INVENTED STRATEGIES CHANGING TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE

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This study investigates how utilizing student-invented strategies in the classroom can inform teachers' pedagogical content knowledge. Two elementary school teachers participated in professional development discussing the benefits of invented strategies. Data was then gathered as the participants implemented this practice in their classrooms. Data was analyzed qualitatively to show the ways in which invented strategies can be useful in a teacher's development of their pedagogical content knowledge, including their Knowledge of Content and Students, Knowledge of Content and Teaching, as well as Knowledge of Content and Curriculum.

Keywords: Teacher Knowledge, Instructional Activities and Practices, Elementary School Education, Curriculum

As one elementary school teacher in this study told me, "In 20 years of teaching I have never come across a math curriculum that encourages teachers to ask students to solve problems with their own invented strategies." In her experience, teaching in three different states, she had not been exposed to the importance of giving students time to develop their own algorithms. After the first day of implementing this practice, I talked with the teacher, expecting to hear validation of the research about students' capacity for thought and their flexibility in their understanding (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Chambers, 1996). I was surprised that the conversation centered on what she, as the teacher, had learned. Her understanding of the students, of her own teaching, and of the curriculum changed after watching students invent their own strategies to solve problems.

The teachers in this study reveal how their pedagogical content knowledge (PCK) increased. Some of this growth was pleasing to them. It amplified their current beliefs and ways of thinking. It resonated with past experiences and harmoniously pushed them in ways that excited them. On the other hand, some of this growth was jarring for the teachers to experience. They found this practice confusing, and claimed that it created dissonance between their past teaching practices and what they wanted to accomplish. This study investigates how teachers can increase their PCK directly from observing their students use invented strategies.

Objectives

We know the value of invented strategies for students (Carroll, 1999). In this study I begin to uncover the value of using student-invented strategies for the teachers themselves. My study is exploratory in nature and seeks out possible ways in which teachers can use this practice to change how they see and understand their students, what they teach and how they teach, and how they use the curriculum in their teaching.

Framework

My research uses Deborah Ball's construct of the mathematical knowledge needed for teaching (Ball, Phelps, & Thames, 2008). As Ball illustrates, there are several components of PCK. Knowledge of Content and Students (KCS) is knowledge that involves the dynamic

between understanding specific mathematical concepts, and familiarity with students and their thinking about those concepts. This includes knowing student conceptions as well student misconceptions. Teachers need to be able to predict how students will perform on a task, and what students will find challenging, what students will find easy, and what students will find confusing. As students are developing their own concepts, even while those concepts are still incomplete, teachers need to be able to understand the students' language and ideas.

Knowledge of content and teaching (KCT), is knowledge that combines knowing about teaching and knowing about mathematics. Teachers need to understand the architecture of a task and what types of responses will be elicited by the design of the task. Teachers need to know how to sequence particular content for instruction. They need to discern the advantages and disadvantages of different tasks, manipulative, and technologies. When developing content with students, they need to make decisions about which student ideas to pursue, when to pause for clarification, and when to pursue a new avenue.

Knowledge of Content and Curriculum (KCC), the intersection between knowledge of mathematical content and knowledge of the curriculum from which it is taught, is not developed in the literature, but Ball theorizes that KCC is a factor in a teacher's PCK. As Hill suggests, currently KCC is an "interim placement…still in need of revision and refinement…it may run across several categories or be a category of its own" (Hill, Ball, & Schilling, 2008)

My study also relies on the importance of student-invented strategies in the classroom. Carpenter's longitudinal study of invention and understanding in multi-digit addition and subtraction with children shows that students benefit from inventing their own algorithms and strategies in a variety of ways (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998). The authors explain that, in contrast to standard algorithms, invented strategies generally are derived directly from the underlying concepts. Therefore, students who are asked to devise and use their own strategies demonstrate knowledge of underlying concepts before students who relied primarily on standard algorithms. Invented strategies demonstrate a characteristic hallmark of understanding, and children who use them are able to use them flexibly to transfer their use to new situations.

In this study, I helped practicing elementary school teachers understand the benefits of invented strategies. They implemented this practice in their classrooms and, in doing so, they learned more about their students, they discovered benefits of and problems with their teaching methods, and changed their relationship to the curriculum.

