OPPORTUNITIES TO LEARN IN CYCLES OF ENACTMENT AND INVESTIGATION

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In recent decades, scholars of teacher education have suggested that teacher educators (TEs) should integrate the development of prospective teachers' (PTs') knowledge with their skills for enacting teaching, characterized in the literature as pedagogies of practice. One way to operationalize pedagogies of practice is through engaging PTs in cycles of enactment and investigation (CEIs). Using an opportunity to learn (OTL) lens, this study investigated one CEI enacted in a secondary mathematics methods course. Analyzing course artifacts and final interviews, we found that the PTs had OTL in all six nodes of the CEI, that OTL differed across the nodes, and that OTL in later nodes depended on knowledge built in previous nodes. Implications include the importance of PTs engaging in all nodes of a CEI to maximize OTL about mathematics teaching practices, mathematics, students, and learning.

Keywords: Preservice Teacher Education, Teacher Knowledge

In recent decades, scholars of teacher education have suggested that teacher educators (TEs) should integrate the development of prospective teachers' (PTs') knowledge with their skills for enacting teaching, which Lampert et al. (2010) described as using a *pedagogies of practice* perspective. TEs who design learning opportunities from a pedagogies of practice perspective focus on specific *decompositions of practice* (Grossman, Compton et al., 2009), which "[break] down practice into its constituent parts for the purposes of teaching and learning" (p. 2058). PSTs interact with *representations of practice* (e.g., narrative cases or video-recorded teaching episodes) and engage in *approximations of practice*, which are "opportunities for novices to engage in practices that are . . . proximal to the practices of a profession" (p. 2058). Theoretically, novices can learn complex practices by engaging in learning opportunities designed from a pedagogies of practice perspective (Grossman, Compton, et al., 2009; Grossman, Hammerness et al., 2009; Kazemi, et al., 2016; Lampert et al., 2010). Some mathematics teacher educators in the United States have taken up this perspective to design learning activities for pre-service teachers that include some form of engagement in approximation of practice (e.g., Lampert et al., 2013; Campbell & Elliot, 2015). Much of the early research regarding these designs has been descriptive in nature; now the field needs research to examine how such pedagogies relate to PTs' understandings and skills.

Theoretical Perspective

Opportunity to Learn (OTL) emerged in the 1960s as a construct for characterizing instructional environments by input variables that might predict student learning as an output (Elliott & Bartlett, 2016). Early works used variables such as instructional time spent on specific content, content coverage, and instructional quality indicators as proxies for OTL (Elliott &

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Bartlett, 2016). Gee (2008) explained that from what he called the mental representations perspective,

> learners have had the same OTL if they have been exposed to the same [content] . . . . If they have been exposed to the same content, then, according to this view, they have each had the opportunity . . . to "learn it." (Gee, 2008, p. 77)

As an alternative to the mental representations perspective on OTL, Gee argues that we should conceptualize OTL from a sociocultural perspective by conceptualizing learning as learning how to act in specific kinds of situations in ways that are aligned with the normative practices of some community. From this conceptualization, acting in some particular situation involves identifying objects in one's environment that one could use or act upon to achieve a desired result. The actor identifies *affordances,* which are defined as the possible actions that the individual can envision carrying out on, with, or in response to those objects. The actor then selects and operationalizes one of those affordances. To do so, the action must fit within the actor's understanding of which possible actions would be consistent with the accepted practices of some community that they identify with, which Cobb et al. (2009) describe as a *normative identity* that the actor has co-constructed with other members of that community. Further, the actor must have *effectivity* with respect to the selected affordance, defined as the capacity to operationalize a possible action (Gee, 2008).

Methods

The context for this study was a semester long methods course for secondary mathematics PSTs at a Mid-Atlantic university. Two mathematics teacher educators (MTEs) taught the course, which met two times a week for fifteen weeks. Seventeen of the 18 PTs in the course participated in this research. The MTEs designed the course from a pedagogy of practice perspective. Specifically, the course involved three cycles of enactment and investigation (CEI) (Lampert et al., 2013; Arbaugh et al., 2020) as described in Figure 1. The focal decomposition of practice in all three CEIs was a set of communication moves: Asking assessing and advancing questions, and using judicious telling (Freeburn & Arbaugh, 2017).

