IDENTIFYING GRAPHICAL FORMS USED BY STUDENTS IN CREATING AND INTERPRETING GRAPHS

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In this paper, we describe a framework for characterizing students' graphical reasoning, focusing on providing an empirically-based list of students' graphical resources. The graphical forms framework builds on the knowledge-in-pieces perspective of cognitive structure to describe the intuitive ideas, called "graphical forms", that are activated and used to interpret and construct graphs. In this study, we expand on the current knowledge base related to the specific graphical forms used by students. Based on data involving pairs of students interpreting and constructing graphs we present a list of empirically documented graphical forms and organize them according to similarity. We end with implications regarding graphical forms' utility in understanding how students construct graphical meanings and how instructors can support students in graphical reasoning.

Keywords: High School; University Math; Cognition; STEM/STEAM; Interdisciplinary studies

Interpreting and constructing graphs that model mathematical or physical contexts is a critical competency across disciplinary fields (Driver et al., 1996; National Council of Teachers of Mathematics, 2000; National Research Council, 2012). While much previous work has examined student difficulties and non-normative reasoning related to graphing (Beichner, 1994; Glazer, 2011; Leinhardt et al., 1990; McDermott et al., 1987; Shah & Hoeffner, 2001), more work is needed that leverages students' knowledge related to creating and interpreting graphs. A new framework has recently been developed that identifies specific types of knowledge resources called *graphical forms*, that permits a finer-grained examination of how students think or reason about graphs (Rodriguez et al., 2019b). The purpose of this paper is to extend the work on graphical forms by empirically documenting and organizing a large set of graphical forms that students used to create or interpret graphs. This work permits researchers greater clarity on the cognitive work involved in constructing and interpreting graphs, and helps instructors know what types of knowledge students can develop or use for productive graphical activity.

Brief Literature Review on Student Graphical Thinking

Past research on graphical thinking has documented students' difficulties (e.g., Beichner, 1994; McDermott et al., 1987), with the consensus being that students' ability to interpret graphs depends on interaction between students' prior knowledge and the nature and content of the graphing task (Glazer, 2011; Leinhardt et al., 1990; Shah & Hoeffner, 2001). Some work has emphasized the nature of assumptions and conventions associated with graphical interpretation (Moore et al., 2019), including work that described students' use of *intuitive rules* to interpret graphs (Eshach, 2014). According to Eshach (2014), students develop a set of *intuitive rules* that share a similar ontology to diSessa's (1993) *phenomenological primitives* (discussed in more detail later) in the sense that they are constructed based on experiences. However, *intuitive rules* are more broadly useful and are not specific to explaining a physical phenomenon. This approach to considering how students interpret graphs is insightful in the way it provides explanatory

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power for students' reasoning that moves beyond identification of misconceptions (Beichner, 1994; Elby, 2000; McDermott et al., 1987).

Much of the literature indicates the role context plays in students' ability to extract information from graphical representations. For example, students tend to perform better when presented with decontextualized graphs in comparison to analogous graphs involving chemistry or physics content (Bollen et al., 2016; Ivanjek et al., 2016; Planinic et al., 2012, 2013; Potgieter et al., 2008). To examine context-specific graphs, recent work by the first author and colleagues has focused on students' graphical reasoning in chemistry, specifically in the context of chemical kinetics (Rodriguez et al., 2018, 2019a, 2019b, 2019c, 2019d, 2020a), which is concerned with modeling the rate of chemical reactions. A limited number of knowledge resources, called graphical forms, have been discussed in these studies, including *steepness as rate*, *straight means constant*, and *curve means change.* In some cases, graphical forms such as *steepness as rate* seem to have a particularly strong cuing priority, which, in part, could be influenced by students' tendency to inappropriately force time onto expressions and graphical representations that do not include time as a variable (Bowen et al., 1999; delMas et al., 2005; Jones, 2017; Popova & Bretz, 2018; Rodriguez et al., 2019d, 2020a, 2020b). In this paper, we build on this work by presenting several graphical forms empirically observed in students' graphical reasoning.

