


Impact of GeoGebra on the Students' Conceptual Understanding of Limit of a Function in Bhutan

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ABSTRACT

The paradigm shift from traditional didactic instruction to technology-enriched teaching and learning environments significantly benefits learners. Educational technology can visualize abstract mathematical concepts contextually and graphically and allow learners to actively construct this knowledge. This study aims to ascertain the efficacy of a computer-assisted instruction method using GeoGebra in further developing the concept of the function limit for grade XI students. This study employed a quasi-experiment static-group comparison design with 60 students from Gongzim Ugyen Dorji Central School at Haa in Bhutan. The students were divided into two equal groups. Group 'A' used the GeoGebra software, while group 'B' used the conventional method to learn the limit of the function. The data was collected through a Conceptual Knowledge Test of Limit Function. In addition, an independent sample t-test was employed using the Statistical Package for the Social Sciences (SPSS 22.0). This study demonstrated that students who were taught using GeoGebra outperformed those who learned through conventional methods. The results confirmed that GeoGebra software could enhance and significantly improve students' conceptual understanding of the limit of the function.

INTRODUCTION

The study of calculus is one of the most fundamental topics in mathematics due to its widespread application in multidisciplinary fields. Calculus is an essential tool employed in the pursuit of biological sciences, physical sciences, engineering, and economics ([Aréchiga Maravillas et al., 2019](#); [Mendezabal & Tindowen, 2018](#); [Sari, 2017](#)), and within it, the concept of limit is considered fundamental to calculus ([Denbel, 2014](#); [Hutkemri, 2014](#); [Zollman, 2016](#)). Indeed, the concept of limit is a prerequisite in defining the concept of derivative and integral calculus ([Denbel, 2014](#)). For instance:

Instantaneous velocity is the limit of average velocities; the slope of a tangent line to a curve is the limit of the slope of secant line; the area of a circle is the limit of areas of the inscribed polygon as the number of sides increases infinitely ([Brokate, Manchanda & Siddiqi, 2019, p.39](#))

Without conceptualizing the vital components of limits, it would be hard to comprehend the subsequent concepts such as continuity, derivative and definite integral ([Luter, n.d.](#)). Additionally,

[Swinyard \(2011\)](#) also stated that a more concise understanding of calculus demands a comprehensive understanding of the fundamental components of the formal definition of the limit function.

Despite its importance, limited concepts have been historically difficult for introductory calculus students ([Cappetta & Zollman, 2013](#); [Denbel, 2014](#); [Gürbüz & Agsu, 2018](#); [Liang, 2016](#); [Muzangwa & Chifamba, 2012](#); [Tall, 1992](#)). For example, they cannot elucidate the role of limit in providing the algebraic definition of derivative and definite integral as the limit of sum ([Aréchiga Maravillas et al., 2019](#); [Orton, 1983](#)). The epistemological obstacles that students encountered in teaching and learning of limits were, “Confusion over whether a limit is reached, the dynamism of limits and static character of limits” ([Thabane, 1998, p.16](#)). Additionally, relating the limit concept to continuity and derivative was challenging for the students ([Thabane, 1998](#)).

Furthermore, the formal definition of the limit of a function is considered problematic for students due to its epsilon-delta approach and abstractness ([Kristanto et al., 2019](#)). Considering the difficulties of the formal definition of the limit of a function, they then created activities employing dynamic geometry software to assist students in exploring the definition of the concept. [Tall and Vinner \(1981\)](#) also affirmed the difficulty of the formal definition of a function limit when the 70 proficient university mathematics students were called to write down the definition of $f(x) = l$. Additionally, [Barak \(2007\)](#) corroborated the difficulty of ϵ and δ definition of limit. In his study, participants were unable to delineate exactly what the ϵ and δ represent. These symbols did not distinctly develop the concept image of the limit of a function in their minds.

