RESEARCH PAPER

Exploring the Alignment Between Pre-service Mathematics Teachers' Beliefs and Espoused Practice

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Researchers in mathematics education have focused on teacher beliefs as an important area of study because of their influence on teaching practices. In this study, we focused on two aspects of beliefs, the nature of mathematics and the teaching and learning of mathematics, with eight pre-service mathematics teachers (PMTs) to explore alignment between these beliefs and their espoused teaching practice. Data were collected through questionnaire prompts and semi-structured interviews. Analysis revealed that the PMTs generally held mixed beliefs about both the nature of mathematics and about its teaching and learning, indicating little alignment within and between the beliefs expressed and their espoused practice.

In trying to understand what contributes to the quality of mathematics teaching, some researchers have attributed the predominant influence of teacher beliefs. For example, studies have shown the influence of teacher beliefs for decision-making and in underpinning teacher actions in mathematics classes (Beswick, 2006). As such, beliefs are increasingly seen as a key component of pedagogical content knowledge (PCK) (e.g., Hashweh, 2005), and even Shulman, the original conceiver of PCK, later recognised the lack of consideration of non-cognitive attributes (including beliefs) in his initial theorisation (Shulman, 2015).

> The affective aspects of teacher understanding are important both because a lot of what teachers 'know and do' is connected to their own affective and motivation states, as well as the ability to influence the feelings, motives, persistence, and identity formation of their students (p. 9).

Whilst some researchers have explored teacher beliefs and teaching practices together (e.g., Beswick, 2012), and others have focused on the consistencies and inconsistencies of teacher beliefs to teaching practices (Roehrig et al., 2009), little is known about the alignment for pre-service mathematics teachers (PMTs). When PMTs enrol in mathematics education courses, they have already developed beliefs about the nature of mathematics, and its teaching and learning from their own experiences of learning mathematics (Beswick, 2019). In this article we aim to better understand how beliefs might account for teaching practices of pre-service mathematics teachers (PMTs) in the early stages of becoming mathematics teachers. We propose that studying the alignment between PMTs' beliefs and their espoused practice can help to understand why they perceive teaching of mathematics in the way they do. Furthermore, the study illuminates the need for teacher educators to be aware of PMTs' beliefs at the beginning of their courses so that they can account for and help to strengthen or change those beliefs and inform the design of teacher education programmes. Our main research question is thus: How do PMTs' beliefs about the nature of mathematics align with their beliefs about the teaching and learning of mathematics?

Literature Review

Several beliefs relating to mathematics, have been identified by researchers, including beliefs about the nature of mathematics, and beliefs about the teaching and learning of mathematics (e.g., Beswick, 2007). These two aspects of beliefs unveil mathematics teachers' views regarding the role of teachers and teaching, and the nature of mathematics activities (Weldeana & Abraham, 2014). For instance, two teachers holding the same knowledge about

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teaching mathematics, may teach differently due to the differences in beliefs they hold, because they use their existing beliefs to interpret whatever comes into their mind (Stipek et al., 2001).

Beliefs about the Teaching and Learning of Mathematics

Beliefs about teaching and learning of mathematics are related to the choices teachers make with respect to their teaching roles, the nature of instruction, and the activities and resources used in their mathematics classroom teaching practice (Ernest, 1989). Yang et al. (2020) related these beliefs to teachers' mathematical understanding and their preferred mathematical activities, teaching approaches, and their conception of how mathematics is learned. The most frequently considered beliefs about the teaching and learning of mathematics are transmissive and social constructivist (Meschede et al., 2017). In the transmissive view, mathematics teachers see effective teaching and learning of mathematics as teacher-centred, where the students' role is to follow their teacher's instructions (Meschede et al., 2017). The emphasis is on memorisation of rules, procedures, and facts.

