

Teacher's Pedagogical Content Knowledge and Students' Academic Performance in Circle Theorem

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Abstract

Aim: The main purpose of this study was to investigate the impact of teacher's pedagogical content knowledge (TPCK) on students' academic performance in circle theorem.

Methods: The study used a mixed-methods research approach and a sequential explanatory design with a sample 210 students selected through the probability systematic sampling technique. The primary data was collected using questionnaire, an interview guide and a circle theorem achievement test. The data collected was analyzed using, the regression tool, and deductive manual thematic analysis, which was used for only the qualitative data collected from the interviews.

Results: The study found a significant relationship between the independent variable teacher's pedagogical content knowledge and the dependent variable academic performance in circle theorem signifying that students' performance in circles theorem depends on the pedagogical content knowledge of the teacher.

Conclusion: Based on the findings that emanated from the data analysed, the study concluded that teachers' pedagogical content knowledge has a significant relationship with students' academic performance in circle theorem.

Recommendation: the study recommends that Ghana Education Service organise training conferences and workshops aimed at improving teachers' pedagogical circle theorem content knowledge.

Keywords: *Teacher pedagogical content knowledge, circle theorem, students' performance*

1.0 INTRODUCTION

Education is essential to human development, and students' academic success is greatly influenced by the efficiency of instructional methods. The value of a teacher's pedagogical content knowledge (PCK) in the field of mathematics teaching cannot be overstated. That is why Yalley et al. (2021) expressed the view that teacher's pedagogical knowledge encompasses how the teacher intends to teach a subject to his or her learners. Yalley et al. (2021), definition seeks to portray PCK as a particular knowledge and comprehension that teachers have regarding the means to teach a given subject area. Laborde (2006) held the view that pedagogical knowledge includes all the strategies and means by which the teacher implements his or her lesson in a way that stimulates learning. Laborde's definition gives the implication that the knowledge of the teachers with respect to the way students learn, the mode of assessing students, and the purpose of education all come together to define the teacher's pedagogical knowledge. The knowledge of how teaching should be done and the science of teaching can be termed "pedagogical knowledge pedagogy" (Laborde, 2006; Sumara, 1998; Sumaya Laher, 2012; Thaanyane, 2021). Circle theorem, which examines the characteristics and connections of circles and their constituent sections, is a well-known field of study in mathematical education.

The circle geometry is an important ground upon which students can ensure effective geometric thinking and their ability to reason (Mohan Chinnappan, 2011). It helps build specific geometric thinking skills that support further learning links within and between geometric forms as expressed by (Behling, Förtsch, and Neuhaus 2022). Geometry has the nature of a thinking tool that aids in the formation of logical thinking and reasoning habits as well as the interpretation and assessment of mathematical arguments (Ansah, Quansah, & Nugba, 2020; Armah & Kissi, 2019).

Various reports from WAEC have revealed that students continue to demonstrate failure on circle theorem-related questions in the West African Secondary School Certificate Examination (WAEC Chief Examiners (WAEC, 2011, 2012, 2013, 2014, 2018). But the reason for which this continuous failure of students in circle theorem has remained persistent is an open but mysterious question. The inadequacy of students not being able to interpret and use deductive reasoning to understand and recognise relevant geometrical properties and also make use of theoretical statements deductively affects their lack of geometrical learning (Aji Wibawa, 2018). Artigue (1999) shared the view that students' difficulties in learning geometry could occur as a result of their challenges making meaningful information from objects both natural and formal.

Difficulties learning the appropriate language of geometry have been extensively discussed by researchers as contributing factors to the difficulties students face when learning geometry (Bledsoe, 2010; Bos, 2009; Bosson-Amedenu, 2018). Unfortunately, most students do not appreciate the fact that their success in school mathematics invariably has a strong influence over their success in life as stipulated by (Chand, Chaudhary, Prasad, & Chand, 2021). As at 2023, the lack of commitment by students to learn mathematics has resulted in many students still having content knowledge difficulties in mathematics, especially in the circle theorem, and this tends to have continual effects on their academic performance in the content and mathematics generally. For instance, the 2012 candidates who took part in the West African Senior High School Examination demonstrated poor factual knowledge and poor achievement in issues involving the circle theorem (WAEC, 2012). The Chief Examiner for Mathematics for May/June 2011 reported that candidates had trouble answering question 3a on the circle

theorem because they were unable to recall the relevant circle theorem relations (WAEC, 2011).