Theoretical Perspective: Graphical Forms

Knowledge-in-Pieces & Symbolic Forms

The construct of graphical forms is rooted in the knowledge-in-pieces (KiP) paradigm, a cognitive model that characterizes the structure of knowledge and the mechanism associated with conceptual change (diSessa, 1993). The salient feature of the KiP view is the manifold ontology of cognitive structure, in which knowledge is conceptualized as a network of fine-grained cognitive units that are activated in concert because of perceptual cuing. These cognitive units, which we call *knowledge elements* and *resources* interchangeably (see also Hammer, 2000), may reflect a variety of types of knowledge, such as ideas related to concepts, epistemology, or ontology. Building within the KiP paradigm, Sherin (2001) introduced the "symbolic forms" framework to describe mathematical resources related to symbolic equations. According to Sherin (2001), this involves associating an idea (conceptual schema) to a pattern in an equation (symbol template). Based on the introductory physics (classical mechanics) context in which the symbolic forms framework was initially developed, the symbolic forms characterized by Sherin (2001) reflected ideas associated with algebraic manipulations such as combining terms, proportional reasoning, and the role of a coefficient in scaling or tuning an expression.

Graphical Forms

The graphical forms framework reflects a natural extension of symbolic forms, providing the language to further characterize students' mathematical resources. Like symbolic forms, reasoning involving graphical forms is characterized by focusing on a structural feature and subsequently associating an idea (Rodriguez et al., 2019b). Whereas the symbolic forms framework focuses on the ideas assigned to patterns in equations, the graphical forms framework augments this work by emphasizing the ideas assigned to patterns in a *graph*. Previously, the specific feature attended to in a representation has been framed as a *registration* (Lee & Sherin, 2006; Roschelle, 1991), which in the context of graphical reasoning can vary in size—an individual may attend to and associate an idea with the entire graph or a specific region of the graph (Rodriguez et al., 2019b).

Although these resources may be activated and applied in less useful contexts, it is important to acknowledge that students have these broadly useful cognitive tools for reasoning that have the potential to guide students in the sensemaking process. Therefore, consistent with the knowledge-in-pieces perspective, research and instruction should emphasize providing insight regarding how we can support students in productively using the resources they have, rather than focusing only on cataloging misconceptions (Cooper & Stowe, 2018). Students seem to commonly draw on graphical forms such as *steepness as rate*, which can result in sophisticated conclusions regarding physical processes. In the context of interpreting graphs, this often involves initially anchoring reasoning in mathematics by drawing inferences using graphical forms and subsequently assigning discipline-specific principles to explain the observed graphical shape (Bain et al., 2019; Rodriguez et al., 2019, 2019a, 2019b, 2019c). In the case of constructing graphs, the reverse is observed in which students consider the physical scenario and subsequently utilize graphical forms as part of the drawing process to create a graphical shape that aligns with the phenomena (Rodriguez et al., 2020a).

The goal of this study is to begin to develop an empirical library of graphical forms, mirroring the current list available for symbolic forms (Rodriguez et al., 2019b); necessitating a clear definition of what constitutes a graphical form. As part of this process, we drew on extant education research related to graphical reasoning and Sherin's (2001) description of symbolic forms to consider the implications for the graphical analog. First, we draw attention to the idea that symbolic forms focused more on meaning than conventions. Second, symbolic forms emphasized the information communicated by an equation, without drawing an explicit connection to what an equation fundamentally *is* in an ontological sense. Moreover, to narrow the scope of the framework we decided to define graphical forms as assigning meaning to the curve itself, as opposed to other aspects of a graph such as the axes and graph labels (Kosslyn, 1989). In summary, our definition of a graphical form was refined to consist of a specific aspect of the graphical curve itself (e.g., a *graphical pattern*) and an intuitive *conceptual schema* associated with that aspect. Thus, our definition excludes beliefs about the nature of the graph, knowledge elements associated with the axes, or general knowledge about functions.

Methods

This paper reports on one set of outcomes from a larger study on students' graphical activity in relation to real-world contexts. In the study, twelve students across two universities at the beginning of first-semester calculus were recruited to participate in two separate interviews that occurred within a one-week timespan. One interview focused on constructing graphs that model real-world situations and the other interview focused on interpreting graphs. For space constraints, we do not present all nine tasks here, but have provided in Figure 1 one graph construction and one graph interpretation prompt that we drawn on in the Results section. The students were interviewed in pairs, and are given the pseudonyms Anna and Aria, Berto and Blaine, Cindy and Caleb, Donato and Demyan, Ellie and Eric, and Fiona and Felicity.

(A) Graph Construction

A homeowner mows the lawn once a week on Wednesday afternoon for 4 weeks in a row. Then the mower breaks and he decides not to mow the lawn for the rest of the summer. Graph the height of the grass as a function of time throughout the summer.