From a social constructivist view, mathematics teaching challenges students' thoughts and guides them towards a complete understanding of mathematical concepts (Weldeana & Abraham, 2014). Students are involved in doing mathematics and developing different ways to solve mathematical problems or tasks, as opposed to being passive recipients of knowledge (Ernest, 1989). Students take an active role by individually processing and constructing knowledge (Meschede et al., 2017) as opposed to merely following procedures. This view is evidenced in the way a mathematics teacher uses teaching and learning resources, provides autonomy to students, and considers varied ways to arrive at the correct answer. In this article we consider social constructivist teaching as an effective teaching practice for mathematics.

Beliefs about the Nature of Mathematics

Beliefs about the nature of mathematics are closely related to the question, "What is mathematics?' and these beliefs are considered to have more impact on mathematics teachers' practices than do beliefs about the teaching and learning of mathematics (Beswick, 2012). Beliefs about the nature of mathematics are generally conceptualised as either static or dynamic (Weldeana & Abraham, 2014). The static position is that mathematics is a body of formulas and mathematical facts that are procedure-driven (Yang et al., 2020), and suggests learning mathematics means an accumulation of facts, rules, procedures, and skills for the fulfilment of some external end, or producing one correct answer (Ernest, 1989). Such beliefs might be seen to produce teachers who teach in a traditional teacher-centred way (Stipek et al., 2001). In contrast, those holding a dynamic view of nature of mathematics understand the nature of mathematics as a process of inquiry (Yang et al., 2020), similar to a problem-solving view (Ernest, 1989) in which mathematics is seen as an active sphere of human invention and creation that is always growing. Likewise, mathematics is regarded as a tool for thought (Stipek et al., 2001). Mathematics teachers who hold this view are likely to employ student-centred teaching often associated with improved students' learning (Baeten et al., 2016).

PMTs' Beliefs and the Issue of Alignment

If we want to help PMTs develop effective teaching practices, then we need to address their beliefs (Beswick, 2006). Research has focused on how PMTs' beliefs about the nature of mathematics and its teaching and learning relate to their teaching practice (e.g., Yang et al., 2020), but little is known about the alignment between these beliefs (Penn, 2012) and how this alignment or misalignment influences their practice. We propose that it is important to investigate these alignments at this early stage of the PMTs' careers to better understand how their beliefs inform their decisions in planning to teach. The expectation is that when a PMT holds static beliefs about nature of mathematics and transmissive beliefs about its teaching, then their beliefs are aligned suggesting a teacher-centred approach. Whereas a PMT holding dynamic and social constructivist beliefs would be expected to align with a learner-centred approach (Baeten et al., 2016). PMTs holding static and social constructivist beliefs, and vice versa, are seen to have misaligned beliefs and we have no expectation about their teaching.

Methodology

A qualitative research method was employed to explore the alignment of eight PMTs' beliefs about the nature of mathematics, its teaching and learning, and their espoused practice. Qualitative research enables analysis of the how and why (Yin, 2009), and here, the how and why of alignment of each of the PMTs' beliefs. In this study, the PMTs were in a teacher education college setting. Detailed information was collected using a variety of qualitative data collection instruments such as questionnaire prompts, and semi-structured interviews.

Participants

Eight PMTs in their second year, second semester (their last year) were purposively selected from one teacher education college in Tanzania. Second year PMTs have participated in their first block teaching practice (BTP) and therefore have some experience of teaching mathematics in classrooms. Twenty PMTs were invited and eight consented to participate. Ethical approval was granted under the guidelines of the Social Sciences Human Research Ethics Committee (SSHEC) in Australia (project number 22979).

Data Collection

Data were collected through questionnaire prompts and semi-structured interviews. Two questionnaire prompts were provided to each PMT, one on beliefs about the nature of mathematics and the other on the beliefs about the teaching and learning of mathematics. PMTs were to agree with the prompts they thought were correct. Semi-structured interviews were conducted with each PMT to gain more information on their beliefs and their perceived teaching of mathematics. The duration of the interviews ranged from 30 to 45 minutes. Questions were based on the beliefs identified and some guiding questions such as:

> In your opinion, what is mathematics? How would you teach your students operations on fractions? (Choose one operation and describe how you would teach it).

