# MODELING TO UNDERSTAND THE WORLD AROUND US AND OUR PLACE IN IT: IF THE WORLD WERE A VILLAGE 

Eva Thanheiser<br>Portland State University<br>evat@pdx.edu<br>Amanda Sugimoto<br>Portland State University<br>asugimo2@pdx.edu

Brenda Rosencrans<br>Portland State University<br>ros8@pdx.edu<br>Torrey Kulow<br>Portland State University<br>kulow@pdx.edu

Brittney Ellis<br>Portland State University<br>bme3@pdx.edu

"The main focus of the day was the "If the World Were a Village" activity/book. I thought it was a really good way to open one's perspective. As an American, I tend to be a bit focused on the US, so to see how much [or how little] of the world is actually represented in my perspective was enlightening."
"Living in the United States ... I was surprised that only $5 \%$ [of the world population] were from North America"
Increasingly, mathematics is being positioned as a tool to support students' understandings of social (in)justice and their own unique social positioning in the world. To this end, this study analyzes the impact of curricular reform efforts in an elementary mathematics content course. The course focused on fractions and statistics, and the course content was taught through tasks designed to support prospective teachers in understanding and critiquing the world. The authors found that through the course, prospective teachers' content knowledge increased and their knowledge of the world's demographics and social inequities increased.

Keywords: Teacher Education Pre-service, Modelling, Social Justice

## Objectives or purposes of the study

The National Council of Mathematics' (NCTM) Catalyzing Change (NCTM, 2018) states that "each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics." (emphasis added, p. 2). If we want students to "function as numerate, critical citizens who are able to use their knowledge in social and political realms, for the betterment of both themselves and society as a whole" (Ernest, 2000, p. 46), we need to teach these students how to understand and critique the world. Some school districts in the United States (for example Seattle Public Schools) are actively working on understanding and critiquing the world in their mathematics classes (Gewertz, 2020).
However, teaching mathematics in a way that enables students to understand and critique the world is challenging. Part of the challenge is due to the tension between focusing on the classical/dominant mathematics goals and on understanding and critiquing the world (Brantlinger, 2013; Gutstein, 2006; Yeh \& Otis, 2019). Integrating real-world phenomena into the classroom requires questioning the status quo so as to not reinforce stereotypes (Esmonde, 2014) which may take time away from focusing on the dominant mathematics. This, in turn, may negatively impact student achievement on standardized assessments (Brantlinger, 2013; Chubbuck \& Zembylas, 2008).
To address this tension, we selected real-world contexts that would allow both, learning mathematics as well as learning about the world we live in. We situated the mathematics learning in the context of shrinking the population of the world to either 100 people (Smith, 2011) or shrinking the US population down to the size of the class. Our research questions were:

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1. What mathematics do students learn when they engage in mathematics tasks based on shrinking populations to 100 or to class size?
2. What do students learn about the world when they engage in mathematics tasks based on shrinking populations to 100 or to class size?
3. How does working on such tasks affect the students' views of mathematics?

