# CHANGES IN PRACTICING SECONDARY TEACHERS' PROFESSIONAL NOTICING OVER A LONG-TERM PROFESSIONAL DEVELOPMENT PROGRAM

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*Much of the research on the development of professional noticing expertise has focused on prospective teachers. We contend that we must investigate practicing teachers as well, and in particular practicing secondary teachers, because they bring with them years of teaching experience and are situated in unique contexts. Hence we studied the longitudinal growth of the professional-noticing expertise of a group of practicing secondary teachers (N=10) as they progressed through a 5-year professional development (PD) about being responsive to students' mathematical thinking. Results indicated that the first half of the PD supported their interpreting and deciding-how-to-respond skills, and the second half of the PD supported their attending skills, which were already strong even before the PD. We compare these results with the activities that occurred in the PD and discuss implications for future research and PD programs.*

Keywords: Teacher Education - Inservice / Professional Development, Algebra and Algebraic Thinking

## Introduction

Professional noticing of students' mathematical thinking is a specific type of teacher noticing expertise, and it occurs when the teacher notices a student's mathematical strategy during instruction (Jacobs & Spangler, 2017). In that moment, the teacher (a) attends to the details of the student's strategy, (b) interprets the student's mathematical understanding, and (c) decides how to respond to the student on the basis of the student's mathematical understandings (Jacobs, Lamb, & Philipp, 2010). These three component-skills occur simultaneously, are interrelated, and are interdependent. Additionally, this noticing expertise distinguishes itself from other types of noticing expertise that only include the component-skills of attending and interpreting, or that focus on issues other than the student's mathematical thinking (e.g. issues of equity, representations, or funds of knowledge; Dreher & Kuntze, 2015; Hand, 2012; McDuffie et al., 2014). It is important to understand the development of teachers' professional noticing expertise because research has shown that teachers who attend to student thinking support student achievement (e.g. Boaler & Staples, 2008; Jacobs, Franke, Carpenter, Levi, & Battey, 2007) and learn from their practice (Sowder, 2007; Wilson & Berne, 1999).

Teachers can differ in their noticing patterns depending on their experiences, backgrounds, and education (Santagata, Zannoni, & Stigler, 2007; Miller & Zhou, 2007). Professional development (PD) on noticing children's thinking can improve teacher noticing (Sherin & Han, 2004; van Es & Sherin, 2010) and has been shown to extend to the classroom (Sherin & van Es, 2009) and support teacher learning after the PD is complete (Franke, Carpenter, Levi, & Fennema, 2001). With respect to professional noticing of students' mathematical thinking, much of the research has focused on prospective teachers' development (e.g. Fernández, Llinarez, & Valls, 2012; Fisher, Thomas, Schack, Jong, & Tassel, 2018; Monson, Krupa, Lesseig, & Casey, 2018; Simpson & Haltiwanger, 2016; Tyminski, Land, Drake, Zambak, & Simpson, 2014). Few have analyzed the development of

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professional noticing expertise among practicing teachers (e.g., Jacobs et al., 2010; LaRochelle, 2018). Practicing teachers differ in important ways from prospective teachers because they bring with them years of teaching experience to PD activities and are situated in contexts that may or may not be conducive to what they learn in the PD (Levin, Hammer, & Coffey, 2009). Hence, it is important for the field to study the growth of this expertise in practicing teachers as well.

### Practicing Teachers' Professional Noticing of Students' Mathematical Thinking Expertise

We found two studies that document the growth of professional noticing expertise among practicing teachers. Jacobs and her colleagues (2010) showed us that for practicing primary teachers, teaching experience alone may not adequately support robust professional noticing expertise, and that long-term, sustained PD may be necessary to develop these skills. They found significant positive differences across all three groups of practicing teachers for each component-skill of professional noticing, with attending reaching a ceiling level after 2 years. Hence, for practicing primary teachers, there is a clear need for sustained PD in order to develop this important expertise.

