EQUITABLE TEACHING PRACTICES: DEVELOPING EMERGENT BILINGUALS' POSITIVE MATHEMATICAL IDENTITIES

PRÁCTICAS EQUITATIVAS DE ENSEÑANZA: DESARROLLANDO IDENTIDADES MATEMÁTICAS POSITIVAS EN LOS BILINGÜES EMERGENTES

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This paper focuses on Emergent bilinguals (EBs) who traditionally face unequal opportunities to learn mathematics, harming their identities. The purpose of this paper is to illustrate how a fifth-grade teacher cultivated the development of her EBs' mathematical identities by giving them opportunities to participate in cognitively demanding activities. Drawing on a conception of mathematical identity as something that changes in response to different situations, we illustrate how a fifth-grade teacher positively impacted her students' mathematical identities. The results reveal that when teachers use instructional strategies such as distributing mathematical authority, positioning students as mathematically capable, and incorporating students' languages as a resource for instruction, their EBs have multiple opportunities to build positive mathematical identities.

Keywords: Equity, Inclusion, and Diversity, Instructional Activities and Practices, Problem Solving.

Most teachers in the United States have or will have Emergent Bilinguals (EBs) in their classrooms, and few are well prepared to teach them (Samson & Collins, 2012). In this paper, the term EBs is used as equivalent of the term English Language Learners (ELs) because it has a more positive connotation of students language abilities; It acknowledges students proficiency in one language who are becoming proficiency in another language (de Araujo et al., 2018). EBs face unequal opportunities to learn mathematics, reflected in their limited access to advanced mathematical courses and experienced and qualified teachers (Flores, 2007; Samson & Collins, 2012). One approach to address the opportunity gap is to provide EBs with meaningful opportunities to learn mathematics (Moschkovich, 2013). Providing equitable opportunities to learn mathematics for EBs entails increasing their access to teachers who focus on developing their reasoning and use language as a means to enhance instruction (Moschkovich, 2013). An outcome of such instruction is positive mathematical identities for EBs (Bartell et al., 2017).

This paper explores how "Ms. Wilson" implemented equitable teaching practices to support students to develop positive mathematical identities. Equitable teaching practices allow teachers to provide all students with opportunities to participate in cognitively demanding activities that enhance their mathematical identities (Moschkovich, 2013). The equitable teaching practices portrayed in this study were implemented to support every student, particularly those historically marginalized by race, class, ethnicity, culture, and language. The research question we explore here is: How do the opportunities provided to EBs during the implementation of equitable teaching practices help students develop positive mathematical identities?

Equitable Mathematics Teaching Practices

Equitable teaching practices in mathematics are defined as teachers providing all students with opportunities to participate in high cognitively demanding activities that respond to students' needs and enhance their mathematical identities (Moschkovich, 2013). Equitable mathematics teaching practices develop students' conceptual understanding, mathematical reasoning, and mathematical identities through discourse by integrating students' previous knowledge, cultures, and languages (Moschkovich, 2013). Bartell et al. (2017) identified nine equitable mathematics teaching practices: Draw on students' funds of knowledge, establish classroom norms for participation, position students as capable, monitor how students position each other, attend explicitly to race and culture, recognize multiple forms of discourse and language as a resource, press for academic success, attend to students' mathematical thinking, and support the development of a sociopolitical disposition. In this study, we highlight three of these practices as characteristics of Ms. Wilsons' classroom that positively impacted her students' mathematical identities: (1) Positioning students as capable, (2) Recognizing multiple forms of discourse and language as a resource, and (3) Establishing classroom norms for participation.

