Think before you act: Thinking time contributes to math problem-solving efficiency

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Chan, J. Y. C., Sawrey, K. B., Hulse, T., & Ottmar, E. (2020, April). *Think before you act: Thinking time contributes to math problem-solving efficiency* [Roundtable Discussion]. The 2020 Annual meeting of the American Educational Research Association (AERA), San Francisco, CA. (Online Presentation)

1. Objectives

One of the beautiful things about mathematics is that there are multiple pathways to solving any problem. However, students often do not take the time to understand the problem or explore alternative and possibly more efficient strategies before solving (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980). Rushing through problems may result in more errors and less efficient strategies, and lead to less positive views of mathematics. The objective of this study is to examine if and how *thinking time, defined as the amount of time from the start of the problem to the students first action to solve,* influences the efficiency of problem-solving strategy. We utilize data recorded in Graspable Math (GM; Ottmar, Landy, Goldstone, & Weitnauer, 2015; Weitnauer, Landy, & Ottmar, 2016), a web-based mathematical tool, to test the associations between thinking time and *problem-solving efficiency, defined as the number of steps taken to solve the problem*. We hypothesize that longer thinking time before solving would predict more efficient strategies.

2. Theoretical framework

According to Polya's mathematical problem-solving process (1957) there are four stages to successful problem-solving: 1) understand the problem, 2) devise a plan, 3) carry out the plan, and 4) check for the correctness of the solution. Polya viewed problem-solving as a process and differentiated between *thinking* and *doing* when solving math problems. As such, mathematical problem-solving at each stage requires procedural, conceptual, and flexible thinking (Star & Rittle Johnson, 2008), and involves cognitive, metacognitive, and affective processes (Schoenfeld, 2017; Mayer, 1998). Encouraging students to think more deeply about the problem and plan different strategies may help students become more proficient, efficient, and flexible problem solvers.

Studies suggest that thinking about and reflecting on the problem-solving strategies before actively solving have positive effects on performance. For instance, adults who paused longer prior to making their first move in the Tower of Hanoi task completed the task with fewer moves, and they reported using that pause time for planning (Welsh, Cicerello, Cuneo, & Brennan, 1995). Similarly, prompting preschoolers to pause and reflect on instructions before making a response leads to substantial improvement in performance on the Dimensional Change Card Sort task (Espinet, Anderson, & Zelazo, 2013). The benefits of pausing also emerge in classroom discourse and student learning. When teachers are prompted to wait a few seconds between utterances, teachers are more likely to ask questions that require application of concepts as opposed to fact retrievals or repeating students' responses (Tobin, 1986); students also ask more questions (Rowe, 1986; Ingram & Elliot, 2014) and show higher achievement (Tobin, 1986, 1987). These studies suggest that longer thinking time may be associated with better problem solving performance.

Affective components of mathematics may also contribute to problem-solving performance.

For example, Hoffman (2010) found that self-efficacy was positively related to problem-solving efficiency, whereas math anxiety was negatively related to problem-solving accuracy. The negative impact of math anxiety is also observed in problem-solving strategies. Students with high math anxiety are more likely to use less sophisticated strategies compared to students with low math anxiety (Ramirez, Chang, Maloney, Levine, & Beilock, 2016). Together, the findings suggest that problem-solving efficiency may be influenced by cognitive as well as affective processes.

Here, we examine (1) whether the amount of thinking time before solving a math problem is associated with steps taken to solve that problem; (2) whether thinking time predicts problem-solving efficiency above and beyond students' algebra knowledge, math self-efficacy and math anxiety; and (3) whether algebra knowledge or affective disposition moderates the relation between thinking time and problem-solving efficiency.

3. Methods

We utilized data recorded in *Graspable Math* (GM), a web-based dynamic mathematics tool where students can explore algebraic notations by performing mouse-based gestures (e.g. moving, combining, and substituting symbols) that apply mathematical transformations to expressions. The GM system responds to the users' gesture-actions by enacting valid transformations (e.g., turning $2x+5x$ into $7x$ when users tap "+") and provides error feedback by shaking (e.g., the expression $2+7x$ shakes when users attempt to add the 2 and the $7x$) and by not enacting the invalid transformation.