## Perspective(s) or theoretical framework

In a teaching mathematics for social justice framework (TMfSJ), a primary goal is for students to become critical members of society who know how to use mathematics to make sense of and possibly change their world (Gutstein, 2003; Raygoza, 2016). TMfSJ tasks involve engaging students in thinking about both the mathematics concepts and social issues relevant to their lives and experiences (Frankenstein, 1983; Gutstein, 2006; Skovsmose, 1994). Therefore, another goal of incorporating social issues into mathematics is for students to both understand mathematics as well as understand and create a more just world (Frankenstein, 2009). To understand and critique the world students often begin by learning about how their own experiences relate to others' experiences in the world in order to understand privilege and injustice. Many prospective teachers (PTs) enter their coursework believing that mathematics is neutral or universal (Greer et al., 2007; Keitel \& Vithal, 2008). Mathematics teacher educators need to address the fact that mathematics can never be neutral and no classroom is a neutral space (Frankenstein, 1983; Gutiérrez, 2013; Yeh \& Otis, 2019). TMfSJ tasks can be a means of engaging teachers in building their sociopolitical consciousness about the political implications of mathematics, and how math can be leveraged to read and write the world (Gutstein \& Peterson, 2005). Gutstein (2003), building on Paulo Freire's (1970) work, distinguishes between reading the world (supporting students in learning about inequities in the world and their own positioning within those inequities) and writing the world (supporting students in developing their own agency to address inequities).
Yet, most PTs have little experience with TMfSJ tasks during their K-12 schooling; therefore, it is important to integrate such tasks into their teacher education courses so they can explore how they may be able to enact TMfSJ tasks in their future K-12 classrooms. This is especially true for content courses so elementary PTs can experience such tasks from a learner's perspective and thus learn to read and write the world (Gutstein. 2006).
In some cases mathematics teacher educators (MTEs) have met resistance from PTs when integrating social justice issues into the mathematics curriculum (Aguirre, 2009; Ensign, 2005; Felton-Koestler et al., 2017; Rodríguez \& Kitchen, 2004). However, MTEs have also found that they are able to broaden PTs' perspectives about mathematics and mathematics teaching to include the idea that mathematics could be a tool for social analysis that supports students in understanding the sociopolitical world better (Bartell, 2013; Ensign, 2005; Felton \& Koestler, 2015; Felton-Koestler \& Koestler, 2017; Leonard \& Moore, 2014; Mistele \& Spielman, 2009). This aligns with Gutstein’s (2003) goal of supporting students in developing their sociopolitical consciousness, and possibly a stronger sense of agency and identity.
Finally, TMfSJ tasks can be designed towards teaching math about, with, and for social justice (Stinson \& Wager, 2012). Stinson and Wager define teaching mathematics about social justice as focusing on reading the world. They define teaching mathematics with social justice as enacting equitable pedagogical practices. Finally, they define teaching mathematics for social justice as focusing on both reading and writing the world. Students benefit from all three forms of teaching mathematics about/with/for social justice. Benefits include a view of mathematics as useful and relevant and can potentially develop agency (Gutstein, 2006).

## Methods or modes of inquiry:

The study took place in two mathematics content courses for prospective elementary school teachers in the United States, one in 2018 and one in 2019. Content courses are typically taken as prerequisites before students enter their teacher education program. The mathematical content of these courses focused on fractions and statistics.
The 2018 course had 26 PTs and the 2019 course had 7 PTs. The task described below was piloted in the 2018 course and spanned one homework assignment and one class day ( 2 hours). The task was then refined and implemented in the 2019 course across over two days of instruction ( 2 hours each). The first part of the task comprised (1) examining www.worldodometers.info and noting how many people are on earth and how some resources are distributed. (2) Shrinking the world population down to 100 people using the book If the World Were a Village (Smith, 2011), the movie https://www.youtube.com/watch?v=QrcOdLYBIw0, as well as the website https://www.100people.org/statistics_detailed_statistics.php?section=statistics to make observations about the distribution of the population and resources. (3) Creating of a poster focusing on one or two elements discussed in the book/movie. Students were given private think time, discussion time, and time to make the poster which was to include the following representations: a table, a hundred chart, a number line, and unifix cubes. Students were asked to connect these various representations to support their understanding of the relationships between fractions, decimals and percent. Sample posters can be seen in Figure 1. After the posters were displayed time was spent to discuss the mathematics (connecting decimals, fractions, and percent) as well as what we learned about the context.
In 2018 parts (1) and (2) were assigned as a homework assignment before the first class session, in 2019 they were done in class. In 2019 PTs were asked to estimate percentages for the village such as population, language, age, etc. For example, they were asked "If the world were a village of 100 people, how many would come from North America?" and "would speak English?" In both years part (3) was done in class and in both years, students responded to online survey questions after those three parts:
Online Survey Questions:

