The Nature of Research on Pre-service Teachers' Mental Mathematics: A Brief Systematic Review

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For pre-service teachers to develop learners' number sense, they need have a broad repertoire of non-standard calculation strategies. In this paper, we present an analysis of selected research literature from 2000 to 2022 on pre-service teachers' mental mathematics published in peer-reviewed journals. The results of this brief review indicate that more research on pre-service teachers' mental mathematics would be beneficial. Of the 12 studies examined, five promoted a deficit view of pre-service teachers' knowledge. The remaining seven differed on the starting point in developing pre-service teachers' knowledge of non-standard mental calculation strategies.

Introduction

Mental mathematics is regarded as central to developing learners' and pre-service teachers' [PSTs] number sense (e.g., Courtney-Clarke & Wessels, 2014). Number sense is regarded as elusive and contested with various authors describing it as "an intuitive feel for numbers" (e.g., Howden, 1989), a set of characteristics (e.g., Berch, 2005) or more broadly a framework for guiding the development of number sense (e.g., McIntosh et al., 1992). Whitacre et al. (2020) maintain that there are three different number sense constructs, each with its own traditions and characteristics. These three constructs include core number sense (e.g., Spelke, 2000), early number sense (e.g., Andrews & Sayer, 2015) and mature number sense (e.g., McIntosh et al., 1992). The focus of this article is on mature number sense. Mature number sense "refers to a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations" (McIntosh et al., 1992, p. 3). It includes "knowledge and facility with numbers", "knowledge and facility with operations" and the competence in "applying knowledge of and facility with numbers and operations to computational settings" (McIntosh et al., 1992, p. 4). Central to the development of mature number sense is the increasing use of various strategies in flexible ways when calculating (Graven et al., 2013).

Whitacre and Rumsey (2018) argued that for PSTs to develop learners' number sense, they need to be able to work flexibly with calculation strategies. It is through the process of calculating mentally that PSTs develop a range of strategies and are able to select strategies appropriate to the particular computational situation (Pourdavood et al., 2020). Verschaffel et al. (2007) distinguished between mental mathematics done *in one's head* and mental mathematics done *with one's head*. Mental mathematics done in one's head focuses on the memorisation of basic facts, whereas mental mathematics with one's head involves the flexible use of appropriate and efficient strategies for calculating. When PSTs are required to solve calculations in writing, they tend to draw on the standard algorithm rather than identifying which strategy is most efficient (Whitacre & Rumsey, 2018). Standard algorithms privilege knowledge of basic facts and implementation of taught procedures whereas mental calculations focus on the structure of number operations and their relationships (Rathgeb-Schnierer & Green, 2019).

For PSTs to develop learners' number sense, they need to be familiar with a variety of strategies for calculating, and these strategies should be "unpacked" in teacher education courses (Westaway & Vale, 2021). A focus on mental mathematics in teacher education

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mathematics courses should enable PSTs to notice, attend to, and respond to learners' strategies when calculating (Westaway & Vale, 2021). This study is based on a brief systematic review of research literature. The question asked is:

> *What insights can be gained from research on pre-service teachers' mental mathematics?*

Due to the constraints of writing a conference paper, a comprehensive review was not conducted. The aim of conducting a brief review was to identify the nature of research conducted with PSTs that could be explored in more depth through the literature in the future.

Methodology

The Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) framework guided the study. The PRISMA framework incudes four stages in the research process: identification, screening, eligibility and inclusion (Moher et al., 2015). A copy of the PRISMA framework generated from the review described in this section is included as Figure 1.