Analysis

The prompts endorsed by the PMTs in the questionnaire reflected two beliefs about the nature of mathematics, and the teaching and learning of mathematics respectively. We employed a deductive thematic analysis process for the data from the semi-structured interviews (Braun & Clarke, 2006), which was managed using NVivo software. The data were coded deductively using elements drawn from the literature as static and dynamic (beliefs about the nature of mathematics, e.g., Ernest, 1989; Stipek et al., 2001), and as transmissive and social constructivist (beliefs about teaching and learning mathematics, e.g., Tatto et. al., 2008). The first author carried out the initial coding, and the other two research team members reviewed sample coding from each participant. There was no disagreement between authors regarding the codes, but there was a considerable discussion concerning the representations and explanations of the examples for teaching. For example, for Francis we discussed the seeming contradiction between a constructivist position of his fraction images and the transmissive position of his interview statements.

Findings: Questionnaires

In this paper, we present data from three PMTs, Francis, Liam, and Shaibu. These three were chosen because they represented a range of different beliefs. The belief prompts that were endorsed by each of the three PMTs regarding the teaching and learning of mathematics are shown in Table 1, and about the nature of mathematics in Table 2.

Table 1
Prompts on Teaching and Learning of Mathematics that were Agreed by Each PMT

Transmissive Prompts	Names*			Social Constructivist Prompts	Names*		
	FR	LI	SH		FR	LI	SH
The best way to do well in mathematics is to memorise all formulas	A**		A	In addition to getting right answer in mathematics, it is important to understand why the answer is correct	A		
Students need to be taught the exact procedures for solving mathematical problems	A		A	Teachers should allow students to figure out their own ways to solve mathematical problems	A	A	
It doesn't really matter if you understand a mathematical problem, if you can get the right answer	A			Time used to investigate why a solution to mathematical problem works is time well spent	A		
Students learn mathematics best by attending to teachers' explanations	A		A	Students can figure out a way to solve mathematical problem without teacher's help			
When students are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed	A			Teachers should encourage students to find their own solutions to mathematical problems even if they are inefficient	A	A	
Hands-on mathematics experience is not worth the time and expense	A			It is helpful for students to discuss different ways to solve problems	A		

*FR: Francis, LI: Liam, SH: Shaibu (note same in Table 2)
** means the participant agreed with the statement (note same in Table 2)
A = Agree

Summarising from Table 1 and Table 2, we see Francis agreeing to all prompts in both tables except for the prompt stating, "Students figure out ways to solve mathematical problems without teachers' help" (Table 1), which suggests he holds mixed beliefs. Liam agreed to only dynamic prompts in Table 2, and only social constructivist prompts in Table 1 (though he chose few prompts in each case). Whereas it was difficult to tell which side Shaibu was based on with respect to the nature of mathematics, as he agreed to two prompts in the static scale and only one in the dynamic scale (Table 2). However, as shown in Table 1, he agreed to only transmissive prompts, which suggests he leant towards this view of teaching mathematics.

Table 2
Prompts on Nature of Mathematics Agreed by Each PMT

Static Prompts	Names*			Dynamic Prompts	Names*		
	FR	LI	SH		FR	LI	SH
To do mathematics requires much and correct application of routines	A		A	Many aspects of mathematics have practical relevance	A		
Fundamental to mathematics is its logical rigor and precision	A			Mathematical problems can be solved correctly in many ways	A		A
Mathematics involves the remembering and applying definitions, formulas, mathematical facts, and procedures	A		A	In mathematics many things can be discovered and tried out by oneself	A		
Mathematics is the collection of rules and procedures that prescribe how to solve mathematical problems	A			Mathematics involves creativity and new ideas	A	A	
Mathematics means learning, remembering, and applying	A			Mathematics helps to solve everyday problems and tasks	A		
When solving mathematical tasks, you need to know the correct procedure, else you would be lost	A			If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts)	A	A	

A = Agree

Findings: Semi Structured Interviews

The beliefs expressed in the questionnaire prompts about teaching and learning mathematics were reflected in the PMTs' interview accounts of how they would teach their students operations on fractions. The three examples are set out below.