It is clear from the previous research undertaken that students’ difficulty in the limit of function stems from learners’ misconceptions ([Denbel, 2014](#); [Muzangwa & Chifamba, 2012](#); [Liang, 2016](#)) and traditional didactic pedagogy, which emphasizes a more computational procedure instead of conceptual development ([Aréchiga Maravillas et al., 2019](#); [Kristanto et al., 2019](#); [Sebsibe & Feza, 2019](#)). For decades, the conventional teaching and learning of differential calculus rely on symbols and notations has been the preferred calculus instruction method. Recent investigation has demonstrated that students were more successful in learning the limit of function operationally rather than relationally; moreover, they memorized the mechanical operation to succeed on the exam. These practices have often been responsible for students failing to comprehend the key concepts of limits. [Aréchiga Maravillas et al. \(2019\)](#) also asserted that students could mechanically calculate the limit but not manifest an underlying conceptual understanding. Thus, the conventional didactic instruction that relied on isolated facts, routine calculations, memorizing algorithms, and procedures was responsible for the student’s difficulty with limits in particular and calculus in general ([Denbel, 2014](#)).

Moreover, numerous reports such as Education in Bhutan: Findings from Bhutan’s experience in PISA for Development ([BCSEA & OECD, 2019](#)) and National Education Assessment ([BCSEA, 2013](#)) have consistently shown Bhutanese students do not meet the minimum educational standards in mathematics and science. They performed relatively poorly when compared with other countries. It necessitates a shift in the educational paradigm from machine learning to relational learning to enhance students’ understanding of mathematics and science.

Despite the wealth of supporting evidence deduced from the studies ascertaining the impact of similar software programs with different populations of learners in the international context ([Aréchiga Maravillas et al., 2019](#); [Liang, 2016](#); [Martinovic & Karadag, 2012](#); [Mendezabal & Tindowen, 2018](#); [Nobre et al., 2016](#)), there is a limited academic discourse on the efficacy of GeoGebra in promoting a conceptual understanding of the limits of function in the Bhutanese context. Thus, there is an essential need to determine the effectiveness of GeoGebra on Bhutanese students’ conceptual understanding of the limits of function.

GeoGebra is open-source software developed by Markus Hohenwarter in 2002 for teaching and learning Mathematics from primary to the university level ([Hohenwarter et al., 2009](#)). It combines many aspects of different mathematical packages and dynamically joins Geometry, Algebra, and Calculus. In addition, numerous calculus-related interactive worksheets and specific methods developed by teachers and researchers are available (www.geogebra.org). Based on the site, GeoGebra users have reached 100 million students, while GeoGebra researchers based on Google Scholar have resulted in 15,800 published scientific papers ([Arini & Dewi, 2019](#)).

Studies have shown that an improvement in conceptual development in the limit of a function in particular and calculus, in general, can be achieved with the integration of educational technology ([Kado](#)

& Dem, 2020). It has the potential to help students visualize the abstract concepts graphically, contextually, and dynamically ([Arini & Dewi, 2019](#); [Martinovic & Karadag, 2012](#); [Nobre et al., 2016](#)). According to Kado and Dem (2020), the integration of educational technology in teaching and learning transforms students into knowledge creators instead of knowledge consumers, as these tools facilitate students' abilities to visualize concepts and actively construct the knowledge. [Aréchiga Maravillas et al. \(2019\)](#) also studied the efficacy of GeoGebra on teaching and learning outcomes of the limit concept. Traditional didactic instruction was employed to a control group, while GeoGebra aided instruction was tailored to an experimental group. The evidence gathered supported the claim that GeoGebra helps to enhance understanding of the limit of a function, but no statistically significant difference in their scores was found. [Kristanto et al. \(2019\)](#) also designed engaging activities using dynamic software to facilitate the students' discovery of the formal definition of the limit of the function.

More recently, evidence suggests that graphical and numerical approaches in teaching and learning the limit of functions can help create a concept image of the limit function ([Kristanto et al., 2019](#)). This fact is supported by the findings that the graphical representation of the abstract limit concept gives an explicit image of the limit function, resulting in its conceptual development. This approach also demonstrates the considerable interest in exploring the concept of the limit of the function, which results in learning the limit of a function with a true understanding rather than simply mechanically applying procedures and techniques without real comprehension. In calculus instruction, graphical representation plays a significant role in helping students to develop their conceptual understanding.

