ENHANCING K-12 PRE-SERVICE TEACHERS' EMBODIED UNDERSTANDING OF THE GEOMETRY KNOWLEDGE THROUGH ONLINE COLLABORATIVE DESIGN

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In this study, we devised research design that provides pre-service teachers to effectively experience embodied geometric thinking with the goal that it will impact teachers' instruction to students in their classrooms. Using a motion-capture video game and design tool, we offered opportunities for pre-service teachers to experience of performing mathematically related movements as well as creating their own directed actions for given conjectures. We hypothesize that these gameplay and co-design activity will reinforce not only teachers' understanding of the embodied nature of geometric thinking, but also their abilities to transfer their understanding to classrooms and the activities and assessments they design for their students. The results showed that after experiencing the interventions including embodied gameplay and co-design activity, teachers' awareness of students' 'sage of gestures was changed and they had better ability to understand and interpret students' gestures as a means of teachers' formative assessment practices.

Keywords: Geometric thinking, embodied learning, gestures, pre-service teacher education

Geometry knowledge is a core component of mathematics curriculum in secondary education. Unfortunately, the formalisms and current pedagogical approaches for instruction in math classrooms are dominated by abstract and a-modal curricula that are often barriers for many students as well as their teachers. It is crucial that geometry instruction connects with students in ways that are intuitive and easier to understand and leverage the inherent, embodied ability of individuals to reason about space and shape.

In order to address such a challenge, our research team created a motion-capture video game, *The Hidden Village* (THV), to both direct and elicit learners' mathematically related movements and enhance geometric understanding. In effect, THV facilitates players' actions (i.e., *directed actions,* ref. Nathan, 2014) that are emblematic of geometric representations and transformations while also providing prompts that elicit gesture production during the process of geometric conjecture reasoning. The current study addresses a practical problem that many teachers often have naïve views of the role of the body in geometric thinking (Walkington, 2019) and fail to understand the cognitive connections that embodiment reinforces in geometric thinking. There is a recognized need for providing empirical guidance for when and how teachers can implement body-based learning activities in curriculum, and evidence-based principles for understanding the usage of gestures in embodied math activities.

Researchers had devised an in-person research program that brings pre-service teachers together to effectively teach embodied geometric thinking with the goal that it will impact teachers' instruction to students in their classrooms. Unfortunately, COVID-19 impacted *in situ* research such that it must be done remotely. The interactions between teachers, the actions they perform, the rationales they provide, the gestures they express, and the designs they co-create must occur virtually. Thus, providing teachers with embodied perspectives on geometry instruction in consideration of how to integrate these practices into their formative instruction

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and assessment. In the current study, teachers play THV to understand how the embodied curriculum connects to geometric thinking, after which they engage in reflective group discussion and co-design new conjectures with directed actions to help learners conceptualize geometric transformations.

Theoretical Background

Embodied mathematics specifically integrates multi-sensory experiences into conceptual understandings. Embodiment grounds (Barsalou, 2008) multi-sensory perceptions of mathematical structures and patterns that may not be accessible from symbolic representations (Gerofsky, 2007; Sinclair, 2005). With math knowledge grounded in body-based and spatial metaphors (Lakoff & Núñez, 2000; Roth, 2011) and action-oriented language (Nathan et al., 2014), learners often express their conceptual understandings in the form of gestures (Alibali & Nathan, 2012; Edwards, 2009; Ng & Sinclair, 2015a, b). In prior work, students' mathematical intuitions and judgments of conjecture veracity (i.e., *always true* or *false*) were reliably predicted by their dynamic gestures (Nathan et al., 2018).

Creating body-based systems for geometric thinking that employ directed actions offers opportunities for learners to physically mimic the spatial dimensions, relationships, and transformations of geometric conjectures (Nathan & Walkington, 2017). In turn, teachers can translate these experiences into effective embodied instruction (Alibali & Nathan, 2007; Roth, 2001) to connect concepts for students. For the current study, the intersubjectivity (Matusov, 2001) of teachers' collective embodied reasoning is distributed (Walkington, Chelule, Woods, & Nathan, 2019) during our collaborative design intervention.

We hypothesize that this co-design activity will reinforce not only teachers' understanding of the embodied nature of geometric thinking, but also their abilities to transfer their understanding to classrooms and the activities and assessments they design for their students. Thus, our main research questions are: (RQ1) How does embodied video gameplay and co-design activity eliciting teachers' gesture production enhance teachers' awareness of students' use of gestures? (RQ2) How do these interventions improve teachers' ability to interpret students' gestures and develop formative instruction and assessment practices?

