CIRCULAR REASONING: SHIFTING EPISTEMOLOGICAL FRAMES ACROSS MATHEMATICS AND CODING ACTIVITIES

RAZONAMIENTO CIRCULAR: CAMBIOS DEL MARCO EPISTEMOLÓGICO EN ACTIVIDADES EN LA INTERSECCIÓN DE MATEMÁTICAS Y CODIFICACIÓN

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STEM integration holds significant promise for supporting students in making connections among ideas and ways of thinking that might otherwise remain "siloed." Nevertheless, activities that integrate disciplines can present challenges to learners. In particular, they can require students to shift epistemological framing, demands that can be overlooked by designers and facilitators. We analyze how students in an 8th grade mathematics classroom reasoned about circles, across math and coding activities. One student showed evidence of shifting fluently between different frames as facilitators had expected. The dramatic change in his contributions gauge the demands of the activities, as do the contributions of other students, who appeared to work within different frames. Our findings have relevance for the design and facilitation of integrated STEM learning environments to support students in navigating such frame-shifts.

Keywords: Integrated STEM / STEAM, Computational Thinking, Geometry and Spatial Reasoning, Middle School Education

Introduction

The "STEM" and "STEAM" labels in education (Takeuchi et al, 2020) signal possibilities for integrative experiences involving multiple disciplines. These experiences can be valued as *workforce preparation,* recognizing that interdisciplinarity is increasingly vital in professional STEM fields (National Science Foundation, 2020; Nersessian, 2017). Or, they can reflect the observation that problems in the world of work are seldom confined to a single school subject area (Lesh, Hamilton, & Kaput, 2007). Alternatively, a case for integrative STEAM activities can be based in goals such as enhancing students' motivation and engagement, and increasing the sense of relevance of STEM subjects (National Science and Technology Council, 2018). Recognizing the motivation for engaging in them, the *value* of such integrative STEM activities hinges on learners' successfully constructing productive relations among the integrated disciplines. Lehrer & Schauble (2020) warn that this can be a challenging proposition indeed, showing how activities that promise to connect mathematics with other STEM disciplines can unfold in ways that diverge from teachers' intended learning goals, or can raise thorny questions that participants may not be equipped to navigate. Connecting with the PME-NA conference theme of *productive struggle*, Lehrer and Schauble's (2020) work highlights the challenges (and opportunities) involved in making struggles over mismatches between disciplinary ways of knowing in integrative STEAM activities into productive inter- and meta-disciplinary experiences for learners.

Charting a course for this line of work calls for rich descriptions of the classroom experience of integrative STEAM activities that engage learners at the intersection of epistemic practices fundamental to different disciplines. Integrative STEAM activities of this kind position learners as *boundary crossers* (Akkerman & Bakker, 2011) with epistemic agency to connect mathematical practices of representation and inquiry with those of other disciplines. The construct of boundary crossing is widely studied in the context of professional and organizational learning. To conceptualize what kinds of learning might be possible by positioning students as boundary crossers, and to calibrate the challenges involved, we draw (with caveats) on that literature of professional boundary crossing and interdisciplinarity. A useful review by Akkerman and Bakker (2011) outlines essential themes that are foundational to our analysis. Boundary crossing research typically studies professional practices in which individuals and groups find themselves at the intersection of communities that are concretely embodied in disciplinary and institutional practices that play critical roles in their work lives. In such settings, boundary crossers can pioneer new directions of organizational and professional growth.
In classroom settings, institutional and disciplinary forces are present in very different ways from how they appear in professional settings. Nevertheless, research from the professional context offers us models for how learners might be supported to negotiate tensions at the intersections of disciplines, models that can offer guidance through target stances and forms of interaction. Table 1, below, describes analogies that we leverage between professional STEM and classroom STEM education contexts.

Table 1: Tracing the key concepts of boundary crossing and epistemic cultures and frames—between professional STEM and STEM education contexts

Key Concept	**Manifestation in Science & Technology Studies and organizational research**	**Manifestation in educational activity designs and analyses**
Boundary Crossing	Shared problems and enterprises create the need for transdisciplinarity. Stable procedures and institutional structures emerge that reflect the interface between distinct disciplinary cultures (Osbeck & Nersessian, 2017).	Activity designs create the need for students to construct connections across subject areas. Diverse participation reflects the interface between distinct ways of thinking. New hybrids prove their *viability* by being useful in practice (Brady, Eames, & Lesh, 2015).
Epistemic Cultures and Epistemic Frames	Epistemic cultures (Knorr Cetina, 1999) have characteristic discourses and representations for concepts relevant to their shared enterprise. Shifts appear in boundary crossing, facilitated by boundary objects and by "creoles" (Galison, 1997) to mediate the boundary.	In talk and interaction, different participants interpret activity settings using interpretive frameworks built out of disciplinary and everyday knowledge resources. Breakdowns in activity can reflect clashes between these epistemic frames (Hall & Stevens, 2015) and provoke repair, negotiations, and shifts.