(B) Graph Interpretation

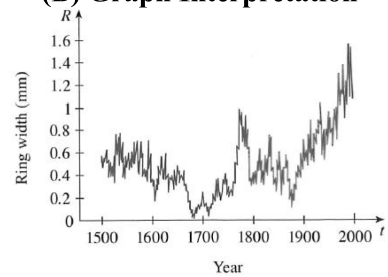

Figure 1: Prompts discussed in this paper.

Following transcription, initial data analysis involved dividing the interviews into bounded episodes based on content discussed to establish a codable unit (Campbell et al., 2013) and providing a narrative general overview of the student discussion within the episode (i.e., narrative coding) (Heisterkamp & Talanquer, 2015; Rodriguez et al., 2020b). Subsequently, we used a line-by-line analysis to analyze each statement within the episodes, focusing on the resources implied by what the student said—and did as they made the statement—also considering the context surrounding the statement, including nonverbal cues such as gestures. The process of identifying graphical resources involved a combination of deductive (previously identified graphical forms from the literature) and inductive analysis (identifying new graphical forms and other graphical resources). To refine our definition of graphical forms, we discussed together the various graphical resources we documented, which involved combining codes and creating new codes, some of which were determined to constitute graphical forms and others which were characterized more generally as "other" resources related to graphing.

Results and Discussion

We begin by providing examples of graphical forms observed in the data that have previously been identified in the extant literature. We then discuss new graphical forms identified and provide an overview list with the various graphical forms identified in the data.

Previously Identified Graphical Forms

Across the dataset various graphical forms were identified, some of which were previously discussed in the literature, such as *steepness as rate,* which involves students associating ideas about rate with the relative steepness of the graph (Rodriguez et al., 2018, 2019a, 2019b, 2019c, 2020a, 2020b). Given that this graphical form has been discussed in detail in previous work, we will not focus too much on it here, except to say that it was the one of the most frequent graphical form observed in the dataset, further building a case for its relatively high cuing priority, phenomenological basis, and its important role in graphical reasoning. For some of the previously identified graphical forms, as part of the process of developing a list, we also built on the prior descriptions, such as modifying *straight means constant* in favor of the more precise language *straight means constant rate*. This was to specify that students were focusing on *rate* as opposed to *values*. To illustrate this, one of the graph creation prompts involved a scenario related to modeling the height of grass over time (Figure 1A). When working through this prompt, Blaine and Berto initially drew the graph provided in Figure 2A, with Blaine describing the straight lines they drew as follows:

> **Blaine:** *So grass grows, um, it grows at a pretty constant rate, and you cut it every once, one or two weeks in the summer, at least where I live. Um, and then you would cut it.*

In this instance, Blaine's reasoning can be characterized as *straight means constant rate*, due to the emphasis on rate. Similar to *steepness as rate,* associations such as Blaine's above that drew a connection between straight lines and a constant rate were frequently observed in the dataset. Revisiting the distinction between specifying what is constant when describing a straight line, later in the interview Berto and Blaine drew a plateau as part of their graph (Figure 2B):

> **Interviewer:** *... what do those horizontal flat points represent to you again?*
> **Blaine:** *No growth.*

Here, Blaine is no longer referring to *rate* being constant, but rather the *height* of the grass being constant ("not growing"), indicated by the horizontal straight line (*horizontal as constant value*).

(A) Berto & Blaine **(B) Berto & Blaine**

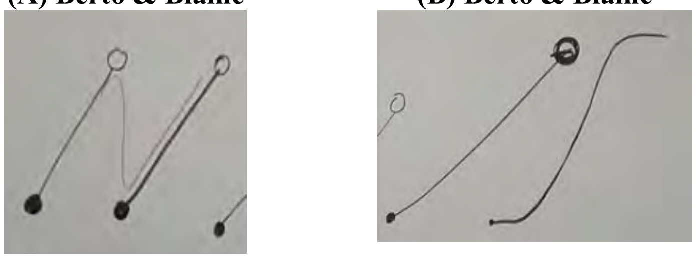

Figure 2: Two Types of "Straight": Linear-Straight as Constant Rate (A) and Horizontal-Straight as Constant Value (B).