LaRochelle (2018) conducted an analysis similar to Jacobs et al. (2010), comparing the professional noticing expertise of prospective secondary teachers, experienced secondary teachers, and experienced secondary teachers who had completed 4 years of long-term, sustained PD about responding to students' mathematical thinking. Secondary teachers differ in important ways from primary teachers because they experience different school structures (Blatchford, Bassett, & Brown, 2011; Ferguson & Fraser, 1998) and exhibit different conceptions about mathematics, students, and teaching (Weinstein, 1989). However, LaRochelle's findings did indicate that for many practicing secondary teachers, teaching experience may not adequately support this expertise. In particular, the experienced secondary teachers exhibited similar professional noticing skills to the prospective secondary teachers, whereas the experienced secondary teachers with four years of sustained PD exhibited stronger professional noticing skills than the experienced secondary teachers.

However, it is unclear from LaRochelle's (2018) study what trajectories of development might exist for practicing secondary teachers, and what activities might support this development. Hence, we build on his study by documenting the longitudinal growth of a group of practicing secondary teachers as they progressed through a 5-year PD program that focused on being responsive to students' mathematical thinking. Specifically, we measure the growth that we saw in each component-skill of professional noticing at various points in time during the long-term PD. Consequently, we answer the following question: What changes in experienced secondary teachers' professional noticing expertise can be seen across 5 years of sustained PD about being responsive to students' mathematical thinking? Answers to this question allowed us to compare the growth we saw with the activities that occurred during the PD, and we share implications of these results in our discussion section.

## Methods

### Participants

Initially, we selected a cohort of 32 master teachers (16 mathematics and 16 science) for a five-year fellowship through a highly competitive application process that included analyzing student work, video clips of teaching and an interview (Nickerson et al., 2018). In addition to evidence of student-centered teaching, we sought teachers who had a disposition as a learner. When they began, the 16 mathematics teachers (who are the focus of this paper) had 2 – 30 years of teaching experience, with an average of 13 years. All teachers came from high-needs school districts in the south-western region of the United States. Due to attrition (e.g., moving, changing content areas, and the like), we have longitudinal data for 10 of the teachers.

## Mathematics Teacher PD

Here we describe the nature of the PD over the five years of the PD program. We shifted the PD activities over the five years, foregrounding some activities in the first few years and others in the subsequent two years. The focus of the PD in the first few years was on further development of content knowledge and pedagogy. In the latter years, the PD was more explicitly focused on developing teacher leaders. We describe the nature of these activities over time.

In the first year, teachers watched and discussed videotapes of teaching and were introduced to the *professional noticing* skills of attending to, interpreting, and deciding how to respond to students' mathematical thinking. The PD began with a teacher educator interviewing a student to illustrate the challenges and affordances inherent in one-on-one interviews. Teachers discussed how to conduct an interview, including the importance of wait time, questioning, and avoiding directing students to a particular strategy or answer; and what one might learn from an interview, such as how students approach problems, what one can learn from incorrect responses and how those are often steeped with kernels of understanding, students' affect, and so on. The teachers, working in pairs, then interviewed secondary students.

During subsequent PD sessions, the teachers engaged in many other activities that allowed them to discuss students' content-specific ideas and how to build on those ideas. For example, in year 1 teachers solved pattern-generalization problems in multiple ways and made connections among the solutions, and discussions of student thinking naturally occurred during these activities. Teachers also discussed interviews that they conducted with students at their schools and brought artifacts of student work across a variety of content areas to the PD sessions.

In addition to activities that focused on individual students, teachers also learned about complex instruction (Cohen, 1994) and discussed issues related to facilitating group work. During the third year, teachers, accompanied by teacher educators, traveled to school sites to observe each other teach and then debrief and reflect on the experience. Teachers also coached each other in team-teaching a group of middle school students. Discussions of students' ideas were always framed using the Professional Noticing Framework.

