PROSPECTIVE MATHEMATICS TEACHERS' DESIGNED MANIPULATIVES AS ANCHORS FOR THEIR PEDAGOGICAL AND CONCEPTUAL KNOWLEDGE

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Positioning teachers as designers of curricular resources invites opportunities for exploration at the intersection of content, pedagogy, and design. As researchers accepting greater responsibility for preparing teachers to maintain a commitment to their pedagogical vision in practice, this work seeks to cultivate the imagination of humanistic forms of mathematics teaching and learning by supporting these explorations. Toward that end, this paper reports on research that examines connections between the pedagogical/conceptual knowledge that prospective teachers embed in the designs of original manipulatives and how those designs mediate the pedagogical moves they make in teaching situations. The promise of this work is that these connections may reveal a viable means to support bolder connections between teacher preparation and practice. Implications of our findings for teacher preparation are considered.

Keywords: Teacher Knowledge, Technology, Preservice Teacher Education

It is an unfortunately perennial problem that teachers often experience considerable challenges in transferring their theoretical knowledge into practice (Ünver, 2014). While teacher education programs that explicitly link teacher preparation coursework to field experiences tend to be more effective than those that do not (National Academy of Education, 2005), colleges and universities have often been criticized for implementing teacher education programs that do not sufficiently engage their students in actual and ongoing practice situated in authentic education settings. Although future teachers tend to craft their pedagogies as they learn about research-supported instructional methods, teacher educators also stress the importance of developing one's practice in real classrooms with real students (Kazemi, et al., 2009). It is with this critical concern in mind that the field seeks to determine the means by which teachers can transform teacher knowledge from theory into practice through approximations of practice (Grossman, et al., 2009) that simulate the work of teaching.

Our work connects with the body of literature that frames teachers as *designers* (e.g., Brown, 2009; Svihla et al., 2015) of teaching and learning experiences and the material resources that mediate them. We conceive of design broadly to include the "intentional activity of transforming ideas and knowledge" (Carvalho et al., 2019, p. 79) into "tangible, meaningful artifacts" (Koehler & Mishra, 2005, p. 135). Our purpose in doing so is to present a novel Making experience within mathematics teacher preparation that we hypothesized would inform their conceptual and pedagogical thinking. Making in this sense is conceived as the creative production of artifacts via activities that include designing, building, and innovating with tools and materials to solve practical problems (Halverson & Sheridan, 2014). Thus, the experience tasks prospective mathematics teachers (PMTs) with digitally designing (using Tinkercad; Autodesk Inc., 2016), 3D printing, and evaluating original manipulatives that are responsive (Akuom & Greenstein, 2021) to the curricular (Dewey, 1990; Pinar et al., 1995) needs and interests of actual learners.

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While there is a considerable body of research on *students'* mathematical Making (e.g., Bower et al., 2020; Valente & Blikstein, 2019), research is only beginning to uncover the benefits that *teachers* experience in Making contexts (Greenstein & Seventko, 2017; Greenstein & Olmanson, 2018; Greenstein et al., 2019). Our prior research (Akuom & Greenstein, 2021) addressed this gap by exploring the conceptual, social, and material resources that mediate (Vygotsky, 1978) the design decisions of prospective teachers' Making of mathematical manipulatives. This paper reports on research that extends that work by discerning whether connections can be made between the pedagogical/conceptual knowledge that prospective teachers construct in teacher preparation and how that knowledge is enacted in their teaching. Specifically, this work seeks to address the question: *As prospective teachers Make new manipulatives for mathematics teaching and learning, can connections be made between pedagogical/conceptual resources for their design decisions and how those designs mediate the pedagogical moves they make in practice?* If connections can be made between the knowledge that prospective teachers construct in teacher preparation, how that knowledge materializes in their designs of physical manipulatives, and how those knowledge-embedded designs mediate their teaching interactions, we propose that these findings can illuminate and subsequently strengthen the relationship between instructional intention and enactment in particular (see Remillard, 2018), and teacher preparation and practice more broadly.