Methods

Participants

Participants were K-12 pre-service teachers (N=16) enrolled in math courses at a midwestern research university. Participants received a $150 e-gift card for their participation. COVID-19 health precautions instituted by the chancellor of the university forced this *in situ* research on embodied group collaboration to take place entirely online, using Zoom, a securely private platform approved by the university's IRB. Fortunately, Zoom enables participants to interact and collaborate with each other in a virtual group setting while recording audio and video of each participant.

Materials

We observed teachers' cognitive processes during THV gameplay. This included their performances of directed actions to embody geometric conjectures as well as their reasoning and explanations. Given the constraints of conducting remote research in the time of COVID-19, both direct and indirect scaffolds for each step of the intervention were developed to ensure adherence to research protocols and preserve fidelity of data collection.

The Hidden Village (THV). THV delivers interactive math geometry curriculum in the form of online motion-capture video game in which each player mimics movements of in-game characters and then reads a geometry conjecture to determine its veracity. Levels of the game are each comprised of 6 parts (*see* Figure 1): (A) Meeting members of the hidden village, (B) the village people implore players to mimic movements (e.g., mathematically relevant directed actions), (C) players receive a math conjecture, (D) players indicate if the conjecture is *always true* or *false* and provide explanation, (EIultiple choice, and (F) players receive rewards and progress game achievement.

Prior to gameplay, participants were provided an introductory tutorial containing instructions for setting up for research participation, a practice trial to familiarize participants with THV gameplay. For gameplay, students were paired into dyads with one person performing the directed actions (i.e., the actor) from the game while the other player observes (i.e., the observer). Midway through the game, players switch roles.

After gameplay, participant dyads rejoined their group to discuss any connections between the directed actions and the conjectures they were proofing. Discussion was guided by three prompts: (1) how in-game directed actions affected teachers' understanding of geometry, (2) how directed actions can be applied in the classroom to support geometry learning, and (3) how teachers can interpret students' spontaneous gestures as a formative assessment of students' understanding to improve their instruction.

Figure 1: The overall structure of THV gameplay

The Hidden Village Conjecture Editor. THV conjecture editor is a tool for participants to create new content for THV. This includes the creation of directed actions for each conjecture in the co-design activity. Since the co-design activity was conducted virtually, a researcher served as a proxy to manipulate the conjecture editor at the participants' behest. The teachers were given a tutorial outlining the features of THV conjecture editor and explaining the mechanics for manipulating the in-game avatar to create directed actions for a given conjecture (see Figure 2). The entire co-design activity was video recorded including participants' gestures and discussions.

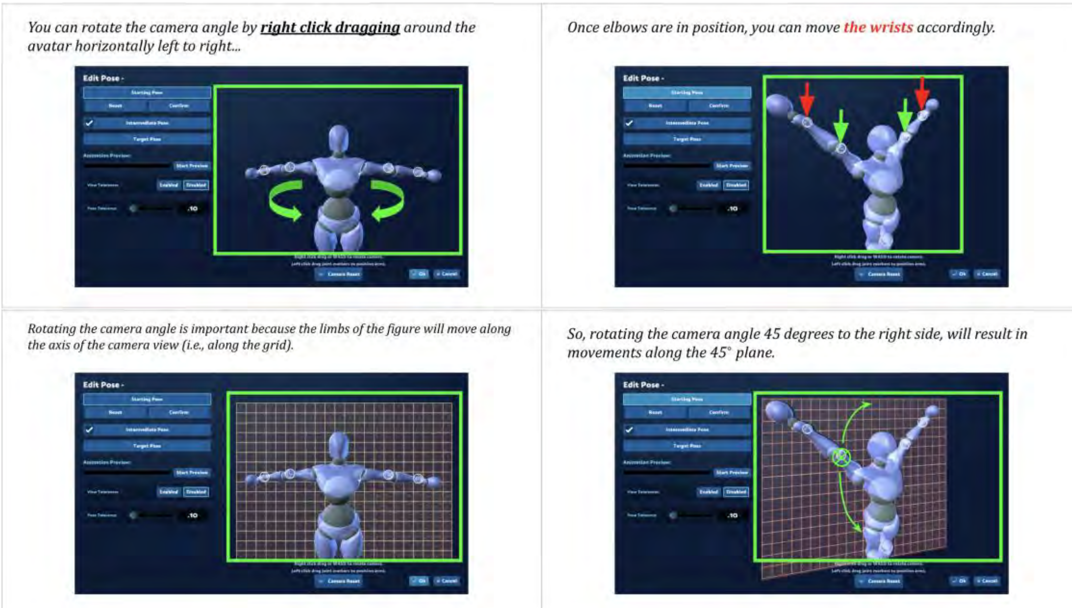


Figure 2: The slides of the design tutorial to inform how to use THV conjecture editor. (e.g., The avatar is posable using computer mouse. The figure can be rotated using a right-click drag or reset back to its origin.)