In this paper, we analyze a classroom episode, in which we describe the distinct *epistemological frames* (Scherr & Hammer, 2009; Thoma, Deitrick, & Wilkerson, 2018) and the *shifts* between such frames, which the facilitators assumed students would navigate. Understanding the demands we are making of students as designers and facilitators of integrative

STEAM activities, and learning how to support students are two critical issues of research and praxis for making such activities scenes for productive struggle in mathematics education.

Theoretical and methodological approaches

The *framing* of a situation or interaction reflects participants' determination of "what is going on" there (Goffman, 1974). Faced with a barrage of information that is overwhelming and often conflicting, humans have to make snap decisions about what "kind" of situation they are in, in order to determine what is relevant, what the rules are, and how they should act. Framing is both interactional and individual; contexts can invite particular frames, but frame signaling can be ambiguous (Wisittanawat & Gresalfi, 2020) or can suggest different frames to different people (Hand, Penuel, & Gutierrez, 2012). It is remarkable, then, that this can be mostly done unthinkingly and without uncertainty rising to conscious experience, especially since framing is a matter of shared agreement and coordination (Goffman, 1974).

In designed or otherwise exceptional environments, however, questions and even disputes about framing can come to occupy the foreground (DeLiema, Enyedy, & Danish, 2019). Novel settings make it possible for multiple candidate framings to emerge, as people look for contextual clues about the tools, participation structures, language, and interactions that are appropriate. Such settings can offer different frames for different people (Hand, Penuel, & Gutierrez, 2012), or make it ambiguous to both participants and outside observers what is actually going on (Gresalfi, Brady, Knowe, & Steinberg, 2020).

Within learning environments, such indeterminacy in framing can be seen as a liability, making it more difficult for individual students to participate or more challenging for teachers to facilitate a student group in activities that require coordination. On the other hand, moments that provoke frame indeterminacy can also offer the potential to bring together different interpretations of shared experience, and thus could also offer powerful learning opportunities. Goffman's (1974) extended analysis of frames and their transformations shows how frame breaks and frame disputes surface fundamental assumptions about the "primary frameworks" that underlie social interactions in various contexts. They thus offer an opportunity to see and discuss the consequences of these underlying frameworks. In the context of frames governed by disciplinary ways of seeing and acting, frame breaks and frame disputes offer a setting where the nature and consequences of epistemic frameworks that are fundamental to the philosophy of a discipline can be made palpable and experiential.

Setting and Participants

We focus our analysis on one session from an 8th grade mathematics classroom at an urban public-school in the southeastern United States. The teacher, Ms. T, has been a co-design partner with the authors in an NSF-funded project (CAMPS, NSF#1742257), to design and study learning environments that integrate mathematics, computer science, and art. To this point in the project, Ms. T and the authors had collaborated in an informal-learning setting, a summer "Code Your Art" camp. In the 2018-19 school year, at Ms. T's initiative, the project team worked to adapt activities and ideas from "Code Your Art" camp to Ms. T's math class on "Code Fridays." Throughout the school year, the research team worked with her to co-design and co-facilitate coding activities on many Fridays, using the NetLogo (Wilensky, 1999) modeling environment. Ms T's school is a community middle school serving a racially and economically diverse population, and the class that experienced Code Fridays sessions comprised 34 students.

On the day in question, Ms. T reviewed practice problems for a high-stakes state assessment before moving on to coding. Facilitators' in-the-moment decisions about how to transition from this phase to the Code Fridays activity created an opportunity for integration across math and computation around *circles* in a sequence of two conversations. Students reasoned very differently about circles across these two instructional moments, leading to the appearance that they did not make connections between the same set of ideas as they switched activities. One possible interpretation is that students demonstrated a failure to "transfer," in that resources and ideas leveraged in one activity were not leveraged in the second. Instead, we argue that the different resources students brought to bear on questions about circles suggested differences in their epistemological framing (Scherr & Hammer, 2009; Thoma, Deitrick, & Wilkerson, 2018) of the two activities, and revealed mismatches between some students' framing and facilitators' expectations. Recognizing the roles of framing and frame-expectations focuses our attention on features (and shortcomings) of our design and facilitation, rather than on shortcomings in students' thinking.

