USING TEACHER AND STUDENT NOTICING TO UNDERSTAND ENGAGEMENT DURING SECONDARY MATHEMATICS LESSONS

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The purpose of this paper is to present a framework that illustrates teachers' and students' ways of noticing mathematical engagement. This framework offers clarity about the complexity of engagement, and it includes three elements: evaluations of the presence of engagement, descriptions of the nature of engagement, and features of the classroom that support or constrain engagement. We interviewed 30 sets of high school math teachers and focus groups of their students and asked them to reflect on students' engagement during a videotaped lesson from their classrooms. Results illustrate cases of how noticing of engagement between teachers and students can be shared. Cases of partially and minimally shared noticing of engagement suggest opportunities for teachers to learn about students' perspectives or how to communicate with students about their intentions to engage them.

Keywords: mathematics engagement, teacher noticing, student noticing, motivation

Engaging secondary students in mathematics lessons is an enduring challenge, as students' mathematics engagement has been found to decrease as they move through years of schooling (e.g., Collie et al., 2019). Engagement is a complex construct, involving affective, behavioral, cognitive, and social factors (Fredricks et al., 2004; Middleton et al., 2017), each impacting student learning and performance differently. In a nationally representative sample of high school students in the United States, behavioral and cognitive engagement explained more of the variance in students' mathematics achievement scores than affective or emotional engagement (Sciarra & Seirup, 2008). This study highlights that the nature of engagement matters for students' learning and performance, even though it was not specific to mathematics learning.

Teaching practice shapes the learning environment that students experience (Anderson et al., 2004), and the learning environment impacts students' engagement (Shernoff et al., 2017). However, according to Pedler et al., (2020), teachers face challenges understanding how to engage students because engagement is such a complex phenomenon. According to Erickson, to teach effectively, "one needed to 'learn' the children one was trying to teach" (Erickson, 2011, p.18). So, perhaps one approach for teachers to learn more about students' engagement is for teachers to become "students of our students" (Ritchart & Church, 2020, p. 11).

In this study, we investigated what mathematics teachers noticed about their students' engagement and how their students exhibited similar or different noticings about their engagement. Building upon research on specialized noticing practices of mathematics teachers (Jacobs et al., 2010; van Es et al., 2017) and research on students' noticing of mathematics (Hohensee, 2016; Lobato et al., 2013), the purpose of this study is to investigate what teachers and their students noticed about mathematics engagement while viewing video recorded events from their classrooms. We offer a framework to demonstrate that noticing of mathematics

Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

engagement involves evaluating whether or not students were engaged (presence of engagement), describing the nature of students' engagement (dimensions of engagement), and interpreting what happened in the classroom to elicit students' engagement (features of engagement).

Teachers' and Students' Noticing

Noticing is a process of identifying events as noteworthy, using evidence to discuss these events, and providing interpretations of these events (van Es & Sherin, 2002). What a person notices influences their reasoning about the event (van Es & Sherin, 2002; Lobato et al., 2013). Across teacher noticing literature (e.g., van Es & Sherin, 2002; Jacob et al., 2010; van Es et al., 2017), researchers consider *interpreting* to be higher quality noticing compared to *evaluating* or merely *describing*.

Different targets for noticing suggest a need for alternative frameworks for illustrating ways of noticing. Researchers have studied what teachers notice about their students' mathematical thinking (Jacobs et al., 2010) or equity in the mathematics classroom (van Es et al., 2017), as well students' different centers of focus of noticing during mathematics lessons (Hohensee, 2016; Lobato et al., 2013). For this study, we investigate a different target of teachers' and students' noticing: mathematics engagement. Additionally, previous research studies examined teacher and student noticing separately, while we investigate them in relation to each other.

