Articles published in the Proceedings are copyrighted by the authors.

## DEVELOPING ASSET-BASED INSTRUCTION THROUGH LEARNING TRAJECTORY-BASED CURRICULAR DESIGN

Jennifer M. Suh George Mason University jsuh4@gmu.edu Theresa Wills George Mason University <u>twills@gmu.edu</u> Sara Kirschner George Mason University <u>skirschn@gmu.edu</u>

Alayna Wearly George Mason University <u>awearly@gmu.edu</u>

Maureen E. Vora George Mason University <u>mvora@gmu.edu</u> Kate Roscioli George Mason University <u>krosciol@gmu.edu</u>

This research report describes a Learning Trajectory-based Curricular Design project that engaged teachers and coaches in the design and implementation process. As the project team, we focused on deepening teacher designers' understanding of the learning trajectory (LT) while situating student learning along a continuum to advance student thinking. Analysis of the design and implementation cycle demonstrated that teacher designers used their professional judgment and knowledge of LTs to assess the quality and appropriateness of curricular resources as they made instructional decisions to meet the needs of diverse learners. School-based coaches used these teaching resources as a type of professional development for identifying student strengths and "packaged" the resources for teachers who were overwhelmed from teaching during the pandemic. We discuss the importance of applying LT research for asset-based instruction.

Keywords: Learning Trajectories and Progressions, Teacher Knowledge, Instructional Activities and Practices, Standards (Strand: Curriculum, Assessment, and Related Topics; Mathematical Knowledge for Teaching)

COVID-19 interrupted teaching and learning in unprecedented ways and presented multifaceted challenges for students and teachers. As educators worked hard to support student learning in mathematics, the field looked for innovative ways to mitigate the challenges. In the spirit of PMENA 44's theme, Critical Dissonance and Resonant Harmony, we share how researchers and teacher designers worked collaboratively in design-based research to move beyond the "dissonance" created by COVID to build a curricular resource framed by learning trajectories (LT) and asset-based instruction to bring "harmony" to educators striving to meet the needs of every student.

# The Need to Translate Learning Trajectory Research to Practitioners

We situate our work in uncertain times, as Ladson-Billings (2021) calls in a "re-set school environment", where she asks educators to use an accurate assessment of what students already know with varied and regular formative assessments to determine how well students are understanding what they are taught. In this way, assessment is not a "punitive tool to 'catch' students but rather a diagnostic and developmental tool that will tell teachers and schools how to adjust their curriculum and pedagogy"(Ladson-Billings, 2021, p.75). This re-set requires teachers to be deeply knowledgeable about the learning trajectory (LT). It is critically important to introduce LT research to practitioners due to the scale of disruption and overwhelm of teachers

in the post COVID-19 learning environment. Teaching that incorporates an understanding of LTs has the potential to make instruction more efficient and supportive of learning mathematics for understanding (Suh et al., 2014; 2021). Clements & Sarama state that a LT describes:

Children's thinking and learning in a specific mathematical domain, and a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domain. (Clements & Sarama, 2004, p. 83)

In many academic and practitioner resources, the terms LTs and learning progressions are used interchangeably with the emphasis on the developmental progression of levels of thinking within a conceptual domain. Confrey's notion of LT/progression is described as:

A researcher-conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e. activities, tasks, tools, forms of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time. (Confrey & Maloney, 2010, p. 1)

According to Confrey (2012), there are five elements of LTs that teachers need to understand: 1) the conceptual principles and the development of the ideas underlying a concept; 2) strategies, representations, and "conceptions"; 3) meaningful distinctions, definitions and multiple models; 4) recognizing coherent structure or pattern in the development of progressively complex mathematical ideas; and 5) bridging standards or identifying the underlying concepts that "bridge the gap" between standards. Focusing on the five elements of LTs can improve instructional planning as teachers anticipate student strategies, representations, and conceptions that can be attributed to students' strengths and resources for building on their understanding. The potential of Learning Trajectory-Based Instruction (LTBI) in professional development (PD) settings has also been explored by the collective work of Sztajn et al. (2012), Wilson et al. (2015), and Myers et al. (2015), who examined how teachers' discursive patterns about students as mathematics learners changed as the teachers engaged with the LT in PD. In particular, they noted that teachers initially voiced expectations about students' mathematical ability related to student age or grade level. But as teachers' understandings of LT developed, their voiced expectations began to acknowledge that students' prior experiences influenced students' performance.