Online videos

In the beginning and the end of the intervention, teachers watched short 1-minute-long videos in which a student reasoned why a certain geometric conjecture statement is either always true or false. The scenes in the videos depicted high school students reasoning geometric conjectures (re-enacted for confidentiality reasons). In a semi-structured interview, teachers were prompted to comment on the videos and asked to explain (1) how they interpreted or assessed the student's understanding of the mathematical concept, (2) what evidence they observed, and (3) what practices they could employ in their classroom.

Online surveys

One week prior to the intervention, participants received online links to provide consent and complete surveys. These survey measures included: (1) the *Diagnostic Geometry Assessment* (DGA; items clustered in three areas: properties of shape (14 items, $\alpha \geq 0.86$), transformations (10 items, $\alpha \geq 0.795$), and measurement (11 items, $\alpha \geq 0.81$)); (2) a truncated version of the *Spatial Reasoning Instrument* (SRI, 15 items clustered around three constructs of mental rotation, spatial visualization, and spatial orientation, $\alpha \geq 0.88$); and (3) a survey of *Teachers' Attitudes about Gestures for Learning and Instruction* (TAGLI; 40 items, Cohen's $\alpha > .70$).

Procedure

This study is a mixed-methods, repeated-measures, within-subject design. The day of the intervention, participants took part in a number of activities over the course of 3.5 hours. First, participant groups were banded together by grade levels (i.e., grades K-5 and 6-12), after which they began a series of activities, including: (1) each teacher previewing and commenting on a 1-minute video of a student considering whether a geometric conjecture is either always true or false (30 minutes), (2) paired gameplay of four conjectures in THV in which players are encouraged to perform the directed actions by mimicking an in-game avatar’s movements (30 minutes), (3) a whole-group discussion and co-design activity in which four pre-service teachers co-create directed actions for given conjectures using THV conjecture editor (90 minutes), and (4) watching and commenting (i.e., interpreting the relationships (if any) between students’ gestures and geometric thinking) on two new 1-minute videos of a student considering whether a geometric conjecture is either always true or false as well as retaking the TAGLI survey (30 minutes).

Data analysis

The activities and measures provided researchers with data to assess changes in teachers’ awareness and interpretation of students’ understandings. The design of the study included pre- and post-intervention measures, teachers’ gameplay and co-design activity. As an ongoing study, all videos of gameplay and co-design will be transcribed, segmented, and coded for teachers’ speech and gesture usage articulating their intuition, insight, and proof production.

Results

Our major predictions were that gameplay and co-design activity that promote teachers’ use of gesture will affect (1) teachers’ awareness of students' ’sage of gestures and (2) teachers’ ability to understand and interpret students’ gestures as a means of teachers’ formative assessment practices. In order to find out how the interventions we designed affect teacher’s ability to interpret students’ gestures, we compared the pre-intervention interviews and post-intervention interviews. The interviews were conducted individually.

Below are the examples of teachers’ gameplay (see Figure 3) and co-design activity (see Figure 4) that teachers experienced. After the gameplay, teachers had a debrief of their experiences, discussing how in-game directed actions affected teachers’ understanding of geometry. Half of the teachers said that they did make the connection between the conjectures they played and the movements they made during gameplay whereas the rest of half said they honestly did not notice the linkage. In common, the teachers who realized the connection during gameplay mentioned that they were able to get the ideas about the veracity of the geometric conjectures by performing the directed actions and they used that information in their reasoning processes.