Methods

After teaching a series of professional development courses at an elementary school, one second-grade teacher, Julie (all names used are pseudonyms), approached me and asked me to look over the text they were using to "see what was missing." I noticed that student-invented strategies were not addressed in the text, nor in her curriculum. She was very interested in exploring the use of this practice. For the second half of that school year, Julie intermittently gave time to students to try their own strategies before teaching them the standard algorithm. She was impressed with what the students gained from this experience, and I was surprised by and interested in what *she* was learning from the experience. This informal interaction developed into a pilot study. I began observing Julie teach each time she used invented strategies in her class and interviewed her afterwards. There was so much she was learning that we decided to continue the study the following school year and to include other teachers at the school.

Julie approached all of the other second and third-grade teachers at her school to see if they were interested in participating. Over the summer I met with four teachers to discuss the research

regarding invented strategies and the benefits for students. Unfortunately, by the end of the summer two of the teachers had left the school. This was the summer of 2020, and it was unclear if we were going to be able to continue the study at all because of the COVID-19 pandemic. However, this particular school district did teach face-to-face for the 2020-21 academic year. So, although the number of participating teachers was down to just two, these two teachers were still willing to take on this new challenge, in the midst of all of the other pandemic-related difficulties, and learn more about student-invented strategies.

I met with Julie, a second-grade teacher, and Kamila, a third-grade teacher, several times during the summer and at the beginning of the school year to discuss general benefits of invented algorithms. We focused on things teachers should know about the importance of and reasons for employing invented algorithms (Trafton, P., & Thiessen, D., 2004). We also discussed content-specific invented strategies such as children's conceptual structures for addition and subtraction (Fuson, Wearne, Hiebert, Murray, Human, Olivier, & Fennema, 1997; Huinker, Freckman, & Steinmeyer, 2003). The teachers decided, completely on their own, how they would use this knowledge to change and enhance their teaching over the following year. Both teachers decided to use invented strategies before each section that involved a new algorithm.

I observed and took notes during any class time during which teachers were using invented strategies. I made copies of materials used by the teacher, took photographs of board work, and recorded other relevant data including occasional student work. After each observed class, I interviewed the teachers. I audio-recorded those interviews as well as took field notes. The field notes, curricular materials, board work, student work, and teacher interviews were used to analyze how the teachers' PCK was being affected by utilizing student-invented strategies.

The analysis was conducted qualitatively. Learning what the teachers noticed, what surprised them, what they expected, and what changes they made to future lessons due to their findings all provided me with an important avenue for investigating what they were learning. I reviewed the materials looking for patterns, and ultimately found evidence of several developments in the teachers. It was not until after I looked for patterns that I began thinking of the teachers' growth as improving their PCK. This became the avenue for how I grounded by research and began analyzing the data

Results

I discovered ways in which students using invented algorithms informed each aspect of the teachers' PCK. The following paragraphs will highlight some of the ways that teachers experienced change in their Knowledge of Content and Students (KCS), Knowledge of Content and Teaching, (KCT) as well as possibly Knowledge of Content and Curriculum (KCC).

Examples of Change in Teacher's Knowledge of Content and Students

Students as imparters and constructors of knowledge. Because students were given more of a voice in the classroom, teachers were able to get to know their students' capabilities on a deeper level. The teacher, as well as the students, began to see the children as 'imparters of information," according to Julie. Everyone in the classroom saw the students as "capable of knowing and sharing things" and not just the teacher. Julie and Kamila frequently described how they watched the students build confidence as they displayed their thinking to others or made connections to previous strategies. As Julie said, "I'd never asked what they already know. I know much more about that now." The teachers were able to see even their struggling students in a different way, focusing not just on what these students did not understand, but also seeing what they *did* understand. After a day of student-invented strategies Julie reported, "One of my struggling students today was able to solve a subtraction problem on her own today, without

having been taught how. I think the flexibility in solving problems in different ways has enabled her to get better at using the strategy that makes most sense to her. Even when posed with the traditional algorithm she uses drawings to help her solve problems and is making great gains. This is so exciting!"

Students spent a lot of time comparing and critiquing other students' strategies. The teachers realized this forced students to explain themselves more clearly and visibly. It encouraged everyone to participate. As Kamila said, "Everyone is working on *something*. No one is just sitting there. They are all willing to come up and share and talk about strategies. They feel successful and proud." Both teachers saw the benefits of students working with partners. They acknowledged that they often used partner-work with reading, but rarely with math. Now they could see how working in partnerships allowed students to help one another, and built confidence and ownership of their work.

