculation procedures of addition, subtraction, multiplication and division of numbers that are traditionally taught in schools (Stacey, 2004). "Algorithms are valuable precisely because they give a routine that works in a prescribed way for all elements of a specified domain. People can use them on any problem that fits the conditions and they can be programmed into machines" (Stacey, 2004, 94). However, Kamii and Lewis (1993) believe that algorithms are harmful because they encourage students to give up their own opinions and ideas and not to use local values in calculations, thus preventing the development of number sense. Children come to school with a huge potential for thinking, and the role of the teacher is to develop that potential instead of 'molding' the students and 'putting all the cards on the table'. Students should use their constructive processes and belief in their own abilities to solve the problems presented to them.

Research on child-friendly methods showed that students were often taught classical written calculation in tasks with four basic mathematical operations where this was not necessary and when other approaches would be possible and more effective (Gravemeijer et al., 2004). At school, written algorithms are taught and automated, and students feel more confident using this method as they are sufficiently trained to use it during mathematics classes. Yet, following an inflexible set of rules is something machines are good at, not people. Furthermore, written calculation often deepens only procedural knowledge, while there is no change in conceptual knowledge. Procedural knowledge is mastery of computational skills and knowledge of procedures in identifying mathematical components, algorithms, and definitions (Lin et. al., 2013) and it often calls for automated and unconscious steps, whereas conceptual knowledge typically requires conscious thinking (Haapasalo, 2003). "An individual task can be performed with only limited understanding. Therefore, the fact that a student gets a calculation correct tells us little about the extent of their understanding" (Barmby et. al., 2007, 45).

Indeed, in written calculations performed in school, evaluation is done only for the correct application of the algorithm and the result, without questioning a deeper understanding of the procedure and conceptual understanding. Mathematical education should be based on understanding, building upon student's prior knowledge and connecting their knowledge. Algorithms can be easily applied without understanding, without even noticing, so students can experience mathematics as an area that is not built on understanding, at least not for them (Gravemeijer et al., 2004).

In fact, most of us do not even use the algorithms we learned at school to solve everyday problems in practice. Standard written 'school' algorithms of four operations are selected from a whole range of possible ways to solve problems (Stacey, 2004), which are often not listed in the curriculum nor in textbooks.

Mental Calculation

The second calculation method to be studied here is mental calculation. Mental calculation (mental computation) is integrated into the child's learning about numbers and refers to calculating the correct result 'in the head' (Sowder, 1988). Rathgeb-Schnierer and Green (2013, 553) define mental calculation as "solving arithmetic problems mentally without using paper and pencil procedures." It is one of the best ways to develop and deepen the students' understanding of numbers and their properties (Reys, 1984). Mental calculation can facilitate the development of the mind and provides more chances for children to learn more easily and get better academic performance (Gómez-Rosales & Mireles-Medina, 2019; Yilmaz, Akyuz, & Stephan, 2019). When manipulating numbers in the head, children look at numbers from different perspective, develop number sense and increase confidence in their own mathematical abilities. It is this confidence that will encourage them to consider mental calculation as an option when solving a task (Haylock, 2007; McIntosh et. al., 1997a).

"Mental computation is the most common form of computation used in everyday life. It is used for quick calculations and estimations" (Pourdavood et. al., 2020, 241). The research showed that 85% of calculations used on a daily basis is mental calculation, 58% of which are estimates (25% of calculations in the context of time, 23% in the context of purchase and money) (Northcote & McIntosh, 1999). Real life requires the ability to perform simple mental calculations quickly; a person must be able to quickly understand relationships and know which calculations to perform (Bruinsma, 1969). It is often enough to get approximate, rather than correct, answers to everyday complex arithmetic problems (Ganor-Stern, 2018).

McIntosh, Bana, and Farrell (1995) conducted a study to obtain information on the mental calculation knowledge of 641 students in grades 3, 5, 7, and 9 in Western Australia. Students concluded that it is important to be good at mental calculation (more than in written calculation) and that mental calculation is what they will use every day when they are adults. It is worth mentioning that written calculation is more learnt in school, while mental calculation is mostly learnt independently.

"Mental computation is an important skill […] which can be developed by practice, to which primary school Mathematics has a decisive contribution" (Baranyai et al., 2019, 8717). Recently, flexible mental calculation has been increasingly considered an important goal in elementary math education (Selter, 2000; Verschaffel et. al, 2009). Heirdsfield (2005) studied the role of teachers in teaching mental calculation. By analyzing the teacher's actions, she discovered two important teaching items that influenced the mindset of students when calculating.

