MOVEMENT MATTERS

HOW EMBODIED COGNITION INFORMS TEACHING AND LEARNING

EDITED BY

SHEILA L. MACRINE AND JENNIFER M. B. FUGATE

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9 Groups That Move Together, Prove Together: Collaborative Gestures and Gesture Attitudes among Teachers Performing Embodied Geometry

Kelsey E. Schenck, Candace Walkington, and Mitchell J. Nathan

Mathematics is a particularly notable domain in which to understand the role of body movement for improving reasoning, instruction, and learning. One reason is that mathematics ideas are often expressed and taught through disembodied formalisms—diagrams and symbols that are culturally designed to be abstract, amodal, and arbitrary (Glenberg et al. 2004)—so that these ideas are regarded as objective and universal. This stems from a Cartesian view of knowledge that separates mental experiences from physical experiences and ways of knowing (Lakoff & Núñez, 2000; also called the "romance of mathematics," p. xv). This Cartesian "duality" carries forth to the various fields touched by mathematics that also strive for objectivity and universality—topics as vast and diverse as the physical and social sciences, business, civics, and the arts. There is a growing appreciation, however, that for effective education, mathematics must be meaningful to novices and that this can occur by *grounding* the ideas and notations to learners' physical experiences and ways of knowing (Nathan, 2012).

Grounding can occur when an abstract idea is given a concrete perceptual referent so that it is more readily understood (Goldstone & Son, 2005). One way that ideas can become grounded is through gesture. Gestures are spontaneous or purposeful movements of the body that often accompany speech and serve as a way to convey ideas or add emphasis to language as well as mathematics (Goldin-Meadow, 2005).

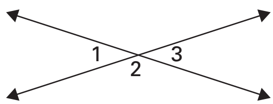
Gestures can act as a grounding mechanism by indexing symbols and words to objects and events, and by manifesting mental simulations of abstract ideas using sensorimotor processes (Alibali & Nathan, 2012). The grounding of novel, abstract ideas and notational systems through gesture, action, and material referents is part of the emerging framework of grounded and embodied cognition. *Grounded cognition* is a general framework that posits that formal notational symbol systems and the intellectual behavior are "typically grounded in multiple ways, including simulations, situated action, and, on occasion, bodily states" (Barsalou, 2008, p. 619).

Nathan (2014) positioned mathematics learning at the intersection of three influences: (1) content, such as numbers and operations, algebra, and geometry; (2) disciplinary practices, such as executing procedures and forming proofs; and (3) the psychological processes, such as spatial imagery and logical deduction, for engaging in disciplinary practices with specific content. The learning experiences are quite different whether from a Cartesian or embodied frame. Consider two experiences for fostering geometric reasoning (figures 9.1 and 9.2).

Figure 9.1, a traditional two-column geometry proof, is a common display from which students (and teachers) are expected to gain an understanding of how to prove that opposing angles formed by intersecting, coplanar lines are always equal. The vertical angles theorem (adapted from proposition 15 of Euclid's *Elements, Book 1*) is widely applied throughout geometry, art, and engineering. The proof poses many obstacles to understanding the content and disciplinary practices, however. The diagram is rich with highly formalized terms, such as $\angle 1$ and $m\angle 1$. Unstated assumptions bound, such as $m\angle 1$ and $m\angle 2$, can be arithmetically added—they are each quantities—but $\angle 1$ and $\angle 2$ are labels that cannot be combined. Another is that operations such as those performed in line 4, which are presented as static, declarative statements—here, the transitive property of equality—hide the processes that enact these

Given: $\angle 1$ and $\angle 3$ are vertical angles.

Prove: $\angle 1 \cong \angle 3$



Statements	**Reasons**
1. $\angle 1$ and $\angle 3$ are vertical angles.	**1.** Given.
2. $\angle 1$ and $\angle 2$ are supplementary $\angle 2$ and $\angle 3$ are supplementary.	**2.** Angles that form a linear pair are supplementary.
3. $m\angle 1 + m\angle 2 = 180$ $m\angle 2 + m\angle 3 = 180$	**3.** The sum of the measures of supplementary angles is 180.
4. $m\angle 1 + \boldsymbol{m\angle 2} = \boldsymbol{m\angle 2} + m\angle 3$	**4.** Transitive property of equality
5. $m\angle 1 = m\angle 3$	**5.** Subtraction property of equality
6. $\angle 1 \cong \angle 3$	**6.** Angles with the same measure are congruent.