Methods of Analysis

We apply epistemological frame analysis to our focal episodes, to understand how students experienced and responded to signals for framing of two successive activities about circles. Data analyzed in this paper include video from two sources, a camera in back of the room positioned to capture the teacher's projected computer, and a second camera set up in the front to capture students' talk, gestures, and interaction at their tables. Through multiple viewings of the record, we narrowed our focus to two brief episodes involving circles—one from the math exam practice and the other from the coding session. We used discourse analysis, including an analysis of gesture, to investigate how different epistemological framings were recruited with respect to expected framings across the two activities.

Findings

We found that across the two focal activities, distinct epistemological framings of circles emerged. We identified one student, Mateo (a pseudonym), who navigated the shift between these two activities successfully (i.e., as the teacher and researchers had expected). We studied the forms of expression and argumentation that he exhibited, as a measure of the difference in framing. We also identified other students in the class, whose contributions appeared to come from frames less well aligned with the expectations of the facilitators. These students did not appear to lack conceptual sophistication or resources; rather, their framing prevented them from participating in the discussion as the facilitator intended. Our goal in the analysis was to gauge the nature of the epistemological discontinuity between these mathematics and coding activities: both the success that Mateo had in constructing compelling accounts across the two settings, and the challenges other students faced, help to characterize this discontinuity between activities.

Mathematics activity: Mapping given numbers to elements of the area formula

The class session began with a review of practice problems for the state exam. Problem 37 asked for the area of a semicircle, given that its diameter measured 6 units. Mateo volunteered to share his work:

Mateo: So, uh, since we know the formula for the area of a circle is, pi times uh radius squared, so for half the circle, we just need to do, uh one-half of pi times radius squared. So, I did…so I did for the radius I found that it was 3 because the radius is half of that,

> the the diameter. So I did uh uh 3? Uh, squared? which is 3 times 3 which is 9, and I did 9 times, uh, 3.14, divided by two, so I got 14.13.

Mateo's contribution suggested he framed the problem as a challenge of *mapping* given elements of the figure to their meanings in a memorized formula, and enacting the operations called for by that formula. His explanation took care to unpack each element in the memorized area formula (the relation of a semicircle's area to that of a circle; the value of *p*; the meaning of *r* and its relation to the diameter; and the meaning of squaring), which was sensitive to classmates who might have missed any of these elements. Moreover, his use of pronouns (e.g., "we know," "we just need," "I did") suggested that Mateo was positioning this mapping against a backdrop of communal and normative mathematical knowledge, which authorize his procedure. Finally, Mateo's manner of pointing and gesture-writing in the air with his pencil as he provided his explanation (Figure 1) is an instance of what McNeill (1992) calls an *observer-viewpoint* gesture. Together these features suggest he is visualizing a figure and that his reasoning was occurring in a mapping between recognized inscriptions and arithmetic calculations.

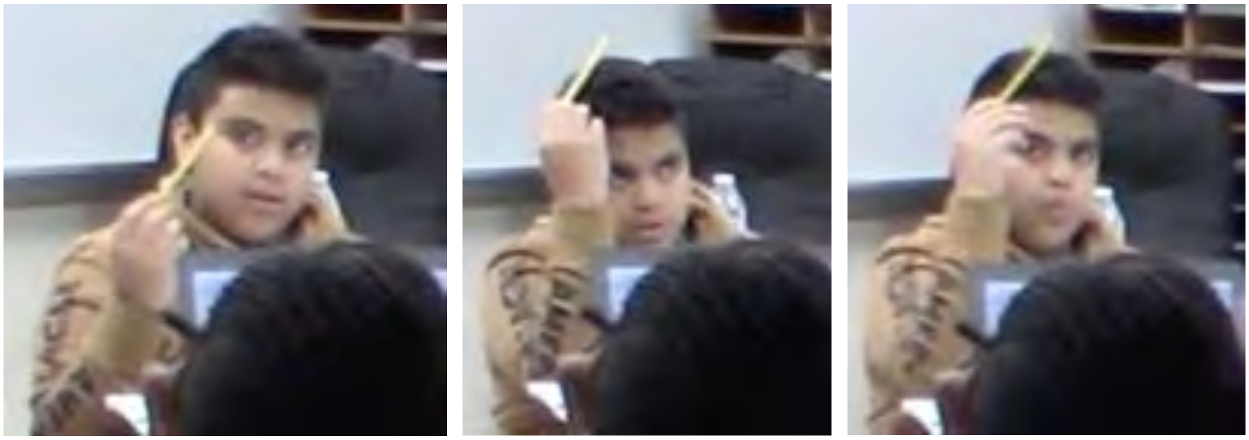

Figure 1. Mateo wrote in the air with his pencil as he described steps to calculate the area of the semicircle.