These are carefully selected tasks which highlight connecting things and encourage strategic thinking. Furthermore, analyzing task-solving strategies gives the teacher information about the level of students' cognitive development, their individual learning style and readiness to adopt a new concept (Sharma, 2001). Certainly, mental calculation should be included as much as possible in formal mathematics education. When students use a learned formula or a procedure to make problem-solving easier, they neglect the underlying concept for deeper understanding (Bowers & Doerr, 2001; Hiebert & Carpenter, 1992; Wilson & Goldenberg, 1998). Ruiz and Balbi (2019) claim that although it is clear that the inclusion of mental calculation in school does not mean abandoning formal algorithms, schools must consider the order in which both skills are taught. Namely, prior knowledge of formal algorithms may hinder the possibility of independently discovering new mental calculation strategies, which will be discussed in more detail later. Boero et al. (1989) studied the transition from mental to written calculation and argued that it is not good to introduce written algorithms early and directly, but to start from (informal) strategies that children have built and gradually direct them towards more efficient, but always meaningful strategies. This way of teaching gives better knowledge. Hickendorff and colleagues (2010) conducted a study with 362 students in grade 6. They compared the use of mental versus written calculation. Students were free to choose their calculation strategy. A total of 20% of students solved the tasks using mental calculation, 40% chose written calculation only, and the remaining 40% applied mental calculation in easier tasks and written in more difficult tasks. It turned out that students with a lower level of mathematical knowledge were more likely to use written algorithms because they felt more confident given the good practice of using this strategy during math classes.

Therefore, the following passages will discuss mental calculation strategies.

Mental Calculation Strategies

"Mental strategies are more about the application of known or quickly calculated number facts in combination with specific properties of the number system to find the solution of a calculation whose answer is not known" (Thompson, 1999, 2). Mental calculation strategies differ from written algorithms because they require more than applying an automated procedure. A key difference is the need for a deeper knowledge of how 'numbers work', i.e. this requires a developed number sense (Hartnett, 2007). The strategy should be chosen according to the numbers in the given problem and their properties and mutual relations, which requires a developed number sense. The chosen path to the solution depends on the observed elements, which are covered by individual calculation skills and which the individual finds easiest, considering the knowledge they feel most comfortable applying (Threlfall, 2002). Some children first notice tens and ones number partition, others round to the nearest tens, or round to the nearest numbers with which they can quickly count, etc. While children are still school beginners, the first thing that comes to their mind is the way, method, representation they were taught in class. Later on, after having developed number sense and gained experience in mathematics, they may be able to engage in other aspects. The ability to connect numbers in different ways is a very useful skill on which much of the later study of mathematics can be built (Howden, 1989).

Carpenter et al. (1997) conducted a three-year survey of 82 children in grades 1–3 who were interviewed five times each to determine and monitor students' development and understanding of multi-digit numbers and operations. The research was based on understanding the concept of number and the problems of addition and subtraction. It was shown that children are able to independently create strategies for addition and subtraction while using different conceptions of number. About 90% of students used their own strategies and it turned out that students who designed their strategies had better knowledge of numbers and flexibly expanded their methods to more complex tasks with multi-digit numbers compared to students who used recently learned standard algorithms at school. It was also found that this is the way to avoid common systematic errors. Moreover, systematic work on mental calculation promotes the development of own strategies (Carvalho & da Ponte, 2013), which makes the teacher's role very important. Heirdsfield (2011) conducted an experiment lasting 10 weeks in collaboration with two class teachers. The task of the teachers was to encourage children in developing strategic thinking skills in mathematics. Teachers worked with children encouraging them to actively participate, explain their strategies, compare them with others and reflect critically. They asked children questions such as, *How did you solve this? Who solved the task in a similar way? What makes your strategy different?* etc. In addition, they helped children establish connections between mathematical concepts paying attention to order and systematicity of their lectures. For example, the task 46+20 was presented before the task 46+24, and 46-29 afterwards. During the experiment, many children developed various strategies when solving tasks using mental calculation, which was found in interviews conducted by the author before and after the experiment. The focus of this research was not only to help children develop mental calculation strategies, but also to develop higher order thinking – reflecting, criticizing and making sense of numbers and operations. Furthermore, Baranyai et al. (2019) conducted research with 239 students, future teachers in primary and preschool institutions. They studied mental calculation skills, focusing on the strategies they use. The results showed that most students do not know a variety of mental calculation strategies; more than a quarter of the respondents did not use mental calculation strategies but calculated only 'in the head' following written algorithms, and more than one-third used only one or two strategies. The results of this research highlight how necessary it is to teach mental calculation strategies to future primary school teachers, and thus to all students. Practicing procedures does not stimulate the development of mental calculation and associated strategies because little time is left for mental calculation. Therefore, Heirdsfiel et al. (1999b) recommend shifting the focus from written to mental calculation and estimation. The authors are also in favor of increasing the time spent teaching alternative calculation strategies and a more individual approach to each student encouraging them to develop their own flexible approach to learning operations and manipulating numbers when calculating.