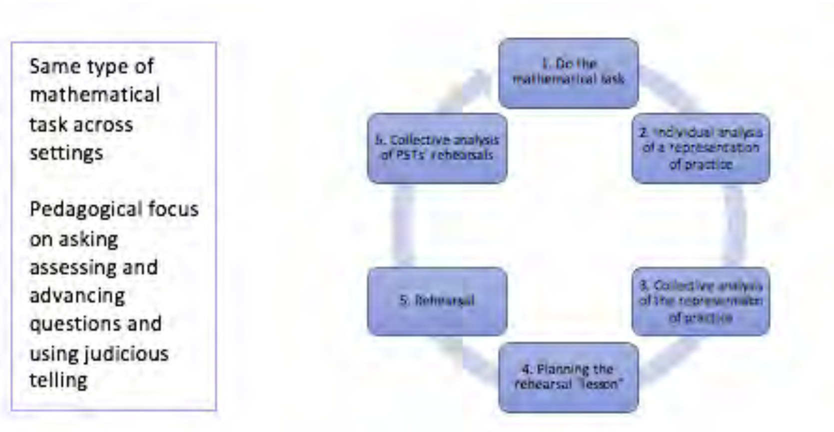

Figure 1: The CEI

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In general, PTs begin a CEI in Node 1 by doing and discussing a mathematical task. In Nodes 2 and 3, PTs analyze and discuss a representation practice (e.g., narrative case) through the lens of a focal decomposition of practice. In Node 4, PTs use the focal practice and mathematics discussed in previous nodes to frame their planning for enacting teaching in Node 5, where they rehearse a teaching episode with simulated students. In Node 6, PTs analyze their rehearsal videos through the lens of the focal practice. This study focused on one of the three CEIs, which occurred mid-way through the course. In Figure 2, we give a brief description of the events that took place in this specific CEI.

Node 1: Doing the Mathematics			
Goal: Defining the mathematical learning goal for CEI focal task by developing criteria for determining if a mathematical argument is a proof.	PTs completed the Odd + Odd = Even Task (Blinded)	PTs analyzed student work for Odd + Odd = Even (Blinded) task to judge whether argument is a proof or not.	Group reached a consensus for criteria to use for when an argument counts as a proof

Nodes 2 and 3: Individual Analysis and Collective Analysis of the Narrative Case		
Goal: Applying PTs' understanding of focal practice to analyze a representation of practice.	PSTs individually coded the narrative case through focal practice	PTs discussed their analyses of the narrative case in small group and whole-class discussions.

Node 4: Planning for the Rehearsal		
Goal: Learning to plan instruction using a focal practice in rehearsal.	PTs completed rehearsal task.	PTs used focal practice to frame their planning guided by a modified "Thinking Through the Lesson Protocol" (TTLP) (Smith, Bill, & Hughes, 2008).

Node 5: Rehearsal	
Goal: Developing skills for engaging in the focal practice and developing deeper understandings of the teaching practices addressed in previous nodes and course activities.	The PTs individually enacted their plan from Node 4 by engaging a "student" in moving towards achieving the mathematical goal of proving a number theory conjecture.

Node 6: Collective and Individual Analyses of Rehearsal		
Goal: Developing skills of analyzing teaching	Using StudioCode©, small groups of PTs collectively coded each PTs' rehearsal.	PTs individually reflected on what

through the focal practice.	Analysis included writing rationales for coding choices as well as an assessment for if the focal practice “worked” or not based on student response.	they learned from engaging in rehearsal and analyses.

Figure 2: Goals for PT learning and descriptions of PT activities in each node of the CEI

Data Collection

Data analyzed for this study include data collected during the multi-day enactment of the CEI. Audio-recordings were collected of whole-class discussions during Nodes 1, 3, 4, and 5 and small-group discussions in Nodes 1, 3, 4, 5, and 6. PTs’ rehearsals in Node 5 were video-recorded. Students’ written artifacts - notebooks, assignments, reflections that occurred during the CEI were collected. In addition, post-course interviews were captured by audio-recordings; the interview data analyzed for this study focused on responses to questions that asked PTs to reflect on how the CEI activities supported their learning.