- What observations did you make when looking over this website http://www.worldometers.info/?
- What observations did you make when you watched the YouTube video "If the world were a village"?
- What observations did you make when you read the book "if the world were a village"?
- [2018] Please reflect on this homework assignment: What did you learn?
- [2019] What did you learn from today's lesson?
- [2019] What did you learn from today's lesson with respect to mathematics?
- [2019] What did you learn from today's lesson with respect to your understanding of the world?
- How can breaking the population down into 100 people help us better understand the information?
- What math do you think could be addressed with this book in a K-5 classroom?

At the end of the term PTs in each class were asked to respond to the prompt "I used to think math is ... now I think math is ..."
In 2019 a second day focused on PTs researching information about their hometown and creating a second poster for their hometown and then comparing across the two posters. Data collected included PTs responses to online surveys as well as detailed field notes of all class sessions and copies of all student work. Data analysis began by filling in the field notes with artefacts from class, and reading
through survey responses to establish initial themes. Two researchers independently read survey responses and identified themes. The researchers then met to discuss and refine themes in order to create a final codebook of themes, and all data was coded with this codebook. Any disagreements were resolved through discussion. After all survey data was coded, similar themes were collapsed to make larger themes. Themes were collected across all questions rather than analyzing individual questions. These can be seen in Table 1.

## Results

## With regard to Research Question 1: What mathematics do students learn when they engage in mathematics tasks based on shrinking populations to $\mathbf{1 0 0}$ or to class size?

One major mathematical focus of the task was to make sense of the meaning of percent and connect percent, fractions and decimals. Several PTs commented that before this day they did not realize that percent refers to "per one hundred". Ellie, for example, stated, "I thought it [percent] was just a word," and Amanda wrote in her notes for the day, "Eva explained that percent actually means per hundred, which I never knew." Candy stated that she learned "that if the world was broken down into 100 people, each person would represent a percent. This allows us to conceptualize the population while understanding the real percentages (which can be translated into the actual population)." In addition, PTs connected decimals to fractions with base ten denominators. This is evidenced by Christa's response that "I learned how fractions, decimals and percentages relate to each other and that they all show representations of part of a whole." One way PTs made sense of percent and made connections was by using color purposefully. As shown in the posters below (Figure 1), PTs used colors to connect across representations. Sabine pointed out how the colors helped her: "It was very helpful for me to see with color and the number 100 ." Jamie reflected on her homework "OMG. As I was completing the homework I realized that a flat partitioned into 100 -sized pieces is the same as a hundred chart!!!!! This was a very helpful connection for me."


Figure 1: Three posters illustrating connections across representations

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In addition to developing meaning for percent, fractions and decimals, scaling the size of the world's population helped the students make sense of the size of our world and the distribution of the world's population. For example, $76 \%$ of PTs reported that scaling down the population to 100 helped then understand the world better (see Table 1 for all survey themes). One way this supported their understanding of the world was the ease of visualizing a certain number of people out of 100 rather than billions of people. For example, Walter conveyed that "when numbers are so large the impact is usually lost. The number can be overwhelming and doesn't necessarily relate to us personally. Yet when we break it down to 100 we have a better visualization of what the population would look like. We can visualize 60 people out of 100 or 10 people out of 100 . Compare that to billions of people... That is much more difficult to grasp." In this way, PTs were able to make sense of the world by considering a total population of 100 people, which provided a relatable scale.

## With regard to Research Question 2: What do students learn about the world when they engage in mathematics tasks based on shrinking populations to $\mathbf{1 0 0}$ or to class size?

