SLOPE ACROSS THE CURRICULUM: A TEXTBOOK CASE ANALYSIS

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This study reviews how slope is developed in expository materials across a seven-textbook series. Slope development is analyzed using a framework of five slope components to describe which components are used and connected, and by investigating accompanying levels of covariational reasoning. Findings suggest that the series describes slope from multiple components, and this development is grounded in various levels of covariational reasoning. While many connections were found between components, occurrences of both visual and nonvisual approaches within components were not prevalent. Suggestions include building connections between Behavior Indicator and Determining Property components through descriptions of covariation as well as more connections to the Steepness component.

Keywords: Algebra and Algebraic Thinking, Curriculum, High School Education, Middle School Education

Slope describes the constant rate of change of a linear function, a notion that can be understood using a variety of representations and applied for different purposes. Even though it is a "universal topic in every country's mathematics curricula," slope has been called *elusive* (Lingefjärd & Farahani, 2018, p. 1188) because a deep understanding of slope is difficult to acquire (Hoban, 2021). Not only does slope involve deeply understanding ratios (Lobato, Ellis & Muñoz, 2003; Walter & Gerson, 2007), students must also develop an understanding of a "function as a process" (Wilkie, 2020, p. 317) that involves covariation (Thompson & Carlson 2017). Students need multiple ways to view situations involving slope (Thacker, 2020); yet research (Styers, Nagle, & Moore-Russo, 2020) suggests that teachers themselves need more experiences with tasks that allow them to build rich, flexible, robust notions of slope.

Slope spans the mathematics curriculum. In algebra, slope is used when considering the covariational contrasts between basic linear and more advanced nonlinear functions (Carlson, Jacobs, Coe, Larsen & Hsu, 2002; Ellis, Ely, Singleton & Tasova, 2018). In statistics, slope impacts linear regression and lines of best fit (Nagle, Casey & Moore-Russo, 2017). In single variable calculus, slope is involved in understanding both average and instantaneous rates of change, as well as working with other key ideas, such as relative extrema and the Mean Value Theorem (Bateman, LaForest & Moore-Russo, 2021). Without a solid understanding of slope, it is difficult to make meaning of derivatives in either single or multivariable calculus (McGee & Moore-Russo, 2015; Zandieh & Knapp, 2006). However, students often struggle to grasp more than rote procedures or mnemonics, such as "rise over run" (Walter & Gerson, 2007). Therefore, it is important to understand how slope is developed in curricular materials.

Framework

This study seeks to describe how slope is developed across a textbook series. The study is informed by past work on textbooks, slope, and covariational reasoning.

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Textbooks

Textbooks reflect "significant views of what mathematics is…and the ways that mathematics can be taught and learnt" (Pepin & Haggerty, 2001, p. 166). Textbooks play an influential role in mathematics education (Fan, Zhu, & Miao, 2013; Pepin, Gueudet & Trouche, 2013), especially in how teachers shape and sequence their instruction (Davis, 2009). Fan and Kaeley (2000) suggest that textbooks send "pedagogical messages" to teachers, since teachers using different textbooks display differences in their teaching strategies. While teachers may have access to a variety of resources, the textbook is typically the only common resource for students (Lepik, Grevholm, & Viholainen, 2015). Textbooks influence how students learn and how they consider and solve problems (Massey & Riley, 2013).

Slope

Stump's (1999, 2001a, 2001b) seminal work brought to light that slope is a multifaceted notion that can be conceptualized in many ways. Moore-Russo, Connor and Rugg (2011) introduced conceptualizations of slope as the ways that people think about and make meaning of the topic. Their 2011 conceptualization categorization has been used in studies of curriculum and standards conducted in Mexico, South Africa, and the U.S. (Nagle & Moore-Russo, 2014b; Stanton & Moore-Russo, 2012; Dolores Flores, Rivera López, & Moore-Russo, 2020). Since then, the 11 categories have been revisited and revised in research that bridges secondary to postsecondary mathematics (Nagle, Martinez-Planell, Moore-Russo, 2019; Nagle & Moore-Russo, 2014a; Nagle, Moore-Russo, Viglietti & Martin, 2013) resulting in a more nuanced conceptual framework using five connected components, each with visual and nonvisual approaches (Nagle & Moore-Russo, 2013b). In Table 1, we adopt a revised framework omitting the *Calculus* component since our study focuses on the development of slope in a precalculus context. Furthermore, we include both the *Ratio* and *Constant Parameter* components of slope to more completely delineate the nuances of slope development around these two closely connected components.