Theoretical Framework

Fundamentally, this research is about the mediating role of conceptual, social, and material resources in design activity. In particular, we seek to extend prior research on the resources and rationales that mediate design decisions when designing a tool by exploring the mediating role of those tools in teaching situations. Accordingly, we take a sociocultural perspective and ground this work in the notion of mediated activity, derived from Vygotsky (1978) and advanced as instrumented activity by Verillon and Rabardel (1995). In terms of instrumented activity, an artifact is a material object that becomes an instrument (e.g., tool, sign) for the subject (e.g., actor, learner, teacher) when the subject has integrated it with their activity. Thus, an instrument is a psychological construct (as opposed to a material one) that "results from the establishment, by the subject, of an instrumental relation with an artifact" (p. 85). What the distinction between artifacts and instruments reveals is the possible range of actions one might take with an artifact and what those actions might implicate about a subject's knowledge. For our purposes, we are specifically interested in PMTs' pedagogical and conceptual knowledge and how their practice is mediated by such knowledge as it is intentionally embedded in their designed artifacts.

In our prior research, we analyzed PMTs' design decisions – and the rationales they gave for those decisions – as they made original manipulatives to teach a mathematical concept. As they designed these manipulatives, it was the PMTs' intention (Malafouris, 2013) to embed their tools with particular affordances (Gibson, 1977) for utilization schemes (Verillon & Rabardel, 1995) that they hypothesized would enable the child to form abstractions, through their sensorimotor engagement (Kamii & Housman, 2000; Piaget, 1970), of the perceptual elements that are the groundings (Nathan, 2014) for target concepts. As this *learning by design* (Koehler & Mishra, 2005; Koehler et al., 2004) process invites occasions for their active inquiry, PMTs made a host of design decisions for a variety of reasons; they drew on a range of conceptual, social, and material resources to mediate them. In order to characterize and organize these resources, we appealed to Sc (1992) design -centered notion of "knowing in action" (p. 2). Sc considers knowledge to be in action as "the designer sees what is 'there'… draws in relation to it,

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and sees what [they have] drawn, thereby informing further designing" (p. 5). This thought-revealing (Black & Wiliam, 1998) process of seeing-drawing-seeing is what Schön means by the phrase "designing as a reflective conversation with materials" (p. 3).

For this phase of the research in which we analyze PMTs' usage of tools in practice, we use the term *embedding* to connote an intentional design element that embeds a PMT's pedagogical and/or conceptual (i.e., mathematical) knowledge. As an example, a PMT named "Moira" designed a fraction tool with a variety of fractional pieces of a whole. She was concerned that if each piece had its own unique color, that might "take away reasoning from children. If a student believes that a yellow ring represents sixths, they will immediately reach for yellow the second that they hear sixths." By giving the pieces the same color and leaving them "unmarked," she intended for children to construct their own meanings for each of the [pieces]. Thus, we say that pedagogical/conceptual knowledge mediated this design decision and refer to the corresponding design element as an *embedding* of that knowledge. In addition, when we infer from a PMT's use of the manipulative in a teaching situation that the tool served as a resource for (e.g., a reminder of) pedagogical and/or conceptual knowledge embedded in the tool, we will refer to that as an *anchoring* phenomenon, as in, "Moira's fraction tool served as an *anchor* for her attention to the pedagogical practice of implementing tasks that promote mathematical reasoning."

Methodology

This study is part of a larger project that aims to test and refine the hypothesis that a pedagogically genuine, open-ended, and iterative design experience centered on the Making of a mathematical manipulative would be formative for the development of PMTs' inquiry-oriented pedagogy. The larger project took place across two semesters of a graduate-level specialized mathematics course for PMTs at a mid-sized university in the northeastern United States. Forty students comprised thirty-four groups. For the study reported here, we took an exploratory case study approach (Yin, 2009) in order to determine what connections could be made between pedagogical and conceptual rationales for PMTs' design decisions and how those designs mediated the pedagogical moves they made in enactment. We did so by taking as the unit of analysis *instances* in PMTs' teaching when the use of their manipulative implicated the pedagogical and/or conceptual knowledge underlying their design rationales. The locus of these particular research efforts among the broader research project is depicted as the arrow from "Design Decision" to "Enactment" in Figure 1. In addition to the PMT's designed manipulative and a video recording of problem-solving interviews with them and their elementary-age focus student, four written project components comprised the data corpus: a "Math Autobiography," an "Initial Idea Assignment," a "Project Rationale," and a "Final Paper/Reflection," which includes findings from their problem-solving interviews.

