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Materialist epistemology lends design wings: educational design as an embodied process

Mitchell J. Nathan[1] · **Michael I. Swart**[1]

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Abstract
Materialist design is presented as an embodied perspective on educational design that can be applied to redesign of classroom-based learning environments. Materialist design is informed by a framework of materialist epistemology, which positions material innovation on equal placement with symbol-based formal theory. Historical examples of Einstein's conceptual reliance on trains for his Theory of Relativity and the Wright brothers' use of wind tunnels in aeronautics illustrate how materialist design drives progress on complex design problems. A key aspect is the application of scale-down methodology, where complex systems are reconceptualized as interactions among nearly decomposable subsystems that can be redesigned and integrated back into the entire system. The application of materialist design is illustrated with the redesign of an embodied video game that uses real-time motion capture technology to promote high school geometry reasoning and proof, following its use in an ethnically and linguistically diverse classroom. Our embodied perspective offers particular insights for understanding and implementing designs of complex learning environments, and assessing their influences on educational practices and student outcomes.

Keywords Complex systems · Educational design · Embodiment · Learning theory · Scale-down

Introduction

We propose an embodied perspective on design. This embodied perspective frames technological design of learning systems as operating within complex systems that are deeply rooted in the workings of the material world. As abstractions of the patterns of behaviors observed in the world, scientific theories are often regarded as superior. In contrast, our embodied perspective places material innovation on equal placement with formal theory. We show how this embodied perspective on design offers novel insights for understanding,

✉ Mitchell J. Nathan
mnathan@wisc.edu

Michael I. Swart
mswart@wisc.edu

[1] University of Wisconsin-Madison, 1025 West Johnson St, Madison, WI 53706, USA

implementing and redesigning learning environments, and for assessing their influences on educational practices and student outcomes.

Materiality in creativity and design

We begin by delving into one of the most prominent innovative acts in science—Einstein's (1916) formation of the Theory of Relativity. In its telling (e.g., Stachel 1982), the reasoning that formed the philosophical basis of Einstein's landmark publication was far afield from the concrete experiences of daily life. Rather, it is often portrayed as the product of Einstein's deeply theoretical mind and his highly abstract grasp of the current scientific theories. Einstein was motivated to resolve some of the most vexing paradoxes of classical physics formed centuries earlier.

To convince members of the scientific community, Einstein (as cited in Miller 1981) offered two *gedankenexperiments*, or thought experiments. These were constructed to explain how any two events can truly be said to take place simultaneously, no matter where observers are located. Explaining this was critical because up to this point, the prevailing theories allowed for the paradox that an observer who was closer in space to an event would believe the event was happening sooner than someone farther away. In reality, however, both observers should understand this singular event actually happened at the same time for both of them.

The first thought experiment was meant to tackle the simultaneity problem. Einstein posited a system of clocks that sent electrical signals along cables, and accounted for "relative time" based on the distances between clocks. His second *gedankenexperiment* was meant to tackle the relativity problem. Einstein sought to resolve the paradox that observers on trains headed toward or away from a lightning strike would appear to experience the same event occurring at different times relative to a stationary observer standing on the railway embankment (Einstein 1916).

Over time historians of science have gained a greater appreciation of the role of materiality in Einstein's thinking, and in scientific theorizing more broadly. Galison (2004) notes that in his role as a patent officer in the early 1900s, Einstein reviewed applications proposing technological solutions to the problem of how to calibrate train travel and reliably coordinate train schedules in different locations. Rather than pure reason, Einstein's creative process drew upon his familiarity with the technological innovations of his day! Electromechanically coordinated systems of clocks and traveling trains were "things to think with" (Resnick et al. 1996). These material artifacts could be used to productively reason about basic questions, time, space, and relativity. Galison (2004) argues these material anchors of creative thought contributed directly to a discovery regarded by many as the prototypical scientific revolution (Kuhn 1962; Posner et al. 1982).

Davis Baird (2004) places material influences of this sort within a framework of "materialist epistemologies" (p. 15), describing their importance across a range of historical technological advancements. A *materialist epistemology* is "an epistemology where the things we make bear our knowledge of the world, on a par with the words we speak" (Baird 2002, p. 13). This is in contrast to *theories*, which are written and spoken propositions that bear knowledge by virtue of being justified, true beliefs.

In this vein, Baird includes Michael Faraday's invention of the electromagnetic motor, a cornerstone of modern technology. (Faraday is the namesake of a fundamental unit of electrical charge.) As Baird (2004) notes, the breakthroughs made possible by Faraday's electric motor contributed to fundamental discoveries about electromagnetic phenomena, including light, and notions of the conservation energy. Faraday's design was accomplished, not as a breakthrough of mathematical theorizing—the appropriate formulas would not be written until several years later—but as a result of his mechanical prowess and iterative persistence. In contrast to a propositional *theory* relating electricity and magnetism to mechanical motion, Faraday's motor functioned regardless of the beliefs held by the bearer. When asked to substantiate his findings, Faraday did not provide a written mathematical derivation, or logical proof of its correctness. Instead, he shipped prebuilt, functioning motor assemblies to those who requested evidence of his claims, much the way scientists share academic papers today.

These brief and influential examples of scientific discovery illustrate a key point about the nature of creativity and the design process: Even some of the most advanced and abstract insights about fundamental properties of the universe are grounded in the material world and the artifacts and events that characterize people's primary experiences. We believe we can expect insights about the design process for understanding and improving learning to be grounded in the material world as well.

Learning and learning environments: assumptions and principles

As we proceed, we wish to state some of our core assumptions and principles about learning. Foremost, is that learning is a process that occurs within a complex system (Cilliers 1998/2002; Jacobson et al. 2016; You 1993). A great deal of the focus of early research on learning focused on internal mental structures like memory and representation during perceptual, verbal and mathematical processing. Pioneering work like Vygotsky's (1934/1978) sociocultural theory and Gibson's (1977) theory of affordances, among others,[1] collectively highlighted that the human ability to learn and apply know-how is a complex interaction between inner workings of our cognitive and affective systems with the environment in which learning occurs.

In *The Sciences of the Artificial*, Simon (1969/2019) regards learning processes and learning environments as complex systems. Although the biological underpinnings of human behavior are governed by natural laws, Simon argued that behaviors are expressed within "artificial" environments—not meaning environments that are *fake*, but ones made of *artifacts*—that are culturally produced (p. 2). Simon characterizes learning as "any change in a system that produces a more or less permanent change in its capacity for adapting to its environment" (p. 100). In this view, any system that is adaptive to its environment is necessarily a *complex* system, since it employs the nonlinearities of feedback and feedforward processes to self-regulate.

This view can be seen as commensurate with Cilliers (1998/2002) view of complex systems, as well as what You (1993, p. 18) refers to as "dynamic, nonlinear systems." These systems are said to defy "complete description" as they adapt in response to interactions with a complex environment. You (1993, p. 20) argues "that fundamentally different concepts are needed to deal with a dynamic, nonlinear system" for instructional design rather than those derived based on principles of linearity, deterministic predictability, and closed, equilibrium-based systems based on negative feedback (i.e., error reduction).

[1] We include: Bruner's (1966) constructivism, Lakoff and Johnson's (1980) conceptual metaphor, Bandura's (2001) social-cognitive theory, Lave and Wenger's (1991) communities of practice, Brown et al. (1989) situated cognition, A. Brown's (1992) call for design-based research, Varela et al. (1992) embodied mind, Hutchins's (1995) distributed cognition, and Barsalou's (2008) grounded cognition.

A second assumption is that one's experiences, memories, and knowledge are subjectively constructed, rather than verbatim, sensorial records of some objective external world. These constructions are highly influenced by a need for people to make sense of their experiences. Theories of embodied cognition (e.g., Clark 2008; Glenberg 1997; Varela et al. 1992) posit that the process of meaning making is constructed through enactment of a continuous interplay of cognitive, motoric and perceptual processes. These operate in a highly dynamic, self-regulating manner, often referred to as the *perception–action loop*. This is illustrated by phenomena such as gesture production during thinking and communication (e.g., Goldin-Meadow 2005).