The Chief Examiner's report for WAEC revealed a significant number of students who failed to attempt circle theorem questions under questions 10 and 11, as those involved the application of geometrical concepts (WAEC, 2013; 2016). The reports revealed that the few students who made attempts on these questions only demonstrated inadequate content knowledge and a lack of understanding of the application of geometric theorems. The 2017 edition of WASSCE revealed a similar outlook where candidates could not sufficiently use applicable geometrical theorems and properties to calculate the values of required dangle problems (WAEC, 2016). The 2021 chief examiner report indicated a good number of students incorrectly solved circle theorem question 3a of the core mathematics paper, affirming their low knowledge and understanding of the circle theorem (WAEC, 2020). The report indicated that part (b) of question 11 was poorly answered by most candidates, which confirms students' dislike for the circle theorem. The 2020 WAEC Chief Examiner reported a downward trend in the academic performance of students in mathematics over that of the preceding years' WASSCE.

Some students choose to skip or avoid making an attempt to encircle theorem-related questions for lack of knowledge in approaching such questions, whereas those that make attempts show no convincing knowledge of understanding the concept (Boyd & Ash, 2018). Taking part in the Trends in International Mathematics and Science, Ghanaian students registered below-average mathematical performance compared to other students (Ansah et al., 2020; Dalbudak & Yiğit, 2021). Participants in TIMSS also revealed a conceptual mix-up regarding geometrical ideas and relationships.

Alex and Mammen (2016) identified that teachers' ineffective teaching methods and a lack of resources prevent them from engaging students in circle geometry lessons leading to students' failure to understand the circle theorem. Bos (2008), also stated that the regular teacher becomes even completely anxious of hearing the word circle theorem. This view of Bos (2008) only suggests that teachers' anxiety about teaching a particular concept hinders effective implementation of teaching and learning activities which affect students' understanding and academic performance. It is undeniable that human development and progress and mathematics relate in such a manner that the latter exhibits influence on the former. Therefore, ways by which students learning of mathematics can be improved both in conceptual and procedural formats so as to facilitate the desired development must be adopted (Dalbudak et al., 2021).

Carlson et al. (2019) shared their view in the context of pedagogy and technology that, teachers who are able to effectively incorporate educational technology into their mathematics lessons could ensure higher chances of creating and increasing high levels of confidence in pedagogical technology skills. These teachers at the same time by utilising student-centered approach to teaching could improve the academic performance of students in mathematics. Looking at the issues of students performing poorly as a result of their incorrect approach to mathematical problems, Bailey (2013) suggested that teachers create a supportive avenue for students to aid them develop the right rules and procedural knowledge required to solve questions correctly.

In the Ghanaian Senior High School mathematics syllabus, many students have perceived mathematics and circle theorem as very difficult concepts; hence, their perception of the subject becomes a barrier affecting their learning of it (Wiafe, Ayensu, & Yeboah, 2023). In almost every year's WASSCE Chief Examiner's report, students' achievement on circle theorem-related questions seems to receive negative comments and outlooks. Except for a few students who are able to demonstrate knowledge about the circle theorem, a significant number of the students demonstrate poor or inadequate knowledge of the circle theorem facts and the

application of its properties to solving problems. Therefore, it is crucial for educational research to examine how a teacher's PCK in circle theorem affects students' academic progress. The gap this study intended to close was that no study has investigated teacher's pedagogical content knowledge and senior high school students' academic performance in the circle theorem in the current study's location, Sefwi-Wiawso municipality and the Western North Region of Ghana.