Table 1: Slope Component Coding (adapted from Nagle and Moore-Russo, 2013b)

Slope Code	Approach	Description
Constant Parameter	Visual (CP-V)	Defining parameter of linear graph (with a *y*-intercept) that indicates a uniform "straightness" of the line's entire graph; no matter which segment of the line is considered the "straightness" is constant due to similar triangles
	Nonvisual (CP-N)	Defining parameter of linear relationship (with a *y*-intercept) indicating constant rate of change between two covarying quantities; slope calculations remain constant between any two points or on any increment of change in independent variable
Ratio	Visual (R-V)	Ratio calculated by rise/run or vertical change divided by the horizontal change between any two graphed points
	Nonvisual (R-N)	Ratio calculated for any two ordered pair points (x_1, y_1) and (x_2, y_2) using the difference quotient $(y_2 - y_1)/(x_2 - x_1)$
Behavior Indicator of line or linear relationship	Visual (BI-V)	Indicator of (increasing, decreasing, horizontal, or vertical) behavior of linear graph; correlates sign of slope to directions of rise and run to determine graphical behavior
	Nonvisual (BI-N)	Indicator of increasing, decreasing, or constant behavior of linear relationship; correlates sign of slope to relationships between *change in y* and *change in x*
Steepness	Visual (S-V)	Measure of steepness of linear graph (how inclined, tilted, slanted, or pitched a line is seen as being); relates slope to angle of elevation of linear graph

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of line's angle of inclination with horizontal	Nonvisual (S-N)	Measure of how extreme a linear rate of change is calculated as being (e.g., relates magnitude of $\lvert y_2 - y_1 \rvert$ with corresponding magnitude of $\lvert x_2 - x_1 \rvert$); relates slope to calculation of $\tan q$
Determining Property	Visual (DP-V)	Property that determines if linear graphs will intersect and how (e.g., if slopes are negative reciprocals, the lines intersect at right angles)
between lines	Nonvisual (DP-N)	Property that determines whether two linear relationships that form a system of equations will have solutions and how many solutions will result

Covariational Reasoning

Covariational reasoning relates to the "mental coordination of two varying quantities while attending to the ways in which they change in relation to one another" (Carlson et al., 2002, p. 354). Slope is a topic that describes the covariational relationship between the dependent and independent variables in a linear relationship. To understand the development of slope reasoning across the curriculum, it is vital to consider how these components are built from an underlying conception of covariational reasoning. Carlson and colleagues (2002) describe five hierarchical levels of covariational reasoning, outlined in Table 2. Within the context of this study, which focuses on the development of slope prior to calculus, we do not code for L5 reasoning.

Table 2: Levels of Covariational Reasoning (Carlson et al., 2002)

Level	Description
L1: Coordination	Coordinate change in one variable with change in second variable
L2: Direction	Coordinate direction of change in one variable with change in second variable
L3: Quantitative Coordination	Coordinate amount of change in one variable with change in second variable
L4: Average Rate	Coordinate average rate of change of function uniform changes in input variable
L5: Instantaneous Rate	Coordinate instantaneous rate of change of function with continuous changes in independent variable

Methods

Data Source

The textbook series for this study was developed by the University of Chicago School Mathematics Project (UCSMP, 2021). This series of textbooks was written to correlate with the Common Core State Standards by emphasizing applications, digital resources, and mastery learning. The seven textbooks that comprise the grade 6-12 series were analyzed. In sequential order, they include: Pre-Transition Mathematics (PTM); Transition Mathematics (TM); Algebra (A); Geometry (G); Advanced Algebra (AA); Functions, Statistics, and Trigonometry (FST); and Precalculus and Discrete Mathematics (PC). The parenthetical letters denote the textbooks abbreviations used in the tables and figures below. Since this study specifically focused on slope of a line or linear function, all textbook coding excluded examples of variable or instantaneous slope, unless explicit connections to linear slope were also made.