Furthermore, [Cheng and Leung \(2015\)](#) explored the efficacy of the GeoGebra-based dynamic applet on students' achievements. Their studies found that effective dynamic visualization has a significant positive impact on students' learning of function limits. As per [Kidron and Zehavi's \(2002\)](#) earlier work, the dynamic visualization of the limit of functions abetted students to associate the algebraic definition with a visual representation to form a definition of the limit. Thus, effective visualization of abstract concepts was crucial in the conceptual development of the limit. [Ayodos \(2015\)](#) also justified GeoGebra as an effective tool for teaching and learning both limits and continuity. Thus, it is clear from the extensive evidence in the literature that GeoGebra abetted students' learning extensively due to its efficacy in the visualization of concepts.

GeoGebra enabled the understanding of limits and continuity in the international student context. On the other hand, the anecdotal evidence revealed that teaching and learning limits in a Bhutanese context are dominated by traditional didactic instructions with little integration of educational technology. Thus, to transform the traditional instruction into the digital-based dynamic learning environment, [Bhutan Education Blueprint 2014-2020](#) recommended a shift in educational systems to leverage the potential of ICT to produce globally competent learners. Subsequently, the Ministry of Education launched its [ICT Master plan-Isherig in May 2029](#) to harness the potential benefit of ICT in teaching and learning by creating a platform for effective communication, enabling access to information and knowledge, and building a conceptually rich learning environment where students construct their knowledge, with visualization and experimentation. These initiatives are both novel and effective.

In conclusion, it is clear from the volume of evidence in the literature that students' knowledge was limited to a simple mechanical component as they knew only how to manipulate the algebraic expression symbolically but struggled in demonstrating any conceptual understanding. Thus, the present study was designed to enhance learners' conceptual or relational understanding of limits. Based on literature research to date, no similar study has been conducted in Bhutan; therefore, my study sheds light on the efficacy of digital tools like GeoGebra in having a significant impact on acquiring a conceptual knowledge of limits. Additionally, this study also serves as a stepping stone for further investigation into the efficacy of using GeoGebra to teach calculus in a Bhutanese context.

METHODS

This study used a quasi-experimental static-group comparison design to compare the effectiveness of GeoGebra software in developing the conceptual understanding of the limit function. A sampling of students received GeoGebra enriched instruction while the remaining students received

traditional instructions. This method was thought to be the best approach as it attempts to study the effect of the instruction on intact groups rather than randomly assigning participants to the experimental or control groups ([Creswell, 2013](#); [Mertens, 2010](#)).

Sampling

A convenience sampling method was used in this study. This non-probability sampling technique gives the researcher the right to pick informants based on identical characteristics ([Tongco, 2007](#)) or realistic requirements such as availability at a certain time due to geographical proximity and ease of access ([Etikan, 2016](#)). Since the concept of the limit of a function was initially introduced in eleven standards of the Bhutanese curriculum, this study involved 60 grade 11 students (35 experimental groups and 25 control groups) studying in one of the higher secondary schools in western Bhutan. For this study, the students were grouped into control and experimental group based on their class sections

Validity and Reliability of Instruments

A Conceptual Knowledge Test of Limit Function (CKTLF), consisting of six questions modified from BHSEC Mathematics Book-II for Class XI students of Bhutan, was used to collect the data ([Malhotra, Gupta, & Gangal, 2020](#)). The instrument was piloted to ensure its reliability and validity. The average Item Objective Congruence (IOC) was 0.89, thereby validating the appropriateness for the study. The instrument's reliability was proven high, with an item reliability of 0.88 ([Cohen, 2013](#)). Comparative statistical analysis was completed using the t-test. The independent sample t-test was used to compare the learning achievement between the control and the experimental group. The inferential t-test with a $p < 0.005$ level of significance, with mean and standard deviation used to infer the results.

Experimental process

For the intervention, researchers treated the experimental group with the GeoGebra enabled learning environment (dynamic graph), while the control group was taught using the traditional didactic instructions, supported with non-dynamic graphs. A detailed explanation of each lesson is given as follows:

At the beginning of the first lesson, a GeoGebra applet was used to develop the concept of the limit function. In figure 1 below, n indicates the number of sides of a polygon. In an experimental group, the teacher elucidated that "as the number of sides of the polygon increases indefinitely, the area of the polygon continually approaches the area of the circle" ([Malhotra, Gupta & Gangal, 2020, p.1](#)). Eventually, the difference in the area between the circle and the area of the polygon becomes negligible. Thus, it will help develop the idea that the limit of the area of the polygon, where the number of sides increases indefinitely, is the area of the circle.

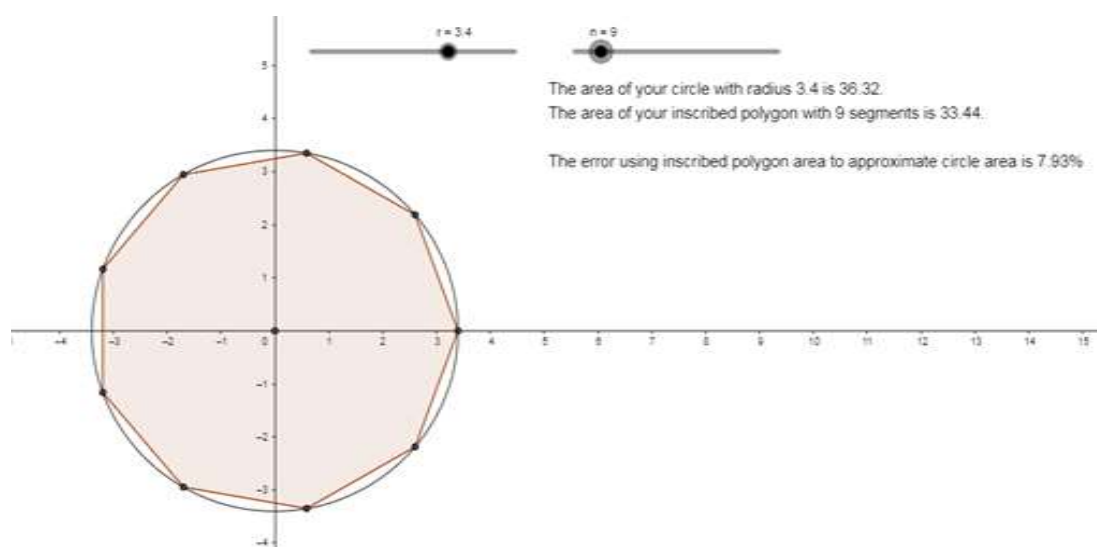


Figure 1. GeoGebra applet for developing the concept of limit

In the second lesson, the GeoGebra applet helped enhance the conceptual understanding of the limit of a function dynamically. First, the researcher delineated that function $y = \frac{x^2-1}{x-1}$ does not exist at $x = 1$, or it is not defined. The researcher demonstrated that x approaches 1 through values greater than 1, and x approaches 1 through value less than 1 (left-hand and right-hand) dynamically using the slider. Thus, as x approaches, one from both sides, y approaches 2. Though the function $y = \frac{x^2-1}{x-1}$ It has no value at 1; it has a limiting value of 2. In the end, researchers stated that the limit of a function, even though the function is not defined at a point, and the value of the limit is not equal to the value of the function at a point. On the other hand, the control group was taught limited concepts through routine algebraic manipulation procedures. Researchers drew the non-dynamic graph of the same functions to elucidate the concept of the limit of functions.

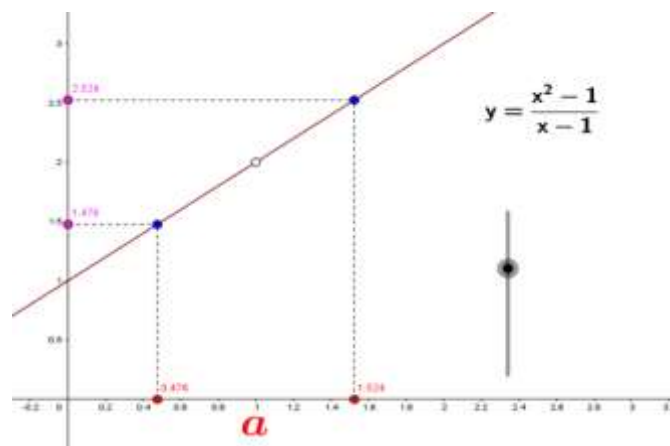


Figure 2. GeoGebra applet to develop the concept of the limit of a function

In the third lesson, GeoGebra software was used to construct the formal definition of a limit. By providing the dynamic geometrical representation in GeoGebra, students could construct the idea of a formal definition of the limit of a function. The function $f(x)$ defined in the neighborhood of point c is said to have a limit 'l' as x tends to c , if for any $\epsilon > 0$ there exists a $\delta > 0$ such that if the distance of x from c is less than δ , then the distance of $f(x)$ from L is less than ϵ . It is illustrated in figure 3.

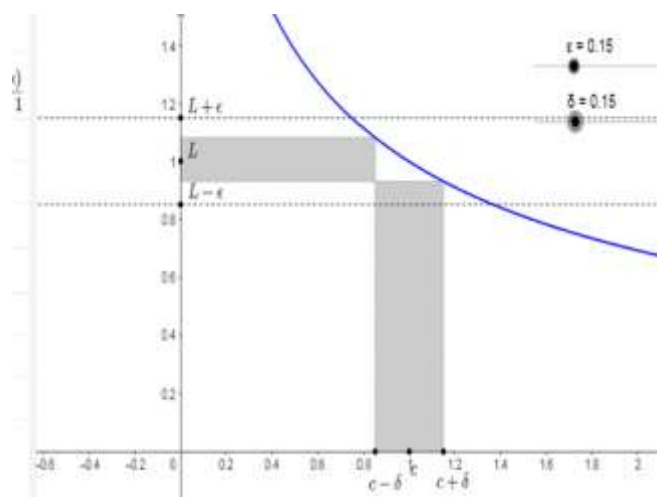


Figure 3. Epsilon- delta definition of limit

RESULTS

Before conducting the inferential *t-test*, the normality test was conducted using the Kolmogorov-Smirnov test, as shown in Table 1

Table 1. Test of Normality with Kolmogorov-Smirnov Test

	Pretest-EG	Posttest-EG	Pretest-CG	Posttest-CG
N	32	32	30	30
Kolmogorov-Smirnov Z	0.978	0.944	0.986	1.406
Sig.(2-tailed)	0.294	0.335	0.285	0.038

The value of CKLF for both the groups were $P > 0.05$. These findings show that the degree of normality assumptions was satisfied for both pre-posttest EG and CG. Thus, the assumption underlying t -test was met.

Table 2 illustrates the evaluation of homogeneity of variance using Levene's test for equality of variances, in which the value of Sig. Value of p should be higher than $p = 0.05$ level of significance to assume that the group variances are equal.

Table 2. Levene's test for equality of variance

		F	DF	Sig
Pre-test	Equal variances assumed	6.571	62	0.78
	Equal variance is not assumed		58.99	
Post-test	Equal variance assumed	17.71	62	0.60
	Equal variance is not assumed		42.73	

Levene's test indicated that the assumption of homogeneity of variance of pre-test was met as p -value is greater than 0.05 ($F(1,62) = 6.571, p = 0.78$). Moreover, the variation of post-test scores for both the groups was the same as p -value is greater than 0.05 ($F(1,42.73) = 17.714, p = 0.00$). Thus, all the assumptions underlying the t -test were met.

Analysis for Pre-Posttest of Conceptual Knowledge Test of Limit Function

An independent sample t -test was conducted to examine the difference in CKTLF between the experimental and control group, as shown in table 3.

Test Group		Mean	Mean Difference	Standard Deviation	Sig (2 tailed)	Effect Size
Pre-test	Control	7.58	0.391	6.10	0.778	0.006
	Experimental	7.59		5.96		
Post-test	Control	23.83	33.125	10.52	0.000	0.91
	Experimental	56.95		4.66		

An independent- samples t -test was conducted to compare the pre-test scores for EG and CG. There was such significant difference in scores for pre-test for Experimental ($M = 7.59, SD = 5.96$) and Control group, $M = 7.58, SD = 6.10; t(62) = -0.283, p = 0.778$ (two-tailed). The magnitude of the differences in the means ($MD = 0.391, 95\%CI: -3.147$ to 2.366) was very small ($\eta^2 = 0.006$). It meant that the data were homogeneous, and treatment could be applied to these groups to identify differences caused by the treatment.