Furthermore, there is reciprocity between cognitive and motor processes based on *transduction processes*. People ordinarily accept that one's thoughts and goals will drive one's actions. Once thinking and acting processes are coupled in this way, goal-directed actions are also capable of influencing cognition. As an example, experimental participants who are instructed to follow specific eye gaze patterns are better able to solve a difficult insight problem (Thomas and Lleras 2007). A transduction account explains that problem-solving performance gains happen because one's eye movements (as well as other types of motor sequences) produce a spatial-motor pattern that influences the cognitive system through their strong interdependencies (Nathan 2017; Nathan and Walkington 2017). Actions become thoughts using the same basic mechanism that enables thoughts to induce and become reified by one's actions. We note that the relation of action sequences to cognitive processes is not subject to a simple one-to-one mapping.

The third assumption is that cognition is embodied and extended beyond the boundaries of the brain and skull. Language-based processes such as conceptual metaphor (Lakoff and Johnson 1980) and conceptual blends (Fauconnier and Turner 1998) illustrate ways we ground more abstract thoughts into physical and spatial experiences. Furthermore, people's thoughts and conceptions about the world result from the influences of the interactions between their physical bodies and their experiences in the world. These *embodied interactions,* according to Melcer and Isbister (2016, p. 4),

> refers to the creation, manipulation, and sharing of meaning through engaged interaction with artifacts (Dourish 2001). This includes our interactions with material objects and environments in the process of meaning making and action formation (Streeck et al. 2011).

People's knowledge is grounded in and distributed across actors, objects and space (Clark and Chalmers 1998) through processes such as cognitive offloading and distributed cognition (Hutchins 1995).

A fourth assumption is that the cognitive system is fundamentally a predictive architecture. Rather than passively waiting for input to act, it continually anticipates the next events in the stream of sensory input, and is already poised to respond. In this sense, transfer is the default mode of a complex adaptive system that is continually striving to act appropriately and opportunistically based on feed-forward predictions of how the environment and other agents will behave (Nathan and Alibali in press). Positive feedback supports system adaptation to ever-changing environments by amplifying deviations, while negative feedback mitigates deviation and maintains equilibration in response to perturbations (You 1993). You (1993, p. 23) argues that "positive feedback should be designed into the [instructional systems design] model in order for the instructional system to continue becoming rather than simply being." Predictive systems need to be responsive to the dynamics of their environment. Here, "becoming" prizes a drive for continuous change rather than stability

Table 1 Education design research as instantiating the ethos of science and engineering

	Science ethos	Engineering ethos
Driving Qs	How are things?	How should things be?
Products	Generalizable theories that lead to predictive models that can be empirically verified	Locally-applied principles for achieving designs intended to meet societal goals that can be empirically assessed
Epistemological basis	Critical rationalism (Popper 1945/2012)	Pragmatism (Dewey 1899/2013)

("simply being"), and relates to Cilliers (1998/2002) notion of disequilibrium as a key trait of complex systems.

Finally, what is considered to be *learning* is influenced by the methods used to investigate and describe it. Methods for investigating learning can be characterized by the degree to which they conceptualize the phenomena as elemental or systemic (Nathan and Sawyer 2014). *Elemental approaches* are best suited for analysis and understanding of the basic elements of learning—facts, procedures, and concepts, or separate investigations of attention, short-term memory, perception, etc.—and generally conform to an acquisition metaphor (Sfard 1998). *Systemic approaches* examine the rich interactions among behaviors and contexts in a holistic manner. They generally conform to participation metaphors and knowledge-creation views of learning. Synthesis is a preferred way to describe the complexities of people and groups learning to participate in authentic practices. Investigations of learning depend on both elemental and systemic approaches (Nathan and Sawyer 2014; Sfard 1998) as complementary perspectives for analysis and synthesis of empirical observations.

Educational design as learning engineering

Our interest is not simply about *design* in its broadest form. Specifically, we are concerned with the processes of educational design of technology-based learning environments that are evidence based and hypothesized to influence student thinking and teaching practices in a range of settings. Thus, it is important that we also consider the nature of educational technology design.

Educational design developed as an interdisciplinary field that investigates the nature of learning as it relates to the design of technology-based learning environments in both formal and informal educational settings (Kolodner 1991; McKenney and Reeves 2014). These investigations are intended to inform both practical educational needs and generalizable theories of learning. As a result, educational design, along with related fields of study such as learning sciences, is subject to tensions between its identity as a *basic science*, on one hand, and, on the other, as a form of *engineering* or *design science* (Nathan et al. 2014). The tension is important because it frames many of the discourse practices that are endemic to this broad, interdisciplinary field.

As contributors to *basic science* (Table 1), educational designers are primarily concerned with uncovering the nature of the world, and documenting how things are. The goal from a science perspective is to produce generalizable theories—explanatory accounts of the properties and dynamics of learning—that lead to predictive models that can be empirically verified. Its epistemic basis as a science is a form of critical rationalism (e.g., Popper

1945/2012). "Truth" is established through the critical evaluation of logically deduced claims falsified empirically or through rational thought.

As a design science (Simon 1969/2019) or form of engineering (Table 1), the emphasis for educational designers, as well as learning scientists, is how the world *ought to be*, and how to build technological artifacts and practices to achieve those goals. Engineering, according to Simon, provides a "home" for research driven by dual goals for basic understanding and applied usage (also see Stokes 2011). The goals from the perspective of design science are to use scientific evidence and theory to produce location-sensitive principles for reliably producing innovations that meet societal goals and that can be empirically assessed. In practice, this creates a commitment to a form of materialist epistemology as described above, which privileges acts of *design* and *making* over proposition-based theory. From this standpoint, knowledge is embedded in the devices and their use, and "truth" is established by demonstrating that things work as expected under the intended conditions. Sandoval (2004, p. 213) makes this point specifically about educational technologies, arguing that each innovation "embodies conjectures about learning within educational designs."

Clearly, there is evidence of both science and engineering practices within educational design and learning sciences (Yoon and Hmelo-Silver 2017; Sommerhoff et al. 2018). As such, it is tempting to look at these fields as a hybrid of the two, or perhaps as *bridging disciplines* between theory and practice (cf. Bruer 1997). However, we advocate rejecting this simple resolution. One reason is that materialist epistemologies and science theory epistemologies are incommensurate. There are several incommensurable points between the epistemologies of theory and of materiality. At its core, knowledge within the epistemology of scientific theory is presented as propositional content that describes a mental state, specifically the state of holding some set of justified true beliefs (Goldman 1986). Fundamentally, the science theory epistemology strives for "semantic ascent" away from the imperfectness of the material world, and toward the universal ideals that only language can provide.

In contrast, within a materialist epistemology, one does not need to believe something for the materiality of a device to exist and operate. The knowledge of a principle on display from a functioning device is not a property of a belief, as it is with science epistemologies, but an objective outcome of the mere existence and behavior of the device. Under a materialist epistemology, directly working with and embracing the imperfect nature of materiality is actually the goal of the intellectual and practical efforts of innovation. This contrasts with the notion of semantic ascent that is so prominent in the science theory epistemology.