Figure 9.1
Two-column proof for the vertical angles theorem.

operations. Most students experience geometry as an amodal topic, disconnected from the sensory systems of the body. It is little wonder that for many students high school geometry is not only poorly understood but an obstacle to advanced studies in math as well (Szydlik et al., 2016).

Figure 9.2 investigates similar content (geometry) and disciplinary practice (proof) through an embodied approach. Rather than static propositions that presuppose logical deduction, we observe psychological processes using body movement and extended social cognition in the form of collaborative gestures to ground the mathematical ideas (Walkington et al., 2019). Instead of a two-column proof, these teachers are engaged in a construction of *transformational proof* (Harel & Sowder, 1998), in which universal claims are investigated using logic in addition to operating directly on the mathematical objects themselves to establish their generality.

Embodied approaches emphasize meaning-making over matching to disciplinary practices. Several scholars have shown that student learning is enhanced when teachers adopt appropriate instructional gestures in their practices (e.g., Alibali et al., 2013; Cook et al., 2008). Unfortunately, teachers and curriculum developers do little to embrace embodied approaches; teachers often exhibit naïve views about the role of the body in mathematical thinking and teaching

Figure 9.2
Forming and transforming mathematical objects collaboratively. Investigating the *inscribed angle* conjecture, “the measure of the central angle of a circle is twice the measure of any inscribed angle intersecting the same two end points on the circumference of the circle.”

(Walkington, 2019). As one teacher put it when asked how the body can be used in math learning, "I haven't really thought about this . . . I assume some students are kinesthetic learners, so movement can help with memory. I also think movement throughout the day helps students stay active and awake." Another reports, "They can use their fingers to count, their feet for measurement, their hands to use manipulatives and draw picture stories." Accordingly, commercial programs such as *Action Based Learning Lab* (https://www.youthfit.com/abl), *MATHS DANCE* (http://www.mathsdance.com), and *Math in Your Feet* (Rosenfeld, 2016) promise "optimal learning" using "brain research" to improve math teaching and learning. As inspiring as these body-based interventions may sound, there is a dearth of rigorous, empirical evidence of their effects on learning and teaching. Few resources for teacher professional development exist that communicate effective strategies for adopting embodied approaches for the teaching and learning of mathematics.

There is a lack of solid research for understanding when and how teachers will adopt embodied teaching practices. Like many new educational practices, we recognize that widespread adoption of embodied instructional practices that use gesture and movement will depend on more than research showing their benefits in laboratory and classroom studies. For teachers to take up new practices, such as effective use of gestures for learning and instruction, the new practices must be presented in ways that are commensurate with teachers' beliefs about learning and instruction and the new practices of interest (Putnam & Borko, 2000). Professional development designers must also understand the role of teachers' content knowledge, pedagogical content knowledge, and teachers' attitudes toward mathematics (Hill et al., 2008).

These needs are discussed in the context of an illustrative case we present in this chapter. In the context of this case, we discuss the relationships between teachers' attitudes about instructional gestures and their actual gesture usage while solving problems. We also discuss how teachers' gesture use during mathematical reasoning is influenced by the collaborative context and describe how gesture production predicts the quality of one's mathematics arguments. Together, these elements form the necessary groundwork for informing future teacher professional development experiences that can bring embodied mathematics practices to scale.

Theoretical Background

The theory of gesture as simulated action (GSA) (Hostetter & Alibali, 2008) posits that gestures arise during speaking when premotor activation, formed in response to motor or perceptual imagery, is activated beyond a speaker's current gesture threshold. This threshold can vary depending on factors such as the current task characteristics (e.g., spatial imagery), individual differences (e.g., prior knowledge), and situational considerations (e.g., instructional context). Hostetter and Alibali (2019) review the evidence that gesture threshold is influenced by cognitive skills, personality, and culture as well as the perceived importance of the information being communicated. They speculate that beliefs about gesture (e.g., whether it is polite) may also influence gestural tendency. GSA is not an account of instruction: therefore, from our perspective, the role of social context and beliefs about the influence of gestures on learning is underspecified in the current theory.