We took a grounded theory (Corbin & Strauss, 2008) approach to analyzing the data. We began by collectively analyzing the written and video components of one PMT's design case to identify instances in their teaching from which we could infer that the PMT leveraged a particular embedding of a design decision in their manipulative to enact a teaching move that was consistent with aspects of their purported pedagogy, which they shared in the written artifacts of their Maker projects. These inferences constitute our conjectures that their designed manipulative served as an anchor for the pedagogical/conceptual knowledge they had been constructing in the course. We generated codes for this design case to characterize connections between embeddings of design decisions and their mediating role in the PMTs' teaching. Next, we collaborated to identify additional instances of anchoring in other design cases. Analysis

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involved the constant comparison of data to ensure coherence is maintained across the generated codes and to get a good sense of the variety of ways in which affordances of the designed manipulatives that were either intended (those that PMTs intended to embed in their tool) or unintended (those that PMTs hadn't intended but realized in practice) could be leveraged to support a PMT's pedagogy.

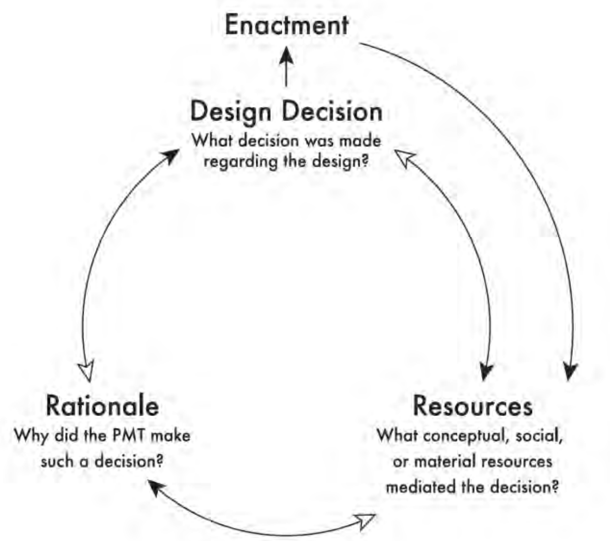

Figure 1: Conceptual resources inform rationales for design decisions and may also be evoked in enactment. Open arrows acknowledge that feedback is reciprocally informing.

Results

Here we present just three excerpts from among the thirty-four task- and tool-based problem-solving interviews that PMTs conducted with the intended user of their manipulative. Findings from our analyses of these excerpts suggests that they are instances in a PMT's teaching when a pedagogical move they made was mediated by the instrumental leveraging of a design affordance whose rationale was explicitly linked by the prospective teacher as designer to their pedagogical and/or conceptual knowledge. In short, these are instances in which a design embedding served as an anchor for a PMT's pedagogical and/or conceptual attention. The manipulatives mentioned in these results are shown in Figure 2.

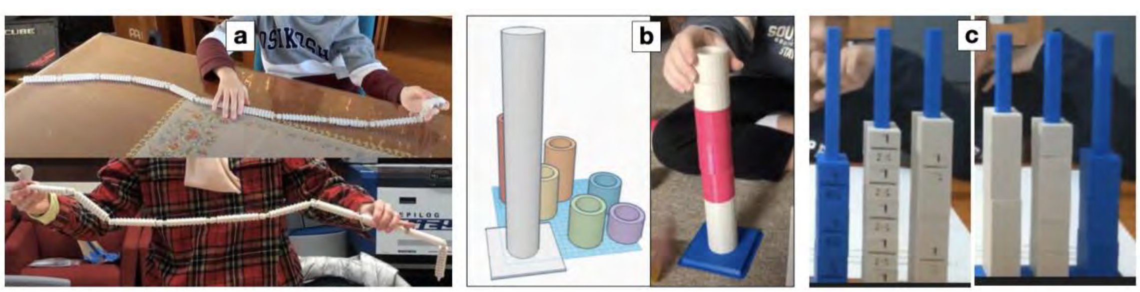

Figure 2: (a) Roda's decimal tool; (b) Kerina's fraction tool; (c) Anyango's fraction tool.