Research Questions

This study seeks to answer the following questions:

1) Which components of slope are emphasized within each textbook and across the series?
2) What connections are made between components of slope across the series?

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3) How is covariational reasoning developed in relation to slope?

Data Coding and Analysis

Data for this study included all the expository material (i.e., the components of the textbook that conveyed information through explanations and descriptions) within the textbook series. The different types of expository material analyzed included: chapter overviews, explanatory dialogue, examples, and activities. Each chapter began with a two-page overview intended to motivate the topics that followed. Within chapters, each section typically followed a similar format of explanatory dialogue punctuated with examples. The explanatory dialogue was text that introduced new terminology and definitions, reviewed foundational ideas, and provided general explanations. The examples were used to illustrate, clarify, and extend the ideas and relationships provided in the explanatory dialogue. They were either fully complete or mostly complete with a few missing details to prompt student thinking. Some sections included activities, often utilizing digital resources, which guided students through a series of steps with embedded explanations and guided questions. The unit of analysis was easily defined for examples and activities, with each example or activity being a single unit of analysis. For the chapter overview and the explanatory dialogue, a unit of analysis was distinguished as all the content included within a single heading or separated by examples or activities. While most units of analysis included one to two paragraphs of mathematical expository content, some were as short as two sentences and others extended to three or more paragraphs.

Two categories were used to code the data: a) slope conceptualization components (distinguishing between visual and nonvisual approaches) and b) covariational reasoning level. Details for the two coding categories are in Tables 1 and 2, respectively. Each unit of analysis was coded for all slope components noted and for the highest level of covariational reasoning present. Therefore, each unit was coded for up to ten possible slope conceptualization-approach pairs and at most one covariational reasoning level. The lead author was the primary coder, meeting weekly for eight weeks with the second author to review coding. Each section of every textbook was coded for all expository material related to slope. Once coding was complete, the data were sorted and prepared for analysis. The sorting was used to study each of the seven textbooks individually as well as to consider longitudinal trends across the entire series.

Results

Across the entire series, 201 units were identified and coded as addressing slope (see Table 3). All seven textbooks in the series addressed slope, even if not explicitly using the term when first introduced. As anticipated by the research team, the number of slope occurrences was highest in the Algebra and Advanced Algebra textbooks.

Table 3: Relative Frequency of Slope Occurrences across the Textbook Series (n = 201)

Percentage of All	Textbook						
Slope Occurrences	PT	TM	A	G	AA	FST	PC
Across Entire Series	5%	14%	35%	10%	20%	7%	8%

Slope Components

Table 4 displays data related to the slope components identified in each textbook, including the number of occurrences with only visual, only nonvisual, or both visual and nonvisual approaches. Across the series, more than two-thirds of all slope occurrences included the

Constant Parameter or *Ratio* component. Moreover, one of these two was the most prominent component identified for each textbook. Table 4 indicates that at least one of these two components was assigned to 50% of the slope occurrences in each textbook, with both components assigned to 50% or more of the slope occurrences in four of the seven textbooks. All other components were assigned to less than 50% of the occurrences in each textbook. Even though textbooks provided a consistent, heavy emphasis on nonvisual approaches of the *Constant Parameter* and *Ratio* components, there were relatively few occurrences linking the visual and nonvisual approaches within either component.