Liam started by emphasising the procedures to be followed when dividing fractions. When asked if there is any other way or representation, he still emphasised the procedures:

> Students will have to follow the division procedures as they are in the textbook such as for $\frac{a}{b} \div \frac{c}{d}$, the first procedure is to invert the $\frac{c}{d}$ to be $\frac{d}{c}$ and multiply it by $\frac{a}{b}$. Thus, $\frac{a}{b} \times \frac{d}{c}$.
>
> For example, $\frac{1}{3} \div \frac{2}{6} = \frac{1}{3} \times \frac{6}{2} = \frac{6}{6} = 1$

When asked if there is any other representation for this, his answer was:

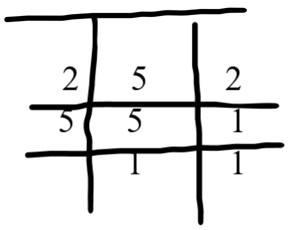
> No, there is no other way, they are supposed to just follow the procedures I explain.

Further, when asked why we invert the second part of fraction when doing division, his response was:

> We invert to change the division sign into multiplication sign, it is the procedure.
>
> The first procedure here is to change those mixed numbers to improper fractions, this will be;
>
> $(2 \times 3) + \frac{1}{2} \div (5 \times 6) + \frac{2}{5} = \frac{7}{2} \div \frac{32}{5} =$, next step is to invert the right-hand side and then multiply by the left-hand side. Thus, $\frac{7}{2} \times \frac{5}{32} = \frac{35}{64}$. The important thing for students is to make sure they follow the procedures as they are.

In his interview, Shaibu started by telling how he would guide his students in learning how to add fractions with the same denominators and those with different denominators, explaining it as follows

> I would like to teach using examples. For example, evaluate the following fractions
>
> 1. $\frac{3}{5}+\frac{1}{2}$ and 2. $\frac{2}{8}+\frac{1}{8}$. …… Here the first thing I will teach my students is to find LCM (lowest common multiples) for denominators 5 and 2 by using prime factorisation procedure because denominators are different. But if the denominators were the same no need for LCM.



$LCM = 2 \times 5 = 10$

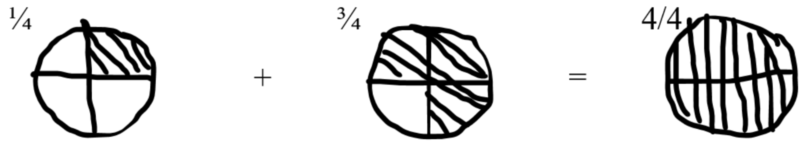
> Therefore, $\frac{3}{5}+\frac{1}{2} = \frac{((10 \div 5)\times 3)+((10 \div 2)+1)}{10} = \frac{(2\times 3)+(5\times 1)}{10} = \frac{6+5}{10} = \frac{11}{10} = 1\frac{1}{10}$, since the numerator is greater that denominator, I will tell them to change it to mixed number.

When we asked Shaibu about why he was controlling everything in the class, his response was:

> Because some students are slow learners, so as a teacher you need to make sure you have covered all that is required.

Francis, however, proceeded as follows:

> I will go with 2 oranges and cut them into four pieces, to teach that $\frac{1}{4}+\frac{3}{4} = \frac{4}{4} = 1$, I will take one piece from the first orange and let them know that's a ¼ and then take 3 pieces from the other and that's ¾, I will then put those pieces together and they will see that it goes back to 1 orange. (Francis).



When asked if there is any other way to arrive at the solution or teach the concept, Francis acknowledged varied representations for arriving at the correct answer. For example, he said:

> … because I will be in the class teaching, I may use students as my real objects let say the class has 60 students where 20 are girls and 40 are boys. Then I may tell them that they are a fraction of:
>
> $\frac{20}{60}+\frac{40}{60}=\frac{60}{60}=1.$