Mateo's contribution expressed a coherent framing, but his was not the only framing possible. In volunteering an alternative solution, ("I have another way") Edgar made a contribution that framed the activity in terms of voicing diverse strategies for sense-making, a framing valued in Ms. T's classroom at other times:

> Edgar: Well, what I do is I multiply uh 6 times the 3.14, and from that I think I got the uh, it was like, yeah, 18.84, then then I multiply it, multiply that number by 6, again, and I get, like, 113.04, and then I divide it by 4 and then half.

Edgar's solution was both correct and well-reasoned. As he later explained, squaring the diameter (twice the radius) and then dividing by a factor of 4 ("since I multiplied twice...times two times two") accommodated the givens of the problem. And with a calculator, his method was no more computationally cumbersome than Mateo's. Yet Edgar's approach appeared to be out of sync with the framing of the activity assumed by the teacher (and the students who followed her lead). Edgar's reasoning was questioned ("Where'd you get the four from?") and critiqued ("you added a bunch of unnecessary steps") by other students. Moreover, Ms. T reinforced these responses, saying "[Edgar], don't confuse yourself. On the test, you don't have that much time to go through all those steps, ok? Stick to the formula…."

Edgar's status in this classroom was quite high; indeed, he had been celebrated minutes earlier for using "a process of elimination" to reason about multiple-choice responses. Yet Edgar's own first-person pronoun use positioned his work as an *idiosyncratic* approach ("What I do," "Then I multiply"), in contrast with Mateo's normative "we." Finally, faced with the responses of classmates and Ms. T, Edgar explained "that's how I've been doing it…because I have no idea…." Overall, the differential rhetorical success of Mateo and Edgar suggest that Edgar's solution was received as less responsive to the "epistemic game" (Shaffer, 2006) of efficiently filling the "epistemic form" (Collins & Ferguson, 1993) of the formula.

Coding activity: Reasoning from the intrinsic perspective of the turtles.

On turning to Coding, the class was introduced to NetLogo *turtles* (agents that can move). The researcher leading the activity, CB, set the stage by creating 100 turtles on the projected computer, noting they were "piled up" at the center of the screen. With students following along, he typed forward 5, to be run by all turtles.

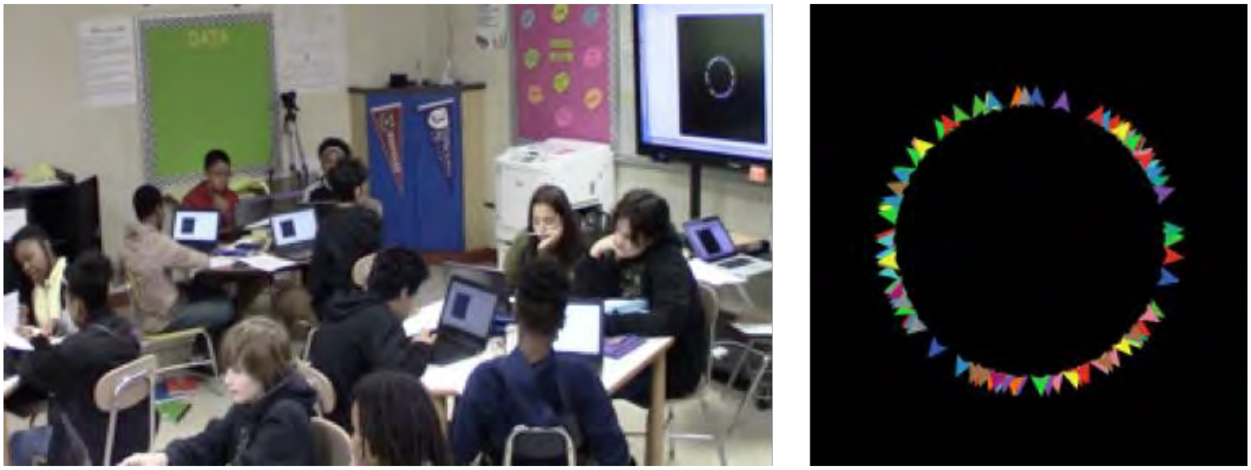

Figure 2. When student created 100 randomly-oriented turtles and executed forward 5**, a circle was formed.**