Toward the end of year 3 and throughout year 4, we began a more explicit focus supporting teachers in learning how to lead PD. Activities included selecting artifacts and rehearsing situations they may face in their leadership practice. In year 5, they engaged in lesson study, and the lesson debriefs in the lesson study were explicitly structured to attend to and interpret student ideas before discussing how to modify the lesson.

The PD activities described above align with what the literature has shown to be productive activities for supporting teachers' professional noticing expertise (Fisher et al., 2018; Jacobs et al., 2010; Monson et al., 2018). In general, these studies have shown that activities such as doing mathematics together, learning about students' mathematical thinking through frameworks and research articles, *decomposing* (Grossman et al., 2009) the practice of professional noticing, and practicing the component-skills of professional noticing with written artifacts, video artifacts, and in one-on-one interviews with students can support teachers' professional noticing skills.

## Data Collection

In May, 2013, we collected (Y0) baseline data; we repeated data collection in year 3 (Y3) and again in Year 5 (Y5). As part of the data collection, participants analyzed and responded to prompts about the Pattern Generalization Video, a video-recording of a class of middle school students engaged in a figural pattern generalization task with beams and rods (National Center for Research in Mathematics and Science Education, 2003).

**Pattern Generalization Video.** In the Pattern Generalization Video, students are considering a set of rods that are being connected in a way that creates a beam (see Figure 1). The rods are connected to form triangles, and the length of the beam is the number of rods that form the bottom. Students begin by finding a recursive pattern based on beams of lengths 1, 2, 3, and 4. Then, students are challenged to find an explicit rule for finding the number of rods for a beam of any length. Near the end of the video, two groups of students share. The first group sees the number of rods that make up the top of the beam (L-1 rods), the number of rods that make up the middle of the beam (2L rods), and the number of rods that make up the bottom of the beam (L rods). They recognize that the total is the sum of these three sections, and write L + (L-1) + (2L) = total. The second group to present deconstructed the beam in a different way; instead, they see a set of 4 connected rods, a triangle with a rod on top, and they iterate this pattern from left to right until they reach the last set of 4 rods, where they have to erase the last rod on top, and write the equation, total = 4L - 1. Participants have opportunities to notice ideas such as quantitative reasoning, use of algebraic symbols, meanings for operations and symbols, and other generalization concepts (Jurdak & El Mouhayar, 2014; Lannin, Barker, & Townsend, 2006).

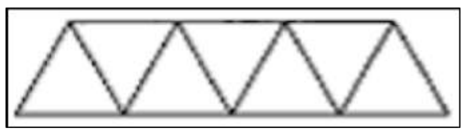

**Figure 1. A beam of length 4 has 15 rods.**

**Prompts.** After watching the video, the participants responded to the prompts listed below. The prompts were adapted from Jacobs and her colleagues (2010), and are related to the three professional noticing component-skills:

1. (*Attending*) Describe in detail what the two groups of students who presented at the board did in response to the task.
2. (*Interpreting*) What did you learn about these students' mathematical understandings?
3. (*Deciding how to Respond*) Pretend you are the teacher of these students. What problem(s) might you pose next, and why?

## Data Analysis

Analysis procedures followed those of Jacobs et al. (2010). Three researchers independently coded the blinded responses for each of the component skills and then discussed codes to resolve any differences. For attending responses, researchers analyzed the extent to which participants attended to the details of the students' strategies. This coding involved identifying the important details of each student's strategy and counting the number of details each participant shared. For interpreting responses, researchers analyzed the extent to which participants interpreted the students' understandings, which involved looking for evidence that participants interpreted specific and nuanced understandings that were consistent with the students' work and with the research on students' pattern generalization skills (e.g. Jurdak & El Mouhayar, 2014). For the deciding-how-to-respond responses, researchers analyzed the extent to which participants based decisions on the students' mathematical understandings. This analysis involved looking for evidence that participants connected their decisions to the students' mathematical ideas, provided specific problems and/or number choices in the problems they selected, provided a rationale that was consistent with the student work and the research on students' pattern generalization skills, and provided evidence of anticipating or building on the students' thinking. Ultimately, each of these component skills was assigned a score of 0, 1, or 2.

