A 2D SHAPE COMPOSITION LEARNING TRAJECTORY OF A STUDENT WITH DIFFICULTY IN MATHEMATICS

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This study explores a spatial reasoning learning trajectory of a student with difficulty in mathematics. Using a teaching experiment methodology across 15 instructional sessions, we observed how the student responded to instruction based on an established 2D shape composition learning trajectory (Sarama & Clements, 2009). A narrative microgenetic analysis identified conditions that were likely to have promoted learning. The analysis shows the student's actual trajectory was similar to that of the hypothesized learning trajectory. Challenges to progress emerged around teacher-guided components of instructional support. We describe how a student-centered approach and mathematizing through specific praise was generative of learning, while explicit guidance was not. We discuss how the guiding conjecture of the teaching experiment evolved and how it is situated within the broader literature base.

Keywords: Learning Trajectories and Progressions, Students with Disabilities, Geometry and Spatial Reasoning.

Students with learning disabilities and math anxiety often experience dissonance in mathematics learning spaces. The "clamor" created for the individual as they interact with mathematics and with others in the mathematics environment can, at times, be intense. This work centers the learning experience of Eva, a nine-year-old student with learning disabilities and anxiety. We wish to take an anti-deficit stance and refer to Eva as a student who has experienced considerable difficulty in mathematics learning situations.

An approach to instruction that has potential for harmonizing the learning environment for students who have experienced difficulty in mathematics is the use of learning trajectories (LTs). We conducted a teaching experiment with Eva that investigated spatial reasoning LTs. Here, we present a narrative microgenetic analysis of one of those trajectories, Eva's actual 2D shape composition LT and describe features of the experience that were supportive of and hindrances to her progress.

Theoretical Framework

In the United States, mathematics interventions are commonly supplemental programs intended to provide more intensive and individualized support (Powell et al., 2013). There is a consensus among researchers that students in interventions benefit most when this support is designed to be systematic (Fuchs et al., 2021). Systematic instruction includes features such as connecting to previously learned mathematics, using accessible number and visual representations, and gradually increasing complexity.

However, there are disagreements around instructional delivery format and what should structure the increasing complexity across lessons. Many researchers contend that instructional delivery must incorporate explicit, direct methods (e.g., Grigorenko et al., 2020). In contrast, other researchers are using carefully structured sequences of tasks that build on students' thinking and direct their attention to mathematical relations without initial explicit guidance (e.g., Hunt et al., 2020; Xin et al., 2016).

LTs offer another potential way to provide structure to support the learning of students who have experienced difficulty in mathematics learning situations. Research has shown positive effects of mathematics instruction based on LTs for young students at risk for later difficulty in mathematics (Clements & Sarama, 2007; Clements et al., 2019; Clements, Sarama, Baroody et al., 2020). Further, research indicates that instruction based on geometry trajectories can support learning in other mathematical domains and support executive functions which have been shown to be important to mathematics achievement (Clements et al., 1997; Clements, Sarama, Layzer et al., 2020; Schmitt et al., 2018).

LTs structure learning through developmental progressions, sequenced learning goals based on the progression, and tasks designed to support progress toward the goals (Sarama & Clements, 2009; Simon, 1994). The systematicity provided by the developmental progression is something that distinguishes instruction based on LTs from instruction that is more typical of intervention contexts for students who have experienced difficulty in mathematics. Whereas a typical intervention approach is to identify components of the target knowledge or skill and provide instruction for each component, instruction based on an LT focuses on intermediary phases of reasoning or acting that culminate in the desired knowledge. Simply put, rather than breaking a mathematical task down into smaller parts in order to learn how to do it, the ability to perform the task is built up "naturally" based on a students' current knowledge. Tasks can be designed to draw on a variety of instructional delivery formats–explicit instructional principles, incorporate multi-modal delivery and response, or first elicit student thinking before guiding that thinking toward more sophistication and abstraction.