We analyzed data collected for a larger study examining the usability and feasibility of GM in three high-school classrooms during algebra instruction across three consecutive days. In the study, students completed a brief assessment on algebra knowledge (e.g., $5(y–2) = –3(y–2)+4$, solve for y; adapted from Star, Rittle-Johnson & Durkin, 2016), and a questionnaire on math self-efficacy (e.g., "I can learn math even if the work is hard"; adapted from Midgley et al., 2000) and anxiety (e.g., "My mind goes blank when I work on math problems"; adapted from Kaya, 2008). Then, students solved math problems using GM. All materials were administered online, and students completed the study at their own pace on their own device.

4. Data sources and materials

Participants

A total of 41 ninth-grade students were included in the following analyses. They completed the algebra assessment and the questionnaire, and were present for at least two of the three study sessions.

Goal State Activity

We used data from the *Goal State Activity* in GM (Ottmar et al., 2015) to examine the relation between thinking time and problem-solving efficiency. In this activity, students were presented with a starting expression and a mathematically equivalent goal state (Figure 1). Their task was to transform the expression into the specified goal state using a series of gesture-actions that were learned and practiced in the sessions. In the example of $6 - 3 + 5 \times 2$ (Figure 2), students can (1) tap "$-$" to subtract 3 from 6, (2) tap "$\times$" to multiply 5 and 2, then (3) drag 3 to the right of 10 to reach the goal state of 10+3. Because GM enacts all valid mathematical transformations, students may take alternative paths to reach the goal state (e.g., (1) multiply 5 and 2, (2) add -3 and 10, (3) add 6 and 7, then (4) decompose 13 into 10 and 3). The unlimited potential paths to a goal state provided an ideal context for examining math problem-solving efficiency.

Because the larger study was designed to test the feasibility and usability of GM, most of the problems that students completed (54 of the 70 problems) were designed to teach the gestures or practice mathematically valid actions, and therefore only required one or two gesture-action steps to reach the goal state. For this paper, we focused on the remaining 16 problems that required at least three steps to reach the goal and were designed to prompt greater variation in problem-solving strategies and efficiency.

Measures

For each goal state problem, we recorded (a) the *total time spent* on the problem from when the problem appeared on the screen to reaching the goal state, (b) the *thinking time*, and (c) the *steps taken* to solve the problem. *Thinking time* represented the amount of time from when the problem appeared to the first action of a student's successful attempt to solve the problem. For each problem, we calculated the *percent thinking time* (thinking time/total time), and used this as the focal predictor in analyses. The *steps* to solve the problem represented the number of steps students took on their successful attempt to reach the goal state. We also calculated *step efficiency* by taking the ratio of the minimum steps required and the number of steps the student actually took to reach the goal state. For instance, if a problem required at least three steps to reach the goal state and the student took four steps, the student's step efficiency on that given problem would be 3/4 or .75. We calculated step efficiency for each problem to account for the fact that the minimum steps varied across problems and that students completed a different number of problems in the study. Finally, we calculated each student's *average percent thinking time* and *step efficiency* across all the problems they completed, and used these values as the predictor or outcome respectively for research questions 2 and 3.

The total score on the algebra assessment, the average self-report rating on the math self efficacy questions, and the average self-report rating on the math anxiety questions were included as covariates (RQ2) or moderators (RQ3) in the analyses.

5. Results

Descriptive analyses

The total number of problems each student completed varied because students solved these problems at their own pace. Most students (*n*=36) completed at least 12 of the 16 problems (*M*=13.98, *SD*=2.35), and all students completed at least 8 problems. The large standard deviations of the *percent thinking time* and the *steps to solve* suggested that there were individual differences in these two measures and provided the premise for examining their associations both within individual problems and overall across students (Table 1).

RQ1: Association between percent thinking time and steps taken to solve a problem

We conducted Spearman's correlations between *percent thinking time* and *steps taken to solve* for each of the 16 problems to examine whether longer thinking time was associated with fewer steps to solve the problem. Next, we conducted an independent sample t-test for each problem to examine whether there were differences in steps taken to solve between students with long vs. short thinking time (based on median split).

The correlations between percent thinking time and steps were statistically significant for 11 problems, suggesting that longer thinking time was associated with fewer steps to solve the problem. Although the correlations for the remaining five problems did not reach statistical significance, they were all in the expected negative direction. The t-tests revealed a similar pattern of results. Students with long thinking time took fewer steps to solve nine problems. Although the remaining seven problems did not reach statistical significance, six of them were in the expected direction (Table 2).