Mathematics Engagement

Academic engagement is a psychological investment in and effort directed towards learning from academic tasks (Jansen, 2020; Newmann et al., 1992). Engagement is a complex meta-construct (Fredricks et al., 2004) that includes affective, behavioral, and cognitive dimensions (Appleton & Lawrenz, 2011; Bobis et al. 2016; Fredricks et al., 2004; Helme & Clarke, 2001; Middleton et al., 2017), and a social dimension (Middleton et al., 2017; Jansen & Bartell, 2013; van Uden et al., 2013; Wang, et al., 2016). Behavioral engagement includes effort or time on task. Cognitive engagement is concentration or connections made while learning. Affective engagement is an emotional state of investment, such as interest. Social engagement is participation in the learning process. When describing student engagement in the classrooms, teachers and students could focus on different *dimensions of engagement* (affective, behavioral, cognitive, or social).

Recent prior research reveals teachers' thinking about what students will find engaging in mathematics classrooms and how these interpretations may guide their instruction (Bobis et al., 2016; van Uden, et al., 2013). We named interpretations of what will engage students as interpretations of *features* that engage students. Researchers have explored whether teachers determine engagement to be present or not (Skilling et al., 2016), or teachers' *evaluations* of the presence of engagement (or disengagement). In terms of describing engagement, when reflecting on engagement generally, teachers tended to focus on relatedness or sense of belonging in the classroom (Herman, et al., 2000; van Uden et al., 2013). However, when teachers were asked to focus on particular students or to consider a situated case of engagement, they tended to focus primarily on behavioral or overt emotional engagement (Bobis et al., 2016; Skilling et al., 2016).

Previous research on students' perspectives have uncovered the motivators that drive their engagement (Daniels & Arapostathis, 2005; Jansen & Bartell, 2013; Middleton, 1995). These motivators range from interest and reward (Daniels & Arapostasis, 2005; Middleton 1995) to interpersonal relationships in the classroom (Daniels & Arapostasis, 2005; Jansen & Bartell, 2013). Cognitive and social engagement appear to be prominent in students' perceptions of their own engagement.

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Research Questions

To understand teachers' and students' noticing of mathematics engagement, we designed this study to answer the following research questions: *What do teachers and their students notice about elements of engagement (presence, dimensions, features) when asked to reflect on a shared mathematical experience? In what ways do they agree or disagree on these elements?*

Methods

Context and Participants

To address our research questions, we analyzed student and teacher interview data collected for the Secondary Mathematics in-the-Moment Longitudinal Engagement Study (SMiLES). SMiLES is a three-year mixed-methods study funded by the National Science Foundation that explored engagement in high school classrooms across two states (one state in the Southwestern region and one in the Mid-Atlantic region of the United States). Data analyzed for this study comes from 6 high schools (3 in each state) collected during the Fall 2018 to Spring 2019 academic year. Data for SMiLES were collected during Algebra 1 or Integrated Math 1 lessons.

Regarding our participants, we recruited teachers for this study by soliciting nominations of teachers from district curriculum supervisors and mathematics coaches. We invited nominated teachers to participate in the study, and 16 teachers participated (11 female, 5 male) in the SMiLES project during the Fall 2018 to Spring 2019 academic year. Teachers self-reported their races: 14 identified as white, one identified as Asian-American, and one identified as Hispanic/Latinx. They averaged 10.8 years of teaching experience with a range of 1 to 27 years. Student demographics for the schools in the Southwest were: 85-94% low income, 2-5% white, 1-15% Black, 74-96% Latinx, and 0-5% Asian, Native American, or Multi-Racial, and student demographics for the schools in the Mid-Atlantic were: 9-30% low income, 24-57% white, 27-46% Black, 7-24% Latinx, and 0-5% Asian, Native American, or Multi-Racial.

Students were selected to participate in focus group interviews from each class period that we observed. The criteria we used to select students for the focus groups was based on an analysis of students' responses to a mathematics engagement survey administered by the research team at the beginning of the semester. A cluster analysis of this data identified motivational profiles of students (Tarr et al., 2019). Three clusters were identified across the sample and the students invited to participate in the interviews: (1) strongly aligned with one of the profiles (2) had parent consent, and (3) had given assent. The average number of students who participated in a focus group was 2.45 with a range from 1 to 3 students.