Asset-based instruction and the use of rigorous mathematics were central to the framing of our project and aligns with one of the key recommendations in *Catalyzing Change in Early Childhood and Elementary School* (NCTM, 2020) to develop "deep mathematical understanding" and build students "as confident and capable learners" (p. 11). Using Asset-based approaches to planning instruction is a conscious way to move away from deficit perspectives (Celedón-Pattichis et al., 2018). Teachers' explicit attention to focusing on strength in students' thinking and what children are capable of doing helps teachers in avoiding biases that impair teaching and learning. According to Gresalfi et al. (2009), what counts as "competent" gets constructed through an interaction between the opportunities that a student has to participate in a particular mathematics classroom and the student's uptake of those opportunities, meaning that structures that promote equitable participation and interaction are key. Positive and discourse-

rich classrooms (NCTM, 2000; Stein & Smith, 2011) allow each student to have feelings of success and pride (NCTM, 2020). The instructional decision to formatively assess and highlight student thinking during discussions has important implications for assigning competence, as it suggests *what* students are accountable for and to *whom* they are responsible for sharing their thinking with (Gresalfi et al., 2009). We believe that by finding strength in students' multiple knowledge bases (Turner et al., 2016; Kobett & Karp, 2020), teachers are better able to assign competence in student thinking, while broadening the notion of what competence means and building student agency and a positive sense of identity (Civil, 2007; Gonzalez, Moll, & Amanti, 2005; Aguirre et al., 2013; Lotan 2003; Cohen et al., 1999; Gresalfi et al., 2009).



Figure 1: Mathematics Learning Trajectory-based Curricular Design Framework

Context for our Math Learning Trajectory-based Curricular Design for Practitioners

The professional development design institute used a LT-based Curricular Design framework (Figure 1) which incorporated LT research, asset-based instruction, and rigorous instructional resources with high levels of cognitive demand. The team of teacher designers curated curriculum modules aligned to state-selected bridging standards. Bridging standards connect content across units within grade levels and articulate prerequisite knowledge for standards in future grades. The modules designed for each bridging standard consisted of five components: a) a zoomed-in LT bridge that illustrated the connection between students' strengths, bridging concepts, and the targeted learning standard; b) "big ideas" about the principles and development of each LT; c) important assessment "look-fors" that included strategies, representations, and "conceptions" to use for formative assessment; d) purposeful questions to assess, clarify, and advance students' mathematical ideas; and e) cognitively-demanding bridging activities, specifically routines, rich tasks, and games.

The LT research embedded in the modules provided direction for teachers to predict their students' potential reasoning, misconceptions, and learning. The modules were designed to support teachers in examining student thinking according to levels of cognitive proficiency rather than age or grade level. The curricular focus on asset-based instruction was intended to challenge and expand what teachers value and consider to be mathematical competence. Based on formative assessment of students' strengths, the modules guide teachers to select targeted activities in response to students' understandings and to support further learning.

### Methods

### **Context and Participants**

Using design-based implementation research, this qualitative case study (Stake, 1995) followed six early elementary mathematics educators from a professional development program to understand how these teacher designers applied a LT framework and asset-based lens while designing and testing curricular materials through two implementation cycles. After each

implementation cycle, teacher designers attended an implementation debrief meeting to share their experiences and change recommendations.

The teacher designers were purposefully recruited for the professional development based on their leadership and teaching experiences. The design process began in the midst of the Pandemic in the summer of 2021 with a 30 hour-one week Design Institute. Teacher designers then implemented the design modules in their classrooms during the fall of 2021 with iterative cycles of refinements. We met three sessions online to debrief each cycle of implementation. In the spring of 2022, we interviewed a core group of our teacher designers to learn about how they continued to use the LT based instructional modules to support diverse learners. This case study followed six total participants: three math coaches, Jana, who taught 11 years in the classroom and 7 years as an instructional coach; Mia, who taught 14 years as a classroom teacher and 14 as a coach; and Sienna, who taught 9 years as a classroom teacher and 18 years as a coach; a first grade teacher, Rebecca, who has taught 3 years, and two second grade teachers, Kara, who has taught 9 years, and Naomi, who has taught for 8 years.