RQ2: Average percent thinking time predicts step efficiency

We conducted two linear regression models using average percent thinking time as the focal predictor of average step efficiency. The baseline model tested the influence of algebra knowledge, math self-efficacy, and math anxiety on average step efficiency, and revealed that they did not significantly predict students' average step efficiency. Next, adding average percent thinking time to the baseline model revealed that average percent thinking time significantly predicted average step efficiency and accounted for 17% of the variance in average step efficiency above and beyond algebra knowledge and affective disposition (model 2), $F(1, 37)=8.18$, $p=.007$ (Table 3). These results suggest that students who spent more time thinking had higher step efficiency.

RQ3: Moderating effects of algebra knowledge, math self-efficacy, and math anxiety To explore whether the relation between thinking time and step efficiency varied across students with different levels of algebra knowledge, math self-efficacy, or math anxiety, we expanded the regression model in RQ2 and added the interaction term between percent thinking time and algebra knowledge (model 3), math self-efficacy (model 4), or math anxiety (model 5). None of these interactions were significant (Table 3) suggesting that the relation between percent thinking time and step efficiency did not significantly differ across students with high vs. low algebra knowledge, math self-efficacy, or math anxiety.

6. Significance

In summary, we found that (a) students who took longer to think about the problems solved individual problems with fewer steps, (b) thinking time significantly predicted step efficiency above and beyond students' algebra knowledge or their affective disposition towards mathematics, and (c) the relation between thinking time and step efficiency was not moderated by students' knowledge in algebra, math self-efficacy, or math anxiety.

The findings provide initial support for the notion that thinking time predicts problem solving efficiency, and have implications research and practice. Future studies should examine the potential cognitive and affective mechanisms underlying the association between thinking time and problem-solving efficiency in mathematics. For instance, thinking time may be associated with students' inhibitory control and pausing to think may be a pathway through which inhibition influences mathematics performance. Based on the research, teachers may consider encouraging students to take time to plan out their strategy before solving a problem in order to improve students' problem-solving efficiency.

(word count: 1996)

Acknowledgements

This work was supported by Institute of Education Sciences: [Grant Number R305A180401] and National Science Foundation: [Grant Number 2142984].

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Figure 1. An example of a Goal State problem.

Figure 2. An illustration of some available gesture-actions in Graspable Math.

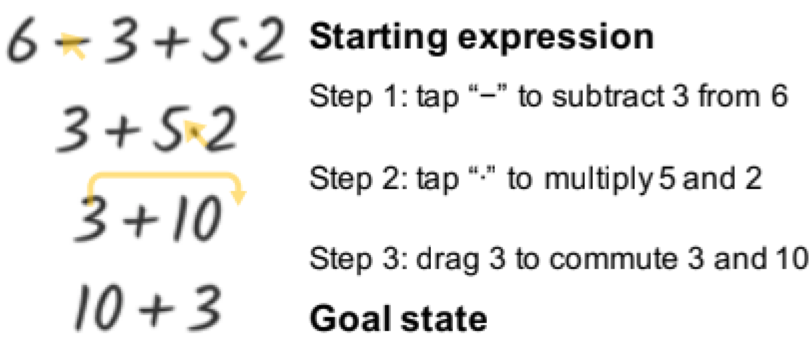

Table 1.
Descriptive results on the percent thinking time, total problem-solving time, and the number of steps students took to solve the individual problems, and the sample size and the minimum steps for each problem are reported.