Data Collection and Analysis

The dataset for this analysis consists of 30 sets of interviews with teachers and their students, with multiple class periods studied for most teachers. Interviews were conducted one-on one with teachers and in a focus group for the students. Prior to the interview, the research team identified a video clip that showed a representative example of student engagement from an observed lesson in the SMiLES dataset for that class. Each video clip was between 90 seconds and three minutes in length and had been experienced by the teacher and students in the focus group. Interviews took place two to three weeks after each observation. We conducted these interviews as video viewing sessions (c.f., Erickson, 2007), during which participants commented upon what they noticed in a video regarding the nature of engagement during that activity. These interviews were not treated as stimulated recall (e.g., Lyle, 2003), as we did not expect participants to be able to recall their experience after multiple weeks and we did not

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intend to capture participants' decision making in those moments.

We employed a case study approach (Yin, 2017) when analyzing these sets of interviews. We defined a case as a teacher's noticing and their students' noticing of a recorded classroom activity. Data for the case consisted of a set of interviews: the teacher interview and corresponding student focus group interview about the classroom activity. We analyzed these interview episodes to identify three elements of engagement reported by teachers and their students: the *presence of engagement, dimensions of engagement* and *features which elicit engagement or disengagement*. We defined *presence of engagement* as the indication by the teacher and students of whether students were engaged or disengaged. We defined *dimensions of engagement* as the type of engagement reflected in how a teacher or student described the nature of engagement, according to six categories of dimensions: affective, behavioral, cognitive, instrumental, social, and relatedness. We defined *features that elicit engagement or disengagement* to be teachers' and students' self-reports of what appeared to support (or constrain) students' engagement (or disengagement). Features are interpretations that the teachers and students provided about what happened in the lesson that engaged or disengaged students. These features were coded using an emergent process (Saldaña, 2013) from listening to voices of both the students and the teachers. We also analyzed for the *presence of engagement*; whether teachers and students determined that students were or were not engaged during the event in the video clip. We then identified ways in which sets of teachers and their students agreed or disagreed about what they noticed with respect to presence of engagement, dimensions of engagement, and features which elicit engagement or disengagement.

The elements of engagement in this study (presence, dimensions, and features) parallel the noticing stances described by van Es and Sherin (2002): *describe, evaluate, and interpret*. The participants' characterization of presence of engagement is a form of *evaluation* of whether or not students were engaged. When teachers or students talked about the ways in which students were or were not engaged, this aligns with *describing* engagement. (We coded participants' descriptions of engagement according to dimensions.) We considered *interpretations* of these video clips to be when teachers or students reasoned about the features that brought about students' engagement or disengagement.

During analysis, it became clear that there were cases in which teachers and their students noticed and agreed about engagement in various intersections of these elements. Our stance was that high quality noticing of engagement between teachers and their students occurred when they shared perspectives on engagement. *Strongly shared noticing* occurred when a teacher and their students agreed on all three elements (presence, dimensions, and features). *Partially shared noticing* occurred when a teacher and their students agreed on any two of those three elements. *Minimally shared noticing* was agreement between a teacher and their students on any one of those three elements. A *disagreement* on noticing any of the elements of engagement could provide an opportunity for teachers' learning about how to engage their students. Either a teacher could learn more about their students in order to engage them or the teacher could communicate rationales more explicitly so that students could learn more about their teachers' intentions for engaging them in particular activities.

Results

Through the process analyzing interview data, we examined ways that elements of engagement (presence, dimension, and feature) intersected and what these intersections revealed about how teachers and their students thought about engagement in secondary mathematics

classrooms. To this end, we built a framework (see Fig. 1) that organizes our results and helps to describe the ways our participants noticed engagement in their math class.