The second reason that we reject a simple resolution of blurring the distinction between basic science and design science is that we wish to advance an explicit agenda to overcome a historical bias that still privileges theoretical/propositional accounts of knowledge and knowing over material/embodied accounts (Nathan 2012). Designers and technicians continue to make technological advancements in the instruments and devices they build and test, absent a theory, with an incomplete theory, and sometimes even with an incorrect theory. Too often society offers a misleading picture of "pure" science driving the so-called "applied sciences." These applied fields, and the professional schools that house them, in turn, experience lower social status on university campuses, and lesser allocations of research funding. This is made eminently clear in sociological studies of science practice such as Latour and Woolgar's (1979/1986) *Laboratory Life*, where the material inputs—chemicals, animals, staff—serve the intellectual output—scholarly papers. As documented by Laurillard's (2001) keen observations, this theory-centric view of scientific advancement has a profoundly debilitating effect on education, which is our primary concern. As Laurillard notes, schooling is largely relegated to *second-order experiences*, which privilege written descriptions and models of actual phenomena. Students seldom—often never—have the *primary experiences* that make up the actual behaviors of the world. Armed only with this distal relationship to the world, they enter a workforce and are poorly suited to tackle real world problems that are completely foreign to them. Given this state of contemporary science and education, to celebrate a middle ground between the epistemologies of materiality and theory, is to dilute materialist progress.

Instead, we offer the position that, at their core, the fields of educational design and learning sciences are forms of *learning engineering*. As such, they enjoy a membership with illustrious fields like civil engineering, computer science, architecture, urban planning, and the like. This is commensurate with Janet Kolodner's (1991) editorial from the inaugural issue of *The Journal of the Learning Sciences.* She describes the emerging field of learning sciences as focused on the design of learning environments and educational practices (p. 4) as applied to a wide range of settings and technological innovations, and assessed based on their influences using multidisciplinary means.

Categorizing educational design, or for that matter learning sciences, as a form of engineering in no way minimizes the contributions to basic science these fields make now and their potential contributions in the future (Klahr 2019; Stokes 2011). For history is replete with examples where the design of innovative technologies preceded the basic science and the formal mathematical models and theories that came to crystalize their contributions. Stokes cites Faraday's rotary engine as just one example. In effect, Faraday's work coiling wire around magnets produced electrical currents while his work passing electrical currents through wire coils produced magnetic fields. Separately and together, these physical innovations demonstrated a clear relationship between electricity and magnetism despite there being no accompanying theory to explain what we now know as electromagnetism.

Educational designers face similar challenges for developing innovations well before the associated learning theory is fully formulated. Sandoval and Reiser (2004) developed a learning environment for fostering inquiry in high school biology. The design of the technological and curricular supports in BGuILE (for Biology Guided Inquiry Learning Environments) embodies conjectures (i.e., provisional theoretical propositions) about the nature of science education and scientific knowledge. One of these conjectures is that conceptual and epistemic scaffolds for inquiry must be integrated so that learners come to understand the purpose of conducting scientific experiments as well as the kinds of knowledge that experiments can produce. BGuILE enabled investigators to design promising systems for advancing science education well before the theory was fully formulated and tested.

Across engineering and educational design, materialist epistemology offers an account of how observations and innovations from the material world inform the theoretical, and thereby provides a blueprint for thinking productively about the design of learning environments.

Design and redesign of complex systems for learning

Near decomposability

A complex systems perspective offers a framework to address the challenges of principled educational design for classroom-based learning. Several scholars have sought to describe complex systems in terms of their structures and behaviors. Cilliers (1998/2002) distinguishes between complicated and complex systems. Such designations are not firm, and have primarily to do with the nature of the interactions in which they participate, and where one draws the bounds of the system. In brief, *complicated systems* are those that can "be given an exact description." *Complex systems*, in contrast, defy description because they are open, self-regulatory, history-preserving systems that exhibit emergent properties that arise from the rich, nondeterministic interactions of a large number of (relatively simple) system elements that perpetuate continual change.

Simon (1969/2019) offers an alternate view where complex systems often naturally develop subsystems to ensure their adaptability and long-term survival. Within the design sciences, developers imbue their system designs with subsystems to ensure their reliability and improve the efficiency of production and maintenance. Simon illustrates this with a thought experiment of two watchmakers, each developing high quality watches with the same number of components, around 1000. One watchmaker uses ten subassemblies of around 100 components each, while the other uses no subassemblies. When all goes well, production is comparable. However, the inevitable interruptions of daily life have devastating consequences for the watchmaker whose design has no subassemblies, and must start anew with each disruption, while the watchmaker whose design utilizes subsystems has a far easier recovery time.

Jacobson et al. (2016) propose a complex systems conceptual framework of learning (CSCFL) as a way to frame theories of learning with respect to relevant processes and conceptual dimensions, such as collective behavior (e.g., sensitivity to initial conditions, levels of performance, nonlinearity, emergence) and the behaviors of individual system elements (e.g., parallelism, conditional actions, adaptation and evolution). CSCFL is offered so that "theoretical considerations of learning as an emergent phenomenon in complex neural, cognitive, situative, social, and cultural systems will yield critically important insights of central relevance" to education design and research (Jacobson et al. 2016, p. 217).

Whereas Cilliers (1998) posits an inviolable integrity of complex systems, both the frameworks proposed by Simon (1969/2019) and Jacobson et al (2016) allow for systems in which complex cognitive behaviors may be viewed as made up of "near-independence of levels" (Jacobson et al. 2016, p. 216) and "nearly decomposable systems" (Simon 1969/2019, p. 474). As with the watchmakers, framing complex systems in terms of nearly decomposable system offers important perspectives on design, though this by no means guarantees design success. Potential advantages can apply to designing new innovations, research methodologies used to study their impact, and evidence-informed redesign to issue forth improvements.

Fully decomposable systems (like the *complicated systems* of Cilliers 1998) are made up of modules that function independently and additively; the larger system behavior being perfectly predicted by the superposition of its constituents. However, such systems are rare in nature, and are often too simplistic to accommodate a broad range of users and circumstances. Far more common are complex systems that exhibit a higher degree of inter-component interactions, and which exhibit emergent behaviors that are not directly predicted by knowledge of the constituent parts. Many biological and social systems, such as ant colonies and traffic flow have these traits, as described by complex systems scholars (e.g., Wilensky and Rand 2015).

Nearly decomposable systems are marked by constituents of a system, where "the short-run behavior of each of the component subsystems is *approximately independent* of the short-run behavior of the other components" (Simon, 1969/2019, p. 474). Because of this, interactions in nearly decomposable systems are relatively strong within the subsystems, while interactions between a subsystem and the rest of the system are relatively weak, even though they cannot be ignored.

Functional decomposition

The thoughtful application of near decomposability of a complex system is apparent in the example from the history of human-powered flight. Aircraft designed for human-controlled flight in the natural world exhibit the behaviors of complex systems. For the Wright brothers, innovations that leveraged nearly decomposable systems enabled two bicycle mechanics to master the design of aircraft well ahead of their competitors who were better funded.

The largest effort in the early 1900′s was the Aerodrome project led by Samuel Pierpont Langley, funded by the US War Department and the Smithsonian Institution (McCullough 2015; Smithsonian 2019). As with many of the efforts to develop flying machines at the time, the designs were based on the equations for lift and drag, based on Smeaton's coefficient, which accounted for the density of air. That this basic physical constant was not correct is an essential aspect to this story, because it shows two important points: advancements in engineering can proceed when the basic science and theory is insufficient, or even incorrect; that the transfer of knowledge flows not only from basic science to the design sciences, but also in the other direction. Langley used existing equations to build entire aircraft, which showed some successes, but had some serious flaws, which resulted in huge losses of entire aircraft, adding to the research and development costs.

All of the competing efforts had their set-backs, owing to the use of incorrect lift equations and the challenges of engineering sturdy yet flexible aircraft. But the effort led by Orville and Wilber Wright was historic because they approached the engineering design challenge through the lens of near decomposability. Bradshaw (2005) labeled their method *functional decomposition,* because it "requires a complex invention (such as an airplane) to be divided into functional parts (wings that produce lift, propellers that produce thrust) that are refined in isolation from the whole." (p. 263). To the extent that the design of complex systems depends on both proficiency in the abstractions that support generalization of the design principles, and technical prowess of getting the design to work, functional decomposition provides a way to manage the complexity of implementation, testing, and theorizing.