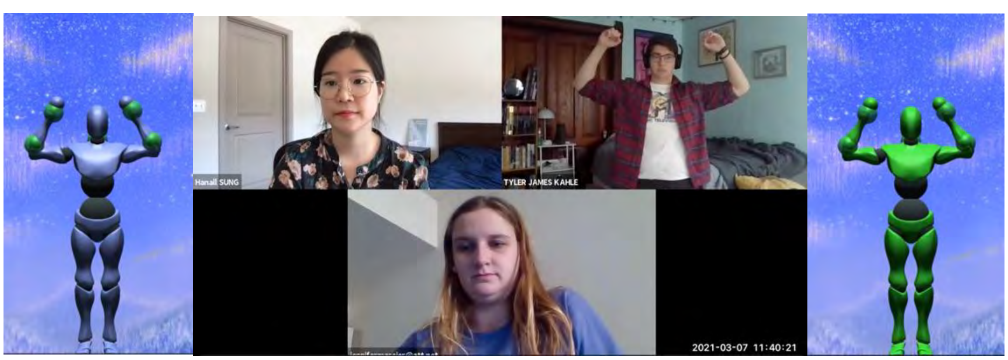


Figure 3: A screenshot of performing a directed action for a conjecture during THV game play

For the co-design activity (see Figure 4), participants were prompted to discuss geometric conjectures and consider how creating body-based directed actions enactive of geometric transformation could foster student learning.

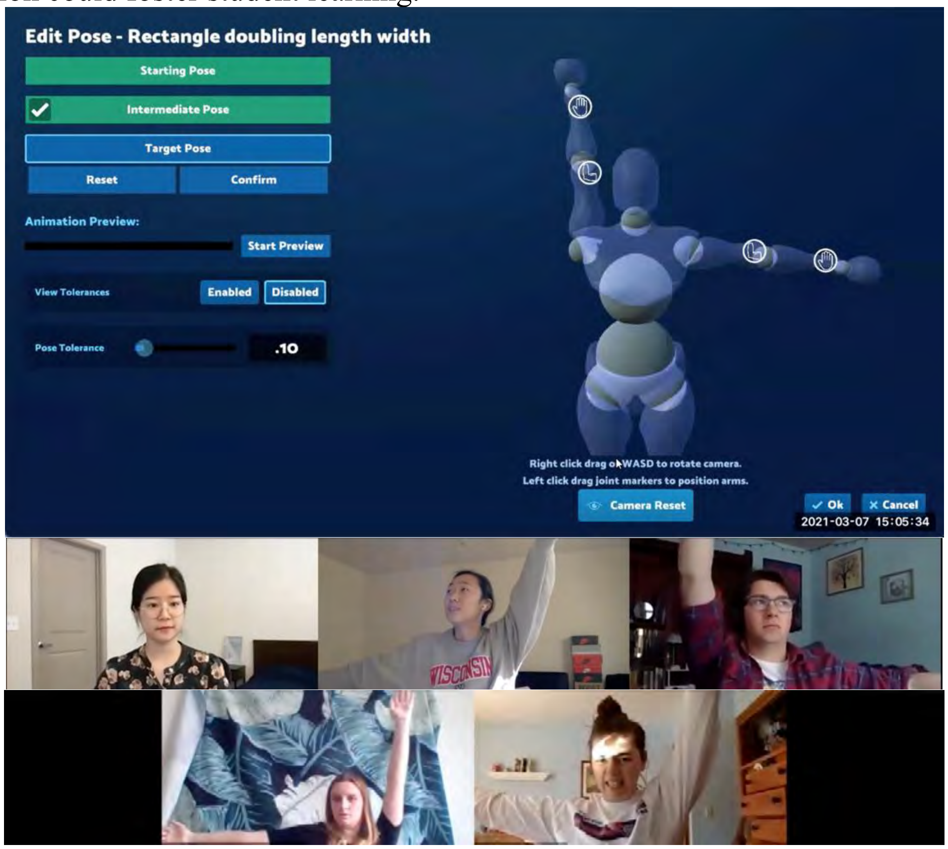
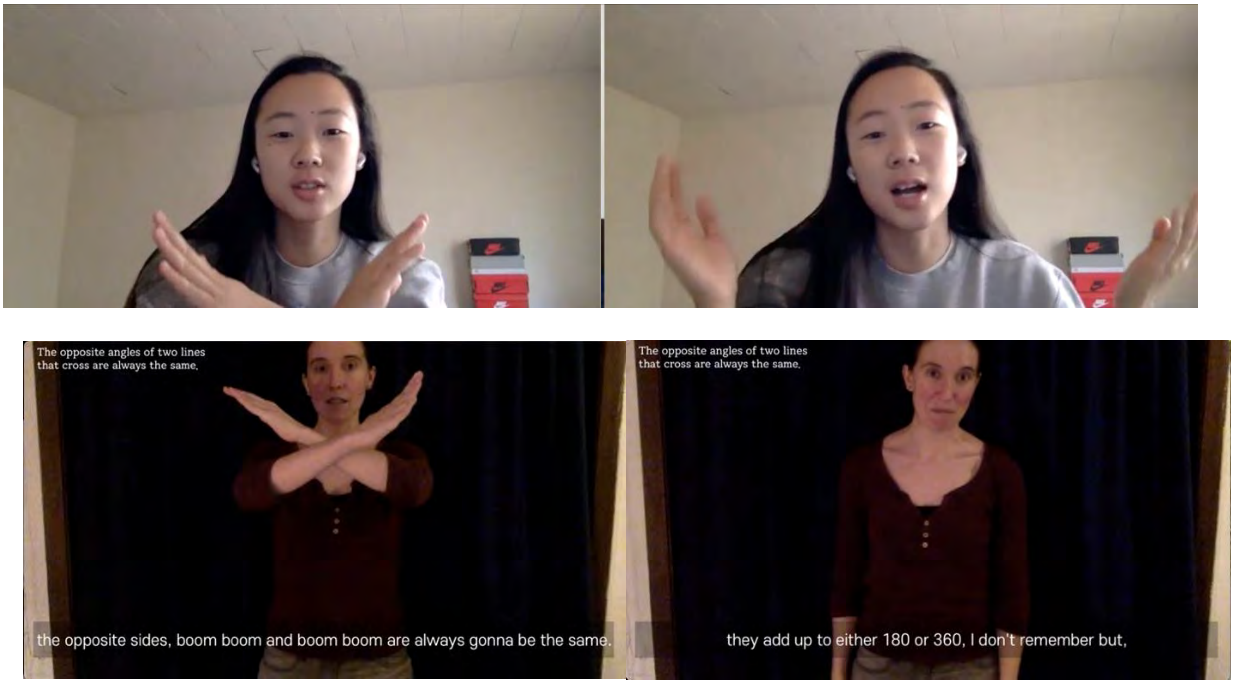