Teachers often use gestures during mathematics instruction (e.g., Alibali & Nathan, 2012; Valenzeno et al., 2003). Teachers can use pointing gestures to indicate different aspects of a diagram or call attention to physical objects and their properties, beat gestures to emphasize particular words or phrases, and representational gestures to directly model mathematical objects, shapes, or relationships using their hands. Studies suggest that teachers use gestures to provide scaffolding (Alibali & Nathan, 2007), and that student learning can benefit when teachers gesture (Valenzeno et al., 2003; Goldin-Meadow et al., 1999). A substantial body of empirical research shows that teachers can modulate their use of gestures to foster learning gains (e.g., Nemirovsky & Ferrara, 2009; Pier et al., 2014; Sinclair, 2005). Students also use gestures to aid their mathematics learning (Alibali & Nathan, 2012; Rasmussen et al., 2004), and gesture use is sometimes correlated with more cogent mathematical reasoning (Cook & Goldin-Meadow, 2006; Goldin-Meadow, 2005; Nathan et al., 2020).

In the realm of education, two important qualities of gestures have emerged. The first is how gestures provide information that is redundant (matched) or complementary (mismatched) to the accompanying speech (Church & Goldin-Meadow, 1986). Pedagogically, children and adults notice information uniquely expressed with mismatched gestures (Kelly & Church, 1997), and learning can benefit more from instruction with gesture-speech mismatches compared with instruction with matched gestures or no co-speech gestures (Singer & Goldin-Meadow, 2005). The second quality is the conditions under which teachers engage in *collaborative gestures,* defined as communicative movements that are physically and semantically co-constructed by multiple interlocutors during social learning interactions in service of learning and instruction. Specifically, collaborative gestures build off the gestures of interactional partners (Walkington et al., 2019).

The illustrative case we present examines teachers' use of gestures during collaborative proofs about geometric conjectures in relation to their attitudes about the role of gestures for learning. Proof is a ripe area for investigation, as it is “a richly embodied practice that involves inscribing and manipulating notations, interacting with those notations through speech and gesture, and using the body to enact the meanings of mathematical ideas” (Marghetis et al., 2014, p. 243). With this chapter, we seek to address the following questions: (1) How are teachers’ gesture behaviors during proof activities associated with their attitudes and beliefs about the role of gesture in learning? (2) When participating in groups, how are teachers’ gesture behaviors associated with the number of collaborators and gesture usage by collaborators? (3) Does group-level collaborative gesture behavior correlate with quality of mathematical reasoning?

To answer these questions about teachers’ use of gestures, we present data from a study with fifty-three preservice and in-service teachers enrolled in a variety of math education courses. Of these participants, 62.3 percent were in-service teachers.

Additionally, 41.5 percent of participants indicated they teach or plan to teach elementary school (grades K-5), 34.0 percent of participants indicated middle school (grades six to eight), and 24.5 percent of participants indicated high school (grades nine to twelve). More detail about the participants and methodology can be found in Walkington et al. (2019). Teachers were arranged in groups to play a video game, *The Hidden Village,* which was designed to support learners’ embodied approaches to proving and disproving middle and high school geometry conjectures (Nathan & Walkington, 2017) (figure 9.3). During the game, the teachers collaboratively produced proofs for up to eight mathematical conjectures. We video-recorded teachers, and we coded both their gestures and the accuracy of the proofs they produced during game play, with each instance of a teacher group proving one conjecture being considered separately.

One important consideration when looking at gestures during mathematical problem-solving is whether gestures are *individual* (i.e., the gesturer made a gesture that was not triggered by or related to the gestures of others) or *collaborative* (i.e., the gesture was spurred by the gestures of others). Collaborative gestures can represent a potentially powerful form of embodied mathematical reasoning. We also determined whether the teachers’ proofs were correct by determining whether the proof (1) was generalizable and held for all cases under consideration; (2) utilized logical inference, progressing through an inferentially sound chain of reasoning, where conclusions are drawn from valid premises; and (3) exhibited operational thought, where the prover progresses systematically through a goal structure, anticipating the outcomes of the proposed transformations (Harel & Sowder, 1998).