Mathematics Curriculum

In most European countries, Mathematics curriculum is an official document, which is often binding. It defines which topics need to be learned, describes education programs and their content, and determines what teaching, learning and assessment materials should be used (Kelly, 2009). The 2030 Global Agenda for Education places learning outcomes at the heart of the international framework for monitoring education. Mathematics curriculum has been revised in all European countries over the last decade, often to introduce a learning outcomes-based approach and/or a key competencies concept. Revisions often seek to improve the way mathematics is taught in the classroom and to connect mathematics more to student's daily experiences. Formation of standards based on learning outcomes is one way of ensuring quality in education while at the same time giving autonomy to educational service providers in defining education programs that meet student needs (CEDEFOP, 2010).

In the following part, we will study the representation of written and mental calculation in the mathematics curriculum in the Republic of Croatia.

Written and Mental Calculations in the Croatian Curriculum

In the Curriculum, mental calculation and written calculation are mentioned only in the part on class teaching outcomes, i.e. for students in grade 1 to 4 of primary school.

A learning outcome for the second grade of elementary school: *the student adds and subtracts in the set of natural numbers up to 100* further elaborates: *adds and subtracts in the set of numbers up to 100 using mental calculation*. Moreover, the document says that "it is desirable for students to master the mental process of adding and subtracting numbers up to 100" (Ministry of Science and Education of the Republic of Croatia, 2019, 25). An outcome for the third grade of elementary school: *adds and subtracts in the set of natural numbers up to 1000* further includes also written and mental addition and subtraction of numbers up to 1000, while the recommendations for achieving this outcome say: "In order to stimulate and develop thinking skills, the student should be continuously encouraged to evaluate the results and to check the solution and the skill of mental calculation" (Ministry of Science and Education of the Republic of Croatia, 2019, 35). In the third grade, written calculation is included in the outcome, *the student multiplies and divides natural numbers up to 1 000 with single digits* (in a long and short way). The content is expanded in the fourth grade, in line with these outcomes: *adds and subtracts in the set of natural numbers up to a million using written calculation* and *multiplies and divides in the set of natural numbers up to a million with two-digit numbers using written calculation*. In higher grades, mental calculation is mostly mentioned only in the recommendations that students should calculate mentally, when possible, without giving accurate and precise instructions, while written calculation is still often used in tasks of adding, subtracting, multiplying and dividing in various sets of numbers.

Let us also indicate which mental calculation strategies are represented in the curriculum, and thus in teaching mathematics at school:

- adding a single-digit or multi-digit number to a single-digit number is done by *making to the next tens*, and by analogy this is also valid for subtraction,
- adding and subtracting two two-digit numbers is done by *sequencing*,

- multiplying a two-digit and a one-digit number is performed by *separation from left to right*, and factorization occurs only by multiplying a number by multiples of 10 and 100, and this factor is further broken into a multiplication of a single-digit number and a ten (e.g. 34·200=34·2·100=68·100=6800),
- dividing a two or three-digit number by a one-digit number is done by *partition division.*

Unfortunately, school practice reveals that often the tasks that should be done mentally are not calculated in this way, but the calculation procedure is written down, and the mentioned strategies are applied accordingly, which then become written, and not mental on the one hand. On the other hand, the practice of written calculation takes up a large part of mathematics schedule intended for arithmetic. Also, it is often only written algorithm that is evaluated and assessed, and thus mental calculation loses importance and students are not motivated to learn it.