### **Data Collection and Analysis**

Three data sources were analyzed for this case study: video clips from implementation debrief meetings that served as focus group meetings, student work, and teacher reflection forms. The implementation debriefs were conducted in focus groups with smaller groups of the entire teacher designers which allowed us to invite individual comments while also situating those comments in context of the group that worked together during the design process (Morgan, 2011). Based on the data analysis from this focus group debriefs, we selected six teacher designers to conduct one on one interviews. The research team used open coding first individually, keeping analytical memos which provided preliminary analysis allowing "processes of discovery in the material" (Morgan, 2011, p.14). Next, the research team employed Knodel's (1993) grid analysis to ensure researcher fidelity to the transcripts during subsequent analysis. Grid analysis allowed the research team to review transcript segments associated with each subtopic and calibrate the codes and categories to ultimately identify recurring themes.

### **Overarching Research Questions**

Our design based implementation research questions included:

RQ1) How do teacher designers use the Mathematics LT-based Curricular Modules during their implementation cycles?

RQ2a) How does involvement in this design project influence teacher designers' future work? b) In what ways did teacher designers' deep dive into LTs translate into their use of LTBI in coaching or leading school districts? c) How does the focus on strengths-based instruction influence teacher designers' approach to their instruction or work with teachers?

### Results

In addressing the first research question, how do teachers use the Mathematics LT Framework/LT-based Curricular modules during their implementation cycles, our findings revealed two themes. First, teacher designers used their professional judgment and knowledge of LTs to assess the quality and appropriateness of curricular resources for supporting students in meeting specified learning goals. Secondly, curricular materials designed using the LT framework supported teacher designers' understanding of students' learning and informed their instructional decision making. Teacher designers' demonstrated their professional judgment by assessing the quality and appropriateness of the curricular materials. Teacher designers implemented formative assessments and bridging activities to assess the quality and requirement criteria of each teaching resource. For example, through implementation and assessment of a teaching resource, Kara determined that it did not meet the requirement criteria of a rich task and recommended it be used instead as a formative assessment. Kara stated that "a lot of those tasks, they feel like worksheets. It was basically just a list of questions. ...That feels like a quick check. No richer. It didn't feel like anything too different than a quiz-like question." Kara questioned the "richness" of the teaching resource and determined that it would not be appropriate as a "rich task" but instead could be useful in other ways. Kara's assessment was affirmed by another teacher designer.

Teacher designers also used their knowledge of their students to assess the value of teaching resources. Rebecca found it difficult to implement a computer-based game with her first grade class due to the technological expertise needed to play it. Although the mathematical concepts of the game were well-aligned to her students' needs, they struggled with the website. Rebecca adapted the game to a paper-based format which was more accessible to her students. Mia used her knowledge of her students in a slightly different way. The substantial changes she witnessed in her students' mathematical confidence helped her realize the "richness" of the teaching resource. She reflected on how the high-quality teaching resources changed the ways students engaged in their mathematics learning, stating that "it was a huge difference just within a week's time. They felt more confident. That was the biggest thing I took away, the kids were not afraid of being wrong anymore, and they were very comfortable with being able to manipulate and do the work."

Additionally, the teacher designers critiqued teaching resources according to their alignment along the LT. After playing the game "Race to 100" with her second grade class, Kara determined that the game was a strong teaching resource for moving her students along the place value LT. Other teacher designers noted that while some games did not align well to a bridging standard, other games could be used for several bridging standards.

Teacher designers' reflections on their students' learning in response to the LT-based curricular modules revealed a second theme and provided a lens for teacher designers to notice their students' understanding and make instructional decisions. Jana, an instructional coach, discussed how the resources helped teachers in her school identify and address unfinished learning from previous grades. Because the curricular design used bridging standards, Kara was able to look for resources aligned to prior grades to support the prerequisite skills and knowledge her students needed for their current grade stating, "If you want to reteach anything, let's say I'm teaching second grade this year, so I'm going to reteach this first grade skill before I teach the second grade skill." Bridging standards were an important element of LTs for both Jana and Kara. This focus on underlying mathematical concepts allowed them to see a progression of complexity and shift along the trajectory as needed to find the appropriate teaching resources for their students.