Finally, we initially gave all the teachers a survey that assessed their beliefs about gesture, the Teacher Attitudes About Gesture for Learning and Instruction (TAGLI) survey (Nathan et al., 2019). This survey assesses whether

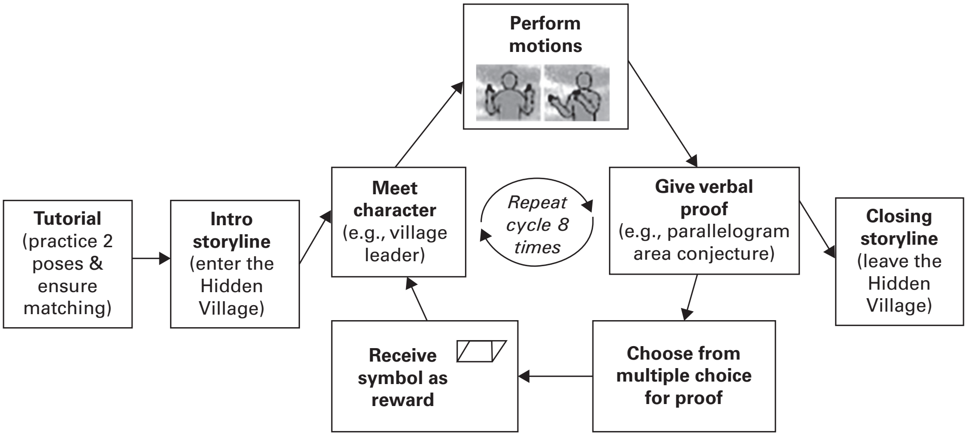


Figure 9.3
Flow of game play for *The Hidden Village*. After an initial tutorial that addresses where to stand and body calibration (box on far left), the game introduces the storyline that the player is a lost traveler who has stumbled into the Hidden Village. Players must interact with the characters (eight in all) who are engaged in village activities (cooking, crafts, etc.) by matching the in-game character movements. With each character, players are prompted to evaluate the truth of a mathematical conjecture and provide a justification for their choice and make a multiple-choice selection, all of which are recorded via audio and video. Players then receive a reward symbol and more area of the map of the village is revealed, indicating players' progress toward leaving the village.

teachers believe (1) gestures benefit classroom learning, (2) gestures are distracting, (3) gestures influence learning because they are redundant, (4) gestures influence learning because they are complementary to the accompanying speech, (5) instructional gestures are due to unconscious processes, and (6) instructional gestures are under conscious control, as well as items addressing the reasons teachers think people gesture, the perceived causes of gesture efficacy, and the frequency of gesture use. We used logistic regression models to perform quantitative data analysis using these variables.[1]

Point 1: Teachers Often Gesture While Solving Math Problems Together

Gestures were ubiquitous in our study as the teachers explored, discussed, and solved problems together. In particular, while they were proving conjectures, we found that teachers made an individual gesture 52.6 percent of the time and made a collaborative gesture 31.5 percent of the time. Figure 9.4 compares two groups of teachers proving the *two sides* conjecture. In the left panel, we see an instance where one group member makes an individual gesture that her group mates do not build upon. In the right panel, we again see one teacher making an individual gesture, but then it is built upon in another teacher's gesture and mirrored in a third teacher's gesture.

Figure 9.4
Two groups proving the two sides conjecture: "the sum of the lengths of any two sides of a triangle is always greater than the length of the remaining side." In the first group, Mary (far right) is performing two individual gestures as she explains to her group members (A and B). In the second group, Karen (middle left) is performing an individual gesture (C). Kristi (far right) collaboratively builds upon Karen's gesture while Tanya (middle right) mirrors Kristi's gesture (D).

Point 2: Teachers of Different Grade Levels May Have Different Gesture Tendencies

We also found that middle school teachers were more likely to gesture than elementary school teachers. There was a marginal difference in the same direction between elementary and high school teachers. It would make sense that middle and high school teachers, who usually teach only mathematics and may have stronger content preparation in mathematics, might gesture more when solving math problems than elementary teachers who are often generalists. Figure 9.5 shows a group of middle school teachers proving the *opposite angle* conjecture. The middle school teachers make a series of alternate and build collaborative gestures to explain that when the length of the side of a triangle increases, the angle across from the side will widen in order to complete the triangle.