1.1 Statement of the Problem

A teacher teaching what he or she knows to students and the ways in which he or she teaches his or her lessons have the power to either positively or negatively impact both the understanding and academic performance of students in certain perceived difficult concepts in mathematics. One of the perceived difficult concepts in mathematics on which students perform poorly is the circle theorem, and this has been confirmed by several reports by WAEC Chief Examiners (WAEC, 2011, 2012, 2013, 2014, 2018). The desire to carry out this study is inspired by students poor academic performance on circle theorem related questions and the numerous reports from WAEC Chief Examiners, which reveal almost every year students' inadequacy in solving circle theorem related questions during WASSCE (WAEC, 2011, 2012, 2013, 2014, 2018). Do teachers teach the circle theorem? How do teachers teach the circle theorem? If the circle theorem is well taught, then why do students continue to perform poorly in it? If teachers do teach the circle theorem and the teaching is effective, then as indicated by Aiyem et al. (2022), students should acquire and use the knowledge, skills, and attitudes acquired to enable them to act in a way that is consistent with the instruction.

The revelations by examination bodies such as WAEC on students circle theorem performance do not fit well with the view shared by Aiyem et al. (2022) that students must act in ways that are consistent with what is taught. The poor teaching techniques employed by teachers in teaching mathematical concepts, including other factors, are attributable to students' dismal performance in mathematics, especially in geometry (Arthur, 2022). Students' academic performance in circle geometry hasn't been sufficient, and the use of ineffective instructional tactics by teachers is one of the factors that have been linked to this underwhelming achievement (Ansah et al., 2020). Chief Examiners commented several times on students' weaknesses and difficulties in solving problems relating to circle theorem questions (WAEC, 2015, 2016) highlighting that most candidates usually avoid solving circle theorem questions, and even the minority of candidates that attempt circle theorem questions only demonstrate inadequate content knowledge. The Chief Examiner for the 2016 WASSCE suggested that teachers should take the necessary steps to make the teaching of mathematical concepts more practical and also relate them to real-life problems so as to help students appreciate what is taught (WAEC, 2016).

Current research studies have focused on pedagogical content knowledge and mathematics in general. However, there is not much research literature on the impact of pedagogical content knowledge on students' academic performance in circle theorem at the current research location where the study is carried out. This study is therefore intended to investigate teacher's pedagogical content knowledge and senior high school students' academic performance in circle theorem at a senior high school in the Sefwi-Wiawso Municipality so as to generate data that will provide grounds for suitable recommendations that will enhance policy formulation in line with helping students improve upon their academic achievement in both circle theorem and core mathematics generally.

1.2 Research Objective

Ascertain whether there is a relationship between a teacher's pedagogical content knowledge and students' academic performance using circle theorem.

1.3 Research Hypothesis

H₀: The teacher's pedagogical content knowledge has no significant relationship with students' academic achievement in the circle theorem.

2.0 LITERATURE REVIEW

The literature review of this study was done based on the research objective. Research studies that are in line with the current study were therefore reviewed in this section of the study. Thaanyane (2021) indicated that the knowledge of how teaching should be done and the science of teaching in that accord can be termed "pedagogical knowledge which is very paramount to excellent performance. To add to that, Kalioldanovna et al. (2022) study found that excellent performance depends on how knowledge is well convey to learners by the teacher. Zeng (2022) stated that the specialised knowledge possessed by teachers which they use to ensure a positive and effective teaching and learning space for every student, regardless of the subject matter, can be described as pedagogical knowledge and this influence the learning outcome greatly.

2.1 Teacher's Pedagogical Content Knowledge and Students' Academic Performance

Emmanuel and Charles (2021) carried out a study to find out the effect of the pedagogical processes of teachers on academic performance in Hargeisa District, Somaliland. The pedagogical process was taken to include all the conditions and circumstances that affect students physically, intellectually, and educationally; knowledge of teaching methods; classroom management; and classroom assessment methods. A cross-sectional survey research design was used with a sample of 160 teachers, and the study found that pedagogical processes, $F_0 = 43.062 > F(2,157) = 3.06$; $p = .000$, have a significant effect on the academic performance of the students. Therefore, Emmanuel et al recommended that the Ministry of Education and Science improve the education of teachers and enhance the training programs that improve the teachers' academic backgrounds.

It is argued that teachers are better able to design interesting, relevant, and fruitful learning experiences for their students when they are well-versed in pedagogical subject (Mohan Chinnappan, 2011; Nel, 2012). Improved student results may result from this, including greater topic knowledge, critical thinking abilities, motivation, and general academic performance as espoused by (Nel, 2012). In science research, the pedagogical content knowledge of the teacher has been classified as having an influence on academic performance and students' learning (Yalley, Armah, Ansah, & Palou, 2021).