Overall, visual (V) and nonvisual (N) approaches of the slope conceptualization components tended to vary greatly with strikingly few occurrences incorporating both aspects of a slope component. Occurrences linking visual and nonvisual approaches of the *Behavior Indicator* (BI) component were more prevalent than the other components. Connections between BI-V and BI-N were often facilitated by explanations that incorporated multiple representations of linear functions (e.g., the equation $y = 3x+5$ and the corresponding linear graph) when analyzing what the slope indicates both about the rate of change of y with respect to x (i.e., as x increases by 1, y increases by 3) and about the graphical representation of that relationship (i.e., an increasing line that goes over 1 unit and up 3 units). Note that in situations such as this, the BI-N code was assigned since the corresponding directions of change of the two covarying quantities were linked (L2 covariational reasoning) and connected to the increasing or decreasing behavior of the linear graph (BI-V). However, these occurrences often stopped short of explicitly relating the direction of change to the increasing or decreasing nature of the function itself (e.g., if $x_1 < x_2$, then $f(x_1) < f(x_2)$).

Table 4: Frequency of Slope Components by Approach within Occurrences by Textbook

Textbook (number of slope occurrences)	Slope Components (by Visual, Nonvisual, or Both Approaches)														
	CP			R			BI			S			DP		
	V	N	Both	V	N	both	V	N	both	V	N	both	V	N	both
PTM (n=11)	0	0	2	0	9	0	1	4	0	0	0	1	0	0	0
TM (n=28)	0	14	3	0	19	0	2	6	5	3	0	0	1	0	0
A (n=70)	5	45	8	4	40	5	7	14	10	4	1	1	2	1	4
G (n=21)	0	11	0	2	10	2	3	0	0	3	0	0	10	0	0
AA (n=40)	3	27	0	0	21	4	1	8	4	1	1	1	8	1	0
FST (n=14)	0	9	1	2	3	0	0	2	2	0	0	0	0	0	0
PC (n=17)	0	6	0	0	9	2	1	2	5	0	0	1	1	0	0
Series (n=201)	8	112	14	8	111	13	15	36	26	11	2	4	22	2	4

Figure 1 illustrates the emphasis of each slope component by textbook. In each cluster, the first bar represents the percentage of total slope occurrences across the series attributed to a textbook. The five subsequent bars represent the corresponding percentage of all slope occurrences where a particular slope component was identified in the textbook. For instance, the first cluster shows that the Pre-Transition Mathematics textbook included 5% of all identified slope occurrences across the series, which included roughly 1% of all *Constant Parameter* occurrences, 7% of all *Ratio* occurrences, 6% of all *Behavior Indicator,* 6% of all *Steepness* occurrences, and 0% of the *Determining Property* occurrences. Uniform distribution of slope components across the textbook series would result in approximately equal percentages of each component for a

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particular textbook. For the most part, this is seen in the relatively equal height of bars within each textbook cluster. However, the *Determining Property* component appears to be heavily emphasized in the Geometry and Advanced Algebra texts (the right most bar in each cluster). The Geometry and Advanced Algebra textbooks included 10% and 20%, respectively, of all slope occurrences but included 36% and 32%, respectively, of the *Determining Property* occurrences. Two-thirds of all *Determining Property* occurrences were identified in these two textbooks even though less than one-third of all slope occurrences occurred in them. Figure 1 also reveals a heavy focus on the *Steepness* component in the Geometry textbook, which might be expected from a geometric (versus algebraic) consideration of lines.

We also considered which slope components were developed together within a single occurrence to determine common component connections. Of the 201 occurrences, 146 included combinations of two more components, while 55 occurrences were assigned a single code. A total of 21 unique coding assignments were made (e.g., *Constant Parameter* only*; Constant Parameter* and *Ratio;* and *Constant Parameter, Ratio, and Behavior Indictor*). Table 5 provides information about each of the coding assignments that were identified in at least 2% of all slope occurrences in the textbooks. Overall, many slope occurrences across the series made connections with the *Constant Parameter*, *Ratio*, and *Behavior Indicator* components. Given the complimentary nature of slope used as a *Behavior Indicator* and *Determining Property* (e.g., recognizing a line perpendicular to an increasing line must decrease), it is also interesting that these two components were linked in only one occurrence and, therefore, were not included in Table 5. *Steepness* was linked with all other slope components at least once throughout the textbook series. However, it had few occurrences across the series, even in the last two textbooks in the series when angles and trigonometry play major roles, and it did not appear in any of the frequent slope component combinations. This is noteworthy since *Steepness*, which can be tied to the tangent of an angle of inclination, is often disconnected from other slope components (Nagle & Moore-Russo, 2013a).