This framework, organized as a Venn diagram, recognizes each element of engagement -- presence, dimension, and feature -- as a set which can intersect and interact with the other elements. What teachers and their students notice can then be described through these elements and whether and how they intersect. Each of the outer circles (no intersections) represents when a teacher and their students notice and agree only on one element. We describe this as "Minimally Shared Noticing of Engagement." If a teacher and their students similarly noticed two out of the three elements of engagement, then they would fall into one of the intersections of two elements: "Partially Shared Noticing of Engagement." Finally, the innermost intersection ("Strongly Shared Noticing of Engagement") indicates that a teacher and their students exhibited shared noticing on all three elements of engagement. The complement of this Venn diagram also exists and would include cases where a teacher and their students did not notice similarly or agree on any of the three elements of engagement. We present two cases here: "Strongly Shared Noticing of Engagement" and "Minimally Shared Noticing of Engagement – Presence."

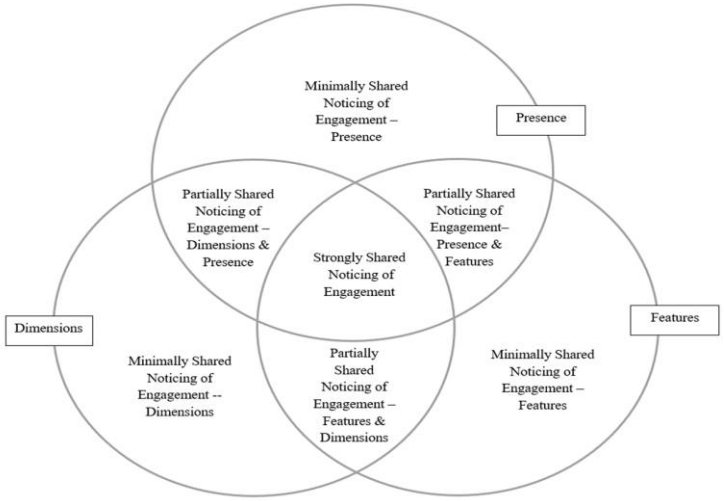

Figure 1: Framework for the Elements of Engagement

Case 1: Case of Strongly Shared Noticing of Engagement

Julie and her students represent a case of a teacher and her students expressing shared noticing of mathematics engagement according to all three elements in our framework: presence, dimensions, and features (see Table 1). In the activity captured on our video recording, we observed that Julie shifted out of a whole-group discussion and had students move into working in smaller groups during the mathematics lesson. When reflecting on the video, Julie and her students evaluated students' engagement similarly; they agreed that the students were engaged. They interpreted the opportunity to work in small groups as the feature which elicited this engagement and in describing this feature, they described engagement in terms of social engagement. Thus, we interpret this case as one of strongly shared noticing of engagement in a secondary mathematics classroom.

Julie attended to the social dimension of engagement (engagement through student interactions and discourse with and around mathematics) when she explained that she knew

students were engaged because they were interacting with each other to make sense of the mathematics. She explained that by providing the opportunity for students to work in groups, she avoided a potential pitfall of whole-class discussion: the same students answering all the questions. She said, "That's why I try to throw it out to them, because they're not communicating with me. I don't want to know just what three people know -- I want to know what everybody knows." She went on to explain that small group work allows her to engage in formative assessment by listening to conversations and to assess each student's knowledge. She said, "That's why I kind of, like I said, threw it back to them, circulated so that I could hear and talk to each group." In her reflection on the classroom video, Julie described students interacting with each other around the mathematics, and she interpreted the students' interactions to mean they were engaged.