Emine Gül Çelebi-Ilhan (2022) in his qualitative study has advocate for consideration of a comprehensive strategy that considers a variety of aspects, including as the calibre of instruction, student involvement, differentiation, formative assessment, and individual student requirements in order to enhance student achievement. Emine Gül Çelebi-Ilhan (2022) Study found no relationship between teacher pedagogical content knowledge and performance. That is why Funda Aydın-Güç (2021) described pedagogical content knowledge as the part of the teacher's knowledge that ensures that the teacher teaches effectively within a discipline. Bernard Fentim Darkwa (2021), indicated that if a teacher has certain content knowledge that allows him or her to assist students in accessing specific content knowledge in a meaningful way, then that knowledge possessed by that teacher is simply pedagogical content knowledge.

Mailizar and Fan (2020), indicated that teacher's pedagogical content knowledge serves as a sole influencer of performance. Other variables used in their study were found not to have any bearing with students' success in mathematics. Pedagogical content knowledge is one such important knowledge that teachers use to transform subject-matter knowledge in such a way that it becomes flexible, ensuring effective interaction between teachers and students in the classroom thus, enhancing performance (Sharafeeva, 2022). Meier (2021) Opined that the performance of students greatly depends on the teacher's ability to convey difficult concepts in a simple and understandable way, which helps students, understand the basics and lay a strong foundation. Sharafeeva (2022) stated that teachers' PCK enables them to adapt their instruction to meet the requirements and skills of students with varied learning styles. To increase performance, they can modify their teaching strategies, resources, and interventions to address the individual students' mathematical strengths, weaknesses, and particular difficulties.

Robin (2015), Indicated that collective pedagogical content knowledge exists along a continuum that ranges from broad, excellent discipline-specific pedagogical content knowledge to more specialised content-specific pedagogical content knowledge to concrete concept-specific pedagogical content knowledge which has a lot of impact on students' performance. Pietro (2013), indicated that teachers' pedagogical knowledge is a prerequisite for excellent mathematics achievement. In relation to the above, Shanley et al. (2019) said that professional development programs can have a big impact on teachers' mathematics PCK, which will give instructors tools and teaching techniques to assist their instruction and ensure student achievement. Di Martino (2014), Indicated that for children to develop their mathematical comprehension, effective instructional approaches must be used. A variety of instructional tactics, such as visual representations, real-world applications, and technological integration, are used by teachers with good PCK in mathematics. Students are given opportunity to explore, reason, and communicate about geometric concepts through these methods, which promote active involvement.

The critical importance of Pedagogical Content Knowledge (PCK) in predicting student achievement in mathematics is highlighted in this literature review. The results imply that teachers with excellent PCK and in-depth topic knowledge are better able to engage students, encourage conceptual understanding, and develop students' problem-solving abilities in mathematics. The review emphasizes the value of offering instructors' opportunity for focused professional development to improve their PCK in geometry education. The need for this study, which explicitly considers performance with the circle theorem, arises from the fact that nearly all studies concentrate on pedagogical content understanding and performance in general.

3.0 METHODOLOGY

The study used a mixed method to investigate how teacher's pedagogical content knowledge and students' achievement in geometry are related. By using this research method, the study employed both quantitative and qualitative procedures for data collection and interpretation. The used of the mixed method strategy gave the researcher the chance to make use of statistical instruments that precisely assess the correlations between the variables. The sequential explanatory design was employed as it adheres to the mixed methods procedure, which allowed the quantitative and qualitative phases to be carried out separately, followed by the integration of results from both phases to achieve a wider perspective and an in-depth understanding of the research questions and phenomena.

3.1 Population, Sample and Sampling Procedure

The accessible population of the study comprised all final year senior high school students in the selected school at Sefwi-Wiawso Municipality of the Western North Region of Ghana. A total of 440 students were present at the time of data collection, although 4.35% of the target population's members were not available owing to absenteeism and ill-health. Additionally, 53% (233) of students were female and 47% (207) were male made up the accessible population. A total sample size of 210 respondents was therefore determined based on Krejcie & Morgan (1970) table for sample size determination.