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Reasoning about the unit whole

Roda designed a "Decimal Snake" in order to teach a child about decimals and decimal comparison. As shown in Figure 2a, her tool consists of ten connected pieces. Each of these pieces is equally partitioned into ten parts. Thus, the decimal snake can be used to represent tenths of tenths, or hundredths, of a whole, that is, any value between 0.01 and 1 to two decimal places. These design features are Roda's embeddings of the concepts of the whole and its decimal parts.

At one point in the interview, we observe Roda asking the child to compare 5.5 and 5.47. [Note that it would not be possible to represent 5.47 if the entire snake represented 1 or even 10.] The child responds, "5.47 is 5 and 47 hundredths, because it's 3 hundredths away from 5 and 5 tenths." Perhaps because Roda is interested in how her tool can support the child's reasoning, she then asks him to "Use the tool to show me?" Over the next sixty seconds, we witness the child struggling to locate 5.5 and 5.47 on the tool. Finally, he locates 5.5 at (what we would identify as) 0.55 (if the entire snake represented 1), and 5.47 at 0.47. Given that several minutes earlier the child established that entire snake is the "whole" and that each piece of the snake is one tenth of a whole, we infer from his solution – locating 5.5 at 0.55 – that he had unintentionally designated each piece of the snake as 1 (as opposed to 0.01) and each partition of a piece as 0.1 (as opposed to 0.01). In doing so, he changed his designation of the entire snake from the whole (1) to 10, and consequently, each piece of the snake now represented 1. Thus, 5.5 would be presented as the 5th partition of the 5th piece.

Roda's next move aims to help the child identify and resolve this confusion. When she asks him to "Show me one tenth," he points to one of the tenth pieces. When she asks for, "Two tenths," he points to the second piece. Then she asks, "Where is 5 and 5 tenths?" And in doing so, she perturbed his thinking and provoked disequilibrium. Soon thereafter, he resolves it and declares, "Oh, wait! This [entire snake] is one whole! 5 and 5 tenths, you can't even make it out of the snake!" In response to this unanticipated move in the child's activity, Roda leverages an affordance of her tool – namely that each piece of the snake could represent either a tenth of a whole or one of ten wholes – and she exploits it to support new ways of thinking for the child as he resolves his confusion about the representational capacities of the tool.

R(oda): You need how many snakes to make 5.5?

C(hild): You need 5– No, 6 snakes!

R: How can we compare [5.5 and 5.47] using 1 snake? Is that possible?

C: We can pretend that each piece is one snake.

In this instance, Roda leverages the embedding of a conceptually resourced design decision that enabled the snake's user to engage in conversations about the unit whole. Specifically, she leveraged a design decision that allows for flexibility in naming the unit whole in relation to the snake and its pieces. And her rationale for leveraging that affordance was a pedagogical one. Rather than correct the child's interpretation, she sought to help him reason through his interpretations in order resolve the confusion himself. In this respect, the tool's capacity for flexible interpretations of quantities (a conceptually resourced design decision) served as an anchor for pedagogical knowledge about the value of revealing student thinking and posing purposeful questions to advance their mathematical reasoning. Worth noting, Roda did not plan for this conversation about the unit whole, nor had she anticipated it. Regardless, her tool mediated activity that made it possible to do so.

Generating a space of inquiry

The second instance we present is from the problem-solving interview that Kerina conducted using the fraction tool she designed for conversations about the meaning of a fraction's denominator (see Figure 2b). Kerina's tool features "*a variety of rings which each represent different fractions (from 1/½o 1/8) that are scaled in relation to the pedestal [whole] that they go on top of."* Each set of like fraction pieces is a *"different color, so it's easy to determine which pieces are the same size."* When fraction pieces are stacked on the pedestal, the tool provides feedback to the child that they can use to determine whether that combination is equivalent to a whole.