The LT-based curricular structure supported the teacher leaders in reframing how they view student learning. Sienna, an instructional coach, found that the LT structure helped teachers see their students' learning as a progression and consider next steps rather than visualizing a gap between students' current understanding and the "final goal of the standard". Kara had a similar epiphany in her second grade classroom. She found that the LT structure allowed her to focus on conceptual understanding in her second grade students' work rather than simply quantifying the number of incorrect answers, stating "When you go to grade something or check it over, I'm not necessarily looking at 'Oh, they got 15 out of 20. They're missing a bunch.' ... I'm really zoning and honing in on what patterns I can find. I feel like that's what this cycle has taught me is that there are patterns in student work." Kara's attention to patterns in her students' work allowed her to identify their position on the LT and plan targeted instruction to support their learning. Kara pinpointed her students' understanding of double-digit addition and made appropriate instructional decisions, stating "We just started teaching double digit addition. I know they know their facts, but they aren't getting regrouping. Okay, well then that's what they need. We don't need to work on the basics. It's not that they can't add... They can add single digit numbers, so how can I break that down when we go to add something more complicated than that." Likewise, Rebecca suggested that the LT approach helps lessons to be more effective because they focus on where students are and their precise needs. The teaching resources were cognitively demanding and grounded in LT research, thus equipping Kara to look beyond the number of incorrect responses, focusing instead on student understanding and instructional decisions.

Naomi found the open-ended formative assessment questions to be valuable for revealing students' prior knowledge. On a formative assessment question asking for a number less and greater than 153, Naomi observed, "that open-ended part of those two questions shows their thinking. Like who's thinking one more or one less. For what's greater than 153, one of my kids wrote 582... That gave them a way to show other numbers they know, not just one more, one less." From the formative assessment, Naomi realized that her students needed more experience with numbers beyond the typical hundreds chart and chose the Mystery Number routine to broaden their exposure. In addition, she appreciated how an open-ended task allowed her to see the edges of student thinking with students naming different magnitudes of numbers that fit the criteria for the Mystery Number.

In addressing research question 2a) "How does involvement in this design project influence teacher designers' future work?", we found that teachers noted the ease of usability as a "grab and go resource". Teachers reported an advantage that all the materials were conveniently organized on a website, saying, "It is truly a one stop shop it really, really is" and "since we were already going to teach this anyways, it's easy and convenient, it is a one stop shop." However, this also led to a disconnect between the original project goal of sourcing pedagogically rich resources and instant implementation of those resources. In several instances, teacher designers initially liked the material, but struggled with implementation. Sienna said, "I really struggled with [the task], to the point where I actually had to create my own little recording sheet." She described a conceptual hands on lesson using base ten blocks, which required monitoring and listening to students working in the moment. The recording sheet helped her keep track and assess various student strategies. While teacher designers attributed successful implementation to the convenience of the materials, that convenience led to a perception that a "one stop shop" did not require the same amount of lesson planning.

With the disrupted instruction due to COVID-19, Jana noticed "gaps" in students' understanding, but found these resources useful for understanding and redefining the specific

nature of the "gaps" using strength-based language. For example, Jana states, "third graders who have first grade gaps and second grade gaps, this LT has been really important for [teachers] and really useful." Similarly Mia transitioned from "gaps" to strength-based language by noticing that teachers were more focused on student work rather than "just number correct." Mia also reported increased student confidence saying, "It was a huge difference just within a week's time and [the students] felt more confident. The kids were not afraid of being wrong anymore, and they were very comfortable with being able to manipulate and do the work." Mia also reported teacher confidence saying, "The teachers were like blown away that our kids can actually do this." This transition from "gaps" to strength-based language was particularly important as teachers implemented previous and current grade standards. Jana explained that "A lot of our teachers are trying to become experts in [prior grade] areas that they're not used to being an expert and having the LT and activities at their fingertips has been super helpful so far" because it helps teachers to "think more about the next steps that the students need."