Through the pragmatics of trial-and-error inherent in the engineering design cycle, the Wright brothers realized it was essential for the enterprise of designing a plane that they get the wing design correct since this enabled pilot control as well as maintaining lift. They also recognized that the prevailing physics equations and the theory that these equations formalized were insufficient. Their approach was also probably tactical in that they lacked the financial resources that would cover expensive crashes. Bradshaw (2005) notes past attempts by them and others showed that very little useful data could be obtained from unsuccessful crashes of the whole plane. Functional decomposition provides the greatest benefit when subsystems do not work well, and the source of system failure is not completely obvious (Bradshaw 1992, 2005). Plato encountered this question of decomposition of complex systems and offered the dictum to "carve nature at its joints" (Plato 1923 in Phaedrus 265d–266a). Pragmatically, designers steeped in the materiality of their designs become intimately familiar with the construction and uses of their systems. From this material stance, they often recognize the candidates for the appropriate places to apply Plato's principle of decomposition designs so as to preserve functionality of a core subsystem that does not compromise the overall system performance.

Rather than test theoretically derived designs exclusively on whole aircraft, as a naïve interpretation of the theory of lift might suggest, the Wright brothers worked on the wing as a subassembly. They built miniature wings and tested a range of wing behaviors in a custom-built wind tunnel. Though the wind tunnel was invented some 30 years prior, the Wright brothers were possibly the first to use one in the aircraft design process (U.S. Centennial of Flight Commission 2020). They also made substantial improvements to the wind tunnel, by incorporating instruments called *balances,* which they developed to directly measure lift and drag. Their rigorous investigation of the various behaviors of the model wings in a wind tunnel allowed them to perform more rapid, less costly, and more informative design cycles on the wing designs. This, in turn, allowed more frequent and efficient evidence-based design corrections, which drove huge breakthroughs over relatively short spans of time.

The Wright brothers successful design process was steeped in materialist epistemology. Their breakthroughs led to improvements to in-flight wing deformation, which improved steering and overall aircraft control. Their efforts also produced reliable measurements of air density, which led to critical corrections of basic equations for lift and drag. The Wright brothers were able to scale up to larger wind tunnels, which enabled them to scale up their wing designs—the crucial subsystems that could then be integrated into full-scale aircraft. This led to successful flights on Dec. 17, 1903, in the Kill Devil Hills of North Carolina, when they performed the first (indeed, the first four!) heavier-than-air powered, controlled aircraft flight in history.

Redesign of an embodied learning environment: The Hidden Village

Functional decomposition provides a way to partially manage the complexity of systems design and refinement. How then does this inform the design of complex learning environments? As described in our analysis of several historically significant designs, one's attention to materialist epistemology serves as the crucial guide to functional decomposition along the most promising joints between nearly decomposable subsystems. Einstein's approach drew on his familiarity with clocks and trains. For Faraday, it was his skill with the construction of motor assemblies that showed the path forward. For the Wright brothers, functional decomposition was guided by their expertise with bicycles and the use of wind tunnels to simulate the conditions of flight. Their specific skills pointed them toward solutions within their areas of expertise, such as the systems of pulleys and cables common in bicycle assemblies. They reframed airplane design as a set of bicycle subsystems optimized for flight. (The parallels to Einstein's familiarity with clocks and trains for theorizing about General Relativity should not be lost here.) The Wright brothers' strategic use of functional decomposition as informed by their well-developed understanding of bicycle mechanics proved paramount to their successes in aeronautics.

Across these different innovations, insights were made by using functional decomposition to reduce the complexity of the overall system and to then grounding general concepts of central importance such as lift, electromagnetism and simultaneity to experiences in the material world. We call this *materialist design* and apply it to our experiences of redesigning the mathematics classroom experience with *The Hidden Village* (THV)*,* an embodied learning environment. We aim to take this design to scale to meet the needs of a variety of learners and learning environments. One way to achieve successful scaling up is the application of scale-down methodology. As described in the next section, *scale-down* methodology draws on principles of materialist design to further manage complexity of complex systems in order to achieve translational goals for implementing evidence-based educational designs.

Instantiating materialist design with re-design of a classroom learning experience: embodied mathematical reasoning with The Hidden Village

We identify the complex system under investigation to be entirety of *The Hidden Village* classroom-based learning environment. As noted by scholars such as You (1993), this system is structured with a multiplicity of interrelationships characteristic of nonlinear dynamic systems. Characteristically, these systems exhibit non-deterministic performance and move toward states of transition (e.g., learning new ways of mathematical reasoning) rather than states of equilibrium. Even so, we went in with expectations—hopes—for how the system would behave and to offer an environment that fostered meaningful mathematical reasoning among the students. While some of these expectations were in evidence, we observed many behaviors that seemed to restrict students' intellectual engagement, and called for re-design of the learning environment. We describe how a materialist epistemology identified potential improvements, which were pursued via functional decomposition to system redesign.

Our efforts at functional decomposition are guided by our stance of the workings of human gestures as a flexible system that contributes to both social interaction (Goodwin 2013; McNeill 1992) and cognition (Alibali and Nathan 2012; Goldin-Meadow 2005). In mathematics education, gestures can mediate complex cognitive reasoning by engaging the body in simulated actions of mathematical operations and relationships (Hostetter and Alibali 2019; Nathan and Martinez 2015; Nathan and Walkington 2017), employ gestures as a means of producing distributed representations of these mathematical ideas (Walkington et al. 2018, 2019), and communicate these ideas in ways that transcend linguistic barriers and foster common ground (Clark et al. 1983; Holler and Wilkin 2011; Nathan et al. 2007). The re-design of a complex learning environment benefits from materialist design when functional decomposition goes hand-in-hand with embodied views, mediated by the materialist epistemologies of the designers.

General Description of the Game

THV is built with the Unity-3D development platform and utilizes the Microsoft Kinect™ 2 peripheral hardware for infrared 3D-motion-capture. All of the game controls are conducted using players' verbal responses into a headset and movements performed at a distance (4 to 6 feet) from the computer screen (see Fig. 1) to advance forward and backward in the game. Data collection includes players' verbal statements, body movements when following directed actions, co-speech gestures while reasoning, and body-based selection of multiple-choice responses. Using Cilliers's (1998) terminology, THV (the platform) is a *complicated system*, while the classroom learning experience with THV is a *complex system*. Within this complex system, we identify the THV game as a nearly decomposable subsystem within the classroom interactions where we exercise functional decomposition in the form of observations and testing, evidence-based redesign, and reintegration back into the complex system of the THV classroom.

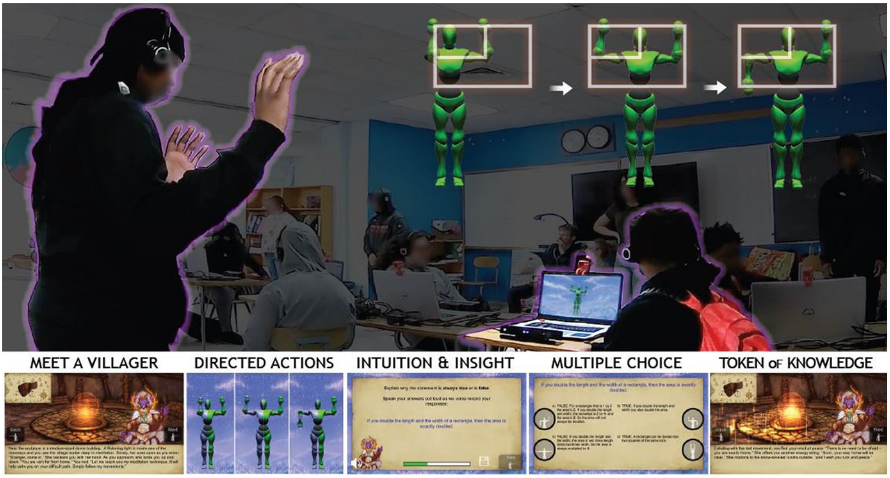


Fig. 1 Game Flow of *The Hidden Village (starting from left):* Meet a Villager, then perform *directed actions*, followed by free-responses to the given conjecture (providing T/F intuition as well as insights and rational for proof), select a multiple-choice response, and finally receive a token of knowledge