Teachers' instructional practices determine the degree to which students develop confidence in themselves and see them as mathematical doers (Aguirre et al., 2013). Teachers can support students' development of positive mathematical identities by believing in their students' capabilities, allowing students to use diverse strategies and forms of communication, giving students a voice, and monitoring students' identities development (Allen & Schnell, 2016). Students' positive mathematical identities are developed when their peers or teachers recognize their mathematical contributions (Wood et al., 2019). Unfortunately, EBs are more likely to face low expectations and be stereotyped, harming their mathematical identities (Flores, 2007). Positioning students as mathematical capable is promoted when teachers move away from stereotyping students based on their cultural backgrounds, select challenging curricula, and share mathematical authority with students (Bartell et al., 2017). Classrooms in which teachers hold high expectations for their students result in students having more opportunities to learn challenging mathematics (Flores, 2007). Teachers provide students with opportunities to communicate their ideas, justify their reasoning, make sense of others' ideas, and transfer their knowledge to other contexts and concepts (Banes et al., 2018; Moschkovich, 2013). Moreover, teachers consider students' cultural and linguistic differences as strengths (Bartell et al., 2017).

As part of their instruction, teachers should choose high cognitively demanding tasks and maintain or enhance that demand during classroom implementation. Cognitively demanding tasks have the potential to develop students' conceptual understanding through oral and written explanations that involve pictures, gestures, diagrams, and formal and informal language (Wood et al., 2019). In a classroom where promoting mathematical reasoning is the goal, tasks must provide opportunities for all students to engage in thinking, make comparisons, establish relationships, make conjectures, validate, and justify their thinking (Vale et al., 2017). Teachers of EBs must provide mathematically and linguistic rich tasks that allow all students to collaborate, interact, think, talk, reason, and use their first language (McGraw & Rubinstein-Ávila, 2009). Instruction of EBs needs to incorporate processes such as explaining, comparing, conjecturing, generalizing, justifying, and exemplifying, referred to as the language of reasoning (Bragg et al., 2016). Teachers should use questions, sentence frames, and prompts to support students in communicating their reasoning (Coleman & Goldenberg, 2009). Tasks serve as resources for positioning students as mathematic doers and consequently enhance their mathematical identities.

Equitable teaching practices for EBs support developing mathematical reasoning by implementing mathematical activities that allow students to use multiple resources to understand, solve, and communicate their mathematical ideas (Moschkovich, 2013). Teachers of EBs need to emphasize the development of everyday and academic English language and focus on culturally diverse and inclusive practices (Samson & Collins, 2012). Incorporating students' first language and cultures creates a school philosophy where home and community are integrated, facilitating access to the curriculum, reducing stress, and maintaining self-esteem (Necochea & Cline, 2000). Teachers are encouraged to use students' first languages for support when possible. For example, prior to a lesson, teachers might preview the content for the students in their primary language and identify vocabulary and different representations that may help students in the communication process (Coleman & Goldenberg, 2009). Additionally, translanguaging, identification of cognates (e.g., equation and ecuación), and use of words in context are strategies teachers can use to help students make meaning of new languages (Coleman & Goldenberg, 2009). Translanguating goes beyond language switching and includes all the linguistic resources to communicate based on social and cultural practices (García & Wei, 2014).

In general, students benefit from teachers establishing sociomathematical norms for participation as an equitable teaching practice (Bartell et al., 2017). Sociomathematical norms go beyond regular classroom norms and are specific norms in mathematics that regulate discussion, encourage students to justify their mathematical ideas, make their reasoning clear, and improve their reasoning skills (Yackel & Cobb, 1996). Sociomathematical norms influence the effectiveness of instruction by providing an adequate environment to support student engagement. In particular, establishing sociomathematical norms helps build a community by setting up and guiding discussions so that all students have a voice and their participation is valued (Bartell et al., 2017). As students become familiar with the accepted norms, they understand their role and the role of others in the classroom (Yackel & Cobb, 1996). In equitable classrooms, students have consistent opportunities to share their reasoning and to assess the reasoning of others; they become a mathematical authority in the classroom (Cobb et al., 2009).