Julie used invented strategies in the previous school year, as part of our pilot study. One day after school, one of the students from the previous year approached Julie and asked, "Did you teach them my strategy yet?" Julie did not immediately remember which strategy the student claimed as hers, but the student quickly reminded her. Julie told me, "It made me realize how powerful invented strategies are because kids take ownership of them. They are the constructors of their own knowledge. For her to remember that a year later made quite an impression on me!" The teachers began to look at their students differently: they were now capable of constructing their own knowledge and sharing it with other students.

Student learning styles. The teachers frequently talked about understanding individual students' ways of thinking more clearly. They did not only view their class as a whole, but the teachers could describe each student, what types of strategies each preferred and which strategies posed a struggle. This understanding helped the teachers group students more effectively. Julie described this process:

> After the lesson I sorted the student work into three categories: 1) students who have at least one strategy and were able to solve all of the problems, 2) students who had a start (part of a strategy or a whole strategy, but may have had a misconception or mistakes, and 3) those who don't seem to have any strategies at their disposal. In thinking about these groups of students, it confirmed for me which students have solid strategies and can work independently and most likely transfer these skills. It also showed me which students would benefit from scaffolding…which students need to work with concrete base ten blocks before working with pictures or abstract problems.

Teachers could describe individual students' learning styles after watching them repeatedly invent strategies because they could see which strategies their students relied on, and which strategies the students were using when they made mistakes. As one teacher reported, "When kids get to the standard algorithm they won't use it if they don't understand it. They will fall back on other strategies they have used." The teachers were more aware of how their students thought, how the students could best express themselves, and what tools helped them learn.

Student misconceptions. Julie and Kamila are seasoned teachers, each having taught their grade level for over 20 years. They were both surprised that letting students use invented strategies revealed student misconceptions of which they were previously unaware. For example, many of the strategies in these grades relied on an understanding of place value. But, as some students used their invented strategies, it became clear to the teachers that the students had unanticipated misconceptions about place value. Frequently, these misconceptions surfaced when students were discussing their strategies and they would use words incorrectly,

demonstrating to the teachers that the students did not fully understand the math concept behind the words.

The teachers were more informed when words such as "equal" or "whole," for example, were concepts that needed more instruction. It was also apparent to teachers when students did not understand *why* they were doing certain algorithms. Teachers described identifying a lack of connection between tasks within a unit that was building to a cumulative goal. There were many times when teachers thought the students understood a topic, but recognized that the understanding was not there when the teachers invited the students to build on that topic for the next topic, using student invented-strategies.

Student transfer. The teachers were very concerned about and interested in transfer as they discussed their students with me. Sometimes the teachers saw the students using previous knowledge to solve new problems more than they had expected. They saw invented strategies as a tool that students used to help make connections between different topics. There were also many times when the teachers expressed discouragement that the students were *not* able to build on what they had discussed previously. In some ways, invented strategies left as many questions as answers when it came to students' ability to transfer knowledge. After a year of using this teaching strategy, the teachers are still interested in invented strategies to investigate transfer.

Examples of Change in Teacher's Knowledge of Content and Teaching

Assessment. The teachers immediately saw invented strategies as a form of assessment. Julie described using invented strategies, and asking the students to articulate their thinking in writing, as one of the best things she could use to see what her students were understanding. "That was more helpful than seeing the problem and how they solved it," she explained. Julie now wants to use invented strategies mid-chapter as a formative assessment in her teaching.

Julie and Kamila also found themselves thinking more about assessment in general after using invented strategies. They examined the type of assessments they were using and realized a need for changes. In a conversation with the two teachers they said, "There needs to be some changes on a school level about how students are assessed. There should be more one-on-one assessment with math. We do it with reading; why don't we do that in math?"

In addition to assessment at the end of, or even during a chapter, the teachers realized invented strategies were an excellent tool to use for pre-assessment. After the very first day that Julie taught with invented strategies she said, "Truly today worked as a pre-assessment and helped me understand what skills my students already have in this arena, as well as helped me identify some action steps for how to address their needs." The teachers were looking to see how students were able to build on concepts they had learned previously and what new boundaries they could investigate all on their own.