Table 1: Julie and her students, Case of Strongly Shared Noticing of Engagement

<table>
<tr><th colspan="2">Elements of Engagement</th><th>Summary</th></tr>
<tr><td rowspan="2">Presence

Dimensions</td><td>Agreed:
Engagement</td><td rowspan="3">This teacher and her students expressed shared noticings for all elements [presence, dimensions, and features]. They agreed students were engaged and have some agreement about what engages students and why.</td></tr>
<tr><td>Agreed:
Social</td></tr>
<tr><td>Features</td><td>Agreed:
Students worked and talked together in groups about mathematics</td></tr>
</table>

Julie's students interpreted engagement in the video similarly to Julie. When reflecting on the video, the students said that the class was engaged, and explained that this was the case because of the interactions they were observing – indicating that they also connected social engagement with the presence of students being engaged. They also noticed instances of engagement similar to Julie's. Katie noted, "I think that the whole class was into the activity just because of all the talking that was going on. We were all discussing what was going on the board and arguing over the correct answer, which is definitely our class." Luna agreed with Katie, "[Student 1] was talking, but then [Student 2] started, and then people were there and over here started talking about what answer was right … yeah … that's when everybody was engaged." Both Luna and Katie interpreted that engagement was evident through discussing and arguing over the answer indicating the social dimension of engagement.

This case is an example of a strongly shared noticing of engagement between an instructor and their students. Both Julie and her students interpreted the video as indicative of social engagement by focusing on the interactions that students had around the mathematics. This indicates that Julie and her students noticed the same elements of engagement: when students have the opportunity to work together on mathematics, the activity can be engaging. Julie's decision to put students in groups suggested that she considered what her students needed to engage. It might be the case that when a teacher and her students have a shared perspective on what engages students (features) and why and how students engage (dimensions), students are more likely to be engaged.

Case 2: Case of Minimally Shared Noticing of Engagement – Presence

Jacob and his students represent a case of agreement on the presence of engagement with disagreement on the dimensions and features of engagement (See Table 2). When reflecting on the video clip, both Jacob and his students evaluated that students were not engaged; however, they associated this lack of engagement with different dimensions of engagement and features of the classroom. This video viewing session illustrates an opportunity for Jacob to improve his teaching practice, as both the teacher and the students agreed that students were disengaged.

Table 2: Jacob and his students, Case of Minimally Shared Noticing of Engagement – Presence

<table>
<tr><th colspan="2">Elements of Engagement</th><th>Summary</th></tr>
<tr><td>Presence</td><td>Agreed:
Disengagement</td><td rowspan="3">Although the teacher and students agreed that students were not engaged [presence], the teacher did not notice similarly to his students in terms of why and how students engaged [dimensions] or what engaged or disengaged students [features]</td></tr>
<tr><td>Dimensions</td><td>Disagreed:
Teacher – Cognitive
Students – Behavioral, Social</td></tr>
<tr><td>Features</td><td>Disagreed:
Teacher – Pressing for explanations
Students – Whiteboards</td></tr>
</table>

During the video recording, we observed that students first solved problems by writing on their desks with dry erase markers while Jacob and the classroom aide walked around to answer questions. Then Jacob brought the class together to discuss the answer to the problem they were working on: *Solve the system of equations: y = -4x -14 and y = 8x+2.* Students found an answer of (1.33, 8.67), but they observed that this ordered pair did not exactly satisfy the equations. In response to this, Jacob spent time explaining to the class that, when plugging an ordered pair in to check a solution, students should use a fraction representation rather than rounded decimals because the fractions are exact. During this whole class discussion, Jacob provided a few opportunities for students to call out answers, but primarily explained through direct instruction.

When Jacob described students' disengagement, his focus was on the challenges the students faced and how he handled them,

> ...they're willing to work until they're done with the problem and then they go away from the engagement. But I think in terms of just trying to explain with the fraction and things like that it was really … I was just challenging them to think on their own. … I was trying to challenge them to think about it and doing some prodding and things like that to steer them in the right direction.