Point 3: Teachers' Attitudes about Gesture Can Have Associations with Whether They Actually Gesture

We also found that teachers who indicated that gestures are distracting and interfere with learning had a lower relative chance of gesturing while proving conjectures. This finding makes sense because if you believe your gestures are distracting, you might be less likely to use them when collaborating. Surprisingly, however, teachers who indicated that gestures were effective because they elicited attention and made connections also had a lower relative chance of gesturing. This finding goes in an unexpected direction (i.e., is a negative effect when it might be expected to be a positive effect). For collaborative

[1] Cynthia: So like the side... so angle A is like bigger than angle B. So the side opposite angle A will be bigger than the side opposite angle B.

((A. Cynthia draws her finger diagonally from a point representing an angle to where the opposite side of the triangle would be and then repeats the motion from the other direction))

[2] Bree: Oh. Yeah. Because it's wider.

((B. Bree spreads her hands apart horizontally a few times))

[3] Cynthia: Yeah.

[4] Bree: Yeah.

[5] Cynthia: Because it's a wider angle.

((C. Cynthia makes an angle with her hands moving vertically. Bree anticipates and makes the same gesture))

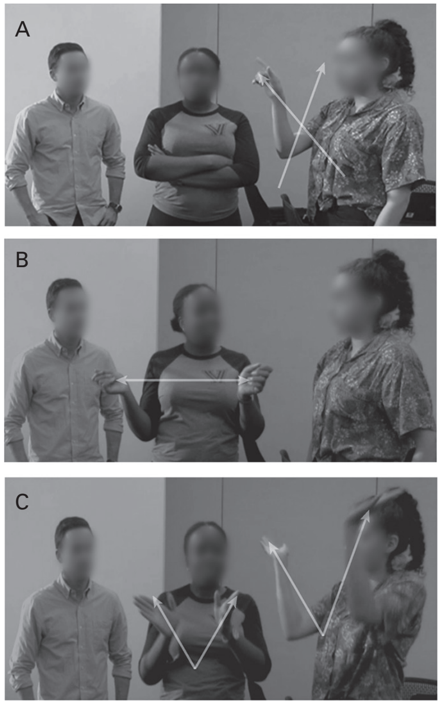


Figure 9.5
A group of middle school teachers performing collaborative gestures while proving the opposite side conjecture: "if one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle." Cynthia is on the right, and Bree is in the middle.

gestures, the results showed that indicating on the TAGLI survey that gesture had a positive effect on instruction was positively associated with performing collaborative gestures.

Point 4: The Characteristics of Collaborative Groups Can Be Associated with Tendency to Gesture

In our study, being in a smaller group while proving the conjectures together seemed to be associated with individual teachers using more gestures. The same relationship held for individual teachers' tendency to use collaborative gestures while solving problems. We also found that teachers were more likely to make collaborative gestures if other members of their group were gesturing, too. Figure 9.6 shows how during the *reflection rotation* conjecture, a group of three each performed their own individual gestures then a series of collaborative gestures. This small group of three teachers performed four individual gestures and three collaborative gestures during this short exchange.

((A. Hayley and Rebecca individually gesture after reading the conjecture aloud. Hayley draws an "R" in the air while shaking her head. Rebecca rotates her right hand away from her left hand.))

[1] Megan: No, because that flips it and you have to rotate it 180 degrees for it to flip.

[2] Hayley: Yeah. Absolutely.

[3] Rebecca: Right for it...yeah.

((B. Megan makes a flipping motion with pointer finger up and away from her body. Rebecca repeats her rotation gesture in anticipation. Hayley mirrors Rebecca's gesture))

[3] Megan: A triangle would be the way to prove this because with a square, you might not be able to tell.

[4] Hayley: Yeah. I always think of an "R".

((C. Hayley redraws an "R" in the air, then rotates her palm))

[5] Rebecca: Oh yeah, yeah. Because then you can like move...yeah.

((D. Rebecca again repeats her rotation gesture but larger in response to Hayley's gesture))

[6] Megan: That's a good idea. That's a good way to prove it. I really like that.