Mathematical Identity

Research on mathematical identity considers students' persistence and interest in mathematics, their motivation to learn mathematics (Cobb et al., 2009), and the relationship between learning and cultural and social issues that influence the learning environment (Cobb & Hodge, 2007). Teachers' instructional choices determine the impact that instruction will have on students and the degree to which they develop positive mathematical identities (Aguirre et al., 2013). As part of instruction, teachers should support students to develop confidence and see themselves as powerful doers of mathematics. Teachers can accomplish this goal by grouping students to work collaboratively, establishing high expectations for students, and creating environments where teachers are facilitators of classroom activities (Bartell et al., 2017). Allen and Schnell (2016) share some strategies that teachers can use to support students' development of positive mathematical identities. These strategies include knowing and believing in the students, reconceptualizing mathematical success (e.g., giving value to the use of different solution strategies instead of already defined procedures), prioritizing students' voices, and monitoring identity formation. Teachers should know and believe in their students' capabilities. In short, "viewing student attributes as assets rather than deficits" (Allen & Schnell, 2016, p. 401). When students believe in one another as problem solvers, they are able to value the positive contributions of their peers.

Meanwhile, teachers can use instructional strategies such as representing new ideas in diverse ways, asking purposeful questions, listening to others, revoicing and paraphrasing students' reasoning, discussing an idea before writing it down, establishing connections, and communicating mathematics ideas using different forms of communication (Moschkovich, 2013). Teachers can prioritize student voice by giving students opportunities to discuss their ideas in small groups and then in the whole class. The incorporation of formative assessments can support teachers to assess student's readiness and understanding of the language and the content of the lesson, as well as tools to monitor identity formation (Allen & Schnell, 2016; Hakuta, 2014).

Beyond these strategies, distributing classroom authority and incorporating students' languages and cultures as resources during instruction are equitable teaching practices (Moschkovich, 2015) that support students' burgeoning mathematical identities. Using language as a resource allows EBs to discuss and share their mathematical reasoning and understand mathematics. An important part of such instruction involves teachers creating a learning environment where students' roles and norms for participation are well defined; specifically, students understand that their ideas are important and need to be publicly shared (Yackel & Cobb, 1996). Additionally, teachers need to promote respectful relationships among students and position them as mathematically capable (Bartell et al., 2017).

Theoretical Framework

The theoretical framework for this study is socio-constructivism, which integrates cognitive, constructivist, and socio-cultural theories to make sense of the different teaching practices and examine students' mathematical reasoning, languages, and identities (Shepard, 2000). Cognitive theories provide the means to look at students' thinking structures as they develop their solutions and specific forms of reasoning (in this case, algebraic reasoning). Constructivist theories guide our analysis of social constructions of knowledge through discourse by looking at classroom interactions. Finally, socio-cultural theories inform our interpretation of students' prior knowledge, experiences, and languages in developing their mathematical identities (Cobb & Hodge, 2007).

Research Methodology

During the 2019-2020 academic year, we gathered detailed evidence of a teacher's and students' experiences during problem-solving lessons (Creswell & Poth, 2018). An explorative case study helped us identify the research question and the procedures to refine our intervention (Yin, 2014). A fifth-grade teacher, "Ms. Wilson" who worked in an urban elementary school, and our research team, collaborated three times to develop the "Discursive Mathematics Protocol," an instructional protocol designed to support the development of students' mathematical reasoning and language competencies (see Kitchen et al., 2020, for additional information about the DMP). The DMP builds on Pólya's (1945/1986) iconic problem-solving heuristic and incorporates research-based "language practices" and essential teaching practices borrowed from the *Principles to actions: Ensuring mathematical success for all* (NCTM, 2014).