Furthermore, in the context of learning and teaching the school subject Mathematics, in the Croatian curriculum it is stated that "regular mental calculation training, determination of a simple percentage or an approximate result develops student ability to calculate 'by heart' and apply the skills of calculation and assessment in life situations" (Ministry of Science and Education of the Republic of Croatia, 2019, 235). Still, from the previous text we can notice that mental calculation is poorly represented. As for evaluating, the Curriculum says that the student does not have to use mental calculation to meet the lowest levels of Mathematics. Also, the benefit of mental calculation is not emphasized in terms of developing number sense and connecting different areas in mathematics. As for mental calculation strategies, they are minimally represented and explained in the Curriculum. Students are forced to independently develop and discover various strategies based on their own knowledge, insights and experience, which, as we already know, is not easy for most students. This is not even possible without the guidance of teachers and appropriate literature (textbooks, workbooks, counting collections).

Method

Research Design

This paper aims to examine the age-conditioned difference in respondents' use of written calculation and the correlation between their mental calculation performance and the use of written calculation in tasks.

In accordance with the above, two hypotheses were set: *There is a statistically significant difference between the groups of respondents with respect to their age in the use of written calculation in mental calculation tasks* and *There is a statistically significant negative correlation between the use of written calculation and mental calculation performance.*

To examine the hypotheses, a test with mental computing tasks and an interview were conducted with each respondent about the mental calculation strategies they used to solve each task. In the analysis of the data obtained in the empirical part of the research, descriptive statistics were used in combination with parametric methods that checked the existence of the differences between age groups in the number of different strategies used. The obtained data were supplemented by analyzing the student's answers from the in-depth interview. Taking into account drawing on qualitative and quantitative data collection procedures, this is mixed-methodology research. The research was conducted in the Republic of Croatia during the 2020-2021 academic year.

Participants

The research sample was composed of 233 students aged 10 to 22 years, i.e. the respondents were students from grade 4 of primary school to the final year of college. To exclude the impact of teacher work, students were selected from different class departments of 6 primary schools, 6 secondary schools and from 4 university departments. Students were voluntarily involved in the research. Parental consent was obtained for the participation of minors in the research. Table 1 shows the number of respondents by age.

Table 1. Number of Respondents by Age

Age	Total
10	19
11	16
12	42
13	46
14	31
15	26
16	23
>17	30
	233

Out of the total number of respondents in grade 4 of primary school, 47% were girls, in grade 5 there were 63% girls, 53% in grade 6, 48% in grade 7 and 71% in grade 8. As for the respondents in grade 1 of secondary school, 62% were girls, while among the respondents in grade 2 of secondary school, there were 49%. Among respondents over 17 years of age, 87% were girls. The total number of female respondents was 139 and the number of male respondents was 94.

Instrument

Two research instruments were applied: a mental calculation test and an interview. The former instrument consisted of 20 mental arithmetic tasks, 5 tasks for each of the four basic arithmetic operations. Assignments were created so as to ensure that the selected numbers in the assignments were appropriate for students of a certain level of education according to the Croatian curriculum. Accordingly, the test was made for three levels: students 10-12 years old attending upper elementary and lower middle school (grades 4-6); students 13-14 years old attending upper middle school (grades 7-8) and students 15-22 years old attending high school or university. Figure 1 shows the number of respondents by groups with regard to the level of mental calculation test.

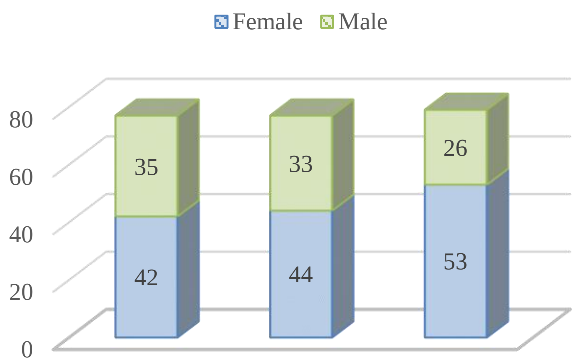

Figure 1. Number of Respondents with Regard to the Level of Mental Calculation Test