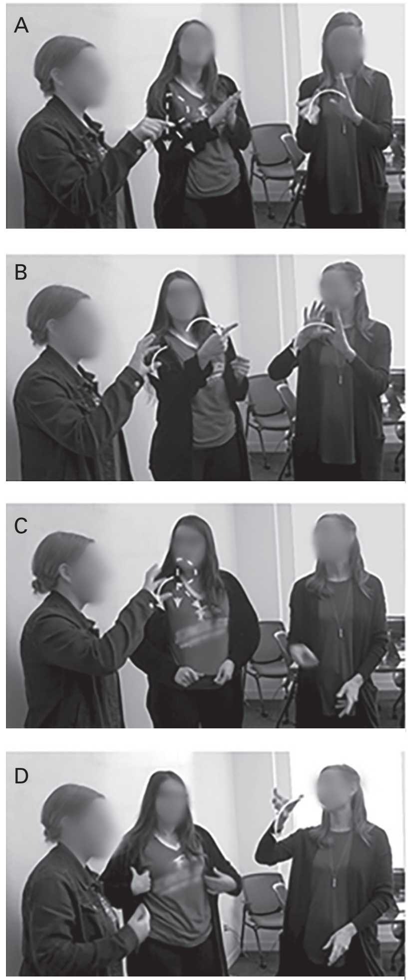


Figure 9.6
A small group of teachers performing individual and collaborative gestures while proving the rotation reflection conjecture: "reflecting a point over the *x*-axis is the same as rotating it 90 degrees." Hayley is on the left, Megan is in the middle, and Rebecca is on the right.

Point 5: Collaborative Gestures Have Potentially Powerful Associations with Valid Mathematical Reasoning

Our study also suggested that as teacher group members made more collaborative gestures, they increased their relative odds of getting their geometry proofs correct. The likelihood of participants producing an accurate mathematical proof (per trial, per group) was 51.8 percent. Making gestures in general that were not necessarily collaborative, on the other hand, did not predict correct proofs. This

[1] Cynthia: Yeah, because a parallelogram is just slanted.

((A. Cynthia draws right arm horizontally across her body while leaning right, then repeats leaning motion))

[2] John: Yeah.

[3] Carole: Is it the same?

((B. Carole holds up two thumbs and index fingers at 90 degree angles and then twists them slightly))

[4] Bree: Yeah 'cause a parallelogram is length times width.

((C. Bree draws finger across horizontally then vertically then horizontally. She did this same gesture silently while Carole was talking above, and was not observing Carole))

[5] John: Yeah, basically if you move… like if you cut off a triangle.

((D. John makes a vertical cutting motion in the air))

[6] Cynthia: Oh, put it on the other side.

((E. Cynthia makes upside-down "U" motion, right hand))

[7] John: And like put it on the other side, it would be the same.

((F. John makes upside-down "U" motion with right hand))

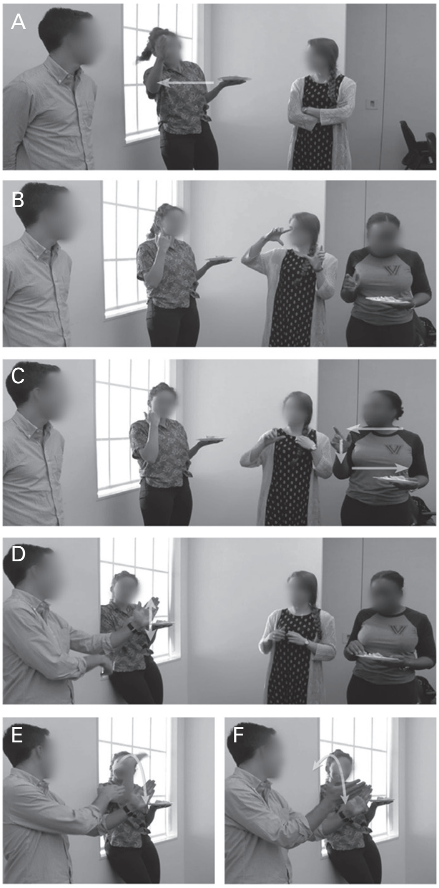


Figure 9.7
A group of teachers performing a series of alternating and anticipation collaborative gestures while proving the *area parallelogram* conjecture: "the area of a parallelogram is the same as the area of a rectangle with the same length and height." In the first image, John is on the left, Cynthia is in the middle, and Carole is on the right. In the second image, Bree is on the far right.

suggests that collaborative gestures might be particularly important to group members' understanding of geometric conjectures. Figure 9.7 shows a group of four in-service middle school mathematics teachers working through the *area parallelogram* conjecture using a series of alternating and anticipation gestures while discussing the veracity of the conjecture, which ultimately leads to a correct proof. Each of the four group members participated in collaborative gestures that both built on arguments when the participants were in agreement, and redirected arguments when a disagreement occurred.