Data Analysis

Our unit of analysis was a segment of communication, which we define as a series of turns of talk with a common focus (Bishop et al., 2016) and with a consistent form of participation (whole-class, paired work, individual work, or group work). We analyzed data sources in three phases. In phase one, we randomly chose three participants’ data corpus and used the four dimensions of Ghoussieni and Herbst’s (2016) Framework for Learning to Teach (FLT) as a priori codes: *Knowledge of Students and Content*; *Repertoire of Practices and Tools*; *Dispositions for Teaching and Learning*; and *Professional Vision* (see Table 1, Column 1 for definitions). At the same time, we began to develop subdimension codes using constant comparative analysis (Miles, Huberman, & Saldaña, 2013) and inductive analysis, and wrote analytic memos that detailed commonalities across the data. Once satisfied with the secondary coding scheme with the limited set of data, we began coding additional participants’ data, refining the secondary codes (e.g., renaming, collapsing similar codes) until all coding was complete. Table 1 contains the resulting subdimensions (Column 2). In phase two, we coded segments for appropriate CEI node (1-6). In phase three, we organized our coded segments (*n*=414) and related analytic memos into a data table that allowed us to sort the instances by CEI node, dimension, or Subdimension. We used the sorted table to identify themes (Miles, Huberman, & Sa , 2013) in the data that allow us to describe the PTs’ opportunities to learn during each CEI node and in each dimension across the nodes. We constructed frequency counts of the coding in each dimension across the CEI’s nodes. Within each dimension, we created frequency counts of the subdimensions identified during phase 2. We then examined the frequency counts and made profiles of the OTL in each node. We then examined OTL across the nodes to arrive at the claims we present next in the findings section.

Dimensions & Descriptions (Descriptions are excerpted from Ghousseini & Herbst, 2016)	**Subdimensions Identified through Data Analysis**
Knowledge of Students* & Content: Understandings of the subject matter, of students as learners, and of ways to support their engagement with this subject matter . . . Teachers not only need to know the content but also understand the kind of reasoning that is entailed in doing mathematics. They should be able to interpret student work in light of what students already know and the tools at their disposal. (p. 82–83)	**Criteria for Valid Arguments -** PTs articulate criteria for a valid argument or criteria for why an argument is not valid.
	Type & Components of Arguments - PTs describe a type of an argument (e.g., proof by induction) or components of proof and reasoning (e.g., identifying a pattern).
	Representations in Arguments - PTs recognize a type of representation and address how the representation is incorporated into an argument.
	Mathematical Ideas and Practices - PTs describe a particular mathematical concept(s) or practice(s).
	***Student difficulties -** PTs identify students' errors in arguments or suggest ways a student could improve an argument.
	***Students think differently about the same task.** PTs explain similarities or nuances among student arguments as well as attributes among student arguments.
	***Validity of Arguments vary depending on grade level -** PTs share ideas about validity or appropriateness of student arguments as a consequence of grade level.
	***Student mathematical understanding -** PTs address student's understanding of mathematical content or capabilities to engage in mathematical practices.
Repertoire of Practices and Tools: Support teachers' beginning enactment of important aspects of instruction. Tools . . . can help teachers translate abstract conceptual tasks into more concrete steps and objectives (p. 83).	**Recognizing practices and tools -** PTs recognize a teaching move or routine in a segment of classroom instruction.
	Attributes of practices and tools - PTs articulate features, definitions, purpose, or characteristics of a teaching move or routine.
	Engaging in Practices - PTs engage in or reflect on their engaging in a teaching move or routine.

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Dispositions for Teaching and Learning: Teachers' dispositions to see students as sense makers and learn the intellectual and professional stance of inquiry are important aspects of teachers' learning in and from their practice (p. 83).	**Honoring Student Thinking -** PTs communicate their stance that mathematics instruction should recognize, incorporate, or build on student thinking.
	Learning mathematics for understanding - PTs communicates their stance towards learning mathematics for sensemaking or understanding.
Professional Vision: The ability to notice and interpret features of practice in ways that are valued by a particular professional group. . . . A vision of practice may also delineate what is possible and desirable in teaching . . . it gives teachers a sense of direction (p. 82)	**Visions of instructional practice -** PTs notice and interpret a component of instruction as desirable based on the PTs' interpretation of a community's considerations for teaching and learning.

