PRODUCTIVE SEEDS IN PRESERVICE TEACHERS' REASONING ABOUT FRACTION COMPARISONS

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Reasoning about fraction magnitude is an important topic in elementary mathematics because it lays the foundations for meaningful reasoning about fraction operations. Much of the research literature has reported deficits in preservice elementary teachers' (PSTs) knowledge of fractions and has given little attention to the productive resources that PSTs bring to teacher education. We surveyed 26 PSTs using a set of 9 fraction-comparison tasks. We report the frequency of complete strategy-arguments and the perspectives (ways of reasoning) used for each item. We further examine incomplete strategy-arguments, noting substantial evidence for productive seeds of reasoning. Using data from interviews with 10 of these PSTs, we identify evidence suggesting these seeds are, in fact, productive in that they provide foundations for further development. We argue that this type of research is needed in order to further mathematics teacher education.

Keywords: Number Concepts and Operations, Rational Numbers, Teacher Education-Preservice

The mathematics education research community is concerned with the mathematics content knowledge of preservice elementary teachers (PSTs; Thanheiser et al., 2014). The research literature has tended to characterize PSTs' mathematics content knowledge as poor (Graeber, Tirosh, & Glover, 1989; Green, Piel, & Flowers, 2008; Putt, 1995; Tsao, 2005; Thipkong & Davis, 1991; Widjaja, Stacey, & Steinle, 2011; Yang, 2007). A synthesis of the literature reveals that there is insufficient research that seeks to make sense of PSTs' conceptions, or that views their conceptions as resources for further learning (Thanheiser et al., 2014; Whitacre, 2013). The literature on PSTs' knowledge of fractions is a prime example of such characterizations (Olanoff et al., 2014). Researchers have found PSTs to be inflexible in their reasoning about fractions, relying heavily on standard procedures, while at the same time having difficulty justifying such procedures and difficulty relating fraction operations to contexts (Olanoff et al., 2014).

Preservice elementary teachers may not typically reason flexibly about fractions when they come to teacher education (Ball, 1990; Yang et al., 2009), but how far are they from doing so? We operate from the assumption that PSTs possess fundamental mathematical resources with which to reason productively about fraction magnitude, but they may not have had sufficient opportunities to exercise such reasoning. In that vein, we analyzed 26 PSTs' responses to fraction-comparison tasks in order to identify the variety of ways of reasoning that they might bring to such tasks. In keeping with our perspective, we went beyond tabulating correct responses and coding for strategies. We also examined how PSTs reasoned through comparisons that they ultimately answered incorrectly or incompletely. Even in these cases, we find evidence of PSTs reasoning about fractions in productive ways. In interviews with 10 of the PSTs, in which we provided low-level support and encouragement, we investigated which strategy-arguments for comparing fractions were readily learnable depending on their current knowledge.

This research highlights the valuable prior knowledge and the potential for growth in PSTs' knowledge of fractions. We believe our findings offer a fresh perspective that contrasts with the vast majority of literature on this topic by highlighting the strengths of PSTs that can be leveraged into effective reasoning strategies for fraction-comparison tasks.

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Background

Knowledge Framework

We have found Smith's (1995) framework useful for categorizing PSTs' ways of reasoning about fraction magnitude (Whitacre & Nickerson, 2016). It consists of four *perspectives*, which are categories of comparison strategies: Transform, Parts, Reference Point, and Components. Below, we briefly describe each perspective.

The *Transform* perspective involves use of procedures such as converting to a common denominator or converting to a decimal. These strategies involve transforming one or both fractions in some way in order to facilitate the comparison (e.g., comparing 6/7 and 7/8 by converting to a common denominator and then recognizing that 49/56 is greater than 48/56).

The *Parts* perspective involves interpreting fractions in terms of parts of a whole. This approach works especially well in certain cases, such as when comparing fractions that have the same numerator or same denominator. For example, 3/4 is greater than 3/5 because 1/4 of a whole is larger than 1/5 of the same-sized whole. Thus, three larger parts are greater than three smaller parts.

The *Reference Point* perspective involves reasoning about the magnitudes of fractions on the basis of their distance from reference points, or *benchmarks* (Parker & Leinhardt, 1995). In particular, Reference Point strategies relates to the number line. For example, to compare 7/8 and 6/7, a student may notice that 7/8 is 1/8 away from 1, whereas 6/7 is 1/7 away from 1. Since a distance of 1/8 is less than a distance of 1/7, 7/8 is closer to 1, and therefore larger.