Sometimes the pre-assessment went well and sometimes it did not. When the pre-assessment did not go well, the teachers learned they might need to go back and address concepts from previous lessons. As Kamila described, "If their foundation isn't there, they can't see it. They can't move forward." Based on how the students performed when trying strategies of their own, the teachers were able to make decisions about the upcoming lessons. The teachers were able to look beyond whether the students performed well or not on the pre-assessment. They were able to decide which concepts they needed to review more, the tasks that would most benefit their students, which students were best suited to work together, and which students needed more instruction.

Manipulatives. At the end of the study, when asking the teachers how they had changed, Kamila expressed that her relationship to manipulatives possibly had changed more than

anything else. She said, "I am the type of person that I like to have everything in its place, but now my intention is to have manipulatives accessible so that students can take any tool they feel will help. I already have this math station envisioned in my mind and am so excited about it!" She was looking forward to a time (past the pandemic) when it would be possible to set up a table of manipulatives for students to use at their discretion.

Because of COVID-19, each of Julie's students had access to their own individual bag of manipulatives that they could use anytime they chose. Even if individualized bags are no longer necessary after the pandemic, Julie plans to continue this practice so that her students have better access to a larger number and a larger variety of manipulatives.

Along with deciding that students need to have more freedom and more choices with manipulatives, teachers were able to tell when, as a classroom, they needed to revert to using a tangible tool instead of proceeding more abstractly. For example, when transitioning from single digit subtraction to double digit subtraction in Julie's classroom, she noticed that many of the students who tried their version of the traditional algorithm were subtracting the top number from the bottom number. She said this helped her understand that "I need to go back to using blocks and have them see that they cannot subtract in any order." Using invented strategies helped her see when it would be helpful to revisit using manipulatives.

The teachers also saw how more use of manipulatives changed the students. Julie noticed that her students seemed less intimidated by new problems that had not been explained to them. The students knew they could find a tool that could help them think through the problems. "Everyone has a starting point now," explained Kamila. The teachers found value in helping students increase their confidence and independence by giving students autonomous access to manipulatives when using invented strategies.

Student-led discussions. According to the teachers, using invented strategies has guided them to a more inquiry-based approach for teaching. Julie learned that getting her students to think deeply about a topic was more easily done as a discussion that was student-led and centered on their own approaches. She realized that when she had been relying on the textbook, the discussions were always teacher-led. She could see that even when the class was not discussing invented strategies, if she wanted deeper thinking, students needed to become more involved. She resolved to plan for times when students would lead the dialogue in all subjects.

Kamila discovered that, when she asked her students a question about their strategy, the students initially would assume that they were wrong. This helped her understand that she was not routinely asking enough questions about how the students were thinking. Both teachers suggested that using invented strategies has taught them to use questioning in a different way. They are more likely to ask questions such as, "What do you already know?" or "What strategy have you used before that could help you?" They describe using these questions as a way to maintain a high cognitive demand in the tasks, allowing students to think more deeply over a longer period of time. In a reflection at the end of the school year Julie said she wants to include "at least three thinking questions in every math class, no matter if it is a textbook lesson or an invented strategy lesson." Using invented strategies changed her teaching style to include more questioning, which elicited student-led conversations.

Examples of Change in Teacher's Knowledge of Content and Curriculum

Previous research is unclear about the details of PCK with respect to the teacher's knowledge of content and curriculum. As I was listening to the teachers talk about their relationship to the textbook and how it was changing, I saw possibilities of how using student-invented strategies could change a teacher's KCC.

Freedom from the textbook. After using invented strategies, teachers decided that they were relying on the textbook too heavily for their curriculum. According to Kamila, "the Common Core was, at least in part, designed to encourage students to think more critically. But we are still teaching systematically. We give students more freedom in reading. We need to do that more with math." Both teachers realized this could not be done if they taught straight from the textbook, as they had previously been doing. In a conversation with Kamila, Julie said, "Teachers need to feel enough freedom, and trust themselves enough, to not rely on the text. There needs to be willingness from teachers and administration to supplement the text so kids are getting opportunities to develop conceptual understanding." She said she felt an expectation to teach with fidelity to the text but that "if we do it with fidelity, we will never create deep thinkers."