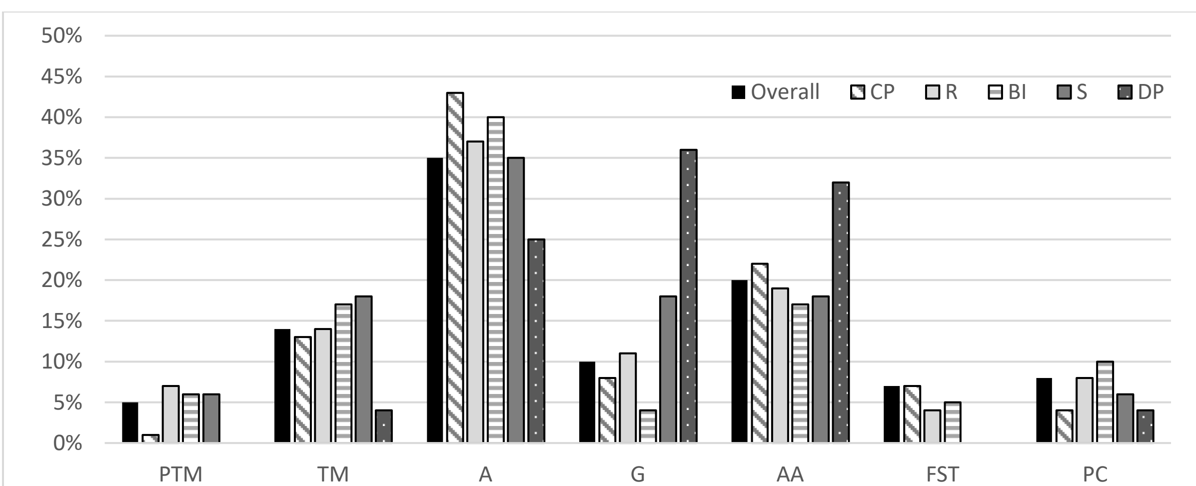

Figure 1. **Relative frequency of occurrences with slope component clusters by textbook.**

Table 5: Prevalent Slope Component Coding Assignments

Individual Component and Component Combinations Codes	% of occurrences (n=201)
CP-R	26.4%
CP-R-BI	12.4%
CP	10.4%
R-BI	10.0%
R	8.5%
CP-DP	6.5%
BI	6.0%
CP-BI	5.0%
R-DP	3.5%

Covariational Reasoning

Table 6 provides the percentage per individual textbook for each of the four levels of covariational reasoning. Nearly two-thirds of occurrences incorporated some level of covariational reasoning. As might be expected based on the definition of slope in terms of quantifying the ratio $\frac{change\ in\ y}{change\ in\ x}$, the majority of the explanations incorporated L3 covariational reasoning coordinating the amount of change in one variable with the amount of change in the other variable. Table 5 illustrates a shift from L1 reasoning in the earlier books in the sequence to L2 and L3 reasoning in the later books in the sequence. The results reveal a shift to a larger percentage of occurrences that include no covariational reasoning at later stages of the curriculum. Early curriculum explanations relied heavily on describing the covariational relationship between two quantities, even with simple L1 acknowledgment that those changes do in fact correspond. In the series, this led to defining slope as a topic that provides the quantification for this rate of change. Later curriculum explanations then frequently used slope as a tool without recounting its interpretation in terms of covarying quantities. Once slope has been formally defined, it seems as though it is often assumed that the covariation exists, but when covariation is acknowledged in later textbooks, it was at higher levels, as would be appropriate.