The *Components* perspective involves noticing additive or multiplicative relationships in the numerators and denominators of the given fractions. For example, in order to compare 13/60 and 3/16, a student may notice that $13 \times 5 = 65 > 60$, whereas $3 \times 5 = 15 < 16$. Thus, 13/60 is greater because the numerator is larger relative to the denominator.

Our coding scheme for fraction-comparison strategy-arguments represents a revised version of that of Smith (1995). Length limits prevent us from providing operational definitions for each strategy-argument here. See Whitacre and Nickerson (2016) for a similar coding scheme.

Previous Research

We note three points that concern us about the state of the literature regarding PSTs' mathematical knowledge: (1) The body of literature tends to emphasize deficiencies, rather than to regard PSTs' prior knowledge as a productive resource (Thanheiser et al., 2014; Whitacre, 2013). This emphasis runs the risk of promoting low expectations of PSTs' abilities to learn. (2) There are few articles that provide specific, qualitative descriptions of PSTs' mathematical thinking that could provide useful information from which to design instruction. The work of Thanheiser (2009) is a notable exception. (3) There is a tendency to overgeneralize about the mathematical thinking of PSTs, rather than to recognize the variety in their thinking.

We view PSTs as sense-makers who are ready and able to improve their mathematical knowledge. Unfortunately, there is scant literature that helps the field to understand *how* PSTs' knowledge of fractions can be improved (Olanoff et al., 2014). Thanheiser et al. (2014) assert that the field of mathematics teacher education needs studies that document successful approaches to improving PSTs' content knowledge and that illuminate the processes by which such changes can occur. We agree. In particular, the field needs studies that find value in PSTs' prior knowledge and that demonstrate how PSTs can and do use that knowledge as they learn, because "The key to turning even poorly prepared prospective elementary teachers into mathematical thinkers is to work from what they *do* know" (Conference Board of the Mathematical Sciences [CBMS], 2001, p. 17). In the literature on K-12 students' mathematical thinking and learning, much attention has been given to students' mathematical conceptions and to the productive ways in which they make use of their prior knowledge as they learn new mathematics (e.g., Carpenter et al., 2015; Clements & Sarama; 2014;

Fuson et al., 1997). Unfortunately, such a perspective has rarely been applied in the literature concerning PSTs' mathematical thinking and learning (Thanheiser et al., 2014; Whitacre, 2013). In this study, focusing on the challenging topic of fraction magnitude, we examined how PSTs made use of their prior knowledge, including to develop new strategies, when comparing fractions.

Theoretical Framework

This study is informed by the notion of the *zone of potential construction (ZPC)* (Norton & D'Ambrosio, 2008; Steffe, 1991). The ZPC refers to the range "determined by the modifications of a concept a student might make in, or as a result of, interactive communication in a mathematical environment" (Steffe, 1991, p. 193). In the case of our work, to say that a strategy for comparing fractions is in a learner's ZPC is to say that the learner can hypothetically extend or reorganize her current schemes or mental operations to compare fractions in this new way.

Informed by the above literature, together with our previous experience, we expected PSTs to approach the fraction-comparison tasks primarily by drawing upon Parts and Transform reasoning. In particular, we expected them to be able to apply Parts reasoning to compare fractions in cases of a common denominator or common numerator. We did not expect many PSTs to compare complements initially, but we conjectured that doing so might be within their ZPCs. We expected many PSTs to default to Transform procedures, such as converting to a common denominator, in cases in which there was not a common numerator or common denominator in the given fractions.

Method

The research questions that we address are the following: (1) How do elementary PSTs reason about a set of fraction-comparison tasks? (2) What productive seeds of reasoning are evident in their responses? (3) Given the opportunity to explore a set of fraction-comparison tasks in an interview setting, which strategy-arguments are PSTs able to construct, and how do these relate to their current ways of reasoning?

This study took place at a large, public university in the Southeastern United States. The participants were a cohort of 26 PSTs in an elementary mathematics methods course. They were senior-level Elementary Education majors enrolled in a credential program.

Collection of Survey Data

Participants were given a fraction-comparison survey early in the semester (prior to instruction related to fractions). The cover page had nine pairs of fractions and asked the PSTs to mentally decide which fraction in the pair was greater or whether the two fractions were equal. The subsequent pages revisited each of these nine comparisons, asking participants for a "Description of Method" and a "Justification" for each comparison. We chose this format in order to encourage the participants to exercise their number sense, although they were free to approach the tasks in any way that they chose.

The same survey was administered at the end of the fraction unit. In both cases, time to complete the survey was limited to 25 minutes. We note that some participants left items toward the end of the survey blank. It is possible that more attempts would have appeared on later items if there had not been a time limit, or if participants had been given significantly more time.