Students need to have an opportunity to select and invent strategies and, in their experience, the text always tells them which strategy to use. The teachers decided that metacognition is not present when students work from their books because students are merely following procedures, compared to using invented strategies which gives students opportunities to think about their thinking. Kamila decided, "teaching the Common core is what we need to do with fidelity, not the textbook."

Thinking critically while using textbook. Even when using the textbook, teachers realized they need to be thinking more about their pedagogical choices. They realized that they could still facilitate student-led discussions and group work, and start a lesson with invented strategies, while still approaching the lesson from the text. In their opinion, most textbook lessons were not complex enough, so they could incorporate activities such as making an argument and critiquing others' work using textbook lessons. One of their conclusions at the end of the study, one they had not previously acknowledged, was that they could omit some parts of the units in the curriculum and add in other lessons. Invented strategies helped them feel freer to make pedagogical decisions. They could decide which units needed to be longer or shorter, and use their assessments to guide their practice. They could use modeling, even if it was not an explicit part of the textbook lesson. They learned that they needed to decide how to balance things like invented strategies versus the textbook's specific algorithms.

The teachers also realized that there were parts of the text they now understood better. By the end of the year, they discovered sections of the text that were intended as inquiry-based class activities, but they were not able to recognize them as such before they started using invented strategies. Kamila described a section in the textbook where the students were asked to write about their thinking. She realized she had not fully utilized that section in previous years, but was now able to see the purpose of the activity. Kamila and Julie both believed that teachers would benefit from professional development regarding textbook use, discussing how to encourage deeper levels of thinking among students and assessing the curriculum as a group.

Discussion

Learning about new ideas and changing teaching practice is both exciting and interesting, as well as difficult and discouraging. Kamila and Julie felt all of these emotions as they were asked to think more critically about their PCK.

Critical Dissonance

It was jarring for the teachers to watch their students fail to see patterns, make connections, or transfer knowledge in unanticipated ways when the teachers asked students to use invented strategies. This practice illuminated problems in ways that were occasionally surprising and disheartening. Teachers sometimes finished a lesson and realized students did not understand the

topic to the extent the teachers had supposed. Teachers sometimes realized that they had been using a word or an expression for some time that the students did not actually understand, but the teachers did not discover this until the students tried to employ their own strategies. These realizations were sometimes painful for the teachers, causing teachers to second-guess past lessons, and wondering what they had missed before. The teachers frequently wished that they had more time.

Resonant Harmony

Of course, much of what the teachers learned resonated with them perfectly. Imparting a stronger voice to their students helped the teachers accomplish their goals. Because of COVID-19 each student was given their own set of manipulatives to use. This was a providential turn of events that led to teachers learning that students need better access to manipulatives. It was not a challenging new idea; the teachers already knew the value of manipulatives. However, they learned how to improve the implementation of this idea.

In addition, learning that their students need to work more with others fit perfectly with the teachers' PCK. They were already implementing this practice in other subjects, but they realized that they were not doing it enough in math. These realizations amplified and improved their teaching, helping them push through the pandemic's obstacles to find a better way.

One of the most remarkable things about teachers such as these is their ability to change. These teachers took in new information about PCK and used it to change the experience of their students. Regardless of whether the change in PCK was difficult or easy for them, these teachers knew that it was necessary to improve their teaching. As Julie said, "Invented strategies are a way to open up the dialogue to include more student-directed discussion, to enable teacher reflection and analysis of student work, and to allow students to build on their conceptual understanding. I will include invented strategies in future lessons to ensure more student voice and encourage a more authentic mathematics community."

Limitations

There were many limitations to the study, specifically because of COVID-19. I had four teachers who were excited about participating and were planning on recruiting others in their grade bands before the pandemic hit. Then two of the teachers decided it was a good time to retire from teaching and others realized that, with all the difficulties that were facing them, it was not a good time to be in a study. So only being able to work with two teachers was very limiting.

Additionally, rules for being in classrooms at this time became a lot stricter. Schools were not interested in having more people interact with students and teachers. In fact, the only way I was able to attend the classrooms was to become a teacher aide. I was not able to conduct interviews with or film the students, but I did interact quite a bit with them and even served in other capacities, such as a running reading groups and going on field trips. This was a major reason for shifting the data collecting to only interviews with the teachers. This seemed permissible because the teachers were truly the participants of the study, but videos of the classroom would clearly have been helpful for data analysis.

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