Another major focus of the task was to create a space for students to realize that the world is much more complex than they had originally thought, and that the US is not the center (or most of) the world. Seventy percent of the students stated this in the survey. For example, Aimee expressed feeling surprised "to see that a majority of the population came from Asia and not the United States," and Sophie also voiced, "I was surprised to learn about the abundance of other cultures. I think in America the media tends to be very self-centered and produce that as the norm, when in reality looking at the world we are a small fraction of diversity in the world." Additionally, Amanda reflected in her notes that "some of the results were surprising such as that $60 \%$ of the population is from Asia and only $5 \%$ is from North America."

Table 1: Themes, larger themes bolded, sub-themes not bolded

| Theme | 2018 N=26 | 2019 N=7 | $\begin{gathered} 2018 \text { and } \\ 2019 \mathrm{~N}=33 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| The world population is large and rising (at a scary rate) | 18 (69\%) | 7 (100\%) | 25 (76\%) |
| The use of smaller numbers ( 100 people in the village) was helpful to make the quantities easier |  |  |  |
| to visualize/data easier to visualize | 21 (81\%) | 4 (57\%) | 25 (76\%) |
| Inequity is larger than I thought | 22 (84\%) | 6 (85\%) | 31 (85\%) |
| Inequity/inequality is larger than I thought | 16 (62\%) | 3 (43\%) | 19 (58\%) |
| Food insecurity is much higher than I thought | 14 (54\%) | 5 (71\%) | 19 (58\%) |
| Education levels are lower than I thought | 8 (31\%) | 3 (43\%) | 11 (33\%) |
| The world is more complicated/goes beyond the US | 17 (65\%) | 6 (85\%) | 13 (70\%) |
| World is complicated/diverse | 13 (50\%) | 3 (43\%) | 16 (48\%) |
| US is not the center of the world | 5 (19\%) | 1 (14\%) | 6 (18\%) |
| I am a small piece in the world/surprised by the small \% of people from NA | 2 (8\%) | 3 (43\%) | 5 (15\%) |
| I am a small piece in the world/There are so many people in the world | 1 (4\%) | 0 (0\%) | 1 (3\%) |
| I am/was America centric/ gained perspective This activity was | 2 (8\%) | 4 (57\%) | 6 (18\%) |
| SHOCKING/jarring/terrifying | 6 (23\%) | 1 (14\%) | 7 (21\%) |
| Need for action/improving the world | 10 (38\%) | 3 (43\%) | 13 (40\%) |
| This activity made me realize the need for action | 6 (23\%) | 3 (43\%) | 9 (27\%) |
| found it helpful to quantify social issues/we can | 6 (23\%) | 0 (0\%) | 6 (18\%) |

use number to improve the world
This activity would be good for elementary students

8 (31\%) $1(14 \%)$ 9 (27\%)

Thus, scaling the size of the world also helped PTs recognize their inaccurate perceptions of the distribution of people and languages in the world. In 2019 when PTs were asked to make predictions all 7 PTs overestimated how many people would come from North America. While the actual value is around 5, PTs' estimates ranged from 7 to 50 with the mean response being 18. All 7 PTs also overestimated the number of people who speak English. Their estimates ranged from 20 to 80 with a mean of 42 and a median of 40 . The actual value is 5 (first language) and 20 (first and second language). Thus, all PTs seemed to overestimate the population of North America as well as the number of English speakers. This activity helped PTs adjust their understanding; Candy, for example stated in her class notes that "learning the proper answer resulted in new understanding of the world. This activity helped me realize that the US isn't the center of the world. Life is very different for people in other countries."
This task helped PTs to grasp the significance of our worlds' population and the current distribution of resources in our world. One of the major themes PTs mentioned across both years was that they learned that the world population is large and rising at an alarming rate (all survey themes are shown in Table 1). Seventy-six percent of all PTs mentioned this theme. Elsie for example stated "There are a lot more people living than I thought," and Sophie stated "The population to me was shocking. When I think about this small realm of people I interact with every day, it blows my mind to picture the world population, let alone this constant rapid increase."
Additionally, almost all PTs (84\%) stated that they learned that the inequities are larger than they thought either in general ( $58 \%$ ) or by mentioning a specific area such as food insecurity ( $58 \%$ ) or education (55\%). To give an example, Autumn mentioned general inequities: "I observed a lot of inequalities amongst all the topics explored." Additionally, Christa noticed that "food insecurity was a way larger number than I thought. It is almost half the world's population which is startling to me," and Sabine mentioned both food and education: "The number of people that always have food to eat is very low. I didn't expect it to be that low, it really surprised me. Also, that there are students that don't get an education. It is very sad that they don't get that opportunity."
For some PTs, their new understanding motivated a sense of urgency for action. Even though taking action was not yet an explicit goal of the course, $40 \%$ of the PTs mentioned the need to take action or need for improving the world. For example, Marcel stated "It is ... important to realize that as Americans, everything is not as we assume it is. We just expect to turn on the faucet and have clean water poured out of it. We pay for tuition and we expect qualified professors. As Americans we must recognize our privilege and use it to help others as much as we can." Gertie said "I learned that there are a lot of things in the world that need our help and attention, and there is a lot of miss-distribution of wealth, food, and resources, and we need to be doing a better job of creating a more sustainable and equitable world.