THV was designed to engage embodied forms of reasoning about shape and space as an alternative to the traditional axiomatic based two-column proof. The central instructional objective (Mager 1997; Reigeluth and Carr-Chellman 2009) after completing THV game-play activities is that the high school student participants will produce mathematically valid responses to four outcome measures for geometry conjectures: (1) Mathematical intuition, measured as the True/False judgment of the truth of the conjecture before the student has engaged in extended deliberation; (2) verbal and gestural reports of their mathematical insight, a measure of students' appreciation of the gist of the relevant mathematical ideas for each conjecture (see Table 3 in Appendix); (3) a mathematically valid proof to justify their truth assessment; (4) the selection of the best response from among 4 multiple-choice options (two True, two False). Regarding the specificity of the instructional objectives from a complex systems perspective, You (1993, pp. 25–26) acknowledges that "[t]he role of [instructional systems design], therefore, is not to provide specific procedures that will produce student achievement of predetermined objectives, but to construct learning environments that stimulate learning which is derived from the dynamic interrelationships among all instructional systems components." Following You (1993), we recognize the non-deterministic nature of the learning and performance behaviors that arise from these complex interactions. Learning, You (1993) notes, is itself an inherently destabilizing process for a complex system. Learning requires subsequent reorganization, such as increasingly sophisticated ways of thinking and acting, to achieve stability. Thus, introducing learning triggers "a fundamental reorganization of the system and its components rather than the mere addition of information" (You 1993, p. 24). Rather than pre-determining the content and degree of correctness for each student's multimodal response (i.e., responses that include both speech and gestures), we conduct detailed coding of these responses to determine their mathematical validity and to describe the nature of students' reasoning in each instance. Details of this coding process are reported in (Nathan et al. 2020).

The player's experience begins with an opening sequence in which a series of illustrated backgrounds accompanied by music displays a remote village. A narrator's voice explains that the player is a traveler, lost from their group, who has stumbled upon The Hidden Village. In order to safely return home, the player must help each Villager they encounter. In exchange, players receive portions of a map and tokens of knowledge in the form of energy strings that will fuel their ship and aid them in their journey home.

For each of the challenges (we typically select 8 for one class period), there is a proscribed sequence (see Fig. 1): First, village characters introduce themselves and beckon the players' assistance. Second, the player is presented with a task that involves a series of arm movements that they must copy. These are empirically derived *directed actions* (Nathan and Walkington 2017), chosen to elicit cognitive processes that activate the appropriate mathematical relations for the forthcoming task. Directed actions are one of the primary intervention methods used by THV to elicit the intended mathematical reasoning. The logic model of THV is that by engaging players' embodied processes through their repetition of the directed actions, we expect to foster players' mathematical reasoning, which is then expressed in both their speech and their own production of depictive gestures (described in more detail, below). The specific directed actions instituted in the game come from mining video data of those in prior sessions who successfully articulated valid mathematical proofs for these conjectures. The directed actions for each of the conjectures, along with their associated mathematical principles, is in the Appendix. Player movements are tracked in real time and a match between players' movements and the in-game directed actions is necessary to proceed (though players can also step through with a key stroke).

Third, the player is prompted to consider intuitively whether a geometric conjecture presented on-screen is true or false and to provide an explanation for their decision, which is audio-recorded via Bluetooth headset and video-recorded via an HD camera cube. Fourth, players are asked to choose multiple choice options that best fit their understanding of the explanation for the truth or falsity of the conjecture. Lastly, players advance by re-encountering the villager who exposes more of the map and gives a token of knowledge. Figure 1 shows students playing THV in a classroom and an example of game flow.

The learning theory related to the design of The Hidden Village

The game leverages *action-cognition transduction* (Nathan 2017) to ground students' understandings of properties of shape and space through the use of their own bodies. For proof, the focus is on identifying *invariant* properties, those properties that persist (e.g., sum of the interior angles) even as other properties (such as size and rotational orientation) change. The hypothesis is that one's geometric reasoning is embodied. Proofs describing invariant properties of objects are expressed, in part, through students' production of dynamic depictive gestures that carry out simulated actions that can reveal these invariant properties (Nathan and Walkington 2017).

When talking about their mathematical reasoning, people regularly generate *depictive gestures*, which illustrate and often represent the mathematical objects and relations among those objects (Alibali and Nathan 2012). Empirically, it has been observed ($N=90$; Nathan, et al. 2020) that the production of depictive gestures during geometry proving sessions reliably predict people's generation of correct mathematical intuitions ($d=0.65$, $p=0.027$) and insights ($d=0.44$, $p=0.017$). Thus, depictive gestures appear to facilitate people's reasoning by engaging nonverbal processes that help them, over and above contributions from their spoken language (Nathan et al. 2020).

During reasoning, people can also produce *dynamic* depictive gestures—gestures that both represent the objects and simulate transformations on those objects (e.g., skewing and dilating the objects). Producing dynamic depictive gestures appears to help people explore the object's generalized properties. They do this by revealing to the speaker what aspects of a mathematical object can change while still preserving key invariant properties. For example, skewing a triangular shape using one's forearm while maintaining a midsegment with one's other flat hand reveals that the midsegment always stays parallel to its base (Nathan et al. 2020, Fig. 1). Dynamic gestures were hypothesized to be especially helpful for predicting overall proof performance. Proofs are statements that establish general truths across a broad range of geometric objects. This prediction was empirically verified ($d = 1.40$, $p < 0.001$), and held even when controlling for participants' spatial ability and math expertise (Nathan et al. 2020).

The action-cognition transduction theory states that directed actions elicited during game play will activate both cognitive and motor processes. These, in turn, lead to the production of the appropriate dynamic gestures. With this in mind, the expectation is that not *all* types of actions are equally helpful for improving reasoning. Rather, it is specifically *mathematically relevant* directed actions that are expected to help. Indeed, this is also what the data show.

When high school students ($N = 85$; Walkington et al., in press) enrolled in a program for first-generation college students participated in a study playing The Hidden Village, they were directed to make both mathematically relevant and irrelevant gestures in a within-subject experimental manipulation. As predicted by the theory, the mathematically *relevant* gestures were significantly more beneficial to students' geometry proof performance, so long as students made some gestures during proof production. The mathematical relevance of directed actions has also been shown elsewhere (Goldin-Meadow et al. 2009). Through multiple gaming cycles, we have evidence that engaging in directed actions can enhance geometric reasoning (Nathan and Walkington 2017; Walkington et al., in press). Still, many facets of the design of embodied learning environments highlight the challenges for translation, and engineering evidence-based learning experiences to scale.

Translational considerations: scale-down and scale-up

An important objective for education research is taking promising approaches to scale in order to reach a wide range of learners. By recognizing some learning environments as complex systems (Jacobson et al. 2019) we also acknowledge the challenges that we face when attempting to scale successful designs to new ecological settings and participants.

We advocate using "scale-down" methodology (Nathan and Alibali 2010; Nathan and Sawyer 2014) to take promising innovations to scale. Scale-down methodology starts with the critical role of the *learning context* in order to make educational innovation and improvement viable at scale (e.g. McDonald et al. 2006; Penuel et al. 2011). Scale-down is an alternative and complement to "scale-up" approaches that have traditionally been defined in terms of the breadth of dissemination and level of fidelity (Glennan et al. 2004). In education, scale-up approaches tend to start with the rigorous study of isolated, elemental learning phenomena (Nathan and Alibali 2010). Taking these successful interventions to scale often involves considerable practical adaptations that can severely alter its success. Consequently, it is rare to implement successful scale-up of research-based interventions to authentic learning settings (McDonald et al. 2006). This is due in part to the realization that many contextual influences that fall outside of the original intervention are significant barriers for successful translation to authentic learning settings (Penuel et al. 2011).