2D Shape Composition as a Developmental Progression

The 2D shape composition LT (Clements & Sarama, 2021; Sarama & Clements, 2009) involves children gradually developing understanding of and attending to geometric attributes and mentally anticipating results of combining shapes. At an early stage of the trajectory, Piece Assembler, when children begin to put smaller shapes together to compose larger shapes, they do so through trial-and-error. At the next level, Picture Maker, children can combine several shapes to make part of a picture but still use trial-and-error. They select a shape based on a general shape or approximate side length, and can fill in puzzles that indicate the placement of the shape (interior lines are present). When children reach the Shape Composer level, they can anticipate shapes that are needed to fill in a larger region and choose shapes considering angles and side lengths. They can also intentionally use rotation and flipping to correctly orient shapes. This is followed by Substitution Composer in which children use trial-and-error to make multiple versions of a composite shape by combining shapes in different ways. The subsequent levels, Shape Composite Repeater and Shape Composer--Units of Units, involve understanding composite shapes themselves as part of larger shapes and working with patterns that combine composite shapes.

Method

We describe a teaching experiment with Eva, followed by a narrative microgenetic analysis. Teaching experiments apply methodological consistency within a naturalistic setting (Confrey & Lachance, 2000; Steffe et al., 2000). A guiding theory or conjecture is tested through intentional teaching, data collection, and on-going analysis. The conjecture that guided this teaching experiment is that an established LT supplemented with individualized supports can facilitate learning in a student who has experienced difficulty in mathematics learning situations.

This narrative microgenetic analysis had two important elements: (a) focusing on identifying conditions that might promote learning (Lavelli et al., 2005; Siegler, 2006), and (b) identifying a plot that links data as unfolding temporal development (Polkinghorne, 1995). The first element, conditions that promote learning, is based on a density of observations during a period of time during which we hope to see change. These observations can describe types of statements or behaviors, how the statements or behaviors change, the rate at which change occurs, possible sources of change, and how widely generalized the knowledge (Siegler, 2006). The second element, the resulting "story," gives shape to a multitude of factors that are at play.

Participants

Eva is a multi-racial girl living in the western United States. At the time of this teaching experiment, Eva was nine years old and in grade 3. Two years prior to this study, Eva underwent a neuropsychological evaluation which identified the following challenges: attention deficit with unspecified impulse-control and conduct disorder, speech-sound disorder, specific language disorder with impairments in written language and mathematics, and generalized anxiety. Eva attends a public charter school in a small city and has received special education services in a pull-out program ("resource room"). Upon meeting with Eva's mother, we discussed Eva's difficulties in mathematics and talked about ways to provide her with additional academic support without increasing her anxiety. We gained informed parent consent and student assent following guidelines established by the university's Institutional Review Board.

We recognize our roles as participants in this research. As a teacher, one of us conducted the teaching experiments in relationship with Eva and her mother, who was present for all the sessions. This relationship involved negotiating goals, activities, what was to be attended to and what was not, as well as negotiating interpersonal interactions and trying to establish trust. As researchers, we both continue this relationship with Eva through our interpretations of the videos, transcripts, and memos. We take the position of bracketing ourselves into this narrative, rather than attempting to position ourselves outside (Connelly & Clandinin, 2006).

Teaching Experiment Procedure

The first author, Angie, conducted all of the teaching experiment sessions. The sessions took place in the winter and spring of 2020 amidst the COVID-19 pandemic. The meeting dates were flexible to allow for family needs and local quarantining requirements. All sessions took place at Eva's home, and Eva's mother was present to support Eva's well-being and engagement. There were 15 teaching experiment sessions, each ranging from 30-45 minutes. Due to the pandemic and to limit factors that might contribute to Eva's anxiety, it was not possible to have an additional researcher present during the sessions. During each session, Angie asked Eva's permission to video record the activity and only recorded when Eva stated she was comfortable with it. We recorded nine of the 15 sessions.

Tasks described in this paper come from the 2D shape composition LT (Clements & Sarama, 2021; Sarama & Clements, 2009). These tasks involved placing pattern blocks (colored, wooden blocks in shapes of triangles, squares, trapezoids, hexagons, and two different rhombuses) into "puzzles" that provided outlines of a composite shape (puzzles may contain all, some, or no interior lines). We were open to adjusting the sequence of tasks and delivery format as needed to support Eva's engagement and documented the nature of and rationale for any changes to the LT.

We used a planning protocol for documenting instructional decisions and reflections on Eva's activity. The protocol included prompts for observations, outcomes, adjustments made, and rationale for adjustments. These plans were shared with two colleagues. One colleague, with expertise in early mathematics instruction for students with learning disabilities, advised on the tasks and supports. A second colleague, with experience with children with anxiety, advised on the plans in light of Eva's affective responses.