Table 6. Relative Frequency of Covariational Levels for Slope Occurrence by Textbook

Textbook	None	L1	L2	L3	L4
PTM (n=11)	9%	82%	9%	0%	0%
TM (n=28)	4%	11%	18%	68%	0%
A (n=70)	33%	1%	13%	51%	1%
G (n=21)	81%	0%	0%	19%	0%
AA (n=40)	45%	5 %	3%	40%	8%
FST (n=14)	50%	0%	29%	21%	0%
PC (n=17)	35%	0%	18%	18%	29%
Series (n=201)	36%	8%	11%	40%	5%

In Table 7, the percentage of slope occurrences assigned a level of covariational reasoning are listed by slope component. The results highlight that *Determining Property* was rarely developed using covariational reasoning. Recall that *Determining Property* and *Behavior Indicator* were rarely combined in occurrences, and that L2 reasoning could provide a foundation on which to build this connection. The lack of *Determining Property* occurrences with L2 reasoning further support this observation. Interestingly, *Constant Parameter* had the next highest percentage of

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occurrences with no covariational reasoning, because slope is often identified as the leading coefficient in a linear equation without discussion of what that represents and often reported in general terms as what makes a line straight without describing the covariation of rise and run on the line's graph. However, *Constant Parameter* also included a high percentage of L3 covariational reasoning when such as description was present. We do not view this as an indicator that the *Constant Parameter* component was developed without covariational reasoning, but that its applications supported many occurrences that did not explicitly denote the covariational relationship it represents.

Table 7. Relative Frequency of Covariational Levels by Slope Component

Component	None	L1	L2	L3	L4
CP (n=134)	36%	4%	7%	51%	2%
R (n=132)	23%	8%	4%	58%	6%
BI (n=77)	17%	5%	27%	44%	6%
S (n=17)	29%	6%	24%	29%	12%
DP (n=28)	93%	0%	0%	7%	0%

Conclusions and Future Work

This study reports the development of slope in a textbook series' expository content, considering slope components and accompanying covariational reasoning. Results suggest that this textbook series provides consistent opportunities for students to develop the various slope components across the series. As expected, slope receives the most attention in the Algebra and Advanced Algebra textbooks, but the previous and subsequent texts in this series carefully build and extend a foundation including all five of the slope components. Furthermore, covariational reasoning frequently accompanied the development of slope components, particularly in the earliest stages when the notion of slope is first being developed from students' intuitive knowledge of covarying quantities. These approaches align with recommendations from the Common Core Standards (Nagle & Moore-Russo, 2014b).

Slope was, for the most part, richly developed as a notion related to the covariational change between two quantities in a linear relationship. One exception is the lack of *Steepness* component occurrences; this is of concern especially in textbooks where angles and trigonometry are emphasized. Meaningful connections to *Steepness* could be created through covariational descriptions of the severity of change in the output variable relative to change in the input variable in contextual situations. Another exception is the *Determining Property* component, which occurred mostly in the Advanced Algebra and Geometry textbooks. The emphasis was on using the previously developed notion of slope as a tool to describe the parallel or perpendicular relationship of lines (often visually in the Geometry textbook). However, comparisons between slopes were seldom interpreted in relation to how the quantities represented by the linear graphs covaried (e.g., equal slopes suggest the same constant rate of change, so lines don't intersect). Connections to the *Behavior Indicator* component of slope utilizing L2 covariational reasoning might facilitate a more connected view of slope from these lenses.

Although slope was developed in terms of covariational reasoning and connections of various slope components, visual and nonvisual approaches within the slope components were only explicitly connected in a few instances. Nagle and Moore-Russo (2013b) describe the importance of developing a robust, flexible understanding of slope consisting of all five slope components

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with meaningful connections within approaches and between components. The analysis of this textbook series suggests that while the links between components were developed, the links between visual and nonvisual representations within a single slope component were often underdeveloped. In particular, the *Ratio* and *Constant Parameter* components were built heavily from nonvisual perspectives and seldom included links between visual and nonvisual approaches. Since this analysis only considered the expository material, it is quite possible that some of the additional connections between these components may come from exercises or other features of the textbook. Future analysis should explore additional elements of the textbooks to see whether opportunities for making connections between the visual and nonvisual approaches to these components might be fostered in the exercises.

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