Scale-down methodology encompasses research on design-based systemic reform (e.g., McDonald et al. 2006; Penuel et al. 2011; Reinking and Bradley 2008). This research suggests that effective reform must conform to the constraints of the local learning environments, and that teachers and other practitioners need to be recognized for the roles they play in implementing reforms (Dede 2006; McDonald et al. 2006; Sutton et al. 2016). Scale-down departs from traditional scale-up approaches in several ways. First scale-down advocates examining learning from a systemic perspective (e.g., an ethnography or case study), in the authentic, cultural contexts in which these complex behaviors naturally occur, such as the classroom (Nathan and Sawyer 2014). Second, analysis at the system level informs the generation of hypotheses for how to improve overall performance by identifying nearly decomposable malleable factors hypothesized to impact system performance. Third, functional decomposition of a malleable factor is applied to preserve the complexities of the authentic contexts (e.g., classroom culture) while reducing complexities sufficiently to allow for relatively rapid and inexpensive testing and iterative redesign—that is, use of educational "wind tunnels." Fourth, elemental analytic methods suited to more fine-grained learning behaviors address causal questions that can verify adequate subsystem performance or inform redesign and subsequent subsystem testing (Sandoval 2004). Finally, following adequate redesign, a critical step in the scale-down method is reintegration of the revised subsystem back into the larger system. An example follows where in situ geometry education is redesigned using modifications of the activity systems that interject embodied approaches to mathematical instruction.

Case study: redesign to support learners

Classroom mathematics learning through embodied game play

We consider how this applies to a case study of students using *The Hidden Village* in their classroom. We take the classroom learning experience as our complex adaptive system. The classroom is in a Midwestern Title 1 high school in the US where 60% of the students receive free or reduced priced lunch, and 33% of the students are white. Furthermore, to focus our analyses, we describe the experiences with one extended episode in a section of an all-ELL (English Language Learners) geometry classroom, whose native languages and cultures included Spanish from Central and South America, French from North Africa, Hmong from Southeast Asia, and Chinese. We witnessed how students dealt with the delivery of the game narrative, instructions, and mathematics in English, and how they employed their bodies and the hands and arms of other students to reason mathematically and formulate their justifications and proofs.

We selected this case for several reasons. First, we are motivated to support mathematical reasoning and engagement for linguistically and culturally diverse student populations in order to provide greater opportunities and access to STEM career pathways and areas of advanced study. Second, participants were highly engaged and showed an expanded view of how THV could be used collectively to foster deep mathematical reasoning. Third, the specific episode revealed limits of the current THV design, and thereby created a compelling motivation for re-design.

As context for this classroom-based intervention, players started in dyads to test the hypothesis of whether the advantages of actually *performing* directed actions for a particular mathematical conjecture during game play differed from *observing* someone else perform the actions. We used a yoked control experimental design. Dyads were initially formed in consultation with the classroom teacher, and adjusted to suit the availability of students as they arrived.

The demands of processing the mathematics in English raised many issues, as students sometimes knew related concepts being referenced in their native language but were not familiar with the words in English. To address these comprehension and production challenges students turned dyadic game play into a larger, collective activity. Students in and across dyads contributed to each other's successful game play by translating narratives and conjectures for each other, and using directed actions to clarify and ground the math. Gestures traveled from the game to students and among students as they served to ground their mathematical ideas. Based on the classroom observations, we introduce redesigns (below) to support students' collaborative co-design that directly connects their game playing experience with enactive processes for grounding mathematical thought. In this way, geometry concepts are less likely to become disembodied theoretical knowledge that are (re-)produced by students through rote verbal reporting. Rather, their collaborative interactions supported intuitive embodied processes for operating with space and shapes.

Our case study deals with students pondering a *false* conjecture: *If you double the length and the width of a rectangle, then the area is exactly doubled.* Selected events in the episode are shown in Table 2. Student-1 (who speaks some English) is paired with Student-2 (who is Hispanic with very limited English proficiency). Student-2 read the conjecture and performed the directed actions indicated in the game (see the images in Table 2, Row 1) but remained confused. His partner, Student-1, offers an example using a rectangle with a length of 3 and width of 2, illustrating with her arms (14:46, seated wearing a grey sweater). Student-1 clarifies that Student-2 needs to think about the effect of doubling length and width on the area inside the rectangle, and uses the laptop keyboard (15:00) as a shared referent in front of them to ground the notion of area of a rectangle, sweeping her open palm across the keyboard 2 times to show the area inside, while she says "here." Student-2 giggles (15:13) and holds his face as he tries to understand his partner. To be responsive, Student-1 asks for help translating words to Spanish (15:19), and beckons Student-3 over (standing, wearing a dark blue jacket) and explains the point she is trying to convey to Student-2 (15:30). Then Student-3 (15:42), as an interpreter between Student-1 and Student-2, restates in Spanish what Student-1 said, while mimicking the arm shape used by Student-1. Student-2 replies "*Todo todo todo*" to Student-3 (16:00) while making several small circle motions between them, and then (16:09) gestures across a nearby tabletop to express area within a rectangle, in a manner that seems to echo the gestures performed earlier (at 15:00) by Student-1.

Scale-down informs learning system redesign

These classroom experiences depict a complex, multimodal set of interactions involving members of an established class speaking multiple languages and using gestures and other actions: Students reenacted directed actions from the game in verbatim and modified forms, used gestures to elucidate mathematical talk, managed turn taking, repeated gestures from other speakers, and directed other students' attentions towards material objects, such as the laptop and a nearby tabletop that served as metaphorical problem spaces.

Table 2 Students of an all-ELL class discuss a conjecture and its relevant actions (figure turns green when players match poses)

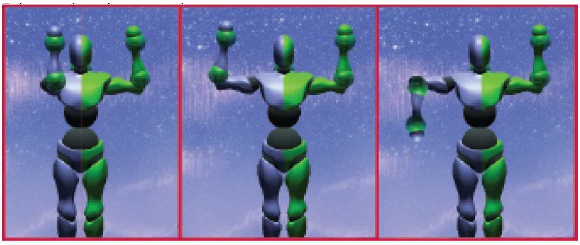
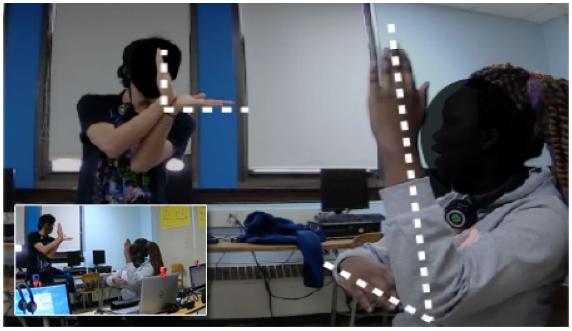
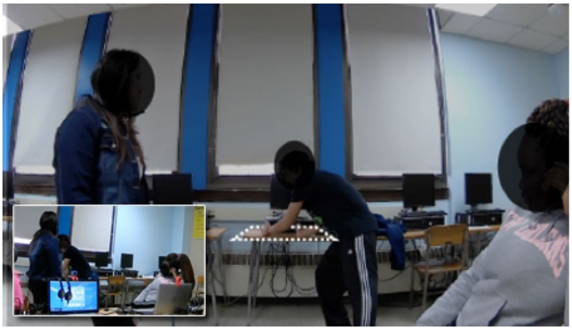
Conjecture on the screen [14:00] If you double the length and the width of a rectangle, then the area is exactly doubled. (FALSE)	
[14:46] *Student-1* (Seated)**:** "Let me show you, if the length were three" *Student-1* raised both arms to gesture two perpendicular lines meeting. *Student-2* (Standing) copies *Student-1*	
[15:00] *Student-1* "The area is here inside the rectangle." *Student-1* uses the laptop keyboard as a rectangle and sweeps her open palm across it two times to show area inside	
[15:13] *Student-2* giggles and holds his face as he tries to understand *Student-1*	