Data Sources

Data sources were the planning protocols previously described, field notes from each session, photographs of student work, videos and transcriptions, and a post-hoc observation protocol. The second author, Aysia, completed the post-hoc observation protocol as a way to triangulate observations and interpretations of Eva's responses. This protocol recorded several features of the interactions: the task description, student behaviors (strategies, demonstrations, comments), teacher behaviors (explanations, providing time to work without interruption, additional supports), and any other observations. Aysia completed the protocol for each video. It was common to pause and re-watch segments of a video file multiple times to capture all necessary data.

Data Analysis

We used a form of narrative analysis adapted for microgenetic studies that can account for the multidimensional nature of learning in individuals (Lavelli et al., 2005). This approach comprises five stages. The first involves viewing videos repeatedly to identify a list of potential "frames," the specific viewpoints or phenomena which could be a focal point for analysis. In the second stage, we constructed descriptive, chronological narratives, one for each session of the teaching experiment to document the sequence of events. To ensure accurate and comprehensive narratives, we each wrote an initial narrative for half of the sessions and then reviewed and revised the narratives written by the other. Thoughts related to frames, explanations, interpretations, questions or reflections were recorded in a parallel set of memos. In the third stage, we used the chronological narratives to discuss possible configurations of frames that seemed profitable ways of portraying learning. This study presents the frames of Eva's activity within the 2D shape composition LT and Eva's response to components of instructional support. The fourth stage involved re-reading the chronological narratives to look for evidence of stability or change over time and to develop stories that synthesize a preliminary "plot." Evidence to confirm or refute this plot was then gathered through an in-depth analysis of the data sources with particular attention to overt behaviors and strategies. In the fifth and final stage of analysis, the stories and evidence were used to create a narrative that synthesizes our views on Eva's experience.

Findings

We conducted a microgenetic study of Eva's learning experience with LTs. We present the frames of 2D shape composition activity and Eva's response to components of instructional support in narrative form, highlighting Eva's responses we believe are representative of Eva's experience.

The Story of Eva's 2D Shape Composition Trajectory

Sessions 1—4: Piece Assembler. In the first session, Eva did not want to solve puzzles, and she abandoned the activity. She returned when asked if she wanted to create her own picture, and she created a figure from a favorite cartoon with each block playing a unique role in the picture (i.e., one shape for each part of the picture, no shared sides). In the next sessions, Eva's activity suggested she recognized triangles, squares, and hexagons, but she did not name them. She used trial-and-error to fill shape outlines. When asked why she picked a particular shape, she would describe the shape as "skinny" or point at an angle or single side without further responses. We used "mini-puzzle" task cards with outlines of 2 shapes to control complexity and encourage Eva's intentional selection of shapes and her use of turning to orient them appropriately. Eva created her own picture of a dog in which three shapes combined to make a part of the picture (dog's head and ears). See Figure 1 for photos of Eva's work at this level.

Figure 1. Images of Eva's work at Piece Assembler level (sessions 1—4).

Sessions 5–7: Picture Maker. Eva placed several like shapes together to create sections of a puzzle when the interior lines were provided. She found this "easy" and asked for harder puzzles. We provided puzzles that had fewer interior lines but with some exterior line "cues" such as clear indicators of a triangle or rhombus. She continued to select recognizable shapes first. For example, she filled a rhombus with two triangles, thus composing without an interior line, but also shared she could have used "the red piece" instead. See Figure 2 for images of Eva's work during these sessions.

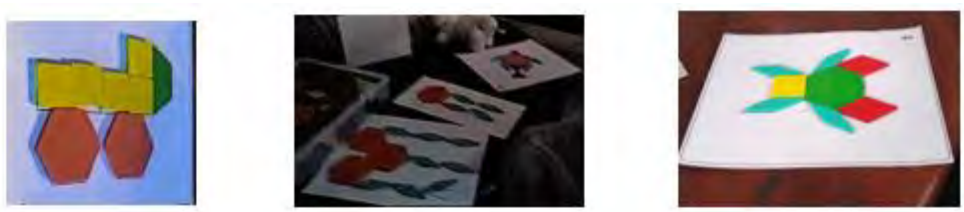

Figure 2. Images of Eva's work at Picture Maker level (sessions 5—7).