Furthermore, the tasks were chosen in such a way as to allow a different mental calculation strategy as the most effective way of solving. After hearing a voice recording of each task (which was used to avoid written calculations), students were given 20 seconds per task to solve it. After the test, an interview was conducted with each of the students. Those who solved the task correctly were asked the question: "What strategy did you use? Describe the calculation procedure in words". Students who did not calculate correctly were asked to try to recalculate the result and to describe the procedure. It should be stressed this part of the research certainly requires an interview, not a questionnaire with multiple-choice questions or open-ended questions. Namely, the skilled examiner will better explain to the respondent what he/she is asking for and will understand what the respondent wants to say and describe because the students were not taught most of the strategies and did not hear their names in school. In contrast, multiple-choice questions would suggest the strategy to be used. On the other hand, open-ended questions may produce many unanswered questions because the respondent does not know how to express, write, describe the strategy he/she has applied. The interview revealed a number of different mental calculation strategies used for each individual arithmetic operation. The prevalence of written calculation was determined, i.e. in how many mental calculation tasks written algorithm was used. Namely, although the students were asked to solve tasks using mental calculation, some still applied written algorithms imagining them in their heads. A pilot study with 17 students aged 12 and 14 students aged 21 was carried out before creating the final version of the test. Based on the results of the pilot study, the number of mental calculation tasks was reduced from 28 to 20.

Data Collection

Testing and interviewing lasted for two months during the 2020/2021 academic year. The entire procedure was independently carried out by the author of the paper because this enables the most credible results of the examination, especially in the interview. The respondents in each grade were clearly presented with the plan;

first, mental calculation test and then the interview. Mental calculation tasks were reproduced using a mobile phone and a wireless speaker, thus enabling all respondents the same test conditions such as volume, duration of the task, number of readings, time for calculation. In this way, the tasks were presented orally, not visually, in order to avoid written algorithms, as mentioned before. The test was anonymous, so each respondent had to come up with their own code and, later on, their test and questionnaire were linked to the strategies examined by the interview. The interview lasted approximately 10 minutes per respondent.

Data Analysis Techniques

The Kolmogorov-Smirnov test verified the normality of the data, and the homogeneity of variance was confirmed using Levene's test for equality of variances. When showing the data for each hypothesis, the parameters of descriptive statistics (arithmetic mean ± standard deviation, median, minimum, maximum) were first presented, followed by test results. All results were calculated using the Statistica 13.5 software.

In the context of the hypothesis *There is a statistically significant difference between age groups in the use of written calculation in mental calculation tasks*, a single-factor analysis of variance (ANOVA) was used. Here, the statistical significance of the main effect was calculated, and Bonferroni's post hoc correction examined the existence of significant differences between individual subgroups of respondents. As a measure of the magnitude of the effect, a partial eta square (η^2) was used, and the F-value and associated degrees of freedom were calculated.

In the context of the hypothesis *There is a statistically significant negative correlation between the use of written calculation and mental calculation performance*, we calculated Pearson's correlation coefficient (r), the coefficient of determination (r^2) and the linear regression model ($y=b_0+b_1x$) of the correlation of variables and the associated statistical significance for each age group.

Results and Discussion

Analysis of Differences between Age Groups in the Use of Written Calculation

In the following part, we will analyze differences between age groups in the use of written calculation. During the interview, the respondents admitted to using written calculation by giving these answers: "*It's like in school… on the board, so I imagine writing*", "*I imagine paper and write one below the other*", "*I imagine it in my head one below the other and subtract*", "*I divide just like in school on the board*" etc. Students who used 12 or less mental calculation strategies, out of a total of 20, resorted to imagining written algorithms in their head. On the other hand, students who used 13, 14, or 15 mental calculation strategies used written algorithms only sporadically by imagining the procedures in their heads. This can be seen in all age groups.

Table 2 includes descriptive statistical indicators related to the hypothesis *There is a statistically significant difference between age groups in the use of written calculation in mental calculation tasks*.

Table 2. Descriptive Statistics on the Representation of Written Calculation

Variable	Age	AM ± SD	MED	MIN	MAX
use of written calculation	10	4.84±3.50	5	0	10
	11	6.25±3.13	7	1	10
	12	4.43±3.93	4	0	15
	13	5.07±3.93	4	0	15
	14	2.81±2.87	2	0	8
	15	3.50±3.44	2	0	13
	16	3.44±3.38	3	0	12
	>17	5.30±3.25	6	0	11