In general, Francis held misaligned beliefs between the questionnaire prompts, when compared to his espoused practice. Liam held a contradicting belief between those identified in the questionnaire prompts, and the semi structured interviews. While this confirmed his leaning towards a dynamic view of mathematics, he shifted from a social constructivist to a transmissive view of the teaching of mathematics. Moreover, looking closely at the example two which he provided, we noticed some mathematical error or procedural/conceptual mix-up. Instead of,

$$\frac{(2\times 3)+1}{2} \div \frac{(5\times 6)+2}{5}, \text{ he wrote } (2\times 3)+\frac{1}{2} \div (5\times 6)+\frac{2}{5}$$

These two representations have two different answers; however, we are not sure if this was just a typo error. Shaibu's interviews confirmed he holds a largely static view of mathematics, and a transmissive view of teaching, as shown in his teacher-centred approach in his espoused teaching.

In summary, the espoused teaching practices of the three PMTs were largely procedural and transmissive in nature, even when there were teaching and learning resources for students to manipulate (e.g., Francis). They acted as a knowledge giver, viewing the students as recipients of knowledge.

Discussion

The purpose of this study was to explore the alignment of PMTs' beliefs about the nature of mathematics and the teaching and learning of mathematics. Through qualitative research, the PMTs were asked to complete open-ended questionnaires and participate in semi-structured interviews. Whilst there is evidence that mathematics teachers' beliefs about the nature of mathematics do align with their espoused teaching and learning beliefs (Ernest, 1993), the findings from the three PMTs presented in this report suggested both alignment and misalignment between the nature and teaching of mathematics. For instance, Francis showed misalignment in both beliefs. His beliefs about nature of mathematics and its teaching and learning through prompts were mixed within and between them, which makes him misaligned. Shaibu, who leaned towards the nature of mathematics as static and the teaching and learning of mathematics as transmissive in approach, suggested alignment in his beliefs. Liam suggested alignment in the beliefs identified via prompts as dynamic and social constructivist, however strongly learned towards transmissive approach in interview. The potential explanation for the misalignment of beliefs may be attributed to misunderstanding of the meaning brought by the prompts, or that the perspective that there are only two (incompatible) ways of understanding mathematics is incorrect. Further, the misalignment might imply that the PMTs were guided by both views in their teaching practices. The other reason might be attributed to the fact that, these PMTs were beginning teachers and therefore had shifting beliefs based on their previous learning experiences as well as still developing pedagogical practices.

The misalignment for Francis and Liam was also evident in their espoused teaching practices where they demonstrated a procedural way of teaching. Francis showed some elements of conceptually teaching or representations. He tried to teach using teaching and learning resources to help students understand the conceptual part of fractions. The teaching and learning materials he used during his espoused practice were figures like circles (mentioned about using oranges). The possible explanations for misalignment, might be they were trying to bring onboard what their tutors taught them (the experiences they went through or saw), but for some reasons they could not apply (if the experiences were student-centred). Maybe it was because they did not understand what they were taught or there was minimal practice on their BTPs. These findings confirm the findings by Penn (2012), who found that the majority of PMTs' beliefs about the nature of mathematics and the teaching and learning of mathematics did not align. Hence, Liam's contradictions in the prompts and Francis's shift from conceptual to procedural in espoused teaching might have resulted from the course expectation to teach in a constructivist way, even though the PMT held a transmissive approach.

These findings collectively suggest that it is possible for a PMT to hold beliefs that are contrary to the expectations we might have for their teaching practice provided by their beliefs (e.g., Liam and Francis). A further question to ask is whether these PMTs were aware of the contradictions in the beliefs that they held, and this remains as a suggestion for further studies.

Conclusion

Beswick (2006) suggested if we want to see changes in teacher' practice, we first need to study their beliefs and help them actualise their fullest potentials. In this regard, this study has endeavoured to study PMTs' beliefs and our findings suggested that PMT beliefs are often misaligned. Ernest (1989) suggested two key reasons that can cause misalignment between beliefs and practice as: 1) powerful influence of the social context, and 2) level of consciousness of beliefs and the extent to which the teacher reflects on their mathematics teaching practice. We therefore argue that these reasons may have caused misalignment between and within PMTs beliefs and these misalignments suggest a more nuanced understanding of PMT beliefs and how they relate to nascent views of effective practice.

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