Figure 1: The A Priori Dimensions and Emergent Subdimensions of the FLT

Findings

One adaptation of the Gousseini and Herbst (2014) framework that resulted from our analyses is a need to separate *Knowledge of Students and Content* into two distinct dimensions: *knowledge of students* and *knowledge of mathematical content*, which more closely reflects a pedagogical content knowledge (Grossman, 1990) perspective. Subdimensions for *knowledge of students* are indicated by asterisks in Figure 1. We present three claims in this paper (see Table 1), and then, due to limited space, we expand only upon Claim 3 to show how the OTL in later nodes depended upon knowledge developed in previous nodes.

Table 1: Three Claims

Doing the mathematics (Node 1) and planning for teaching (Node 4) created opportunities to develop knowledge of content.
Analyzing the narrative case (Nodes 2&3), engaging in the rehearsal (Node 5), and reflecting on the rehearsal (Node 6) created opportunities to develop a repertoire of practices and tools.
Doing the mathematics (Node 1), planning for teaching (Node 4), and reflecting on rehearsals created opportunities to develop knowledge of students.

Doing the Mathematics, Planning for Teaching, and Reflecting on Rehearsals Created Opportunities for PTs to Develop Knowledge of Students

Evidence of opportunities to develop knowledge of students was much more prevalent in Nodes 1, 4, and 6 than in Nodes 2, 3, or 5. Further, the number of PTs providing that evidence is higher in Nodes 1 (n = 12), 4 (n = 11), and 6 (n = 7) than in Nodes 2 (n = 3) or 5 (n = 2). For those reasons, we conclude that OTL in the *knowledge of students* dimension are primarily accounted for in Nodes 1, 4, and 6 and we next describe the progression of the OTL across these nodes in two subdimensions: *student difficulties, errors, and areas for improvement* and *differences* and *students have different ways to think about the same task.*

Student difficulties, errors, and areas for improvement. The discussion of the student work samples in Node 1 provided OTL for PTs to develop their knowledge of the kinds of errors that students might make when attempting to engage in argumentation. For example, as PT10 read the Student Work Sample A, she noticed a similarity between the argument from Student A and the argument that PT10 had constructed, namely that both Student A and PT10 had used one variable (n) to represent two different odd integers. The instance is evidence of OTL for identifying an error that might occur as a student engages in constructing an argument. Similarly, as PT10 and PT13 examined Student F's argument, PT10 stated, "I don't know how you would judge what they know from this." T13 stated, "Well, they have some errors." The PTs agreed that the algebra of the argument is wrong, and that the argument lacked coherence. As PT13 said, "[“he] statements are not connected to the ones before it." In Node 4, as PTs responded to the elements of the TTLP, they discussed possible errors that students might make or misconceptions that they might have when attempting to argue for the given claim. These errors fell into two broad categories: Errors that were general to argumentation, and errors that were specific to possible approaches to arguing for a particular claim. For example, PT7, PT8, and PT9 suggested that a student might not understand definitions of *even* and *odd*, that a student might consider examples sufficient justification for a general claim or might not use enough evidence to justify the claim. These potential areas of difficulty are more general across claims. However, they also examined errors for each of five different possible approaches to proving the specific claim that they were assigned.

In addition to opportunities to consider both difficulties at the general level and at the specific level, PTs’ statements again gave evidence that their OTL was mediated by their experiences in previous nodes. Specifically, in Node 4, while discussing potential errors related to how students might argue for their claim, PT3 anticipated that students might use a table of values to present examples in support of the claim that the product of two squares is a square. PT3 connected that anticipated response to their experience in Nodes 2&3: "Kind of like the students in [the case narrative]. Using examples to prove, but we need to get them to do a general case. Not just use examples."

Students have different ways to think about the same task. In Node 1, PTs had the OTL to see that students (themselves, their peers, and the students represented in the student work samples) had different approaches for writing an argument that the sum of two odd numbers is always even. In Node 1, PT12 and PT15 briefly discussed their arguments for the task - –T15 explained that his argument involved stating that odd numbers are even numbers plus 1 and that the sum of the two odd numbers then will be an even number plus 2. PT12 replied that she had argued for the claim using the same approach. Each of the groups noted strong similarities between the arguments from Student B and Student C and interpreted the differences as a distinction between a valid argument (Student B) and a not valid argument (Student C). The

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activity also included opportunities to recognize similarities between the arguments in Student Work Samples and their own attempts at proving the claim--for instance, while reviewing Student D's argument, PT10 stated that the argument was valid because it was similar to an argument that she and PT13 had previously identified as valid (though she did not clearly indicate which Student Work Sample she was referencing) as well as to the argument that PT10 had made in her own attempt to prove the claim.

The main areas of OTL within the Node 1 activities were in the dimensions of Knowledge of Content and Knowledge of Students. Given that the activities were explicitly intended to engage PTs in conversations about criteria for valid arguments, it is unsurprising that the plurality of instances coded in Knowledge of Content dimension were related to the domain of Criteria. However, in the context of those conversations about criteria there was also OTL about types of arguments, the components of arguments, and to compare and contrast students' arguments toward a claim as well as the errors or areas for improvement in students' arguments. These domains of knowledge are important for establishing learning goals for students' argumentation, for anticipating the kinds of arguments that one might encounter in a secondary learning environment, and for framing how one determines, of the affordances he or she recognizes as possible actions to take in response to student argumentation, which affordance to attempt to transform into action.

In Node 4, the OTL about students' mathematical understanding was primarily a consequence of PTs anticipating students' solutions in response to an element of the TTLP. Drawing on the PTs' arguments in Node 1 as well as the Node 1 student work, PTs discussed arguments students may make for their assigned number theory task. PTs also raised questions about what knowledge students might be expected to already have, and whether that would change which parts of the argument would need to be supported rather than assumed. For example, PT8 and PT9 wondered whether students could be expected to know that N^2 is even if and only if N is even, and if so whether that would mean that students could draw on that fact without justifying it. Evidence in Node 4 indicates that PTs drew upon their experiences in Node 1 as a resource to support their anticipation of student thinking. For example, PTs referred back to the student work samples from Node 1 for ways to represent even numbers and odd numbers.

Connecting to the theoretical perspective. Viewed through the sociocultural perspective, the PTs had OTL in Nodes 1 and 4 about how to act as a teacher who knows how students think mathematically in ways that are aligned with what it means to prove, which is a normative practice of the mathematics community. In doing the mathematical task and analyzing student work in Node 1and then anticipating student responses in Node 4, the PTs had the opportunities to build the kinds of knowledge that will allow them to preplan possible actions they can choose from in Node 5 (rehearsal).

Discussion and Conclusion

This study contributes to the field's understandings of what PTs and the opportunity to learn from engaging in pedagogies of practice, extending the work of Arbaugh et al. (2019; 2020), Ghousseini and Herbst (2016), Tyminski et al. (2015), and Baldinger et al. (2016) and adding to current evidence of the impact of CEIs on PTs' building of knowledge about teaching mathematics. What makes this research unique is that we studied OTL through the content of what PTs took up and discussed in small groups, large groups, and in reflective interviews. Much OTL work has been done from a researcher-down perspective – what we, as researchers, intend for PTs to learn. Considering OTL through a PT lens provides powerful indicators of the possible impacts of engaging in CEIs. One implication from this research is that it is important to engage PTs in a whole CEI – not just choose to do parts of it (e.g., planning and then enacting practice in rehearsals). Our findings indicate that opportunities to learn occurred in all CEI nodes and, perhaps more importantly, OTL in latter nodes depended upon knowledge built in previous nodes. We have also come to understand the power of having PTs analyze student work samples in Node 1. Simply doing the mathematical task itself would not have offered the same kind of OTL about student thinking that doing the task and analyzing the work samples did.

This study joins very few others who are examining (possible) outcomes for PTs who learn to teach through engagement in pedagogies of practice and learning cycles. Much work is to be done before the field has a solid understanding of this kind of pedagogy in ways that are convincing about its effectiveness.

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