3.2 Data Instruments

Inferential statistics was used to analyze the quantitative data. The responses from the respondents were compiled and entered using SPSS. The association between the main variable and the outcome variable was determined. Students' Performance in Geometry was measure using Geometry Achievement Test which was made up of twenty-five (25) multiple-choice questions on both circle theorems chosen from WAEC past questions. Hence the instrument was valid. A questionnaire made up of 10 item statements were formulated to measure the teacher's pedagogical content knowledge by exploring students' views on what happens in their circle theorem classroom. The interview guide was used as a follow-up to the data collected and analysed from the questionnaire to seek an explanation for any unexpected outcomes. Based on the situations that were identified as anomalous or contradicting the questionnaire results, only a small number of students were interviewed

A pilot study was conducted to ascertain the reliability of the instrument. An introduction and ethical evaluation clearance letters were presented to the managers of the school that participated in the study. The management, after reviewing these letters, granted permission for the data collection to be carried out. The study's goal was explicitly communicated to the participants and they verbally agreed after being given the assurance that any information they provided would be kept in complete confidence. Participants were thus urged to refrain from using any identifiers or other information that could be used to identify them. The exam was the initial step in the data collection process. The questionnaire was given out a second time, and an interview was held based on cases that were observed and came from the information gathered through the questionnaire. In order to ensure coherence, participants were given codes to use when responding to the questionnaire After examining the data, a Cronbach's alpha reliability value of 0.848 was discovered. This shows that the instrument was trustworthy and implies an outstanding performance. Expert researchers in the field of the study evaluated and validated the instrument.

3.3 Study Area

According to the Ghana Statistical Service (2014), there are 139 200 people living in the Sefwi Wiawso Municipality, with men and women making up respectively 50.1% and 49.9% of the population.

4.0 RESULTS

H₀: There is no significant relationship between Teacher's Pedagogical Content Knowledge and Students' Academic Achievement in Circle Theorem.

In order to determine rejection or otherwise acceptance of the null hypothesis, the study performed a regression analysis on students' views regarding the teacher's pedagogical content knowledge and the students' circle theorem achievement test scores. The results are shown in the table below.

Table 1: Results of regression analysis on teacher’s pedagogical content knowledge and students’ academic achievement in the circle theorem

| Std. error | df | R ² | Beta | F | T | Sig. |
|------------|----|----------------|------|-----|------|-----------|
| 1.72 | 1 | .00 | .043 | .39 | 8.59 | P < 0.001 |

Source: Field data (2023)

The result indicated a p-value of 0.00 which is less than the alpha value of 0.005 suggesting that there is a significant relationship. There is no significant relationship between Teacher’s Pedagogical Content Knowledge and Students’ Academic Achievement in Circle Theorem. As a result, the null hypothesis H₀ was rejected. This means that a change in the independent variable (the teacher’s pedagogical content knowledge) will result in a corresponding shift in the dependent variable (academic achievement).

The manual deductive thematic analysis method was used to explore the data collected from students via qualitative interviews. This method was ideal because the study had pre-determined codes that guided the qualitative data collection. This method was also used because the study intended to obtain explanations and clarifications from students on unexpected results that emanated from the quantitative data. There was a summary of key recurring meanings of the data to organise themes, which were mapped against the complete data set. The table below shows the qualitative interview information.

Table 2 is structured in a column and rows. The column contains all the various codes which guided the qualitative data collection phase and are linked to the objective of the study. The rows of the table contain all the themes that were created from the codes which were used to summarise the key ideas and recurring meanings from the qualitative data. These themes were reviewed and mapped against the complete data set.