Sessions 8-14: Shape Composer. Eva described puzzles with interior lines provided as easy, and now she began to fill in those with fewer interior lines and more shared sides. During the first sessions at this level, Eva was showing more evidence of intentional actions by looking at the picture and then selecting a particular block. When a selected shape did not work at first, rather than try to turn and orient it differently, Eva would return to the container and look for something else. She showed more perseverance, trying several blocks before abandoning the task. Once she placed a block that appeared to fit but in combination with additional blocks no longer did so. Rather than replace any, she added a block that overlapped others to fill the empty space.

In later sessions at this level, Eva spent more time thinking about what would fit before selecting a block. Eva began to intentionally turn the blocks to orient them. Also, she began to see herself as an expert and wanted harder puzzles. We introduced pentomino puzzles, and Eva returned to trial-and-error and resisted guidance or suggestions. In session 14, though she still used trial-and-error, she did use rotations and flips to try different orientations of the pentomino to see if it fit. Images of Eva's work at Shape Composer level are shown in Figure 3.

Figure 3. Images of Eva's work at Shape Composer level (sessions 8—14).

Session 15: Substitution Composer. Despite her previous confidence, on this day Eva did not feel ready for "expert" puzzles. She did complete a pentomino puzzle with two pieces and no interior lines which shared a two-unit long side by applying turns and flips. She filled a pattern block puzzle two different ways, but she became frustrated on the third attempt and said she "tried her hardest."

Figure 4. Image of Eva's work at Substitution Composer level (session 15).

The Story of Eva's Response to Instructional Supports

Having learned that Eva was very anxious about math and that math instruction usually focused on numbers, we decided to focus on spatial reasoning tasks. Perhaps unsurprisingly, a challenge arose during the first session. Eva tried a few pattern blocks to fill in a puzzle and did not find they fit and said she wanted to do something else. Angie suggested looking at another section of the puzzle, but Eva got mad, stood up from the table, grabbed a scooter, and rode around her home's patio. Eva returned when Angie said they would do something different and brought out a whiteboard and markers. At the end of the session, Angie asked if Eva would make any picture she wanted with the pattern blocks. Eva made an image of a cartoon character she liked.

Based on interactions like that just described, Eva's engagement was a primary factor in task selection and instructional decisions. We selected activities that would be accessible to her and provide her with quick successes. For example, in the second session, we gave Eva a puzzle with no shared sides and easily recognizable shapes (i.e., square, triangle, and hexagon). For the next few sessions, we gave Eva mini-puzzle cards, each with two blocks, gradually increasing the proportion of the shared sides and gradually introducing less recognizable shapes (i.e., trapezoid and two rhombuses). Eventually these cards only provided the outline with no interior line. After Eva completed a few of these mini-puzzles, we offered her a larger puzzle that incorporated the shapes she had just worked with and asked if she wanted to try it. Using this approach of offering accessible puzzles and choices, Eva began filling in more complex shapes, perhaps due to increasing confidence or awareness of attributes or both.

We initially planned to discuss mathematics involved in tasks and to mathematize her activity to support her naming of shapes, noticing and describing attributes, and orienting shapes to compose a larger shape. However, when Angie discussed how many blocks were used in a section or tried to show the difference between angles of different shapes, it elicited avoidant behaviors–stopping participation, leaving the table, and expressions of frustration, anger, or sadness. This was even more evident when Angie asked Eva for a verbal response in return. Eva expressed that she didn't know how to answer Angie's questions or answered by repeating what

Angie had just said. The following transcript illustrates one of the times when Eva was upset by Angie's approach to providing guidance:

> Angie: But you know what I see? I see lots of gaps and holes. I wonder if there's another way that might work better. Is there another way that will work better?... [Eva starts making humming noises] I just want you to show [Angie pauses]…. Does that work? …[Eva makes a noise like "grrr"]
> Eva: This is right.
> Angie: This is right? [Eva makes a noise like "grrr"] How do you know that?
> Eva: [sounds as though she is growling out the words] It looks just like it.
> Angie: It looks just like it?
> Eva: [loudly] Yes it does.
> Angie: Okay. Okay, all right.
> (Simultaneously) Angie: Eva, I'm gonna use that description that you just used… Eva: I'm in the yellow.
> (Simultaneously) Angie: I'm going to ask you to…. Eva: I'm in the yellow!
> Eva: I…am...in…the…yellow!
> Angie: Yes, I heard you say that.
> Eva: You don't continue if you're in the yellow.