Discussion and Implications

The illustrative case presented in the previous section helps to provide some insights in answering important questions about teacher gesture. Working backward from important educational outcome measures, we learned that making more collaborative gestures was associated with better proof performance. Thus, identifying individual factors and malleable environmental factors that elevated gesture production could lead to superior mathematical reasoning in an area that is vital for future educational advancement.

Teachers were more likely to produce any gestures and collaborative gesture sequences during proof activities when they were members of smaller groups. We also observed that teachers are more apt to produce collaborative gestures when those around them are gesturing. These social influences on gesture production signal potentially important and practical implications for teacher educators and designers' professional development interventions as they consider group size and group composition as factors directly under their control. Whether this plays out the same way for K-12 students is a subject for future research.

Teachers were also less likely to gesture during proofs when they believed gestures to be distracting. Given that gestures help with performance, the suggestion that negative attitudes toward gestures may show up in teacher behaviors may provide valuable diagnostic information that can inform future interventions targeted at teachers' belief systems. Believing that gestures are effective for learning also was negatively associated with overall gesture production, a finding that went in an unexpected direction. However, we also found that these same attitudes about gestures were *positively* associated with collaborative gesture production. This second finding may be more consequential because it is *collaborative gesture* that is ultimately predictive of proof performance among these teachers. While this invites further study, it points to the value of documenting gesture attitudes and the possibility that interventions targeted at gesture attitudes could positively influence mathematics reasoning, mediated, perhaps, by the collective gesture behaviors of one's collaborators.

The discussion here suggests several potentially fruitful directions for future work. First, proof production and geometric learning for smaller versus larger collaborative groups could be experimentally varied, with individual gestural tendency as a mediator. It would further be interesting to simultaneously examine how participation in the group's reasoning via talk moves changes as groups become smaller or larger. It may be that participation structures for gesture production are quite different than those for speech.

Second, future studies could test whether purposefully placing low-gesturers in groups with high-gesturers might increase low-gesturers' collaborative gestural production and increase their proof performance. Social and dispositional factors have been identified as important for determining the threshold for a speaker's resistance to overtly producing a gesture (Hostetter & Alibali, 2019), but little research has specifically examined how to increase the tendency to gesture as a way to increase learning and understanding. Creating a social situation where learners feel comfortable gesturing and feel like their contributions will be meaningful, and thus have lower gesture thresholds, may be key to promoting math learning for each individual participating in a group dynamic.

Third, interventions where group members are all explicitly encouraged to make collaborative gestures could be tested to see if they improve problem-solving outcomes. In the present study, we told the students they could not use writing implements and that their hands should be empty, but other more direct approaches could be used to encourage gesture. We can also explore how positive effects from collaborative gesture may carry forward and show a gestural trace in mathematical reasoning outside of the collaborative setting.

An interesting avenue for future research would be interventions that attempt to change people's beliefs about gesture—like those indicated on the TAGLI survey—and then examine how changing those beliefs impacts gesture usage and problem-solving. Many teachers may not be aware of the importance of gesturing or may not think gesturing or paying attention to student gestures is a particularly important element for them to be focusing on. Interventions that seek to increase gesture usage may not be successful unless they take into account underlying beliefs about teaching and learning.

Our chapter paints an optimistic picture of how understanding attitudes and social considerations influence gesture production and performance on advanced areas of mathematical thinking (see Megowan-Romanowicz et al., chapter 11 in this volume; Tancredi et al., chapter 13 in this volume). This invites new opportunities for embodied educational innovation as well as new areas of research on the embodied nature of teaching and learning.

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Note

1. For regression tables and additional descriptive information, please contact the authors of this chapter.

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