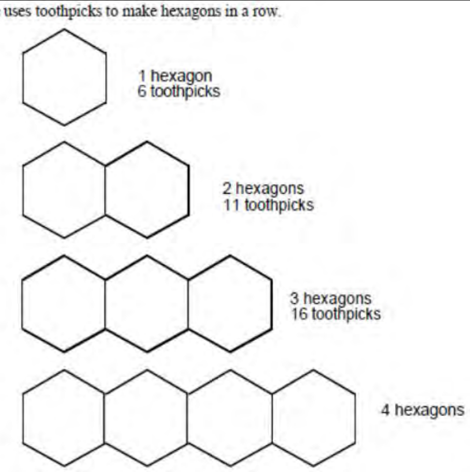
This paper examines how Ms. Wilson's implementation of the equitable teaching practices with the aid of the DMP contributed to developing her students' mathematical identities. Ms. Wilson is a bilingual (English-Spanish) White woman certified as an English as a Second Language (ESL) teacher. When this study was undertaken, Ms. Wilson had taught for five years, attending a significant number of EBs because of her ESL expertise. For each problem-solving lesson, the research team engaged Ms. Wilson in a planning session conducted via Zoom prior to the lesson and in a debriefing session after the lesson. The focus of the planning lessons was on the language and cognitive demand of the task, possible strategies and challenges, and the formulation of questions to elicit students' thinking and participation. The debriefing sessions serve as a space for reflection and improvement. The teacher and the team discuss what went well in the class and what can be improved to support students' participation and development of mathematical reasoning and language. During the problem-solving lessons, the three research team members and Ms. Wilson followed a co-teaching approach (Cook & Friend, 1995) to teach the lesson (roles and amount of teaching vary across lessons). Team teaching served as a strategy to directly support Ms. Wilson to plan, deliver, and reflect on the problem-solving lessons.

We gathered information to gain insights about the equitable teaching practices and how the different opportunities provided to EBs supported them to develop positive mathematical identities. We collected data from two problem-solving lessons that included students' work, videotapes of interactions among students and Ms. Wilson and just the students, and videotapes of Ms. Wilson carrying out each lesson. In addition, we interviewed Ms. Wilson to learn directly from her about the teaching practices she implemented to support EBs in developing positive mathematical identities.

We interpreted videotapes, student work samples, and the interview conducted with Ms. Wilson using interpretative methods (Creswell & Poth, 2018). The emergent themes summarize Ms. Wilson's beliefs and teaching practices during problem-solving lessons that supported students' development of positive mathematical identities. In the findings, we discuss each indicator by examining Ms. Wilson's narratives, her teaching practices, and her students' work.

For the problem-solving lessons illustrated here, the "Hexagons in a Row" task (See Figure 1) was implemented. In the first, second, and fourth questions, students start by finding near generalizations (patterns that can be found by drawing or making a short table) and then directly apply the derived functional relationship to find a far generalization (patterns that require to find arithmetic or algebraic expressions) (Callejo & Zapatera, 2017). In questions 3 and 5, students apply the inverse of the functional relationship derived (reverse thinking) (Callejo & Zapatera, 2017) to find the number of hexagons made with a given number of toothpicks.

Joe uses toothpicks to make hexagons in a row.



1. How many toothpicks does Joe need to make 5 hexagons? Explain how you figured it out.
2. How many toothpicks does Joe need to make 12 hexagons? Explain how you figured it out.
3. Joe has 76 toothpicks. How many hexagons in a row can he make? Explain how you figured it out.

Extension questions

4. How many toothpicks does Joe need to make 100 hexagons? Explain how you figured it out.
5. Joe has 1001 toothpicks. How many hexagons in a row can he make? Explain how you figured it out.

Adapted from Mathematical Assessment Resources Service
https://www.insidemathematics.org/sites/default/files/materials/hexagons%20in%20a%20row.pdf

Figure 1: Hexagons in a Row task

Research Findings

We identified three major themes related to equitable teaching practices enacted in Ms. Wilson's classroom during the mathematics lesson to support EBs' development of positive mathematical identities: Sharing mathematical authority, positioning students as mathematically capable, and incorporating students' languages as resources for instruction. Table 1 summarizes the different indicators we identified in the data. All the vignettes are instances of EBs' oral participation in the class or written work.

Table 1: Themes Derived for Mathematical Identity

Theme/	Indicator	Practice code
Sharing mathematical authority	Students had consistent opportunities to develop and articulate their solutions to tasks and assess the solutions of others (Cobb et al., 2009)	Exchange and assessment of ideas
	Students acknowledge the importance of making their ideas public shared (Bartell et al., 2017).	Making ideas public
Positioning Students as mathematically capable	Teachers believe their students are mathematically capable (Bartell et al., 2017).	Teacher's beliefs
	Choose and implement high cognitive demand tasks.	Tasks
Incorporating students' languages as resources	Incorporate of students' languages as resources during instruction (Moschkovich, 2015)	Language
	Review vocabulary and develop mathematical language (Banes et al., 2018).	Vocabulary
	Engage students in mathematical discourse (Banes et al., 2018).	Discourse

Ms. Wilson Shared her Mathematical Authority. She distributed the authority in the classroom, letting students share their ideas and assess the ideas of others. She engaged students in productive discussions that included asking purposeful questions, using and connecting multiple representations, interpreting diagrams, and discussing ideas in small groups and whole-class discussions. Throughout, Ms. Wilson encouraged students to build arguments and to reject or accept the strategies or solutions presented in the class. For example, three students shared the expressions they use to calculate the number of toothpicks to make 12 hexagons. Students discussed the similarities, differences, and validity of those expressions to answer the problem. Most of the students could indicate what each number in the expressions represented (number of hexagons, number of toothpicks to make one hexagon, and number of touching sides) and applied those expressions to respond to questions in the task. Laura described the three different expressions provided for her classmates giving the arguments for their validation (expression 1: 12 x 5 + 1, expression 2: 6 x 12 -11, expression 3: 5 x 11 + 6)

> Me and Sandra said that for the first one, 12 times five plus one; Ryan was looking at it like they were all five [the number of sides in a hexagon]. And then he saw that one of them has six sides while the others have five, so he had to add that one to show that one is six [has six sides]. The second one, six times 12 minus 11, shows that it was easier for them to think of them [the sides] as all six and then because only one of them was six and there was 12 hexagons, they needed to subtract 11 because one still has six [sides]. [For] five times 11 plus six; we thought of it as the two different groups, the groups of the fives and the one six, and so it just depends on how you group them.

Laura's explanation showed a clear understanding of the different strategies. She acknowledged her peers' ideas and tried to make sense of those ideas. Students presented their solutions, while

Ms. Wilson asked them to indicate the meaning of each term in their expressions. We can see that through this exchange, Laura was able to summarize the different strategies indicating how many sides were counted, how the hexagons were grouped, and why some expressions included addition and others subtraction. Laura was able to connect multiple ideas and restate others' thinking. In Ms. Wilson's class, students were constantly asked to make their ideas public and assess others' ideas to engage them in learning mathematics and as a means to demonstrate that their ideas were important contributions. Students in Ms. Wilson's classroom valued all the mathematical ideas presented to solve the task. Ms. Wilson let students decide when a strategy or solution was wrong.

Positioning Students as Mathematically Capable. Ms. Wilson's high expectations were reflected in her students' positive attitudes and participation in the class. She believed in students' capacities to solve challenging tasks. In the following excerpt, Ms. Wilson talked about the hexagons in a row task and the possibilities it offers to the students.

> … when I was thinking about [choosing a task] I was trying to think more in line with the content that I was doing, but I think this one was broader, and so it allowed for more thinking and problem-solving, which is something I would like to be focusing on more…how does this [content of the task] connect to a bigger mathematical idea or what can you take away from this and use later because I feel like there's so much good in math and the hexagons task that maybe some of them [the students] will say I can look for these different patterns next time.

When implementing cognitively demanding tasks, Ms. Wilson was challenged by some of her students who had negative attitudes toward mathematics or lower expectations for themselves. Some of her EBs were not confident about their abilities. We asked her how she dealt with EBs when they did not participate in the class and how she helped them realize that they could solve the task.

> Students like Samuel [an EB student], he definitely wants teacher direction first, even if he does understand. He wants to like to clarify everything with me. So I'm working on it with him of like, we can talk through it, but I'm not going to tell you what to do. And same with today, he really likes to just know that he's doing the right thing. And so, for them, it's just like building confidence.

Building students' confidence increased their participation and helped them develop positive mathematical identities. Ms. Wilson created a learning environment that empowered her EBs. She constantly compared their work to the work of mathematicians. At the conclusion of the problem-solving lesson, she stated the following to the whole class:

> What does complete [a task] look like? It is having an answer for each question, showing your work for each question, and then explaining your thinking. Each place [questions in the task] has explain how you figure it out. You will not be able to communicate with them [the researchers] after this task is over, right? So, you have to put it in writing. Great mathematicians put their ideas into writing.

Ms. Wilson had high expectations for her students and prioritized their voices. Students talked about mathematics, compared their solutions, tried other strategies, and assessed their thinking and other students' thinking.

Incorporating Students' Languages as Resources in Instruction. Ms. Wilson recognized and valued students' backgrounds as a resource to learn mathematics (Aguirre et al., 2013). Her acknowledgment of students' languages helped her EBs develop positive mathematical identities. As a whole class, they discussed the meaning of some words, used the words in context, and put the problem in their own words. Mathematical discourse played an important role in supporting students to understand the task and share their ideas using multiple forms of communication. Students in Ms. Wilson's class mainly increased their vocabulary by talking. Gestures were widely used to describe mathematical patterns during which students pointed at different parts of the diagram or showed how to count the toothpicks and the shared sides. Gestures combined with speech helped students assess their thinking by noticing disagreement between the description provided and the elements pointed to in the figure. One EB, Samuel, explained that there were four shared sides in four hexagons in a row. He used the diagram as an aid to explain his understanding of the task. When counting and pointing to the shared sides, Samuel realized that there were just three of them. The use of gestures supported him and other students to communicate their ideas. Ana, another EB, used her pencil and the figure representing four hexagons to explain how she found the number of toothpicks needed to make five hexagons:

> What I did was I got 6 as like a whole, so there is 6 right there [circle the first hexagon and write down 6], and then there's 1, 2, 3, 4, 5 [she outlined the second hexagon using the pencil] and then there's 5 [counting the third hexagon], and then there's 5 here and then is 5, 5, 5, and since there are five hexagons is another hexagon right there [sketching the fifth hexagon] so then 5 times 4, because is 4 right then plus 6 because is the hexagon here [She pointed out the first hexagon in figure 4] and then 26 which is the answer.

Consistently communicating her ideas and combining linguistic resources (gestures, diagrams, language) helped Ana and other students to assess their solutions and become more confident about their ideas.

Discussions and implications

In this study, we explored how the implementation of equitable teaching practices provided EBs with opportunities to develop positive mathematical identities. Ms. Wilson's implementation of equitable teaching practices, such as opportunities to solve cognitively demanding tasks, use multiple forms of discourse and language, position students as mathematically capable, and share mathematical authority, helped her students develop positive mathematical identities. She valued the ideas of her students and held high expectations. The excerpts show that representations, discourse, gestures, and language served as resources for students to communicate their reasoning, increasing their opportunities to participate in class to share and assess their ideas (Banes et al., 2018; Cobb et al., 2009). Students like Laura recognized multiple solution strategies and were able to justify the validity, similarities, and differences between them. Students in Ms. Wilson's class had the authority to evaluate others' thinking and were positioned as mathematical capable. Their linguistic approaches were seen as strengths instead of deficiencies contributing to the development of positive mathematical identities (Bartell et al., 2017).

Incorporating equitable teaching practices in the mathematics classroom benefits all students, increasing participation and enhancing students' mathematical identities (Cobb et al., 2009; Yolcu, 2019). This study demonstrates that incorporating equitable teaching practices is beneficial for EBs to develop positive mathematical identities. Specifically, our analysis of equitable teaching practices shows the benefits of providing opportunities for EBs to simultaneously develop mathematical reasoning and language competencies.

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