problem	n	% think time M (SD)	total time M(SD)	min. steps	steps taken M (SD)
2+15+6+3 => 9+17	40	.41 (.16)	12.64 (4.84)	3	3.25 (0.78)
2*y*3*3 => 18*y	40	.30 (.12)	14.47 (4.38)	3	3.27 (0.51)
13-4+6*2 => 12+9	41	.38 (.21)	25.87 (29.83)	3	3.71 (1.38)
5+5*2-2 => 3+10	41	.31 (.19)	23.93 (23.89)	3	3.95 (1.24)
4+10+5 => 6+13	36	.29 (.22)	73.18 (54.06)	3	5.89 (3.82)
3+3+3+3 => 4+4+4	34	.13 (.09)	83.36 (138.45)	5	8.18 (5.40)
5-5*5 => -15-5	29	.17 (.17)	105.64 (109.31)	3	6.41 (6.59)
24/20 => 12/10	33	.10 (.10)	63.52 (47.65)	3	8.67 (4.61)
4*(2*x-1) => 8*x-4	38	.21 (.16)	14.57 (11.72)	3	4.42 (0.68)
(3*x+2)*4 => 12*x+8	38	.29 (.22)	22.67 (25.30)	3	5.37 (2.34)
20+30 => 5*(4+6)	37	.20 (.14)	40.08 (29.72)	3	5.41 (4.06)
36+48*x => 6*(6+8*x)	30	.16 (.12)	63.97 (69.09)	3	6.87 (10.54)
3*(5+4*x)-10 => 12*x+5	30	.22 (.13)	46.14 (34.18)	5	8.83 (4.24)
(5-4*x+3)*2 => 16-8*x	31	.19 (.13)	33.38 (17.21)	5	8.61 (1.50)
16+12+42 => 20+50	18	.22 (.13)	35.19 (34.14)	3	5.50 (4.42)
7*3+x+7-5*x => -4*x+7*4	15	.22 (.18)	50.91 (44.36)	4	6.73 (3.69)

Table 2.
Spearman correlations and independent sample t-tests on the association between percent thinking time and steps for each problem.

problem	*n*	r_s for % thinking time and steps	average steps		*p*
			short % think time	long %think time	
2+15+6+3 => 9+17	40	**-.41****	**3.50**	**3.00**	**.047**
2*y*3*3 => 18*y	40	-.11	3.26	3.28	.891
13-4+6*2 => 12+9	41	**-.36***	4.05	3.38	.131
5+5*2-2 => 3+10	41	-.33	4.35	3.57	.052
4+10+5 => 6+13	36	**-.53*****	**7.72**	**4.06**	**.004**
3+3+3+3 => 4+4+4	34	-.36*	9.71	6.65	.106
5-5*5 => -15-5	29	-.34	8.86	4.13	.071
24/20 => 12/10	33	**-.57*****	**11.82**	**5.31**	**<.001**
4*(2*x-1) => 8*x-4	38	-.15	4.53	4.32	.349
(3*x+2)*4 => 12*x+8	38	**-.43****	**6.26**	**4.47**	**.020**
20+30 => 5*(4+6)	37	**-.47****	**6.95**	**3.78**	**.016**
36+48*x => 6*(6+8*x)	30	**-.41***	**10.60**	**3.13**	**.060**
3*(5+4*x)-10 => 12*x+5	30	**-.46***	**10.67**	**7.00**	**.020**
(5-4*x+3)*2 => 16-8*x	31	**-.54****	**9.38**	**7.80**	**.002**
16+12+42 => 20+50	18	**-.60****	**7.78**	**3.22**	**.037**
7*3+x+7-5*x => -4*x+7*4	15	-.43	**8.63**	**4.57**	**.031**

* p<.05, ** p<.01, *** p<.001

Table 3.
Regression models predicting step efficiency

Outcome: step efficiency	Model 1: baseline		Model 2: baseline + % think time		Model 3: % think time * algebra		Model 4: % think time * efficacy		Model 5: % think time * anxiety	
Predictors	***β***	**Std. Error**	***β***	**Std. Error**	***β***	**Std. Error**	***β***	**Std. Error**	***β***	**Std. Error**
algebra score	.019	.181	-.017	.166	-.014	.167	-.065	.167	-.008	.165
self-efficacy	.284	.178	.270	.163	.277	.164	.131	.188	.291+	.162
math anxiety	.247	.196	.346+	.183	.341+	.184	.323+	.181	.281	.187
% think time	--	--	.430**	.150	.428**	.151	.474**	.151	.334+	.167
% think time*algebra	--	--	--	--	.117	.160	--	--	--	--
% think time*efficacy	--	--	--	--	--	--	-.287	.201	--	--
% think time*anxiety	--	--	--	--	--	--	--	--	-.248	.193
R^2	*.0827*		*.2526*		*.264*		*.294*		*.287*	

+ < .10, * *p*<.05, ** *p*<.01, *** *p*<.001