From Table 2, we can read the average number of tasks in which the respondent used written calculation, which means they did not use one of the mental calculation strategies when solving the task but imagined in their head that calculation was written on paper or a board. It is interesting to note that respondents over 17 years of age are just below the students aged 11 in the use of written calculation. In 11-year-olds, there is even a minimum of 1, which means that each respondent used written calculation in at least one task. In all respondents, except 14-year-olds, a maximum of 10 to 15 was reached. Thus, out of 20 tasks, some respondents used written calculation in a minimum of half of them. As we have seen in the passage on the Croatian curriculum, written calculation is taught in formal education in grades 2, 3 and 4 of primary school, so we can be surprised by the fact that respondents who are high schools students and older used written calculation even in 13, 12 and 11 tasks regardless of the fact that in higher grades of primary school this strategy is used less and new knowledge that contributes to mental calculation (and not written) is taught. In order to examine the statistical significance of differences in arithmetic means of the variable *use of written calculation* among the observed age groups, a one-way analysis of variance for independent samples (ANOVA) was used. The results will be presented in Table 3.

Table 3. Variance Analysis Results

	F	p	η^2
age	2.592	0.013	0.074

According to the results shown in Table 3, it can be observed that the main effect is statistically significant (0.013<0.05). This confirmed the hypothesis, i.e. there is a difference in the use of the written calculation strategy in different age groups in mental calculation tasks, and the results of the Bonferroni post hoc test show us in more detail where the differences are (Table 4).

Table 4. Bonferroni Post Hoc Test Results

	11	12	13	14	15	16	>17
10	0.24	0.67	0.81	(0.04)	0.21	0.19	0.66
11		0.08	0.25	0.01	0.02	0.02	0.38
12			0.39	0.05	-0.29	0.28	00:30
13				0.01	0.07	0.07	0.78
14					0.46	0.52	0.01
15						0.95	0.06
16							0.06

Differences between age groups are present in 21% of couples. Fourth graders (age 10) and fifth graders (age 11) certainly use written calculation more than older students, which is expected because formal education mostly focuses on written calculation. However, what is interesting to note is that in respondents aged 12 and older ones, only two differences were observed, one in those aged 12 and 14, with younger students using written calculation more, and in respondents aged over 17 and 14-year-olds, showing that older students use written calculation twice as much. Thus, although there are differences, they are not present among older respondents, starting from grade 6 (age 12) and are not always in favour of older respondents, i.e. we have shown that in some cases older respondents use written calculation more in mental calculation tasks. Indeed, it has been shown that students prefer the paper and pencil method in calculation tasks, i.e. written calculation involving accurate and exact results, but this does not help them in acquiring the competence of assessment and developing number sense (McIntosh et. al.,1997c). Also, in the research on mental calculation and conceptual understanding, it was shown that inflexible students mostly use the automated procedure, i.e. written calculation. This was partly explained by “blind faith” in the written algorithms taught by the teacher, and partly by reduced knowledge that could not support more effective mental strategies. These students even managed to solve the tasks of mental calculation in part, but they did not show a developed number sense (Blöte et al., 2000). Teachers should focus on encouraging students’ thinking, rather than teaching them written procedures that do not support thinking, they should stimulate the development of number sense and expect students to use their self-developed strategies. In this way, students would apply their own strategies, instead of resorting to the ‘teacher’s’ actions without thinking about the given numbers or the context of the task (Heirdsfield & Cooper, 2004).

Analysis of the Correlation between Use of Written Calculation and Mental Calculation Performance

This passage will analyze the correlation between the use of written calculation and mental calculation performance. Table 5 shows descriptive statistical indicators related to the hypothesis *There is a statistically significant negative correlation between the use of written calculation and mental calculation performance.*

Table 5. Descriptive Statistics Indicators

variable	AS ± SD	MED	MIN	MAX
mental calculation performance	14.59±4.21	15	1	20
use of written calculation	4.41±3.60	4	0	17

According to the results in Table 5, we can see that in 50% of respondents, written calculation is represented in as many as 4 tasks with a maximum of 17, although 20 mental calculation tasks were given in the questionnaire. Thus, a part of the respondents cannot solve some of the tasks with one of the mental calculation strategies, but they must resort to imagining a written algorithm. Furthermore, the results of the correlation analysis used to verify the correlation between the variable *use of written calculation* and the variable *mental calculation performance* will be presented in Table 6, followed by a scatter diagram and a direction of regression (Figure 2).