Figure 4: A screenshot of teachers' discussion to create new directed actions for conjectures during co-design activity

In the pre-intervention interviews with teachers, we found that teachers provided both gestures and verbal utterances of the student in the video as their evidence to assess the student's mathematical understanding of a certain conjecture (i.e., *The opposite angles of two lines across are always the same*). However, teachers' interpretation was limited—they were only able to share their superficial impressions of whether the student has the correct idea of the geometric conjecture, speculating from the student's tone, attitude, and vocabularies.

Presented here is a representative example of the pre-interviews with teachers (see Figure 5). Included is a photo transcript of Teacher 1's gestures and speech while explaining their interpretation of the student's mathematical understanding.



Teacher 1: She understood the opposite rule [mimicking the student's X pose], but when she said "it either adds up to 180 or 360" and then was like I don't know really [making a pose to portrait 'I don't know'] at the end that just tells me that she doesn't fully understand the role, because if you understood the rule that would be like the two side by side angles would equal 180, so she doesn't have like the full understanding, but she has like the very basic core understanding of what the rule should be.

Figure 5: An example of pre-interviews (top row: Teacher 1, bottom row: the pre interview video clip)

By mimicking student's particular gesture representing vertical angles (Figure 5, top row), Teacher 1 showed that they perceived the student's basic understanding of the geometric conjecture, but their focus quickly moved to the utterance and interpreted the student's level of understanding based on how the student verbally described the utterances ("It either adds up to 180 or 360").

On the contrary, we found that teachers in the post-interviews were more focused on the connection between the student's gesture and their reasoning processes. For example, Teacher 2 interpreted that the level of the student's mathematical understanding of the conjecture is poor and provided several rationales. First, by integrating the information from student's gesture and speech (Figure 6, bottom row), Teacher 2 noticed that the student in the video was trying to provide reasoning with proof by contradiction (Transcript in Figure 6, "The logic she was trying to use was almost like a contradiction, like a proof by contradiction I felt like"). Next, Teacher 2 pointed out that the ineffectiveness of the student's gesture in their reasoning process by saying "the gesture, she kept using, which is this [mimicking the student's poses], but like she didn't'really do anything with it" (Figure 6, top row). That is, Teacher 2 paid attention to not only the meaning of the student's gestures but also the function of the gestures in their geometric understanding.