## Findings

In Table 1, we provide the mean scores of the ten participants, at three time points, in response to the Pattern Generalization Task, described above. Attending scores started high, remained stable at Y3, and increased to a ceiling level in Y5. Interpreting scores were modest at Y0, grew in Y3, and then remained stable. Means for deciding how to respond started low (most participants provided lack of evidence for deciding how to respond on the basis of students' mathematical ideas), then rose relatively dramatically by Y3, and remained stable.

**Table 1: Mean Scores in the three component skills for Y0, Y3, Y5**

| | Attending | | | Interpreting | | | Deciding how to Respond | | |
|---|---|---|---|---|---|---|---|---|---|
| Year | Y0 | Y3 | Y5 | Y0 | Y3 | Y5 | Y0 | Y3 | Y5 |
| Mean | 1.4 | 1.4 | 1.9 | 0.7 | 1.0 | 0.9 | 0.3 | 0.9 | 0.9 |

### Examples

In this section, we share examples of two participants who exhibited significant improvements in scores on Attending to the details of students' strategies and on Deciding How to Respond from Year 0 to Year 5.

**Adam**. In Year 0, Adam's responses provide insight into both the relative importance of students' ideas and the detail with which they were shared (see Table 2). For example, in Year 0, Adam recognized that both groups of students connected their formulas to the model, but did not provide details of the strategy. For example, beyond saying that each student used a formula, the actual formula and the students' explanations were not shared. The descriptions were so general that one would be hard-pressed to be able to recreate the strategy, or be able to use what was shared to plan next steps for instruction. In contrast, in Year 5, Adam was able to share the specific formulas that each student used, and provide details about how students connected their formulas to their physical representation of the pattern. This response signifies a change in response from Years 0 to Year 5, not only in the number of details provided but, also implicitly, in the value that Adam placed on the students' strategies. We conjecture that because Adam valued students' ideas and had learned more about students' thinking about pattern generalization, he paid close attention to them and was thus better able to recall those details.

**Table 2. Adam's Responses to Attending to the Details, Year 0 to Year 5**

| | |
|---|---|
| Year 0 | *Lack of Evidence*<br>*Student 1:* I do not remember. I'm going to assume Tristan is the last girl who presented. The students presented their formula algebraically. Next, they connected the symbols with their model. The students who were part of the group were there for support. Before they presented, they were engaged in the discussion.<br>*Student 2:* She also presented her formula and connected the symbols with the model they had built. |
| Year 5 | *Robust Evidence*<br>*Student 1:* She explained her formula L+(L-1)+2L= total by connecting each part of the formula to the beam diagram. She explained the bottom of the beam was "L", the top of the beam was the length of the bottom minus the one rod and the total middle rods was 2 times the length of the base of the beam.<br>*Student 2:* She also explained her formula which was 4L-1. The teacher asked her to explain how [the formula] was connected to the diagrams. She pointed at the rods beam diagram showing how the rods enclose 4 complete triangles but at the end you are missing one rod to complete the figure. [draws two figures and notates each as follows: 4(1)-1 and 4(2)-1]. |