Kerina's fraction pieces have no identifying attributes other than color, so if a child wanted to determine what fraction of a whole is represented by a pink piece, for example, they would make that determination by seeing how many pink pieces it takes to "fill" one pedestal. If 6 pink pieces fit on a pedestal, then each pink piece would represent ⅙. This finding would give meaning to the 6 in the denominator of fractions of the form $n/6$. As she designed her manipulative, Kerina was mindful that students tend to struggle with symbolic representations of fractions, particularly in the context of adding fractions and "finding least common denominators." As an alternative, she proposed that "students' brains will work in more creative ways than we can anticipate." Accordingly, she wanted to design her tool that would accommodate such diversity and enable students to "visualize" concepts and avoid the "frustration" that purely symbolic approaches to fractions often cause.

With these intentions in mind, Kerina embeds a particularly salient feature of her pedagogy in the design of her tool that is made evident in one task that challenges a child to use the tool to "Find three different ways to make a whole." Operating in tandem with a tool that requires its users to construct their own meanings for each of its pieces, the task generated a space (Stroup et al., 2004) for the child's active, creative, and playful inquiry and insight into fraction meanings and relationships. Indeed, Kerina designed her tool for such an imagined utilization scheme in which the child, at least initially, uses trial and error to stack different pieces onto the pedestal and then "see how much space is left" before adding on more pieces to make the whole. These accomplishments would be seen as groundings (Nathan, 2014) for connections she would subsequently help the child make to symbolic representations of their tool-based activity.

In practice, we observed Kerina's commitment to her design intentions. At one point, when she posed her "Find three ways" task, the child selected pieces of the same size to place on the pedestal in order to form a whole. Kerina notices this strategy and asks the child to "Try to use ones that have different denominators." Note her use of "different denominators" as opposed to "different sizes," even though she's referencing physical objects. In doing so, she is cultivating a connection between physical and symbolic representations of fractions. At the same time, it's also important to note that Kerina had written the symbolic names of each fraction piece on its interior where they could be concealed from the child's view. Thus, she seems to have a trajectory in mind for the meaningful development of fraction proficiency from physical to symbolic representations of collections of different unit fractions. Her tool and tasks are anchoring pedagogical and conceptual knowledge that mediate her response to the child's initial activity at this moment as she supports his construction of procedural fluency on a foundation of conceptual understanding. Specifically, design elements of her tool embed conceptual knowledge relevant to that trajectory (e.g., a "complete" stack of pieces represents a sum of unit fractions equal to 1), and design elements of both the tool and the task embed pedagogical knowledge about the value of enabling multiple solution strategies in order to generate a space for open and productive inquiry.

Noticing in action

Anyango designed a fraction tool "to help the student visualize and deepen their understanding as they explored fraction relationships." Her tool looks similar to Kerin's and appears in Figure 2c. In contrast, however, Anyango emphasizes a different purpose for a similar affordance. She explained that her design decision to stack fraction pieces on vertical pegs rather than lining up those pieces horizontally would enable her to use those pieces to represent "height as value and amount." "What was most important to me," she wrote, "was having all the fractions mounted on one platform with the 1 (whole) always being visible, so that the student could begin to grasp how all the smaller parts can equate and compare to the whole." Also in contrast to Kerin's design, Anyango engraved the name of each piece on one of its lateral faces.