Analysis of Survey Data

Note: We do not assume that the participants performed all of their work mentally and then reported that work in writing. In fact, some participants explicitly noted that they changed some answers. Whenever a participant gave one answer on the cover page but gave a different answer when describing or justifying their method, we regarded the latter response as the final answer.

Whitacre and Nickerson (2016) used a modified version of Smith's (1995) framework to code fraction-comparison strategies. In this study, we further developed that coding scheme to capture the wider variety of strategies that we observed. Two authors separately coded every response for perspective and strategy. The data were coded in batches (e.g., data from 8 participants) and inter-rater reliability was checked after each batch. By coding in batches, we were able to make revisions and additions to the coding scheme along the way and to code each subsequent batch with an updated scheme. This approach also enabled us to identify any interrater reliability issues early and to clarify our interpretations. Overall, the authors initially agreed on the perspective for 91% of the participants' responses and agreed on the specific strategy for 89% of the responses. Consensus was reached through further discussion of the disagreements until the authors were satisfied with the final coding decision.

In addition to coding for a perspective and specific strategy-argument, we also focused on comparisons that fell short of being complete strategy-arguments, yet demonstrated what we judged to be productive seeds of reasoning. Thus, all comparisons were coded into one of five categories: (a) complete strategy-argument and correct solution, (b) complete strategy-argument with minor errors, (c) incomplete strategy-argument with productive seeds, (d) incomplete strategy-argument with no apparent productive seeds, and (e) no strategy-argument evident. Incomplete strategy-arguments were given credit for productive seeds if the characteristics of the work, together with the perspective, were consistent with a complete strategy-argument (i.e., the participants' reasoning was headed down a productive path but stopped short of the complete argument). We believe that this approach to the study of PSTs' mathematical thinking provides a more comprehensive picture than that which has typically been reported in the literature.

Collection of Interview Data

Prior to the fraction unit in class, PSTs were invited to participate in one-on-one interviews. In contrast to the typical interview style in which the interviewer refrains from providing any form of support, these interviews were designed to allow for minimal support. The purpose of the interview design was to investigate which strategy-arguments were in participants' ZPCs. Thus, we specified in the interview protocol allowable types and levels of intervention. The intervention strategies that interviewers used included emotional encouragement, requests to solve a comparison task in a different way, requests or encouragement to continue down a path of reasoning, and offering counter arguments or pointing out evidence that was relevant to determining whether a solution was correct or incorrect. Ten of the 26 PSTs participated in these video-recorded interviews. The first and second author each interviewed five of the participants. In these interviews, PSTs were given nine fraction-comparison tasks that each mapped closely to the nine comparisons on the survey, but with different components (see Table 1).

Analysis of Interview Data

To analyze the interview videos, we targeted comparisons in which participants activated a productive seed in their work. We first identified comparisons from the videos that demonstrated use of productive seeds, we then wrote short narratives on what transpired in each case, and finally chose representative cases that highlight successful progressions in reasoning from productive seeds with low-level support. In our analysis, we applied three criteria as evidence that a strategy-argument was within a participant's ZPC: (1) the PST had not previously used that strategy-argument, as determined from pre-assessment and interview data; (2) the PST produced the strategy-argument during the interview with no more than minimal intervention from the interviewer; (3) the PST later used the same strategy-argument independently.

Table 1: Fraction-comparison Tasks from the Survey and Interview

Item	Survey Comparisons	Interview Comparisons
1	2/8 vs. 3/8	4/6 vs. 5/6
2	3/4 vs. 3/5	5/8 vs. 5/9

3	6/7 vs. 7/8	7/8 vs. 8/9
4	14/13 vs. 13/12	11/10 vs. 12/11
5	8/24 vs. 13/39	6/24 vs. 13/52
6	13/60 vs. 3/16	17/42 vs. 8/23
7	7/28 vs. 13/50	4/20 vs. 11/56
8	2/7 vs. 12/43	5/12 vs. 30/71
9	35/832 vs. 37/834	25/287 vs. 28/290

Results

Survey Results

We summarize the survey results in terms of three themes: (a) PSTs know what they were expected to learn in school, (b) when encouraged to do so, PSTs explore different perspectives and strategies, and (c) PSTs exhibit productive seeds for reasoning about fraction comparisons.