**Ella**. We share the example of Ella to highlight changes from Year 0 to Year 5 in deciding how to respond (see Table 3). First, in Year 0, the problem Ella selected serves a funneling function toward a correct answer (Andrews & Bandemer, 2018) when Ella wanted her students to answer a question about whether the two formulas are, in Ella's words, "the same." Then, in the rationale, Ella appears to use directive language by sharing that she wanted students to *see that* the generalizations are equivalent, rather than, for example, *explore whether* the generalizations are equivalent. Her language "see that" speaks to funneling toward a correct answer rather than an openness to exploring and building on students' ideas or anticipating other student responses. In contrast, in Year 5, Ella suggests four questions, two that are specific to Tristian's and Beverly's formulas, and two that reflect an openness to learning about students' ideas. Two of the questions that Ella posed are specific and specifically related to the students' previous approaches. Further, her question, "What do your results tell you about both generalized formulas?" coupled with her rationale about questioning reflects a stance that values students' ideas and provides students with opportunities to reflect on the connections between the two formulas, rather than funneling students toward a single response. These kinds of changes reflect the sorts of responses we would hope to see and that teachers can eventually enact with their students in the classroom.

**Table 3. Ella's Responses in Deciding How to Respond, *Year 0 to Year 5***

| | |
|---|---|
| Year 0 | *Lack of Evidence*<br>Problem or Problems: I may ask students if these 2 formulas are the same.<br>Rationale: The purpose being for students to *see that* even though the problems were deconstructed and generalized differently that the end result, the generalizations are equal. [italics added] |
| Year 5 | *Robust Evidence*<br>Problem or Problems: Can we use Tristan's equation to find the number of rods in a beam of length 10? What about Beverly's? How do the answers compare? What do your results tell you about both generalized formulas?<br>Rationale: This questioning would hopefully lead students to the idea that these 2 generalized expressions are equivalent. |

**Change in Responses**. Looking across both sets of responses provides the reader with the opportunity to see how responses for attending and deciding-how-to-respond changed over time. In both cases, the responses became more detailed and reflected a valuing of and curiosity about students' ideas. This orientation is one that we know is generative. That is, teachers who are attuned to and curious about their students' ideas continue to grow in their mathematics teaching practice long after PD has ended (Franke et al., 2001), and so improved professional noticing is an outcome we seek not only for the specific expertise that teachers develop, but also for their orientation toward students that continues to aid their own learning for years to come.

## Discussion: The Development of Practicing Secondary Teachers' Professional Noticing Expertise and Related PD Activities

In our study we found growth from Y0 to Y3 in the Interpreting and Deciding How to Respond component-skills. In the first three years, the teacher educators spent much time engaging teachers in activities and discussions around individual students' ideas, including interviews, analyzing video and written artifacts, and observing each other teach with a protocol that focused on the students' thinking. As other studies have shown (e.g. Jacobs et al., 2010; Monson et al., 2018), these experiences provided many of our teachers with opportunities to learn about and build on student thinking in a rich way and supported them to develop a disposition to build on student thinking, both of which may have helped them to develop their interpreting and deciding-how-to-respond skills. We believe that the opportunities to consider the research on student thinking in pattern generalization and investigate a single student's (or small group of students') ways of reasoning allowed many of our teachers to develop their understanding of students' learning trajectories within the domain of pattern generalizations, which we posit is an important component of developing one's interpreting and deciding-how-to-respond skills (Nickerson, Lamb, & LaRochelle, 2017).

However, our results indicate that many teachers' interpreting and deciding skills had room for improvement. This differs from Jacobs et al.'s (2010) study of primary teachers, wherein most had demonstrated robust professional noticing skills after 4+ years of PD. As Nickerson et al. (2017) point out, there is a difference between the student thinking frameworks for secondary mathematics and for primary mathematics, in that the student thinking frameworks for primary mathematics are much more explicit and well-connected than those available for secondary mathematics. It is likely that well-connected and detailed learning trajectories at the secondary level could help teacher educators further support teachers' professional noticing expertise. For example, we noticed that the video artifact we selected showed students constructing explicit symbolic generalizations that also made connections to the figure, which is a high level of sophistication of generalization skills (Jurdak & El Mouhayar, 2014; Lannin et al., 2006). During the PD activities, teachers talked about some of the earlier stages of students' generalization skills, such as recursive thinking. However, what fruitful directions should teachers pursue after students demonstrate sophisticated generalization skills?