The turtles each moved forward 5 steps from the center of the screen and created a circle (Figure 2), which surprised the class. Making a connection to the first half of the session, CB asked, "By the way, since you guys were just talking about circles, what's the RADIUS of this circle?" Students shouted out three answers: "Three-sixty!" "Five!" and "Two point five!" CB then asked the class to discuss their reasoning in groups and different groups came to different conclusions. Marissa and Elena shared first and then Mateo joined the discussion:

Marissa: 2.5
CB: 2.5? And why?
Elena: The the the diameter is 5, all the way across. And the radius is half of that.
CB: Ok, so IF the diameter was 5, then the radius would be 2.5, for sure. How…what is the diameter of this guy?
Mateo: 10.
CB: 10. Why?
Mateo: Because if all the patches are going forward five, all facing in different directions/

CB: /turtles//
Mateo: //Ah turtles. So, they're all going 5 in every direction. The diameter's going to be 10.

<u>Figure 3</u>. Mateo gestured to show the movement of two oppositely-oriented turtles and the diameter they made.

The class's discourse about the circle of turtles illustrates the discontinuity between the two activities and the CB's expectations about how students would shift their framing to participate. On one hand, Marissa and Elena's group reasoned within the epistemic frame of mapping given values to formulas, as provided by the prior math activity. They interpreted the number 5 as mapping to the turtle-circle's *diameter*, as had happened earlier in Problem 37. Thus they argued for a radius of 2.5. In contrast, Mateo's explanation revealed a different epistemic frame and a new form of reasoning, distinctive to the context of agent-based programming. From the group of 100 turtles, he selected an imagined pair facing in opposite directions. With two hands, he gestured to simulate their movement, and then gestured (Figure 3A-B) to show how they would produce a line segment passing through the center of the circle, 10 steps long. Finally, (Figure 3C) he interpreted this to be the diameter of the circle formed by all the turtles.

Each of Mateo's moves arises from successfully constructing and operating a mathematization of the computational agent-based environment. First, the selection of two turtles from the *agentset* of 100 relies on a characteristic feature of computational simulations, which use randomization to present a finite sample of an infinite outcome space. The "circle" is only *suggested* by the turtles' bodies, and yet the "professional vision" (Goodwin, 1994) of a mathematically-attuned user of this representation can reason from the particular sample of turtles to imagine two of them oriented with precisely opposite headings. Next, Mateo uses a peculiar species of communicative character-viewpoint gesture (McNeill, 1992; Ochs et al. 1994), in which he embodies the *pair* of turtles with his hands, positioning his own head as the invisible center of the constructed circle. This gestural achievement stabilizes the mathematical objects (center, radial points, radius, diameter) that are necessary to align the situation and enable a link with the forms of reasoning about circles used Problem 37 can be applied effectively.

Conclusions and implications for future work

Mateo was successful in reasoning across the two activity contexts. But to do so, he had to make a substantial leap between epistemic frames. The differences in reasoning showed in the "embodied modeling" approach he employed (cf, Wilensky & Riesman, 2006) and in the different gestural resources that he recruited. The connection that CB assumed would be straightforward, in fact required a significant conceptual reorganization. Many students in this class exhibited strong and flexible resources for reasoning about circles, across each of the two activity contexts, as shown in Edgar's example. Nevertheless, differences and shifts in the discourse and forms of reasoning demanded by the two activities suggested that the "circle" in the math problem and the "circle" formed by the NetLogo turtles were substantially different kinds of objects. We take this example as indicating a challenge for the design and facilitation of activities that aim to provide STEM integration. Specifically, we must recognize that in moving across disciplinary contexts, we may be unknowingly asking students to bridge between epistemic frames, to carry ideas and resources from one domain to the other.

STEM integration has high potential. Indeed, treating the circle from an agent-based perspective offered Mateo a significant resource for mathematically conceptualizing it. Mateo's gestures suggest that he was able to use the turtles to imagine a circle as a *set* or a *locus* of *points* (turtles), and to infer that relations among those turtles/points gave rise to a property of the circle (its diameter). Utilizing a "point-set perspective" is typically viewed as a notable achievement in mathematics. Reasoning in this style, (Figure 3) Mateo leveraged the relation between two turtles and the vacated "center" (his head), as an embodied support for describing the emergent circle's diameter. His description was compelling in the classroom discourse, but it is not clear that his turtle-based reasoning and means of bridging computational and mathematical worlds were fully shared. Providing disciplinarily hybridized learning environments where students can reap the benefits of bridging the disciplinary divide between mathematics and computer science is a challenge for both research and praxis.

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