Table 6. Correlation of mental Calculation Performance and Use of Written Calculation

	r	r^2	t	p
mental calculation performance use of written calculation	-0.466	0.217	-8,011	<0.001

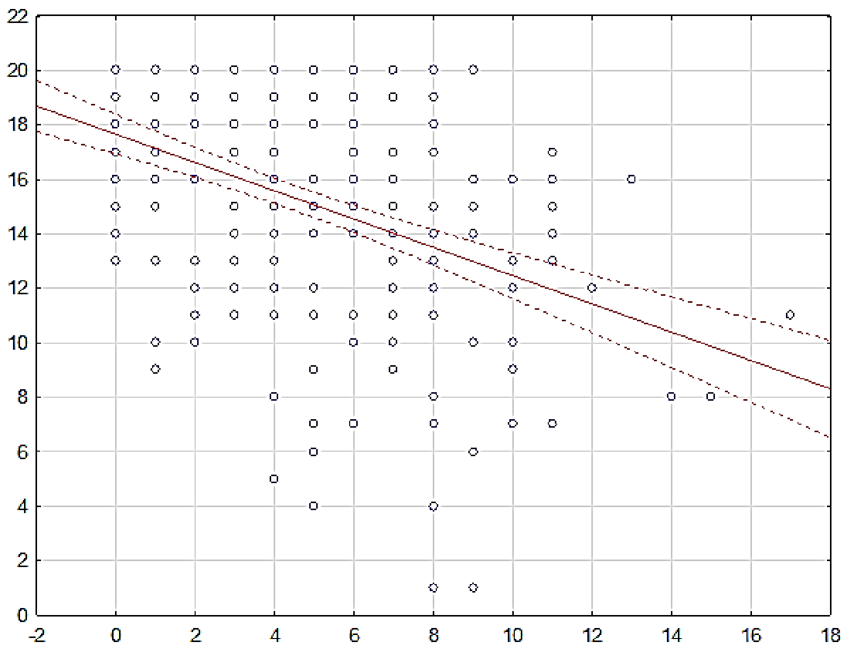

Figure 2. Scatter Diagram and the Direction of Regression of the Set

Pearson's correlation coefficient (r=-0.466) shows a moderately to good negative statistically significant correlation (p<0.001) of the variables *use of written calculation* and *mental calculation performance*. It can be observed that 21.7% of the variability of the criterion variable is explained by the predictor. The results point to the fact that mental calculation performance is correlated with the use of written calculation. Therefore, we confirmed the hypothesis, and we can say that respondents who use written calculation less are more successful in mental calculation. Consequently, the recommendation is that during formal mathematics education there should be less insistence on written calculation and more on mental calculation strategies to make students more successful in mental calculation. We can also say that students with better knowledge of mental calculation will benefit less from written calculation, which for most of the time is not useful in everyday life of adults.

Conclusion and Recommendations

Allowing today's youth to be able to respond to the demands of society in the future requires a constant change of the education system. Globalisation, the rapid advancement of technology and the wealth of information we receive every day set us new goals, challenges and problems that we have to cope with. All of the above is easier for us to handle and successfully solve if applying logical thinking, correlation, careful and meaningful organization. Mathematical literacy is recognized as one of the most important prerequisites for the development of individual life skills, the application of mathematical knowledge, lifelong learning, openness to the use of new technologies and the realization of one's own potential. Learning and teaching the subject of Mathematics encourages creativity, precision, systematicity, abstract thinking and critical thinking that helps in identifying and solving problems from everyday life and social environment. The task of school and formal education is to train the individual to become a participant in society and successfully cope with everyday life problems. Mental calculation is a tool that has helped us to develop number sense, cognitive skills and abilities since our childhood. Relying on professional literature and experience, it was found that mental calculation is the best way of brain training as it improves concentration, memory, visualization, logical reasoning, focus, that is, everything we need in everyday life. An analysis of classes of Mathematics in Croatia and the Curriculum reveals that written calculation is more used than mental. Unfortunately, school math is mostly written math. The above prompted us to examine the impact of the use of written calculation on mental calculation performance.

We have statistically confirmed that written calculation has a negative effect on mental calculation, and this is the scientific contribution of this paper. Namely, overtraining and automation of algorithms and procedures make students resort to written algorithms even when not necessary and this does not contribute to the development of mental calculation. What is more, students do not use mental calculation at all.

Regarding the practice itself, i.e. the professional contribution, this paper should help both class and subject teachers to formulate and plan the teaching of mental calculation and to transfer the focus from written to mental calculation. Teachers cannot be guided rigidly only by a curriculum and a textbook, but they should be informed and taught daily so that in class practice, they can respond adequately, answer students' questions and teach in a quality way.

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