First, unsurprisingly, we find that the participants tended to use Parts and Transform strategies. As expected, those two perspectives are most familiar to PSTs, and they were the perspectives most commonly used. Figure 1 tabulates the number of instances of strategy-arguments for each perspective, broken down by item. Parts was a common perspective across items involving smaller, easier fraction comparisons. Transform was also a common strategy perspective across most of the items, with many PSTs frequently converting both fractions to a common denominator explicitly or using cross multiplication. Components strategies occasionally appeared on the later items, but were predominant on the last item with many PSTs noting the common difference of two in that comparison. Reference Point strategies were rare. Strategy arguments coded as "Other" were not developed enough to code, and comparisons coded as "None" had no work shown (either a simple answer or completely blank).

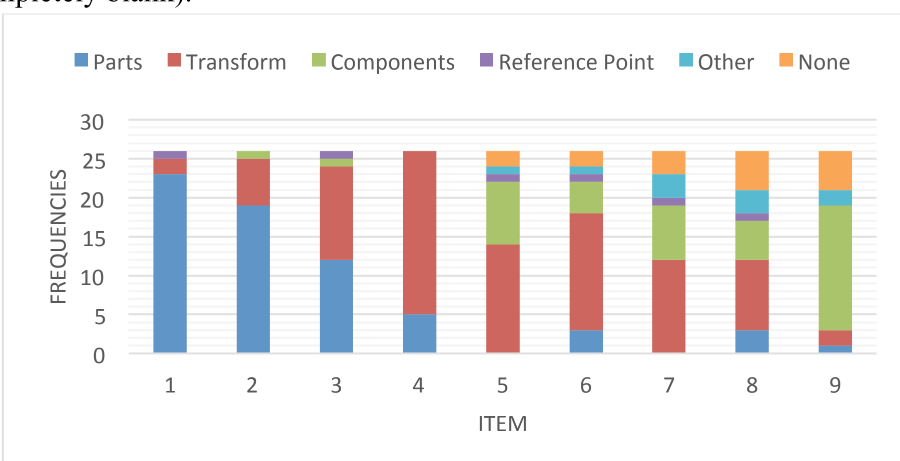

Figure 1: Perspectives used by comparison item

Second, in contrast to descriptions in the literature, the participating PSTs did exhibit flexibility in their reasoning about fraction magnitude. Recall that the instructions for the survey asked participants to first make comparisons mentally and that the numbers chosen for the comparison items lent themselves to different strategies. Nonetheless, the participants could have defaulted to converting to a common denominator for every task. They did not. Instead, PSTs used an average of 5.5 distinct strategies across the 9 tasks, and they averaged 3.73 distinct strategies that were accompanied by complete arguments. Of the 26 participants, 20 used five or more distinct strategies. All of the participants used at least three distinct strategies.

Third, we see substantial evidence of productive seeds of reasoning. The comparisons items ranged from easy to difficult for the participants, especially given that they were instructed to make their judgments mentally and that the survey was administered under time constraints. Each participant was given a correctness score for each item: correct answers scored 1 point, and incorrect answered scored 0 points. The mean total score was 5.96 (of a maximum of 9 points) with a standard deviation of 1.66. The average number of correct answers accompanied by complete arguments was 4.38 with a standard deviation of 2.17. Thus, the participants answered most items correctly, but there was substantial room for improvement in correctness and especially in producing complete arguments.

Candidate responses to be coded for productive seeds were those that did not constitute complete arguments. Of the 87 incomplete arguments, 39 (approximately 45%) included evidence of productive seeds. (There were another 6 responses with no written work provided, and there were 17 items left blank, which may have been due to time constraints.) Thus, even in cases of incomplete arguments, the participants often reasoned about fraction comparisons in productive ways. This result indicated the potential for the interview participants to construct new strategy-arguments with minimal intervention during the interviews.

Interview Results

Due to length constraints, we focus here on the interview participants' responses to the third comparison item, 7/8 vs. 8/9. This item was intended to invite PSTs to consider the complements of the given fractions (i.e., 1/8 vs. 1/9) and to construct a strategy-argument based on comparing complements (e.g., 1/9 is smaller than 1/8, so 8/9 is larger than 7/8 because it is closer to whole). Indeed, 8 of the 10 interview participants were able to construct a complete argument that involved comparing the complements and reasoning from a Parts perspective. Most participants did not compare complements initially. Instead, they began with a more familiar strategy such as converting to a common denominator. They then compared complements in response to an interviewer's request, such as to try to find a "different way" of making the comparison. Alternatively, the interviewer followed up on something that the participant had mentioned (e.g., the possibility of thinking in terms of "parts" or "pies"). Given such requests and encouragement, 80% of the participants constructed a complete Comparing Complements strategy-argument, supporting the correct conclusion that 8/9 was greater than 7/8. By contrast, only 1 of the 10 participants had compared complements for the corresponding item (6/7 vs. 7/8) on the pre-survey. On the post-survey—without assistance and free to choose any strategy they wished—7 of the 10 interview participants used comparing complements for 6/7 vs. 7/8.