New Graphical Forms

Although there is not space to provide student examples of all the new graphical forms identified, based on the contexts associated with the graphs provided, we discuss some of the graphical forms that emerged from the data that have not yet been discussed in the literature. For example, the nature of the grass prompt discussed previously (Figure 1A) resulted in students discussing ideas related to discontinuity, which we characterized using the graphical form *jump discontinuity means sudden*. Here, the graphical pattern of a jump/break in the graph was intuitively associated with a sudden event, such as when cutting the lawn results in a sudden decrease in height. As with other graphical forms, the name selected is intended to be descriptive for ease of communication and presentation. Another example of a new graphical form associated with the prompt in Figure 1A is *open/closed dot pair as existence*, as exemplified by Dontao and Demyan:

> **Demyan:** *... I want to show that the height is like, this is something continual, like grass didn't like stop, uh, existing there [i.e., at the discontinuity] for like a very split microsecond while it was cut…*
> **Donato:** *Yeah. I think that, in that case, you would do the like open circle here, closed circle there, but yeah, again, I don't think that happens-*
> **Demyan:** *And if that is what, what, what counts, like if that makes it clear in mathematical terms that the grass is still around, it's just, you know, cut from the edge or from the bottom at three inches, then yeah, I'm down for that change.*

The graph drawn by the students did not initially have open and solid dots (only slanted lines), which bothered Demyan because it seemed to imply that the grass was no longer there because the graph was not connected (continuous). After discussing it with one another, they adopted the

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solid-open dot notation utilized by other students in the sample (Figure 2) to express the concept of existence at a particular point.

When analyzing student responses to the construction and interpretation prompts, we also noted graphical forms related to how features in the graph suggest realism or indicate the graph involves empirical data. For example, revisiting the grass prompt with Aria and Anna:

Interviewer: *And what does that mean that it's like a straight line segment and then a straight line segment, like as opposed to like a curve?...*
Anna: *Then we would assume it just grows at a constant rate over time, but I guess that's not true either. Cause there's a lot of factors that can affect the growing that isn't just time.*
Aria: *Yeah. Like bugs.*
Anna: *Yeah. But we're not looking at that. We're literally just looking at if grass grew in terms of time and not in terms of other things. So realistically that's probably not what it looks like. It probably is more gradual because of other factors.*

Here, the students above, as well as other students for multiple prompts, were hesitant to draw straight lines because that implies a direct linear relationship between the variables. We characterize this reasoning as *curves mean realistic*, in which students opted to draw curved lines to account for unknown factors and avoid making assumptions about the relationship between the variables. Moreover, this graphical form was complemented with *jagged means data*, which is related to *curves mean realistic* in the sense that a jagged graph with multiple sporadic increasing and decreasing regions is far from an "ideal" and "clean" linear plot. This idea was observed when students were asked to interpret the graph provided in Figure 1B:

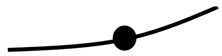
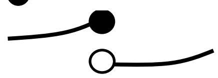
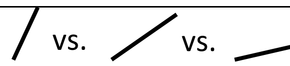
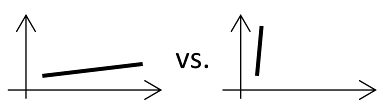
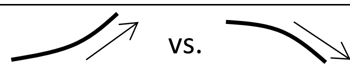
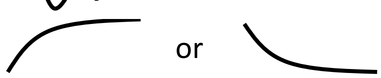
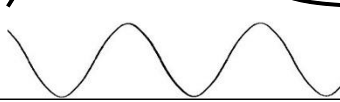
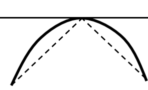
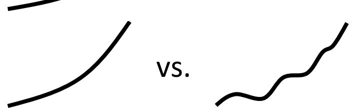
Sally: *Well, it's varying changes [Figure 1B]. It's not I guess constant in a way.*
Samuel: *Yeah, it's not like a smooth function, it's staggered in a way, I guess.*
Interviewer: *What do you mean by staggered?*
Samuel: *It was drawn like, like that [draws a graph with rigid lines]. ... I feel like it's just data plotted on the graph.*
Sally: *Yeah, and it's more abrupt, I guess.*

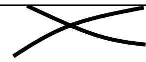
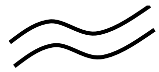
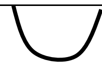
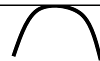
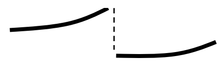
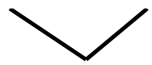
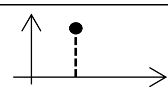
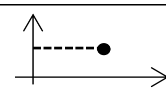
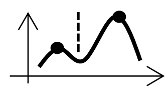
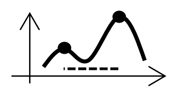
For the students, the jagged nature of the graph indicates the plot involves empirical, collected data. Combined with *curves mean realistic*, *jagged means data* reflects a productive idea for thinking about the relationship between variables and what is expected when collecting data.