## With regard to Research Question 3: How does working on such tasks affect the students' views of mathematics?

At the end of the term PTs in each class were asked to respond to the prompt "I used to think math is ... now I think math is ...." Four themes emerged: (a) a shift from math as politically neutral to math as a place for the integration of social issues/social justice [8 PTs in 2018, 2 PTs in 2019], (b) a shift from rom math as boring/uninteresting, to math as interesting/useful [8 PTs in 2018, 4 PTs in 2019], (c) a shift form math as rules and procedures to math as sense-making [12 PTs in 2018, 3 PTs in 2019], and (d) shift from difficult to possible [7 PTs in 2018, 0 PTs in 2019]. Some PTs mentioned more than one theme [ 9 in 2018, 1 PT in 2019]. Ellie illustrated theme (a) in her response "I used to
think math is the most politically neutral subject taught in school. Now I think math is a great place to bring up the issues that are going on in our world while developing a better understanding of the material because of the connections that students can make to their own lives."

## Discussion and/or conclusions

Reflecting on the data analyzed for this study with respect to content it seems like PTs learned about the meaning of percent as well as the connection between fractions, decimals, and percent.
Reflecting on the data analyzed for this study with respect to the world, it seems the PTs learned about social justice (reading the world) but not quite yet for social justice (writing the world). PTs seemed to have a better understanding about various characteristics of the world as a whole, which is important for future elementary school teachers because these future teachers need to understand characteristics of the world themselves before they can enact TMfSJ tasks in their own future teaching. Additionally, being aware of their own unique social positions may support more reflective practice with elementary students who differ from them in social position.
The PTs also learned how to use math to make sense of the world. Moving forward, the authors intend to modify the tasks to move more towards learning for social justice. In 2019 the PTs compared what they learned about the world to what they learned about their own local city which allowed them to (a) learn about their local place and compare it to the world, and (b) refine their understanding of how they are situated in the world. Christa, for example, stated "My topic was shelter and that was an underestimate of one in 20 people are homeless [in local city]. ... we obviously have issues in [local city]. But then comparing it to the real world numbers. It's still a very privileged place to live. So I think that's important to recognize." The act of comparing their new understandings of the world to the makeup of their local city appeared to make inequity more pressing for PTs. In the future, the authors will continue to develop ways to connect TMfSJ tasks to PTs local reality, e.g., local issues of homelessness, gentrification, in an effort to prompt PTs toward taking action and learning mathematics for social justice.
Reflecting on the data analyzed for this study with respect to the students' views of mathematics we reported on students' views at the end of the term, thus this tasks may only have played part of the reason for shifts, however, all students mentioned shifts in how they viewed math as either not neutral anymore, or more interesting, engaging, focused on sense making, and doable.

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