In addressing research question 2b) "In what ways did teacher designers' deep dive into LTs translate into their use of LTBI in coaching or leading school districts?", we found that mathematics specialists and school-based coaches used these resources as a type of professional development to look at LTs using student strengths. Sienna described this as, "looking at more about what is the next step, the thing that [students] need, what is it that our students do know and because of that we're thinking more about the progression of the students, rather than the end." Mia explained that these resources were especially beneficial after the disrupted learning due to COVID-19 because they "gave me a tool, because I kind of felt as lost as they did. [sic] we've never done this before, we've never faced it... I feel like anybody I've told about it, I feel like it makes so much sense, the fact that it's [strength-based] like this is, this is what they need to come in with versus where they're going." However, the way the coaches implemented the resources varied. Mia created classroom-ready Google Slides for a routine. "I just kind of developed these little snippets of little activities of Google slides that I would use with the students and the teachers." Mia also explained the need for creating supplemental resources because "my teachers [sic] are stressed, they are worn out" and by "packaging" the resources, she can share them with teachers and the many long term substitute teachers in her school. Further, these resources have been accepted and distributed at multiple leadership levels from teachers, principals, and even division levels. Leaders at Jana's school and district were also eager to distribute the modules. She stated, "Our curriculum supervisor at the division level is the one who's really pushing it out for our division [sic] and in our curriculum unit guides that we have for each of our units k-5 with the PDFs from the bridging site have been dropped in so teachers have access to the site. They know about it, they've had to go through video training on it."

In addressing research question 2c) "How does the focus on strengths-based instruction influence teacher designers' approach to their instruction?", we found that teacher designers began to have a more holistic view of student knowledge and they were less overwhelmed. Making the connection to growth mindset, Sienna noted that the teachers she works with were

"thinking more about the next steps that the students need rather than 'they can't get to the standard' or 'they're not doing the standard.' It's more about 'this is where they're at, this is the next step they need to know."" For example, Mia shifted focus from percentage correct to patterns evident in student work, reporting, "there are patterns in student work, and you can often find that they're doing one thing correctly and then they're just misinterpreting whatever next step." This perspective of highlighting strengths reduced teacher stress, because "it just kind of helps it not seem as overwhelming because you can see progress in a student, even though they're not achieving the standard, if you're looking at their strengths and what they do know."

### **Implication of our Study**

This design-based research suggests several practical applications for teachers, coaches, and curriculum developers. Firstly, this research showed that teacher designers' knowledge of LTs supported their instructional decision making. The teacher designers who participated in the professional development program were recruited for their level of experience and recognized mathematics education leadership. And yet, after participating, they felt more equipped to identify and implement rich strength-based lessons. Specifically, teacher designers knew more about LTs and how to interpret student work in order to identify their strengths on the LT.

Secondly, this research showed that teacher designers who were coaches or leaders distributed their understanding about strength-based LTBI on both a small scale, such as coach to teacher, and large scale, such as school or district settings. While the structure of the modules provided support for teachers' understanding of the LT, coaches needed to constantly refer back to the resources until teachers used consistency when implementing them. All leaders reported that the materials were organized, accessible, easy to modify, and invoked discussions about student strengths and differentiated lessons.

Finally, this design research gives insight on the important aspects to consider when designing curricula and resources. The resources in this project were sourced by a recruited group of teacher designers. Through multiple cycles, the teacher designers vetted and implemented the resources, gathered student work, and discussed the implementation to validate the choice of activities. This cycle allowed teacher designers to propose, explore, discuss, and finally edit those resources in order to make them more user-friendly and applicable to the standards. This research demonstrated significant ways that practitioners can engage with learning trajectory research to transform "post pandemic pedagogy" (Ladson-Billing, 2021), reframing the predominant conversation from "educational gaps" to student progress along the LT continuum.

### Acknowledgments

The work reported here is supported by the Virginia Department of Education

### References

Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The Impact of Identity in K-8 Mathematics: Rethinking Equity-based Practices*. Reston, VA: National Council of Teachers of Mathematics.

Lischka, A. E., Dyer, E. B., Jones, R. S., Lovett, J. N., Strayer, J., & Drown, S. (2022). Proceedings of the forty-fourth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Middle Tennessee State University.