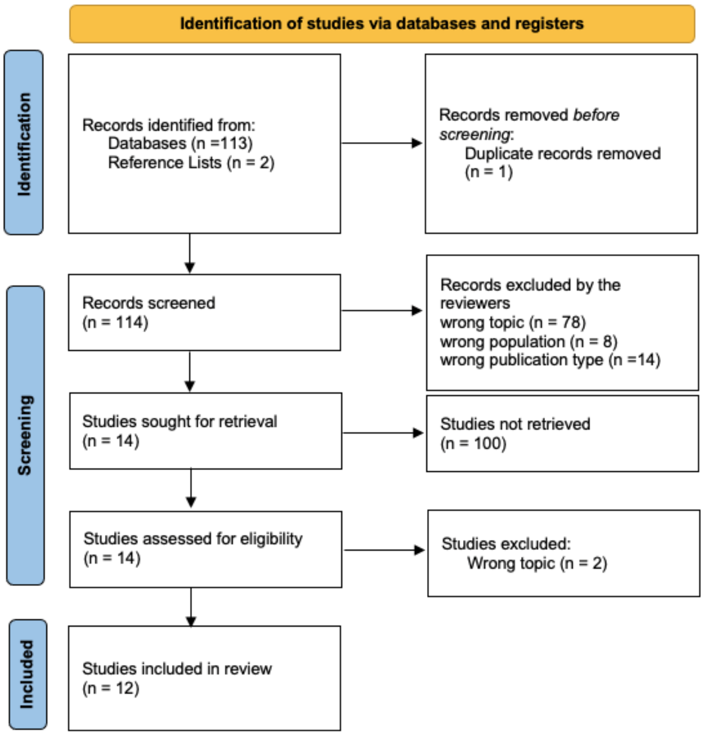

Figure 1. PRISMA Framework (Moher et al., 2015).

To initiate the process, the first author conducted a review on the EbscoHost database (Academic Search Premier, APA PsychInfo, APA PsychArticles, Eric and SocIndex) to identify articles that would relate to the research question. The keywords for the review included: "mental mathematics OR mental calculation OR mental computation OR mental arithmetic" AND "pre-service teachers OR preservice teachers OR prospective teachers OR initial teacher education OR student teachers". We specifically sought articles that were peer-reviewed and published between 2000 and 2020. The search identified 113 results. These were imported into Rayyan, an open-source software specifically designed for systematic reviews (rayyan.ai).

The authors read independently through the titles, abstracts, and keywords of each of the 113 papers and decided which of the articles should be retrieved. This screening process required that we develop a set of inclusion and exclusion criteria. The inclusion criteria focused on PSTs' mental calculations with whole numbers. The exclusion criteria included: wrong topic, wrong population, and wrong publication type. For the most part, the literature excluded focused on practicing teachers and/or students or mathematics topics other than whole number Examples of the wrong publication type were conference papers, books and professional articles. These publications were excluded because they do not always follow rigorous blind peer review processes. Through this process twelve of the initial 113 articles were deemed relevant to the study and were retrieved. A further review of the reference lists of the twelve articles identified another two articles. This brought the total number of articles retrieved to 14. These were read to assess their eligibility for inclusion in the review. Two articles were discarded at this point as they focused on rational numbers. The final number of articles included is twelve.

Results

Research on PSTs' mental mathematics between 2000 and 2022 is descriptive and illuminating. Table 1 shows the years in which each of the identified articles was published. Most prolific in authorship was Whitacre, who contributed to Whitacre (2015, 2018) and Whitacre and Rumsey (2018), three of the twelve articles reviewed.

Table 1
Research on PSTs' Mental Mathematics

2004	2008	2009	2013	2014	2015	2018	2019
2	1	2	1	1	1	2	2

Non-intervention Studies

A large proportion of the papers reviewed (5 out of the 12) presented case studies in which PSTs' ability to apply mental calculation strategies was assessed, with no intervention. These papers presented a deficit-focused perspective on PSTs' knowledge and ability without revealing any possible strategies to address the knowledge-gaps identified.

Tsao (2004) assessed the number sense, mental computation performance and written computation performance of 155 participants and concluded that the three were significantly correlated. They reported that the mean scores on the mental computation test and the written computation test were low. Courtney-Clarke and Wessels (2014) similarly used a number sense assessment with a written computation and a mental computation assessment to assess 47 final-year primary PSTs. Their study aimed to explore whether teacher knowledge was a possible factor explaining the poor performance of Namibian primary school children. They found "very poor performance on the mental calculations questionnaire" (p. 6) and concluded that the participants were not proficient in number sense and claimed that this "lack of a sound foundation in the domain of numbers and operations may be the root cause of the low standards of performance of Namibian learners" (p. 8).

Yang et al. (2009) assessed the number sense strategies used by PSTs through a written test in which the participants were instructed to estimate or mentally compute the answer. Despite instructing the participants not to use a written algorithm, their findings indicated that a low percentage used number sense-based methods and most preferred to use written, rule-based methods to compute the answer. