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Considerations for Developing Effective Math Coaches: A Content-Driven Perspective

Region 7 Comprehensive Center

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Overview

Coaching is a complex endeavor that blends a myriad of concepts and skills. Practical experience, content knowledge, and coaching knowledge are instrumental to becoming an effective math coach. In order to design a professional development system to best support and build the capacity of coaches, we must identify what specific content and coaching knowledge is needed to become an effective coach. While there is a great deal of literature specifying coaching knowledge generally, for math coaches, it is critical to have a content-specific target within their professional learning. There are various types of knowledge, specific to math coaches, that are important to consider in relation to both coaching and content.

Mathematics Coaching Knowledge

In the last decade, research has sought to describe the math coaching knowledge necessary for effective coaches. Eight domains of mathematics coaching knowledge (see Figure 1) were defined as a part of a five-year research project, Examining Mathematics Coaching (EMC), which examined the effects of math coaches' knowledge for coaching on K-8 teachers (Sutton et al., 2011). These researchers note that states and local educational agencies planning to utilize math coaches can use these domains to identify and select coaches as well as determine necessary supports and training for coaches. Much of what is included in the definitions for the eight domains aligns to the National Council of Teachers of Mathematics (NCTM) guiding principles of effective mathematics programs (NCTM, 2014). An effective coach must master a multitude of skills, but the foundation for a math coach must be content expertise. Just as teachers cannot teach what they do not know, math coaches cannot coach teachers without having subject matter expertise to identify pedagogical, instructional, and assessment needs.

Figure 1

Domains of Mathematics Coaching Knowledge

- Assessment
- Communication
- Leadership
- Relationships
- Student Learning
- Teacher Development
- Teacher Learning
- Teacher Practice

> *"Perhaps most interesting to us has been evidence that teaching may require a specialized form of pure subject matter knowledge—"pure" because it is not mixed with knowledge of students or pedagogy and is thus distinct from the pedagogical content knowledge identified by Shulman and his colleagues and "specialized" because it is not needed or used in settings other than mathematics teaching. This uniqueness is what makes this content knowledge special" (Ball et al., 2008, p. 396)*

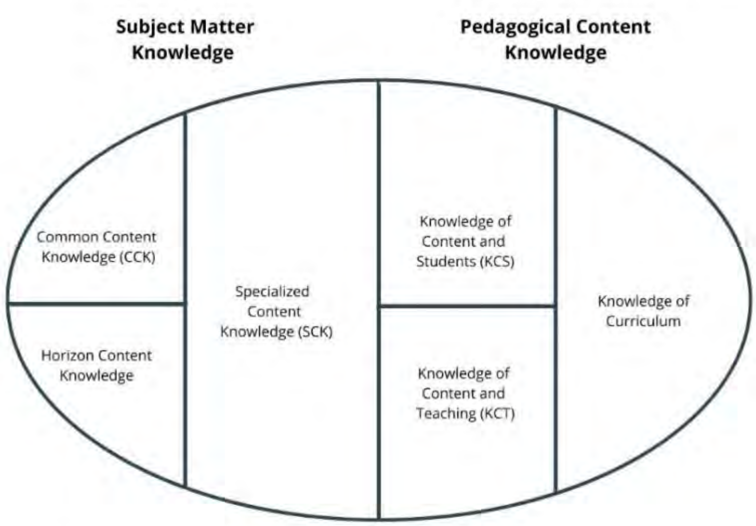
Mathematics Content Knowledge

Content knowledge is critical for math coaches to effectively support teacher and student learning. There is no debate regarding whether or not a math coach must possess strong mathematics content knowledge. The seminal work of Ball et al. (2008) highlights the complex nature of math teaching based on their analysis of math teachers' practice. Their research finds that, "mathematical demands of teaching are substantial. The mathematical knowledge needed for teaching is not less than that needed by other adults. In fact, knowledge for teaching must be detailed in ways unnecessary for everyday functioning. In short, a teacher needs to know more, and different, mathematics—not less” (p. 396). Based on their research, they specified, “mathematical knowledge for teaching” (MKT) as the mathematical knowledge inclusive of common content knowledge (knowledge used in settings other than instructional) and specialized knowledge (unique to teaching) needed to carry out the work of teaching mathematics. Their work goes further to unpack what is unique about MKT, identifying the construct as multidimensional with subdomains that help to illuminate the knowledge required for teaching as they relate to subject matter knowledge and pedagogical content knowledge. Based on their analysis of mathematical demands for teaching, the authors extend the work of Shulman (1986) by further defining subject matter knowledge and pedagogical content knowledge. As a result of their research, Ball et al. created a conceptual map of four domains of MKT that highlight critical intersections between content knowledge and knowledge for teaching, as shown in Figure 2.

- Common content knowledge (CCK): “The mathematical knowledge and skill used in settings other than teaching” (p. 399), referred to as common because it is not necessarily unique to teaching but may be math knowledge that is used in other settings.
- Specialized content knowledge (SCK): the mathematical knowledge necessary and unique to teaching (unlike CCK, it is not necessarily used outside of teaching).
- Knowledge of content and students (KCS): knowing students and knowing about mathematics. It combines “specific mathematical understanding and familiarity with students and their mathematical thinking, including common student understandings and misconceptions about content” (p. 401).
- Knowledge of content and teaching (KCT): knowing about teaching and knowing about mathematics combines “specific mathematical understanding and an understanding of pedagogical issues that affect student learning” (p. 401).

Figure 2

Domains of Mathematical Knowledge for Teaching



Note. This figure was adapted from Ball et al., 2008.

In addition to these four domains, the authors proposed two additional domains of knowledge within their map of MKT. This includes knowledge of content and curriculum based on Shulman's work from 1986 and horizon content knowledge, which is knowledge of "how mathematical topics are related over the span of mathematics included in the curriculum" (p. 403) or a kind of "mathematical 'peripheral vision' needed in teaching, that is, a view of the larger mathematical landscape that teaching requires" (Hill & Ball, 2009, p. 70). The authors note that these two additional categories may overlap with others already specified and will be continuously investigated and refined.

The domains of MKT describe the depth of knowledge math teachers need to be effective. Research suggests there is a correlation between teachers' level of MKT and their practice; teachers with strong MKT indicated a higher quality of mathematical instruction (Hill et al., 2005). Yopp et al. (2019) note that it is important to determine when a teacher might lack specific types of mathematics teaching knowledge in order to inform how a coach might support that teacher to improve mathematics practices and learning. However, that assumption rests on the notion that coaches themselves also have strong MKT to provide effective diagnosis and support. Yopp et al. (2019) state, "to assist teachers in leveraging that knowledge when planning and delivering lessons, coaches must possess that knowledge as well" (p. 9). Rigorous research has pointed to the positive impact that highly trained elementary math specialists (provided numerous courses specifically designed to enhance their content knowledge) have on student achievement (Campbell & Malkus, 2011). However, the authors stress that their findings are relevant only to other contexts in which coaches have extensive content expertise. Given this, one key use of this conceptual map is to inform opportunities to support coaches to develop ALL facets of MKT. Professional learning systems can use the domains of MKT to organize the learnings for both teachers and coaches to more clearly identify and target the focus for developing expertise in math knowledge and skill.

National Shortcomings in Mathematical Content Knowledge

Our American school system erroneously assumes that elementary teachers have the content expertise to teach mathematics at the elementary school level. Extensive research indicates otherwise, as described by Mewborn's (2001) review of literature in which she concludes, "that many elementary teachers do in fact lack a conceptual understanding of the mathematics they are expected to teach" (p. 30). American teachers at the elementary level are weak in mathematics content expertise. Findings from the National Council on Teacher Quality's 2018 review of graduate teacher preparation programs note, "the systematically poor preparation of elementary teachers in mathematics may stand as one of the most staggering weaknesses in teacher preparation, contributing to the chronically low standing of American schoolchildren in mathematics internationally" (Rickenbrode et al., 2018, p. 16). Most pre-service programs typically include one elementary math methods course, which usually focuses on teaching strategies rather than content improvement. Only 13% of undergraduate teacher preparation programs nationally require "no less than one course in the methods of teaching elementary mathematics to young children and a minimum of three courses that cover at least 75% of topics identified by mathematicians as critical. Many programs require no elementary math courses" (Lubell & Putman, 2016, p. 5). As a result, teachers enter the workforce equipped primarily with the mathematics they were taught in school as students themselves. Compounding the problem is the fact that most teachers tend to overrate their content expertise. In a survey conducted with a national sample of elementary school teachers, researchers observed that teachers regard themselves as having adequate or exceptional mathematics knowledge (Hill, 2010). Hill reiterates the fact that teachers cannot provide accurate descriptors of their own level of mathematical knowledge. Thus, in their minds, they know the mathematics extremely well, when in reality, they may not. The problem is compounded up the leadership scale resulting in teachers, coaches, and administrators who do not know what they do not know. This results in shortcomings in math content expertise that tend to fly under the radar and fall short in teacher evaluation systems or in continued professional development.

> *"In order for teachers to have opportunities to learn MKT, those who prepare teachers and provide professional development will themselves need to have adequate support. Better materials, more specific guidance focused on the teaching of MKT, and better design of opportunities to learn from practice are essential" (Hill & Ball, 2009, p. 5.)*

Complexities of K-6 Mathematical Content Expertise

Mathematics in K-6 is not elementary mathematics. The term "elementary" is synonymous with the term "simple," but even at the K-6 level, fundamental mathematics is deeper and more complex than most educators realize. Any reform effort to improve mathematics learning and instruction must have the improvement of content expertise as its primary goal. Using the most effective teaching strategies will accomplish little if the mathematics taught consists of surface-level routines, procedures, and processes in place of deep mathematical knowledge. It makes sense that teachers will not only tend to teach *how* they were taught but also tend to teach *what* they were taught, which often consists of the shallow rules, algorithms, and procedures previously mentioned.

Weak content expertise can manifest in the quality of instruction and assessment. Without a deep conceptual understanding of mathematics, a teacher may not recognize such things as the errors in student thinking, the validity of a student's invented strategy, or the recognition of how current knowledge of one topic, such as operations with whole numbers, may create obstacles in a future topic, such as operations with fractions. Another example would be to recognize and anticipate how prolonged use of one-step word problems might hinder students' ability to master multi-step word problems in the future. Likewise, with assessment, a teacher will be limited on the depth and types of questions to ask as part of formative assessment or the types of problems or activities that will actually expose the depth of student understanding. For these same reasons, the content knowledge of the coaches is paramount. In fact, researchers posit that implementing coaching as a policy intervention lacks promise if the selection of coaches lacks an assessment of coaches' level of content knowledge (Hill, 2010).

The Power and Role of Conceptual Understanding in Coaching

The content expertise of elementary school coaches is a necessary foundation and should be an integral component of any program focused on supporting fundamental mathematics instruction. Research underscores that effective teaching and learning "requires teachers to have a deep understanding of the mathematical content that they are expected to teach and a clear view of how student learning of that mathematics develops and progresses across grade levels… [and that teachers should also] be skilled at using instructional practices that are effective in developing mathematics learning for all students" (NCTM, 2014, p. 4).

The section below illustrates the power and role of conceptual knowledge of math, using key examples of what superlative content expertise, or strong MKT, will do for elementary coaches and their teachers. The examples are organized into three sections: Teacher Development, Learning, and Practice; Student Learning and Assessment; and Communication, Leadership, and Relationships. These sections are related to the domains of mathematics coaching knowledge defined by Sutton et al. (2011) and MKT, highlighting the intersection between the knowledge of coaching and knowledge of math content that are critical for effective coaching. After each section, we provide key concepts aligned to evidence-based practices in teaching and learning for mathematics. These key concepts exemplify activities that should be part of a broader system of professional learning for elementary mathematics coaches to ensure they have the necessary MKT to support teacher and student learning.

Teacher Development, Learning, and Practice

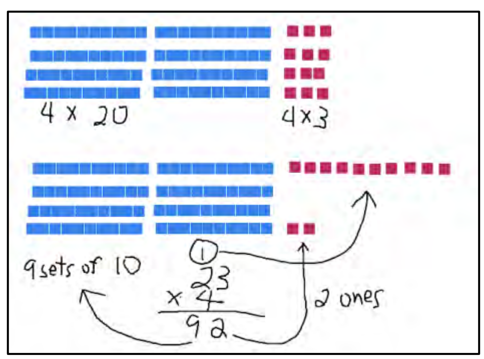
It is imperative for a coach to develop a conceptual understanding, make connections to instructional methods, have a strong knowledge of fundamental mathematics, as well as standards, in order to support teachers to set students up for future success in mathematics. To provide these supports, a coach must combine MKT (specialized content knowledge and knowledge of content and teaching) with knowledge of "various models of teacher stages of development, adult change, and the continuum of learning that teachers often experience in exploring content knowledge, pedagogy, beliefs, and management" (Sutton et al., 2011). Aguilar (2013) notes, "deepening a content coach's knowledge of specific instructional practices and curriculum is important for content area coaches, but they also need to learn coaching skills"

(p. 268). Coaches also need to have a deep understanding of effective teaching practices and strategies to support teachers (Knight, 2015). Coaches must be capable of supporting teacher learning through reflective practice, goal setting, as well as recognizing how practices and resources translate into teacher actions for effective teaching and learning (Sutton et al., 2011). **The intersection of MKT with the knowledge of coaching related to teacher development, learning, and practice is highlighted in the following examples:**

Key Concept: *<u>True conceptual understanding helps coaches recognize connections between or among seemingly unrelated ideas.</u>* Shulman (1986) argues, "to think properly about content knowledge requires going beyond knowledge of the facts or concepts of a domain. It requires understanding the structures of the subject matter. Teachers must not only be capable of defining for students the accepted truths in a domain. They must also be able to explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions, both within the discipline and without, both in theory and in practice" (p. 9). Refer to Figure 3 involving the concrete modeling of 4 x 23. The odds are that most teachers will stop with the first example and do an area model with partial products of 80 + 12 = 92. However, an astute coach will recognize that if one combines 10 of the single units to form another block of 10, this will have the effect of visually connecting what happens with the standard algorithm, thus bridging and connecting the concrete with the abstract, something that teachers often neglect to do.

Figure 3

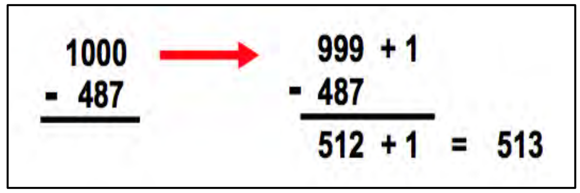
Example of Multiplication Problem



Key Concept: *An effective coach needs to ensure that teachers understand that the bulk of fundamental mathematics involves taking things (numbers, geometric shapes, etc.) apart, putting things together, or rearranging them.* When coaches address teacher content knowledge in mathematics, this can include both a teacher's own mathematics content knowledge and a teacher's understanding of students' mathematics content knowledge (Sutton et al., 2011). This will enable them and their students to be less limited in their methods and strategies. As an example, subtracting from a multiple of 1000 can be problematic because of the regrouping involved. A coach with strong expertise will identify or develop a valid alternative process that utilizes fundamental mathematical properties, as shown in Figure 4. Coaches may often need to explicitly illustrate important content for teachers in order to directly improve teacher content knowledge (West and Staub 2003).

Figure 4

Example of a subtraction problem.



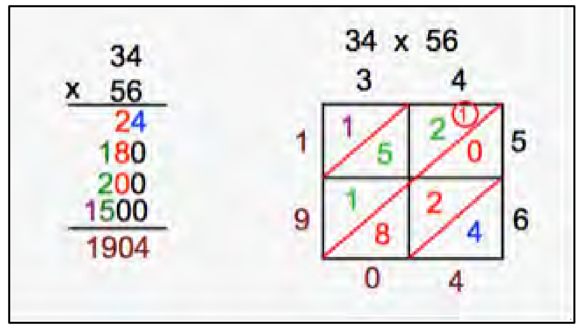
Key Concept: *Coaches must be cognizant of teachers' reliance on textbooks, "canned" curricula, and especially on the state content standards. Coaches must understand that mathematics standards are insufficient.* Although standards are a useful tool, there are multiple shortcomings that the coach must help teachers address and overcome. Since standards are more skills-based and focus on what students must know for accountability exams, it is up to the teachers to teach the real mathematics, the *whats* (i.e., definitions of concepts), and the *whys* regarding processes that help students see why they work the way that they do. Again, it is incumbent upon the teachers' content knowledge to fill in those gaps. Standards do not address critical obstacles or transitions, such as how to transition from additive reasoning to proportional reasoning. The standards do not illustrate how a teacher can lay an informal foundation for a topic that will be taught in a later grade. Improving the content expertise of teachers under a coach's tutelage will assist them in addressing and overcoming these challenges and shortcomings of state standards.

"Teachers without mathematical knowledge cannot provide explanations, justifications, or make careful use of representations. But just as troubling, and perhaps even more so, is our observation that a lack of MKT leaves teachers unable to navigate common and necessary elements of even very basic instruction"(Hill et al., 2008, p. 468)

Key Concept: *One of the problems with mathematics education is that there is a proliferation of new exciting methods that really are neither new nor exciting.* For example, the lattice method (Figure 5) of multiplication is not a different method at all. It is simply a different way to exhibit the partial products in multi-digit multiplication; it is just a variation of the distributive property. An astute coach will recognize that, although using the lattice method is satisfactory, it is essential that the teacher and students understand how and why that method works and how it connects to other ways to represent the partial products. Another inherent problem in math education is the tendency to give different names to the same method or process. For example, the process of partial quotients is also referred to as the "chunking method" or "the Big 7 method." A good coach will not only be cognizant of this fact but will also recognize that in this particular example, this process of partial quotients is actually the application of the distributive property to division, a property that many associate with only multiplication.

Figure 5

Example of the Lattice Method



> *"Productive mathematics assessment is a process that is coherently aligned with learning goals and makes deliberate use of the data gathered as evidence of learning and provides guidance for next instructional steps and programmatic decision making" (NCTM, 2014).*

Student Learning and Assessment

A math coach must be proficient in student learning in order to create and manage mathematical learning environments, analyze student thinking and conduct mathematical error analysis, and provide instructional formats and strategies to engage students in challenging and meaningful mathematics problems and tasks (Sutton et al., 2011). Mudzimiri et al. (2014) note, "a content-focused coach makes student mathematical learning the central focus of coaching sessions…Whenever possible, the coach brings evidence about student learning such as student comments, examples of student thinking, student assessment data, and samples of student work to the coaching session" (p. 5). Coaches need to be able to make connections and see relationships between mathematical practices and concepts across multiple grade levels in order to provide the foundational knowledge needed for students to progress. This is true for teaching and learning practices, as well as when designing and utilizing assessments to collect formative and summative data to drive instructional decision-making and professional development. Part of a coach's role is to facilitate the use of data to help educators understand how data can be analyzed to make informed decisions about improving instruction and inform, change, and improve student outcomes (Barr et al., 2003, Sweeney & Harris, 2020). They must also understand "formative assessment as a research-affirmed process for student and adult learning [and that it] serves as a catalyst for successful mathematics content implementation" (as cited in Kanold & Larson, 2015, p. 3). Therefore, coaches must have mathematics coaching knowledge of both student learning and assessment (Sutton et al., 2011) as well as MKT (specialized content knowledge, knowledge of content and students, and horizon content knowledge) to build the capacity of their teachers to apply a conceptual understanding of mathematics to best support student learning. Through structured observation, a coach must be able to diagnose teachers' current understanding of assessment and teachers' capacity to apply student assessment data to make informed instructional decisions to design individualized and group professional development. Coaches must also be well-versed in the selection, adaptation, and alignment of curriculum with assessment design to improve the accuracy of student data and the instructional decisions made on that data (Sutton et al., 2011). **The intersection of MKT and the knowledge of coaching related to student learning and assessment is highlighted in the following examples:**

Key Concept: *Mathematics coaches must use their deep mathematics understanding to develop their observation skills regarding student thinking.* Part of a coach's repertoire must include strategies to reveal what a student is thinking during their learning process. All too often, a teacher only has a jumble of written computation to judge how and why a student approached the solution to a problem. This is insufficient, and teachers typically have to resort to oral questioning to determine a student's line of reasoning. This is effective but time-consuming. The coach should be able to suggest written approaches that will reveal that information. For example, suppose that the task is to find how many hours there are in 13 days. How does a student end up with 13 x 24? A typical response would be with the oral explanation, "Well, it just came to me." One strategy from a coach would be to insist students write out their thought process, which would reveal information beyond written computation. In this case the progression might look as follows: 13 days = 13 sets of 1 day each = 13 sets of 24 hours each = 13 x 24. Such an approach would be helpful to both teachers and students.

Key Concept: *Improved conceptual understanding can boost a coach's ability to help teachers improve their assessment.* We tend to gauge student understanding with quizzes/tests that consist of 10-20 problems that are limited to one classroom or one grade level. A content expert can suggest one simple question/problem that should be given across several grade levels to measure the progress of a "big idea."

- **Example 3a.** Give the problem 8 + 7 = ___ + 5 across several grade levels in the school as a warmup activity. Teachers can get together and examine student work (a practice often neglected that a math coach can support via assessments like this one). If many students answer incorrectly with 15, that indicates that students have lost the real meaning of the = sign, and instructional changes must be made schoolwide to address that.
- **Example 3b.** Utilize the following problem in a manner similar to example a). 26 x 24 is more than 25 x 24. How much more? Please show/explain how you determined your answer.

This, too, should be given across several grade levels in the same school as a warmup activity. Teachers can get together and examine student work. In this scenario, if students arrive at their answer by multiplying and then finding the difference between the two products, there is a real issue with students' understanding of the concept of multiplication because they do not recognize that you just have one more set of 24, which would be similar to comparing 26 cats to 25 cats. This is also a great example of a "false positive" where a student can know how to successfully apply a concept yet have no true understanding of the concept itself.

Key Concept: *<u>Deep understanding of mathematics across multiple grade levels will help coaches recognize potential pitfalls that may escape the eyes of most teachers.</u>* As an example, breaking down an addition problem into partial sums is an effective approach. However, in the Figure 6 example, an experienced coach will recognize a problem with the symbolism utilized. Note the pattern of the partial sums where the partial sums "slant" from left to right, which is contrary to what happens in later instruction with the standard algorithm for multiplication. The coach could suggest a better way to express the same idea of partial sums, but do it symbolically in a way that does not cause any future confusion and reinforces the idea of place value (which must be continually reinforced) as indicated in Figure 7. Coaches can engage teachers in "disciplinary investigations to challenge teachers' own specialized disciplinary knowledge and their view of disciplinary norms as they engage in rich explorations of mathematics... to support teachers to engage with the content in deeper ways than would be required if the goal were to solve tasks by developing a single strategy" (Gibbons & Cobb, 2017, p. 417).

Figure 6

Example of Addition Problem - Problematic

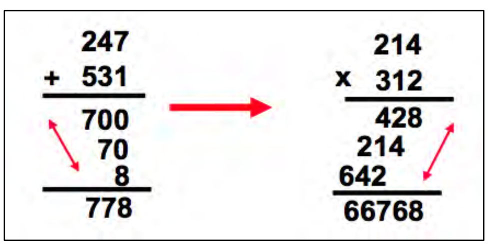
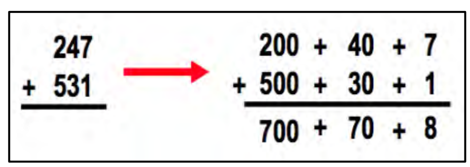


Figure 7

Example of Addition Problem - Symbolic



Communication, Leadership, and Relationships

Coaches are often thought of as "a knowledgeable colleague with a deep understanding of mathematics and how students learn, as well as pedagogical expertise, to serve as an on-site resource and leader for teachers" (Campbell & Malkus, 2011, p. 431). Research supports the notion that professionalism in mathematics education encourages "a tangible sense of the professional imperative to grow personally and collectively, to hold one another accountable for this growth… [and to] cultivate and support a culture of professional collaboration and continual improvement that is driven by an abiding sense of interdependence and collective responsibility" (NCTM, 2014, p. 5). This emphasizes the importance of communication, leadership, and relationships in mathematical coaching best practices and their connection to professionalism when making decisions around procuring proper curriculum and guiding effective teaching and learning practices. With this in mind, it is crucial for a coach to understand that the "coaching relationship is grounded in content and how to use that relationship to support self-directedness in teachers" (Sutton et al., 2011). Coaches must have strong MKT to recognize and communicate that the shortcomings in knowledge or practice are within the system and not within the teachers. Coaches can support the development of teachers serving as a "mechanism to support change, to foster implementation, to promote reflection, to applaud efforts and to challenge further growth" (Campbell, 1996, p. 462). They must also encourage a growth mindset for learning mathematics for teachers and students, and they must understand the implications and components of effective observation and feedback practices specific to mathematics. With these understandings, coaches must also support teachers by addressing challenges regarding instructional decision-making and have a strong ability to "advocate for, negotiate with, and influence others" while keeping in mind how "autonomy, issues with authority, and socio-cultural aspects of class, race, and gender for students and teachers influence relationships and influence perceptions and models of help and authority" (Sutton et al., 2011). Effective coaching is based on the coach's ability to identify and react to gaps, misconceptions, and cultural and systemic barriers in order to successfully communicate and influence administrator understanding, teacher practices, and student learning outcomes. **The intersection of MKT and the knowledge of coaching related to communication, leadership, and relationships is highlighted in the following examples:**

Key Concept: *The weak knowledge of mathematics is a shortcoming of the system, not of the teachers, and it is important that coaches communicate that to their teachers.* Establishing trust between coaches and teachers is essential, so it is important that the aforementioned gets communicated so that teachers do not feel slighted. Campbell (1996) notes, " instructional change is not easy; it is demanding, threatening, and risky…Individual teachers must reconstruct their own teaching style, in light of their increased knowledge of mathematics and their emerging understanding of children learn mathematics" (p. 454). Mathematics coaches serve in a leadership role in schools to provide professional development for teachers on mathematical content, pedagogy, and curriculum (Campbell & Malkus, 2011).

> *"Students who do not engage in conceptual thinking and instead approach mathematics as a list of rules to remember are not engaging in the critical process of compression, so their brain is unable to organize and file away ideas; instead, it struggles to hold onto a long list of methods and rules. This is why it is so important to help students approach mathematics conceptually at all times. Approaching mathematics conceptually is the essence of what I describe as a mathematical mindset" (Boaler, 2016).*

Key Concept: There are two critical messages that coaches must impart to their teachers. *The first is for a teacher to constantly deliver the message that contradicts the American cultural belief that only a select few can master mathematics.* This must be countered with constant reminders to students that anyone can learn math, that it is not a matter of ability, but rather more about confidence, effort, and persistence. Research on the brain indicates that the difference between successful and unsuccessful students is less about the content they learn and more about their mindsets; this includes the mindsets of the adults who model that thinking (Boaler, 2016, p. 55). *The second message is that the teacher's job is to teach mathematics, not efficiency.* There are misconceptions that students should be shown perfect examples. This trains them to exercise their thinking in isolation, practicing the same methods over and over again, which sets them up to fail when faced with applying math in the real world (Boaler, 2016, p. 55). All too often, the focus is the learning of standard algorithms, which are more about efficiency, not conceptual understanding of mathematics. An algorithm should be taught as the last step and only after students understand everything about the processes and concepts on which the algorithm is based. Establishing both those conditions will greatly enhance the effectiveness of any mathematics classroom.

Key Concept: *Classroom observation is a difficult skill to master. The preparation of coaches in this area is paramount to their success.* Coaches need all the help and experience possible regarding actual classroom observation. Real observations or virtual classroom videos where coaches observe a lesson and then discuss what they saw should be an integral part of their preparation. Traditionally, classroom observation protocols or instruments contain little information on the observation of teachers' content knowledge. It is easy for coaches to focus on the teaching strategies (how things are taught) and neglect the content (what was taught). In addition, the observer's content expertise is critical here because an observer with little content expertise might rate a lesson with a lot of "bells and whistles" highly, whereas a content expert might rate that same lesson much lower if the mathematics taught in the lesson was shallow or incomplete. Recognizing good teaching is not the same as recognizing good learning.

Key Concept: *Just as critical as classroom observation skills is the ability to conduct effective debriefs with teachers after the observations.* In addition to the judgment of teaching methods/strategies, the coach must be prepared to comment and make suggestions regarding the mathematics taught. As part of the partnership dialogue (Guskey, 2000), a coach should be skilled at asking probing questions during debriefs to discuss the impact of instructional practice, student learning, coaching support, and the next steps. As an example, the coach must be prepared to successfully diagnose why students are struggling with multiplication of mixed numbers and the role of the teacher's depth of knowledge on the topic. In this situation, it would be key if the coach determines that the teacher only knows the standard algorithm because that lack of expertise severely limits the alternate approaches/methods that the teacher could employ. The debriefing skill comes into play when tactfully addressing content shortcomings.

In sum, an excellent elementary mathematics program requires that all students have access to effective teaching and learning, which is obtained through building the capacity for teaching mathematics. Yopp et al. note, "Underlying this construct is the notion that there exists mathematics knowledge that is unique to the profession of mathematics teaching. To assist teachers in leveraging that knowledge when planning and delivering lessons, coaches must possess that knowledge as well" (Yopp et al., 2019). The key concepts described above are backed by research that clearly indicates that a focus on math content is critical to support the understanding of how mathematics learning develops and progresses across grade levels (NCTM, 2014). The examples in the key concepts can inform the development of a training system to help ensure that math coaches have a strong conceptual understanding of math as well as the ability to use this content knowledge to effectively provide teacher development, reflective practices, and support for the implementation of effective teaching, which will drive the proper use of assessments and data to inform decision-making for teacher and student learning.

"To understand mathematics at a deeper conceptual level, students need to develop a strong foundation by learning to define basic concepts, make connections, and unearth relationships; an understanding of the language, symbolism, and visual representation of mathematics is integral to this process" (Molina, 2012, p. 9).

Critical Considerations

Data-Driven Coaching

The systematic use of data to assess student learning and drive instructional decision-making has long been accepted as critical to supporting growth and achievement for all students. Hamilton et al. (2009) state, "Armed with data and the means to harness the information data can provide, educators can make instructional changes aimed at improving student achievement" (p. 5).

However, the authors also note, "the skills that educators need in order to use data to identify achievement problems and develop instructional solutions are complex. To enhance data-literacy and data-use skills in a way that is consistent with school goals, it is essential that schools and districts provide ongoing professional development opportunities for administrators, principals, teachers, and classroom support specialists" (p. 35). One way to support a culture of data use is to ensure coaches are also armed with the knowledge of data as a means to drive a decision-making process for supporting teachers.

An effective coach needs to know how to diagnose teachers' needs, how data and assessment of student thinking inform instruction and work with teachers, and how to assess and use teacher content knowledge and pedagogical content knowledge to inform, grow, and support teachers (Sutton, 2011). One state educational agency, Alabama, recently created a statewide coaching framework that describes coaching responsibilities associated with effective use of data. The Alabama Coaching Framework (Alabama State Department of Education, 2020) notes, "effective coaches collect, use, and analyze data to identify specific and targeted issues for continuous instructional and learning improvement. Coaching through the use of data creates coherence within a district and/or school, as well as provides the evidence for focusing more intentionally on identified areas of need versus personal opinions about areas of need" (as cited in Annenberg Institute for School Reform, 2004, p. 31). The Alabama Coaching Framework describes the important role of embedding the collection and analysis of teacher data (e.g., observation, diagnostic, fidelity) and student data from multiple assessment sources (screening, progress monitoring, diagnostic, and outcome) within the coaching cycle as a means to facilitate continuous improvement. A coach's use of data in their coaching practice is important to best support teachers, and it can increase the supported teachers' use of data for their own instructional decision-making (Shoemaker, 2014). It is critical to consider the knowledge and skills coaches bring to their practice regarding data use. A math coach's ability to effectively use data cannot be dissociated from their level of content knowledge or MKT. Using data to drive math coaching depends on strong content knowledge and knowledge for coaching.

Systems for Professional Learning

The Alabama Coaching Framework (ALSDE, 2020) notes that training for coaches should "expand their knowledge of professional learning in areas such as content, instruction, assessments, adult learning, core resources, and intervention. Content knowledge should include understanding the standards, the vertical progression, and the rigorous application of learning the standards in the classroom" (p. 17). Based on what we know of the critical importance of both MKT and the domains of math coaching knowledge necessary for effective coaching, how do we build a system of professional learning to support math coaches?

An effective math coaching model must address gaps in the mathematics content expertise of math coaches. Thus it is imperative to build organizational and system capacity to develop effective coaches who possess the optimal level of content expertise and skillsets to positively impact teacher practice. Suppose the pool of candidates encompassing high levels of content expertise is limited. In that case, a robust system must be developed and effectively implemented to support teacher growth, professional learning, and a continuous cycle of capacity building.

In addition to content expertise, coaches must understand how to drive implementation practices not only in a one-to-one coaching session but within school ecosystems through fostering teacher teams that function to build the teachers' collective capacity both vertically and at grade level. Coaches must have the knowledge and skills to address the gaps "beyond the product of standards themselves by providing…[the] guidance, support, and leadership tools necessary to help the adults in their system to achieve mathematics program greatness within the context of higher levels of demonstrated student learning and performance" (Kanold & Larson, 2015, p. 1). Research suggests that a strong professional development system for coaches and teachers alike should provide strategic, ongoing, and sustained rather than episodic professional development. It should be collective rather than individualistic. It should be job-embedded so that the learning is at the point of use and should encompass results-oriented activities that have a clear link to improved student achievement (Kanold & Larson, 2015). In conclusion, it is crucial to take inventory, assess, and analyze the current system to ensure that the coaches are provided professional learning and supports to improve upon their content knowledge, become competent in the collection and application of data for decision-making, and become experts in the facilitation of evidence-based best practices, specifically in mathematics coaching, in order to foster teacher growth and improve student outcomes.

The mathematics coach is a change agent, but it is incumbent upon the professional learning system to determine the scope of those responsibilities and ensure that the necessary preparation for those expectations is included in the system. Will a coach be responsible for one teacher or a whole department? In most scenarios, a coach might be tasked with improving mathematics instruction and the performance of an entire school. With that assumption, part of the coach's preparation must include how to effectively work with an entire staff rather than just one or two teachers. Implementation of what is learned from the mathematics coach is critical in order to build capacity. Coaches must assess the level, or fidelity, of implementation of their teachings and assure that the effort goes beyond building individual capacity to ensure sustainability. That school or district reaps little benefit if trained teachers leave and take new knowledge with them. "Teacher movement out of schools and out of teaching creates costs for the schools they leave behind. Estimates exceed $20,000 to replace each teacher who leaves an urban school district. Most importantly, high turnover rates reduce achievement for students whose classrooms are directly affected, as well as for other students in the school" (Carver-Thomas & Darling-Hammond, 2017, p. V).

Coaches can incorporate their efforts into a true professional learning system so that the learning becomes embedded in the culture and routines of the school or district. Building school- or system-level capacity should involve group processes such as how a department can function to collectively examine student work or how the school can incorporate strategies such as lesson study to develop lessons for the entire math department rather than for individual teachers. Carver-Thomas and Darling- Hammond recommend "to stem teacher turnover, federal, state, and district policymakers should consider improving the key factors associated with teacher turnover: compensation, teacher preparation and support, and school leadership" (Carver-Thomas & Darling-Hammond, 2017, p. vi). By building capacity at the system level, the district is closer to achieving sustainability and mitigating turnover.

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Additional Resources

There is a considerable amount of research and information available regarding the mathematics content expertise of elementary teachers. Some of the better-known researchers in this arena are Deborah Ball, Heather Hill, Lee Shulman, Steve Schilling, Liping Ma, John Van de Walle, and Roger Howe. A particularly relevant area of math expertise revolves around the work of Deborah Ball and Heather Hill at the University of Michigan. As described in the overview, they have coined the phrase Mathematics Knowledge for Teaching (MKT), which refers to the type and depth of mathematics that a teacher needs in order to effectively teach fundamental mathematics. Information on MKT should be of use in examining the necessary knowledge of mathematics coaches. Researchers have released examples of MKT in sample survey items that measure mathematical knowledge.[1]

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[1] http://www.umich.edu/~lmtweb/files/lmt_sample_items.pdf

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