Thus, we see evidence that the strategy of comparing complements was in the ZPCs of the majority of the interview participants. This was especially the case for those who took the size of the parts into account in comparisons involving a common numerator (5/8 vs. 5/9 in the interview). Those who explained that 5/8 was greater than 5/9 because eighths are larger than ninths (using Parts: Denominator Principle) appeared to be ready to reason in terms of complements for 7/8 vs. 8/9, even if doing so was novel and somewhat challenging. For example, Jane used the Denominator Principle to correctly compare 5/8 and 5/9. When she was posed 7/8 and 8/9, she noted that "the numerators are each one less than the denominator" and that eighths were larger than ninths. However, she was not immediately sure what conclusion to draw. She made rectangular area drawings of 7/8 and 8/9. Her drawings were sloppy and actually made 7/8 appear to be greater. However, despite her drawing, Jane reasoned that the missing piece from 8/9 must be smaller than the missing piece from 7/8, and therefore 8/9 was greater. Even after constructing a complete strategy-argument, Jane expressed doubt, so the interviewer invited her to explore the idea further. She created her own example, using 1/2 and 2/3, which bolstered her confidence in this new way of comparing fractions.

The two participants who did not construct a complete strategy-argument involving complements during the interview conspicuously ignored piece size in their reasoning. Both focused on the number of parts, rather than their size, when thinking in terms of Parts (and otherwise relied on converting to decimals). For example, Kimmy described 5/8 as missing 3 pieces and 5/9 as missing 4 pieces, without making any mention of the size of said pieces. As best we can tell from the data, even with interviewer probing, the size of the pieces did not enter into her reasoning. Like Jane, she noticed that both 7/8 and 8/9 were "missing one piece," but unlike Jane, she was unable to arrive at a complete strategy-argument using complements.

Discussion

We have begun the fine-grained work of identifying strategy-arguments for comparing fractions that are within the ZPCs of some PSTs, depending on their current ways of reasoning about fractions and given low levels of intervention. This finding is encouraging. Our work also reveals substantial diversity in PSTs' reasoning about fraction comparisons—a theme that is underemphasized in the literature. In the absence of documented distinctions, the literature might encourage mathematics teacher educators to treat all PSTs as if they think similarly.

We have shown that certain, nonstandard strategy-arguments, such as Comparing Complements are readily learnable by some PSTs, given their current ways of reasoning. Note that we are not distinguishing PSTs based on supposed ability. Our data do not speak to their mathematical abilities in general, and we do not claim that some of our interviewees were more mathematical capable than others. Instead, we are concerned with how they were thinking about fractions at the beginning of the course, in relation to the progress that they were able to make during the interview. Those PSTs who took the size of the parts into account when reasoning in terms of Parts were able to compare complements with lows levels of interviewer intervention. Those PSTs who consistently ignored the size of the parts did not appear to be readily able to construct Comparing Complements on the day of the interview. However, later on, having first constructed Parts: Denominator Principle, they may have become able to do so. Thus, in making claims about PSTs' ZPCs, these are limited to what was readily learnable *during the interview*.

Conclusion

In our review of the research literature concerning PSTs' knowledge of fractions, we pointed our three problems with the emphasis on negative generalizations. We frame our contributions in response to these problems: (1) Whereas the emphasis on deficiencies runs the risk of promoting low expectations of PSTs' abilities to learn, our approach is concerned with documenting learning and identifying the conditions under which it is readily achievable. (2) Whereas literature that emphasizes deficiencies fails to provide useful information from which to design instruction, our approach identifies PSTs' particular conceptions (in the form of strategy-arguments, in this case) and charts the terrain of viable reorganizations. (3) Whereas generalizations about deficiencies in PSTs' content knowledge fail to distinguish PSTs from one another, our approach focuses on the diversity of reasoning that can be found among PSTs. This research has illuminated our own understanding of PSTs' reasoning about fraction comparisons and has helped to inform instruction in the courses that we teach.

This line of research values PSTs' prior knowledge and identifies desirable mathematical understandings that are readily learnable, given favorable conditions. In this way, we identify PSTs' particular conceptions and chart the terrain of viable reorganizations. Once they move beyond default approaches like converting to a common denominator, we find considerable diversity in the mathematical thinking of PSTs, and we discover what they are ready to learn.

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