Table 2: Showing qualitative data analysis on pedagogical content knowledge and students’ academic performance in the circle theorem

| Codes | Organising Theme |
|---|--|
| Pedagogical content knowledge and students’ academic achievement in circle theorem | |
| <ul style="list-style-type: none"> Teacher moves fast when teaching | <ul style="list-style-type: none"> The pace at which teachers teach influences students’ academic performance in circle theorem |
| <ul style="list-style-type: none"> Teacher is Confident teaching circle theorems | <ul style="list-style-type: none"> The confidence level of the teacher in demonstrating mastery over the content of circle theorem influences students’ interest and academic performance in theorem. |
| <ul style="list-style-type: none"> Able to use alternative ways to teach circle theorem Engagement of students in hands-on activities | <ul style="list-style-type: none"> Teachers must use alternative techniques including practical activities to enhance students learning of circle theorem Involving students in practical teaching and learning activities enhances students understanding and academic performance. |

| | |
|--|--|
| <ul style="list-style-type: none"> • Focus on only few students | <ul style="list-style-type: none"> • Teachers with the right pedagogical content knowledge ensures an all-inclusive classroom environment where all students feel loved and motivated to learn. |
| <ul style="list-style-type: none"> • Teacher showed mastery of circle theorem content | <ul style="list-style-type: none"> • When teachers demonstrate mastery of the subject matter, it motivates students to learn more. |
| <p>Teacher:</p> <ul style="list-style-type: none"> • Is approachable • Is not approachable | <ul style="list-style-type: none"> • When teachers are not sociable, it affects students' participation in teaching and learning activities. |
| <ul style="list-style-type: none"> • Show responsibility | <ul style="list-style-type: none"> • showing responsibility for students learning makes them feel loved and motivates them to learn hard. |
| <ul style="list-style-type: none"> • Competent • Incompetent | <ul style="list-style-type: none"> • An incompetent teacher does not encourage learning so when teachers exhibit signs of incompetence in the classroom, it affects students learning and academic performance. |

Source: Field data (2023)

Findings as expressed by the views of students indicate that there is an influence of the teacher's pedagogical content knowledge on students' academic achievement

5.0 DISCUSSION

The results of the study showed that teachers' pedagogical content knowledge accurately predicted students' Circle's Theorem performance. This indicates that when students believe their math teachers are knowledgeable about the circle theorem from a pedagogical perspective, they will be more interested in and have a deeper comprehension of the subject than students who believe their teachers are lacking in this area. This is corroborated by one of the key informants: "The teacher plays a significant role in teaching and learning. The success of students in any area in mathematics depends largely on the teacher's knowledge of this area and his or her ability to convey this knowledge".

The findings also corroborate with Emmanuel (2021) when he carried out a study to find out the effect of the pedagogical processes of teachers on academic performance in Hargeisa District, Somaliland. The pedagogical process was taken to include all the conditions and circumstances that affect students physically, intellectually, and educationally; knowledge of teaching methods; classroom management; and classroom assessment methods. A cross-sectional survey research design was used with a sample of 160 teachers, and the study found that pedagogical processes, $F_0 = 43.062 > F(2,157) = 3.06$; $p = .000$, have a significant effect on the academic performance of the students.

This finding is also consistent with that of Nel (2012) who revealed a substantial association between instructors' pedagogical knowledge and students' success in mathematics in a quantitative examination of the predictive determinants of students' performance in mathematics. The pedagogical content knowledge of the teacher was classified as the major influence of academic achievement and students' learning. Without a doubt, the teaching and learning processes greatly benefit from the content understanding of math teachers. A teacher needs to have quality content and professional knowledge in order to be effective. Therefore, teachers' subject-matter expertise has a significant influence on pupils' academic performance.

However, Emine (2022) in qualitative research found no association between teachers' knowledge of pedagogical content and students' performance. However, given that Emine did his research at a primary school with less difficult curriculum than Senior High School levels, it's possible that this cannot be generalized.

6.0 CONCLUSION

Based on the findings that emanated from the data analysed, the study concluded that teachers' pedagogical content knowledge has a significant relationship with students' academic achievement in the circle theorem. This means that a positive or negative change in a teacher's pedagogical content knowledge will result in a corresponding shift in the academic achievement of students.

7.0 RECOMMENDATION

- The Ghana Education Service should organise training conferences and workshops that are aimed at improving Teachers' Pedagogical Content Knowledge. An improved teacher's pedagogical content knowledge will result in improved academic achievement for students, as found by this study.
- Schools must hire mathematics teachers who have the necessary pedagogical content expertise to teach mathematics in order to increase students' performance in geometry.

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