- Celedón-Pattichis, S., Peters, S. A., Borden, L. L., Males, J. R., Pape, S. J., Chapman, O., Clements, D. H., & Leonard, J. (2018). Asset-Based Approaches to Equitable Mathematics Education Research and Practice. *Journal for Research in Mathematics Education*, 49(4), 373–389.
- Civil, M. (2007). Building on community knowledge: An avenue to equity in mathematics education. *Improving Access to Mathematics: Diversity and Equity in the Classroom*, 105-117.
- Clements, D. H., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning*, *6*, 81-89.
- Cohen, E. & Lotan, R. & Scarloss, B. & Arellano, A. (1999). Complex instruction: Equity in cooperative learning classrooms. *Theory Into Practice*, 38, 80-86. 10.1080/00405849909543836.
- Confrey, J. (2012). Articulating a learning science foundation for learning trajectories in the CCSS-M. In Van Zoerst, L. R. Lo, J. J. & Kratky, J.L. (Eds.). Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology Mathematics Education (pp. 2-20). Kalamazoo, MI. Western Michigan University.
- Confrey, J., & Maloney, A. (2010). The construction, refinement, and early validation of the equipartitioning learning trajectory. In Gomez, K., Lyons, L., & Radinsky, J. (Eds.) *Learning in the Disciplines: Proceedings of the 9th International Conference of the Learning Sciences* (ICLS 2010) – Volume 1, Full Papers. International Society of the Learning Sciences: Chicago IL. Retrieved from http://dl.acm.org/citation.cfm?id=1854484.
- Gloria Ladson-Billings (2021) I'm here for the hard re-set: Post pandemic pedagogy to preserve our culture. *Equity* & *Excellence in Education*, 54(1), 68-78, DOI: 10.1080/10665684.2020.1863883
- Gresalfi, Melissa & Martin, Taylor & Hand, Victoria & Greeno, James. (2009). Constructing competence: An analysis of student participation in the activity systems of mathematics classrooms. *Educational Studies in Mathematics*, 70, 49-70. 10.1007/s10649-008-9141-5.
- Knodel, J. (1993) The design and analysis of focus group studies in social science research. In D. Morgen (ed.). Successful Focus Groups: Advancing the State of the Art (pp. 35-50). Sage.
- Kobett, B. M., & Karp, K.S. (2020). Strengths-Based Teaching and Learning in Mathematics: Five Teaching Turnarounds for Grades K–6. Corwin.
- Lotan, R. (2003). Group-worthy tasks. *Educational leadership: Journal of the Department of Supervision and Curriculum Development*, *N.E.A.*, 60, 72-75.
- Moll, L., Amanti, C., Neff, D., & Gonzalez, N.(1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory into Practice*, *31*(2), 132–141.
- Morgan, D. L. (1997). Conducting and analyzing focus groups. In *Focus groups as qualitative research* (pp. 46-65). SAGE Publications, Inc., https://dx.doi.org/10.4135/9781412984287
- Myers, M., Szajin, P., Wilson, P. H., & Edgington, C. (2015). From implicit to explicit: articulating equitable learning trajectories based instruction. *Journal of Urban Mathematics Education*, 8(2), 11–22.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2020). *Catalyzing change in middle school mathematics: Initiating critical conversations*. Author.
- Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26(2), 114-145.
- Simon, M. A., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning*, 6(2), 91–104. https://doi.org/10.1207/s15327833mtl0602 2
- Stake, R. E. (1995) The art of case study research. Sage.
- Stein, M. K., & Smith, M. (2011). 5 practices for orchestrating productive mathematics discussions. Reston, VA: National Council of Teachers of Mathematics.
- Suh, J. M. & Seshaiyer, P. (2014). Examining teachers' understanding of the mathematical learning progression through vertical articulation during Lesson Study. *Journal of Mathematics Teacher Education*, 18(3), 217-229.
- Suh, J. M., Birkhead, S., Frank, T., Baker, C., Galanti, T., & Seshaiyer, P. (2021). Developing an Asset-Based View of Students' Mathematical Competencies Through Learning Trajectory-Based Lesson Study, *Mathematics Teacher Educator*, 9(3), 229-245. Retrieved May 29, 2022, from https://pubs.nctm.org/view/journals/mte/9/3/article-p229\_1.xml
- Sztajn, P., Confrey, J., Wilson, P. H., & Edgington, C. (2012). Learning trajectory based instruction: Toward a theory of teaching. *Educational Researcher*, 41(5), 147–156.
- Wilson, P. H., Sztajn, P., Edgington, C., & Myers, M. (2015). Teachers' uses of a learning trajectory in studentcentered teaching practices. *Journal of Teacher Education*, 66(3), 227–244.

Lischka, A. E., Dyer, E. B., Jones, R. S., Lovett, J. N., Strayer, J., & Drown, S. (2022). Proceedings of the forty-fourth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Middle Tennessee State University.