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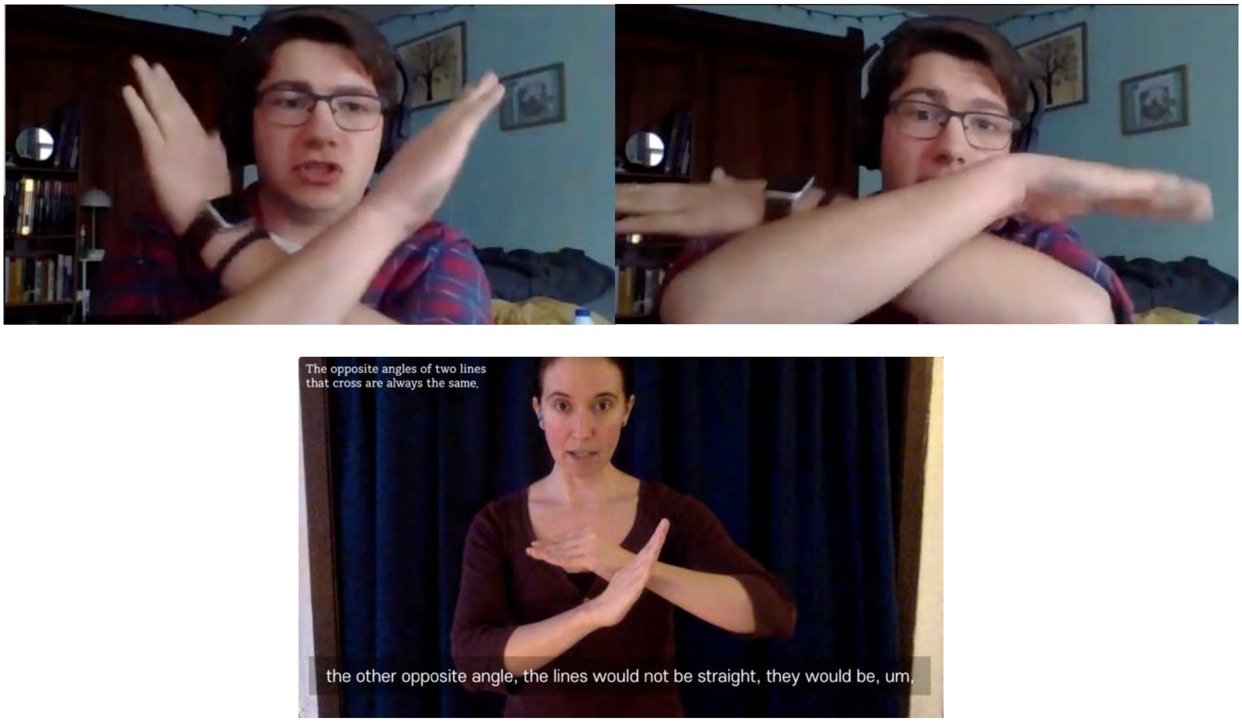

Teacher 2: I thought she did a kind of poor job on that one. You very much could see the problems. The logic she was trying to use was almost like a contradiction, like a proof by contradiction I felt like. She was trying to say “okay, well, if the lines aren't straight than the angles won’t work”. And the gesture, she kept using which is this [mimicking the student’s sequence of poses], but like she didn't really do anything with it.

Figure 6: An example of post interviews
(top row: Teacher 2, bottom row: the post interview video clip)

Discussions

The qualitative results of this study demonstrated that the interventions that include gameplay and co-design activity facilitated teachers’ use of gesture had impact on changes in teachers’ awareness of students' ’sage of gestures. Although teachers mimicked the student’s gestures that they observed in the videos in both pre- and post-intervention interviews, the level of information that teachers were able to extract from the gestures were different—they were more likely to focus on how and why students used gestures in their proofings in the post-interviews. Moreover, the results showed that the embodied interventions affected teachers’ ability to understand and interpret students’ gestures as a means of formative assessment practices. After experiencing the interventions, teachers were more likely to assess students’ geometric thinking better by focusing on the function of gestures in their reasoning.

This study has several limitations. First, as an ongoing study, we do not provide any quantitative results yet. Second, the recordings of gameplay and co-design have not been fully analyzed, so we were not able to articulate teachers’ learning processes during the interventions. Third, the sample size is pretty small because it was conducted as pilot study. We plan to deal with these limitations in our future work.

Despite these limitations, the preliminary results of the study suggest that the embodied intervention has reasonable potentials to change teachers’ belief and attitudes toward gestures.

Considering an average 14.2-year career of teachers, we expect the potential changes in these 16 teachers could influence over 15,000 students. In a broader consideration, the findings of this research are for the benefit of pre-service and in-service teachers who provide instructional practices at the forefront as well as professors of mathematics education and can be extended to professional development.

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