An independent sample t -test conducted for comparison of post-test scores between experimental and control groups showed the significant mean difference between EG ($M = 56.95, SD = 4.66$) and CG ($M = 23.83, SD = 10.52$); $t(42.730) = 16.276, p = 0.000$ (two-tailed). The magnitude of the differences

in the means (mean difference= 33.125,95% *CI*: 29.057 to 37.193) was very high (eta squared =0.91). It indicates that there was a statistically significant difference in post-test scores between experimental and control groups. The test scores of the experimental group were significantly higher than the test scores of the control group. It implied that using GeoGebra is a significantly better method to teach the function's limits than without using it.

DISCUSSION

Results in the Table 3 showed that the experimental group outperformed the control group in CKTLF, indicating the effectiveness of GeoGebra in enhancing the conceptual understanding of the limit of function compared to traditional didactic teaching and learning. Furthermore, the significant mean difference between the experimental and control groups was due to the intervention and not chance.

These findings are consistent with [Sari \(2017\)](#), who used both the GeoGebra and Realistic Mathematics Approach among the school students in Indonesia, and found that geometric representation of the limit concept using GeoGebra and providing contextual examples assisted in developing a complete conceptual understanding of the limit of the function. Moreover, the other significant rise in post-test scores can be attributed to the ability of GeoGebra to represent the abstract concept graphically. It has been experimentally demonstrated that GeoGebra was an effective tool for teaching and learning calculus as it allows students to actively construct their knowledge in a multidimensional visualization of concepts and experimentation ([Nobre et al., 2016](#)). The GeoGebra dynamic-based applet facilitates students' construction of the formal definition of the limit of a function. More specifically, it will help them connect the abstract formal definition of the limit of a function and its graphical representation ([Cheng & Lueng, 2015](#)). They also concluded that dynamic visualization has a significant impact on comprehending the limit of the function.

One possible reason for significant improvement of the results was attributed to the potential of GeoGebra to make the connection between concept image and concept definition of the limit function. The study conducted in Taiwan for the first-year calculus students at the university level revealed that students' development of visualization ability increases students' performance in solving the problems in calculus. The findings also indicated that students taught through multidimensional approaches, like visualizing concepts both graphically and algebraically, were able to connect the concept image and concept definition, leading to significant improvements in their learning. Additionally, [Hutkemri & Zakaria \(2014\)](#) also claimed that GeoGebra helps teachers present the abstract limit concept contextually and allows students to construct the knowledge instead of passively receiving the knowledge actively. It is in line with my study, where concepts were developed with computer-assisted graphing tools, and students used GeoGebra to build the concept of the limit function.

CONCLUSION

This study designed and developed an instructional unit on the limit of the function using the GeoGebra. Our findings established that the 5E GeoGebra aided instructions significantly enhanced the students' understanding of the limit of function than the traditional lecture method. The students in the experimental group who were taught using the lesson designed using GeoGebra.

The instructions based on the GeoGebra have further excited the students to construct the knowledge instead of passively receiving the knowledge actively. The students had the opportunity to visualize the abstract concepts dynamically and contextually to narrow the gap between the concept image and concept definition of the limit of the function. In short, this study indicated that the student's understanding of the limit of function was enhanced using the GeoGebra. In other words, the traditional method of teaching that was largely teacher-centered and textbook-oriented revealed a minimal improvement in enhancing the students' conceptual understanding of the limit of the function.

In summary, this novel study in the Bhutanese context suggests that the GeoGebra as an educational technology can be used in teaching and learning mathematics concept that often challenges and create difficulty for the students. Though the empirical findings of this study limit its generalizability, they are consistent with the literature in suggesting the positive implication of the GeoGebra for developing the conceptual understanding in the limit of the function. Hence, mathematics

teachers can consider integrating GeoGebra in teaching and learning to enhance the relational understanding of mathematical concepts.

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The author declares that there is no funding and conflicts of interest for this research.

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