List of Empirically Identified Graphical Forms

Having discussed a few graphical forms in detail in the previous section, in this section, we now present the various graphical forms observed from our students as they created or interpreted graphs (Table 1). We have organized the graphical forms into "clusters" in terms of which forms deal with similar aspects of a graph, such as points or slopes.

Table 1: List of graphical forms, organized into clusters of related graphical aspects (e.g., patterns)

Graphical form	Graphical pattern	Conceptual schema
Point Cluster		
1. Point as instance		A single point on a curve is a single instance
2. Big dot as focal point		A large dot indicates a special instance or event
3. Connecting as transition		Connecting dots transitions from one instance/event to another
4. Open/closed dot pair as existence		The closed dot defines “exists”, open dot defines “nonexistence”
Slopes Cluster		
1. Steepness as rate		The steepness of the graph indicates the rate of change
2. Straight means constant rate		A straight line indicates the rate is constant
3. Curve means changing rate		A curving graph indicates the rate is changing
Cardinal Direction Cluster		
1. Horizontal as constant value		Horizontal implies a constant (“*y*”) value
2. Vertical as constant value		Vertical implies a constant (“*x*”) value
3. Vertical as simultaneous		Vertical means simultaneous “*y*” values at one “*x*” value
4. Running along axis		The more parallel the graph is to one axis implies more change in that axis’ variable
Global Trend Cluster		
1. Shape directionality		Up-right means increasing and down-right means decreasing
2. Wavy means variation		Lots of up/down implies a lot of variation
3. Plateau as levelling off		Plateauing means variable is “levelling off” to a stable value
4. Periodic means repeated		A periodic graph means a repeating situation
Smoothness Cluster		
1. Straight lines as idealized		Straight lines give rough approximation of graph segments
2. Curved means realistic		Curved graphs are more “realistic” for real-world quantities
3. Smoothness as strength of relationship		A smoother graph implies a stronger relationship between “*x*” and “*y*” (and vice versa)
4. Jagged implies data		A jagged graph implies it depicts real-world data
Two Graphs Cluster		

1. Intersection means same		Intersection means same values at that instant
2. Transformation as same	 or 	Shifting or stretching does not change the basic *x-y* relationship
Local Feature Cluster		
1. “U” as max/min	 or 	Top of a hill or bottom of a valley is a max or min
2. Jump discontinuity means sudden		Jump discontinuity implies a very sudden change
3. Cusp as event		A cusp implies an event occurred at that instance
Position Cluster		
1. Distance as value (horizontal or vertical)	 or 	The distance from an axis indicates the other variable’s value
2. Displacement as difference (horizontal or vertical)	 or 	The displacement between points indicates the difference in values

Implications and Conclusion

Building on Sherin’s (2001) work related to exploring how students use knowledge resources to reason about equations, our work has detailed many resources students used to reason about graphs. It is important to note that context likely plays an important role in the activation of these graphical forms (diSessa et al., 2016; Elby, 2000; Hammer et al., 2005). Thus, if students were provided different graphical shapes or alternative coordinate systems, we would likely observe additional graphical forms. In this way we do not claim Table 1 to be an exhaustive list of all graphical forms, but we do believe it represents many important forms. Further, additional types of graphing knowledge resources exist that do not fit the strict definition of graphical forms. For example, we also saw students use knowledge about the axes or functions in creating or interpreting graphs. Beliefs about the nature of graphs also were important resources the students drew on (see Hammer et al., 2005; Hammer & Elby, 2003 for more on belief resources). However, the point of this work is to better understand the conceptual schemas coupled with specific graphical patterns (such as steepness or points) that students used in both creating and interpreting graphs. Future work will unpack the additional resources we observed and how graphical forms and these other types of resources worked in concert when students created graphs or interpreted graphs.

Our work has important theoretical and pedagogical implications. Theoretically, we have extended the initial work on graphical forms (Rodriguez et al., 2018, 2019a, 2019b, 2019c, 2020a, 2020b) to an identification of a large set of graphical forms. Such identification allows researchers to see finer-grained aspects of student reasoning when creating or interpreting graphs. It can also help researchers code for these specific knowledge resources when studying students’ graphical activity, or in examining how or when specific resources might be used. Pedagogically, our work is useful for instructors in identifying knowledge they may wish to help their students develop or to draw on during in-class graphical activity. It also helps instructors gain insight into the thinking their students might be doing in-the-moment as they interpret graphs or model a situation with a graph. Lastly, our results are important in demonstrating productive knowledge resources that students have and can use to create or interpret graphs. In other words, our work helps show what students can bring to problem-solving tasks in terms of graphical reasoning, rather than focusing on what they lack.

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