Table 2 (continued)

[15:19] *Student-1*: "Hey <name> Can you explain in Spanish what I mean?"	
[15:30] *Student-1*: "Here they doubled the length." With her raised arms, *Student-1* again gestures two perpendicular lines meeting	
[15:42] *Student-3* acts as an interpreter, restating in Spanish what *Student-1* said while making the arm shape	

Table 2 (continued)

[16:00] *Student-2*: "Todo todo todo." *Student-2* makes several small circle motions between them with his right hand with the index finger extended	
	
[16:09] *Student-2* uses table top behind him to indicate area inside the rectangle	

One notable observation was how students' embodied forms of mathematical reasoning acted to bridge students' language barriers in at least two ways. First, gesture production seemed to help Student-2 to access mathematical ideas even when he was unable to produce the English words, while supporting deep insights about explorations of the generalizable properties of space and shape. Second, in this multilingual environment, gestures helped facilitate peer interactions. These productive exchanges drew students into the mathematical ideas and showed how students engaged in rich discussions as they strive to make meaning in ways that support generalized geometric properties.

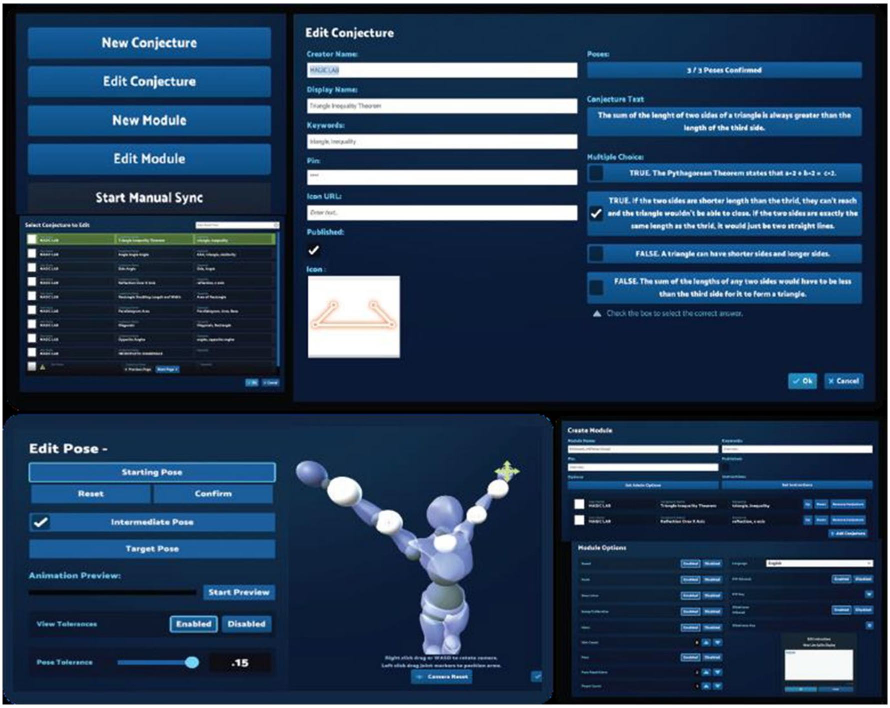


Fig. 2 The Conjecture and Module Editor of *The Hidden Village:* (*top-left*) Main Menu and Conjecture Selector; (*top-right*), Conjecture Editor; (*bottom-left*) The Pose Editor; (*bottom-right*) Module Builder and Admin Panel

"Wind tunnels" for learning environment design

As designers of educational environments, we observed a complex tangle of elements and interactions, and hypothesized about the design changes in this complex system that would support students' learning. Our "wind tunnel" is the experimental classroom, which takes place with developmentally appropriate participants within the current school culture and curriculum sequencing. Findings that arise from this engineered setting can be implemented in classrooms at large.

In the process of testing THV in the classroom, we also identified nearly decomposable sub-components. These can be altered in environments functionally similar to high school classrooms. In these settings, we can collect data on the relative efficacy of the intervention for improving geometric reasoning. The most dramatic observation for us was how students combined our pre-specified directed actions from game-play with their own customized gestures to reason through the mathematical relations. This proved to be very valuable for their engagement and mathematical success.

To support this in future interventions, we created a new directed actions module (recently implemented Pose Editor) that allows players to create, edit or remove movements from The Hidden Village game (see Fig. 2). User-created movements made by students (or teachers) can then be shared with any other players via a cloud-based conjecture library. This re-design allows us to experimentally compare whether it is more effective to have students observe and copy directed actions that we created ahead of time (as observed in Row 1 of Table 2), or allow students to create their own directed actions.

We also envision (and have partially implemented) an administrative “control panel” that allows researchers to customize the settings for players’ experiences (i.e., treatment conditions), including set the language, and control the number of players, number of conjectures, number of times the directed actions must be performed, and so on. Methodologically, these design options may be viewed as factors that the researcher team can add, subtract or edit for A-B experimentation during game play. These various design features then need to be tested with developmentally appropriate students in classroom-like settings.

Discussion

Originally, we designed the classroom learning experience with THV like a 1000-piece watch with no sub-assemblies. The system worked well when everything proceeded as planned, but changes in the gaming experience required costly system-wide revisions to the technology and the activity structure. Our early observations revealed many shortcomings that were outside of the scope of the nascent theories of learning and student use. These shortcomings could be reliably addressed only through iterative cycles of empirical observation and redesign.

To guide the redesign of this video game intended to promote collaborative and embodied learning in the classroom, we adopted a *materialist design* approach, influenced by scholarship on materialist epistemology (Baird 2004) and embodied conjectures (Sandoval 2004). Because materialist epistemologies are broad, we recognize that other philosophical interpretations may suggest different approaches. Our approach is offered as a starting point for educational designers and learning scientists to explore ways to develop and refine theory-based interventions that are responsive to local constraints of a given learning setting. We believe that there are insights to be gleaned from our presentation that point out the potential benefits of an embodied perspective to educational design.

First, a materialist epistemology orientation enables a shift away from symbol-driven models of learning and supports theorization even when existing theories are inadequate. For example, despite a theory of embodied mathematical reasoning that influenced the initial design (Nathan and Walkington 2017), we initially lacked sufficient theoretical understanding of the kinds of collaborative and extended peer interactions that could be elicited from game play with *The Hidden Village* (Walkington et al., in press). Fortunately, the existing technology served as an “embodied conjecture” (Sandoval 2004, p. 215) of an emerging understanding of embodied collaborative learning that was subject to refinement through iterative redesign.

Second, near-decomposability combined with functional decomposition helps manage the complexities of the design of complex learning environments and provide for less costly and less time-consuming redesigns. It proved to be beneficial to reconceptualize the game architecture in terms of modules that mapped to classroom use.

Third, educational designers can benefit from the Wright brothers’ ingenious usage and enhancement of the wind tunnel. We found it valuable to evaluate critical design innovations in a controlled, scaled-down environment of the experimental classroom (our “wind tunnel”) that sufficiently emulated the ecologically valid learning conditions of the classroom. Maintaining this microcosm of complexity enabled us to still obtain high quality data of rich learning interactions that could inform future design changes. These changes could then be re-integrated back into the complex system of the authentic educational setting, contributing to a new design that is more responsive to the needs of the learners and the constraints of the learning environment.