In practice, Anyango posed the following task to an intended user of her tool: *Jack and his two friends each had the same size pizzas for lunch. Jack ate 5/8 of his pizza. Judy ate 2/3 of her pizza. And Sam ate 3/6 of his pizza. Who ate the most pizza? Who ate the least?* In response, the child stacks five one-eighth pieces, two one-third pieces, and three one-sixth pieces, each on their own pedestal with their labels facing him (as shown in the image on the left of Figure 2c) and says nothing further. Following up on the child's activity, Anyango asks, "So, if we just look at this, who ate the most?" We interpret this pedagogical move by Anyango as one that leverages her design decision to represent fractional values in terms of height by directing the child's attention to the relative heights of the three fraction pieces. In other words, she's prompting the child to decide which person ate the most pizza by choosing the fraction piece that is the tallest, and which person ate the least by choosing the piece that is the shortest. Counter to her expectations, the child attended exclusively to the symbolic representations engraved on each piece and not their heights. This led him to decide that, "It's Jack" (represented by the ⅝ piece) who ate the most. He justifies his answer by saying that "5 out of 8 is the biggest of all of them… 2 out of 3 is smaller and 3 out of 6 is… kind of small." When Anyango asks, "What makes you think it is small?" he explains that, "The top is two and the bottom is three." We infer from this response that the child is basing his comparisons on interpretations of fractions not as parts of a whole but as two separate whole numbers. This would explain why, for the child, ⅝ is greater than 3/6, which is greater than ⅔.

We interpret Anyango's next move as a noticing one (Sherin et al., 2010) that leverages her pedagogical knowledge about the efficacy of attending to, interpreting, and responding to student thinking. Indeed, the design of her tool embeds this knowledge, as a primary rationale for its design was to enable a child to compare fractions without having to rely on the overhead of a symbolic representational infrastructure. In a move that we interpreted as unplanned and that was therefore striking for each of the researchers to observe, Anyango turns her tool around (see Figure 2c, right) in order to hide the symbolic labels on each piece.

A(nyango): If I turn this [*pedestal*] around [*so that the child's gaze can no longer be restricted to the fraction labels on the pieces*], who has the most?

C(hild): This one [*points to the stack of two one-third pieces, which corresponds to Judy's share*].

A: Who has the lowest?

C: This one [*points to the stack of three sixth-pieces, which corresponds to Sam's share*].

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What we find remarkable is that while Anyango made the intentional design decision to label each of her pieces, this "flipping" move leveraged an *unintentional* design affordance, that the opposite face of each piece is *not* labeled. In this regard, we suggest that Anyango's tool served as an anchor for a pedagogical *knowing* in action mediated by that affordance. Translating Sc oncept of knowing-in-action as a noticing-in-action, we suggest that in this instance, Anyango sees what is there, makes a move in relation to it, and sees what that move accomplishes, thereby informing her next steps. In those next steps, she returns the tool to its initial, label-facing orientation so that she can connect the physical representation of amount to the symbolic one, and asks the child, "Who ate the most?" "Judy," he says with a smile, and points to her stack of fraction pieces.

Concluding Discussion

This work set out to explore teacher learning at the interface between theory and practice by discerning whether connections can be made between the pedagogical/conceptual knowledge that prospective teachers construct in teacher preparation and how that knowledge is enacted in their teaching. The following question framed the inquiry: "*As prospective teachers Make new manipulatives for mathematics teaching and learning, can connections be made between pedagogical/conceptual resources for their design decisions and how those designs mediate the pedagogical moves they make in practice?*" We pursued this inquiry by analyzing approximations of practice in order to identify instances in PMTs' teaching when their manipulative served as a mediating anchor for pedagogical and/or conceptual knowledge acquired in teacher preparation and subsequently embedded in their designs.

Findings from previous work that explored the conceptual, social, and material resources that inform the rationales for PMTs' design decisions suggest that engagement in an open-ended and iterative design experience centered on the Making of a mathematical manipulative can be formative for their conceptual and pedagogical thinking. Findings from this work extend the value of that experience by considering the use of made manipulatives in practice. Specifically, the identification of instances of anchoring phenomena suggest that the experience can also yield material epistemic scaffolding (in physical manipulative form) that supports teachers and their commitments to the models of knowing and learning they construct in teacher preparation. Relative to theory, these findings suggest the analytic value of our design, rationale, resource, and practice (DRR-P) framework for revealing the promise of such an experience. Relative to practice, they suggest that the experience offers a viable means by which more robust connections between teacher preparation and practice can be nurtured.

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