Jacob explained that he noticed that student engagement was low, especially after they finished the problem they were working on. He described how he tried to engage students cognitively by pressing them to think about why the approximation did not yield the same answer as the exact fraction. When these justifications were not correct, he tried to steer them in the correct direction. Jacob's description of engagement focuses on the cognitive dimension; students could be engaged when they are asked to actively think about their own work.

When Jacob's students were interviewed about if they thought their classmates were engaged, they expressed that they did not think so, and both described evidence of this disengagement in terms of behavioral and social dimensions. Nikia said, "I feel like most of them wasn't [engaged]…you know how he said, 'So what'd everybody get?' Only one person answered. Then everybody was just looking down like this, playing with their markers." She decided they were not engaged because they were not paying attention. Ashiya agreed with Nikia on the presence, dimension, and feature of engagement, but provided an example from before the class discussion, "Whiteboards, yeah, it's a lot of talking. But you'll do the work. While you're doing the work, you'll talk to somebody. Then when you're done, you'll sit there and wait for [the teacher or the aide] to come around and say that's correct or they'll help you if it's not correct… but what Nikia was saying, it is a lot of talking." Although both students recognized their classmates' behavioral disengagement, Ashiya mentioned that sometimes when Jacob and the class aide were busy answering questions, students sometimes helped each other, recognizing the potential for social engagement, but she did not comment on whether or not helping each other was engaging.

Although Jacob and his students did not agree on the features or dimensions of engagement seen in the video clip, they did agree on the absence of student engagement. This indicates that Jacob and his students had a shared understanding of when students are not engaged, even if the dimensions and features were different. This is a case of minimal shared understanding, but we recognize the potential for teacher learning if there is a shared noticing about presence of engagement. If Jacob used this opportunity to learn about what students thought about how and why they were disengaged, he potentially could find ways to increase engagement in the classroom.

Discussion

We offer a framework for teacher and student noticing of mathematics engagement aligned with noticing stances of evaluating, describing, and interpreting (van Es & Sherin, 2002), as illustrated by these cases. Both teachers and students were capable of going beyond evaluating whether or not students were engaged (presence) to describe the nature of students' engagement. Both teachers and students also articulated features of engagement to interpret what may have elicited engagement in the classroom.

This study extends previous work on noticing by investigating noticing of engagement in contrast to noticing mathematical thinking (Jacobs et al., 2010; Hohensee, 2016; Labato et al., 2013) or noticing related to equity in mathematics teaching and learning (van Es et al., 2017). Additionally, previous research on teachers' noticing (e.g., Jacobs et al., 2010; van Es et al., 2017; van Es & Sherin, 2002, 2008) and students' noticing (Hohensee, 2016; Labato et al., 2013) did not compare what teachers noticed with what their students noticed. This study also demonstrates that teachers are capable of noticing a range of dimensions of engagement beyond behavioral and affective engagement, as seen in previous research studies (Bobis et al., 2016; Skilling et al., 2016), as these teachers also noticed cognitive and social engagement.

We conjecture that when a teacher and their students have a more strongly shared understanding of engagement, students' engagement is likely to be stronger, but this could be explored in future research. To establish a shared understanding of engagement, a teacher could (a) strive to understand their students' perspectives and adjust their teaching to align better with students' views or (b) more explicitly provide meaningful, explanatory rationales to students. When perspectives on engagement are not shared, this is an opportunity for teachers to learn

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about their students. When a teacher provides an explanatory rationale for their instructional choices, this can support students' autonomy and motivation (Reeve, 2009).

This study offers a framework for characterizing teacher and student noticing of mathematics engagement, and it investigates the potential for examining whether and how students and their teachers share noticing practices. The evidence provided in this study shows that teachers and students *can* share common descriptions, interpretations, and evaluations across this framework, and that differences in shared noticing can align with different elements of engagement. Our framework illustrates how elements of mathematics engagement can provide insight on the complex construct of engagement and how it may reveal opportunities for teachers to learn how to further engage their students in the future.

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