Angie stopped and looked to Eva's mother who explained that when at school 'in the yellow' means that Eva is upset, and it is a time to bring in tools to help Eva regulate her emotions.

Eva consistently responded negatively to guidance in the form of explicit explanations, discussion of math involved in the tasks, or corrective feedback. However, if mathematizing Eva's activity came in the form of specific praise, such as, "Wow! What great problem solving! I saw you *turn* the block to find out if it would fit another way," it had a much more positive effect. Angie would mimic Eva's activity while giving the praise, emphasizing important terms and motions. Eva would then continue to draw on that behavior or strategy in future puzzles. Eva also appeared to learn the names of some of the pattern block shapes after hearing them in the context of praise across several sessions.

We settled on an approach that offered Eva choices of puzzles that very gradually increased in complexity and which were likely to elicit some of the behaviors or strategies we hoped to see. We then used praise to draw her attention to those elements. Thus, our conjecture needed to be revised: engagement and progress would be supported by a gradual, systematic increase in complexity allowing for student choice and mathematizing through specific praise.

Over the course of the sessions, Eva began to express confidence. In session 7, Eva said she wanted "a very, very hard one." When Angie asked what she would do if it was too hard, Eva said she would "think hard about the shape." In session 8, she said, "Give me all challenge." In session 11, she picked a puzzle she said "would be hard for other people." By session 14, Eva said she was an expert and didn't know why she had to do the puzzles again. For this session, we had also planned to switch to puzzles that involved pentominoes rather than pattern blocks to encourage her attention to "turning" and "flipping." Eva's interest in the new puzzles was piqued, but it was clear when these were introduced, that her persistence with these still depended on early successes.

Discussion

This microgenetic study provides information on several different aspects of Eva's learning: presence of change, rate, stability, sources of change, hindrances, and generalization (Siegler, 2006). Eva's actual trajectory in 2D shape composition aligned with the established LT (Clements & Sarama, 2021; Sarama & Clements, 2009). Eva demonstrated increasing ability to use multiple pattern blocks to compose larger composite shapes with fewer lines as interior cues. She spent more time working in Shape Composer level than in the previous levels as she became more intentional in selecting and rotating blocks to complete the puzzles. Rather than suddenly demonstrating multiple actions that comprise a higher level in the progression in a single session, Eva demonstrated gradually more sophisticated actions-on-objects across sessions. These changes also appeared to be stable. In fact, Eva found tasks of a similar level of difficulty "too easy." Eva also appeared to be close to generalizing some use of attributes and transformations (i.e., turning and flipping) to solve problems with pentominoes.

A teaching experiment intentionally applies and tests a conjecture in real teaching and learning interactions (Confrey & Lachance, 2000; Steffe et al., 2000). Teaching, data collection, and analysis are responsive to the students, and conjectures are evaluated and revised accordingly. We found that Eva quickly abandoned tasks for which she did not possess a ready solution strategy. We also found that directing Eva's attention prior to activity, providing explicit explanations or demonstrations, or drawing attention to challenges were ineffective in engaging her with the task. Our conjecture evolved to state that a gradual, systematic increase in complexity which allowed for student choice and mathematizing the student's activity through specific praise would support engagement and progress. As we adjusted instruction to align with this conjecture, we found Eva expressed growing confidence and willing engagement in 2D shape composition tasks. We ensured she had the opportunity to choose puzzles and begin to solve them without any guidance at first, allowing her the opportunity to approach the task in whatever way made sense to her. We ensured systematicity by carefully selecting or creating puzzles that gradually increased in difficulty. We supported her continued use of strategies or vocabulary by providing specific praise when strategic thinking was observed and naming the shapes she had used.

A noteworthy aspect of these findings is that the revised conjecture problematizes recommendations that instruction intervention contexts should incorporate explicit explanations and specific corrective feedback (Grigorenko et al., 2020; Fuchs et al., 2021). A direct, explicit instructional approach did not encourage Eva's engagement with mathematical tasks. Rather, this study adds to other research which has shown students who have experienced difficulty in mathematics learning situations can benefit from instruction which elicits and builds on student thinking (e.g., Hunt et al., 2020; Xin et al., 2016). In Eva's case, that guidance was most successful when it also celebrated her successes. We are continuing to analyze Eva's responses to better understand how guidance impacted her engagement in other LTs. We believe these findings add to the literature which suggests LTs can provide a structure for establishing more "harmonious" mathematics learning situations for students like Eva.

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