With respect to the attending component-skill, we did not see growth from Y0 to Y3. This also differs from Jacobs et al.'s (2010) study of primary teachers, wherein the primary teachers reached a ceiling level of attending skills after 2 years. However, we noticed that many of the secondary teachers in our study exhibited strong attending skills prior to engaging in the PD. Our teachers were specially selected from a large pool of applications to participate in the 5-year PD program, which may have contributed to our results (Nickerson et al., 2018).

From Y3 to Y5, we saw growth in teachers' attending skills, but not their interpreting and deciding-how-to-respond skills. During these years, teachers focused on issues of coaching and becoming leaders in their respective teaching communities. In year 5, they engaged in a year-long lesson study activity in which they collaboratively planned, taught, re-taught, and debriefed about a lesson. We hoped that lesson study would become a PD structure that they could bring to their respective teaching communities that focused other teachers' attention on student thinking. During discussions about becoming a teacher-leader, we maintained a focus on being responsive to student thinking because instruction that attends to student thinking has many positive benefits for both students and teachers (e.g. Franke et al., 2001; Jacobs et al., 2007). Hence, student thinking was still an important component, but it may have been back-grounded by the other discussions regarding being a teacher-leader. We wondered if the growth in attending may have resulted from the intense focus on attending to evidence of student thinking, in both the lesson study and the discussions of coaching. During lesson study, teachers were required to gather as much evidence of student thinking as possible for the debrief and lesson modification, which may have given them more opportunities to practice attending to student thinking and supported in them a belief that one should attend to student thinking.

These results differ from studies of prospective teachers, which found significant improvements in all three component-skills in much shorter amounts of time (e.g. Fisher et al., 2018; Monson et al., 2018). However, there are many factors that make comparing our results to the results reported in their studies tenuous. First, it is unclear to what extent the surrounding contexts support or constrain teachers' professional noticing skills. Practicing teachers do not often have the luxury of attending a PD on a weekly basis (as do prospective teachers), and the classroom contexts in which they work may or may not be conducive to supporting this expertise (Levin et al., 2009). We imagine that the competing obligations of practicing teachers (and especially our emerging teacher leaders) may have influenced the development of their professional noticing expertise. Second, it is unclear how the selections of artifacts influence the results of these studies. In particular, we wondered if we had overly challenged our teachers by selecting an artifact that exhibited sophisticated generalization skills (Nickerson et al., 2017). Third, we recognize that the field does not have an agreed-upon standard for high quality professional noticing skills. For example, while Monson et al. (2018) used the term "emerging ability" to represent the highest levels of noticing in their study, Fisher et al. (2018) and Jacobs et al. (2010) used the terms "robust evidence" to represent the highest levels of noticing. How do the responses that received the highest scores compare across each study? What we can say for certain is that significant growth was documented in each of these studies. However, comparing the amount of growth with other studies is less obvious.

In this study, we documented the longitudinal changes of a group of practicing secondary teachers as they progressed through a long-term PD about being responsive to student thinking and becoming teacher-leaders. Our results indicated that different PD activities supported each component-skill differentially. Specifically, the activities in the first half of the PD that supported discussions about individual student thinking and how to build on student thinking seem to have supported our teachers' interpreting and deciding-how-to-respond skills. In addition, the activities in the last half of the PD that supported discussions about coaching others and using evidence of student thinking seem to have supported our teachers' already-strong attending skills. These results have implications for both researchers and teacher educators. In particular, our study can inform teacher educators about the nature of the activities that contribute to various components of professional noticing through explicit or implicit practice. Additionally, our study provides researchers with insight about (a) artifacts they might use to measure professional noticing and (b) rubrics for categorizing responses, as well as factors that may support or inhibit practicing secondary teachers' development of professional noticing expertise.

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