It may seem tempting to regard educational design as merely an application of scientific learning theory. But doing so would put us in the same trap as marginalized aeronautical engineers who have preceded us and made enormous contributions to both engineering and basic science. Educational designers are often guided by formal psychological and sociological theories of learning and social interaction. Formal learning theories are always inadequate to completely specify classroom-ready designs, especially as one strives for scale. Inevitably, there are myriad design features that likely matter for which there is no theoretical guidance (Nathan 1998). Conversely, learning environment designs are reifications of conjectures about learning (Sandoval 2004) that are as yet not empirically established. Even with uncertainty and inaccuracies in the theories, people still need to learn and teach, classrooms must still convene, children must continue to advance; and so educational designers must move forward. Materialist design frees the design process from a restrictive paradigm that would have designs strictly follow formal theory to applied theory. Materialist design recognizes the value of materialist epistemological approaches that underscore the primacy of our physical and lived experiences to drive design, improve learning and ultimately inform evolving theories of knowing and learning.

Taking an embodied approach to learning environment design freed us up to think about ways people already engage with space and shape via body structure and movement. It highlighted how barriers to mathematics understanding in the form of language proficiency and mathematical formalisms (Nathan 2012) can be transcended through embodied forms of movement and interactions– principally gesture production and comprehension.

This work also strives to place educational design in terms of complex systems theory. You (1993), Cilliers (1998), Jacobson et al. (2016), and others, note that complex systems cope with a continually changing environment by exhibiting two emergent traits, representation and self-organization. First, complex systems need to *represent* and *retain* information about the environment and its relation to the internal states of the system in some form of non-localized, distributed representation. One especially notable way this was evident in the THV learning environment was how geometry knowledge was elicited through interactions with the game. Knowledge showed up as “extended” representations, as when students spontaneously and flexibly use their bodies, the bodies of others, and the objects and spaces in their environment to depict, communicate, and explore (their thoughts of) geometric relations. Walkington and colleagues also observed this form of extended, embodied representation among high school students (Walkington et al., in press) and mathematics teachers (Walkington et al. 2019).

The second emergent trait is *self-organization* in service of adaptation to changing conditions. From a complex systems perspective, such self-regulatory behavior is decentralized, arising from the nature of the interactions of the simpler elements in light of environmental demands. Here it is important to reflect on the boundaries of the system under investigation. In one sense, THV-based learning is adaptive because it is embedded in a cycle of evidence-based design-test-redesign, as described in the case study. A key adaptation to observing the broad range of distributed, body-based and materialist mathematical representations exhibited by members of the classroom learning environment using THV was for the design team to expand the capabilities of the game to create new movements from players. The re-design allowed players to add new conjecture-movement combinations to the game, which are then disseminated to the user community for future game play (a design change that has since been implemented; Swart et al. 2020). In another sense, learning analytics on players’ movements provided feedback directly to the motion-capture component. Together, this enhanced THV’s capacity to adapt to the range of movements players may execute, as well as the variety of body types among our players (another design change that is already implemented and in use).

We also are pursuing future adaptation capacities. For one, as the conjecture library grows, we foresee users (both students and teachers) offering multiple instances of individual conjectures paired with different player-created directed actions (as already observed on a small scale). This offers new opportunities to enable the motion-capture component of THV to adapt its performance to optimize the intended types of embodied mathematical reasoning based on player performance. Movement sequences that lead to higher rates of matching across multiple body types, and that yield the intended level of performance on metrics of mathematical reasoning would receive higher activation than other directed actions, thereby increasing the likelihood these are selected for inclusion in modules for subsequent classroom activities.

The authors were invited to speculate on how this materialist design approach can be nurtured. To nurture this process, designers will likely benefit from interrogating their own educational designs for implicit material epistemologies about knowledge and learning. Along with the pragmatic choices they may make about their designs, this will provide guidance for places in their complex learning systems to make functional decompositions. Educational designers also need versions of their own wind tunnels; metaphorically, environments that offer microcosms of the full range of complexities that are present in the target settings, yet which allow for the kind of data collection that can meaningfully inform system redesign. Ultimately, the likelihood of scaling these technological innovations will depend on the fit of this microcosm to the authentic learning environment.

Finally, designers must see themselves as material theorists. In accordance with materialist epistemology, educational designers must recognize that the learning environments they design and use in authentic environments *are* the learning theories that they are investigating (Sandoval 2004). The linguistic re-descriptions of them are abstractions that background important qualities and foreground others. This makes efforts to take abstractions of designs to scale challenging as local constraints influence implementation and learner performance. An embodied perspective on learning environment design positions *learning engineering* on par with attempts to articulate a science of learning as a means toward improved learning experiences.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflicts of interest.

Appendix

See Table 3.

Table 3 The 8 conjectures the relevant directed actions used in *The Hidden Village,* an embodied video game, along with the corresponding mathematical insights

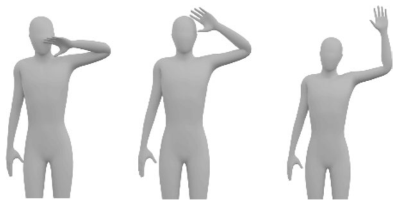
Conjecture	Relevant actions	Insight
Angle angle angle [False] Given that you know the measure of all three angles of a triangle, there is only one unique triangle that can be formed with these three angle measurements		Refers to similar triangles or infinitely many/family of triangles Shape or size may change and preserve angles
Triangle side [True] If one angle of a triangle is larger than the second angle, then the side opposite first angle is longer than the side opposite the second angle		Relation of angular measure to opposing side
Area of rectangle [False] If you double the length and the width of a rectangle, then the area is exactly doubled		Refers to formula for area of a rectangle

Table 3 (continued)

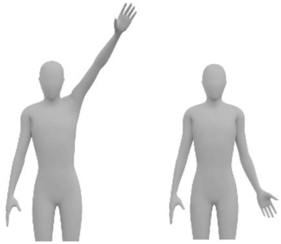
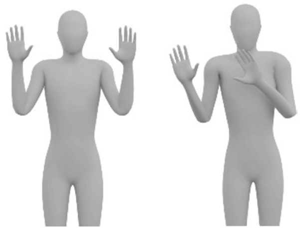
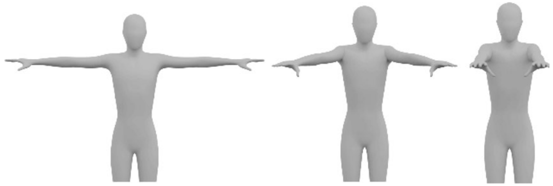
Conjecture	Relevant actions	Insight
Reflection [False] Reflecting any point over the x-axis is the same as rotating the point 90 degrees clockwise about the origin		Posits it is true based on specific case(s) (e.g., 180 degree rotation)
Parallelogram [True] The area of a parallelogram is the same as the area of a rectangle with the same base and width		Says a parallelogram is a rectangle skewed, tilted, pushed over Says a parallelogram and a rectangle have the same formula for area
Diagonals [True] The diagonals of a rectangle always have the same length		Refers to similar triangles

Table 3 (continued)

Conjecture	Relevant actions	Insight
Triangle inequality [True] The sum of the length of two sides of a triangle is always greater than the length of the third side		The sum of the sides cannot meet
Vertical angles [True] The opposite angle of two lines that crosses are always the same		Refers to supplementary angles

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Mitchell J. Nathan is Vilas Distinguished Achievement Professor of Learning Sciences, in the Departments of Educational Psychology, Curriculum & Instruction, and Psychology at the University of Wisconsin-Madison, and a Fellow of the International Society of the Learning Sciences. His research focuses on how cognitive, social, and embodied processes are employed in teaching and learning in mathematics, engineering, and integrated STEM. Dr. Nathan is a founding member of the EMIC (Embodied Mathematical Imagination & Cognition) Consortium, embodiedmathematics.com

Michael I. Swart is an artist and designer working in the Learning Sciences as an Associate Researcher at The University of Wisconsin-Madison. His work specializes in designing virtual inter-activities that make learning fun, interesting, accessible and equitable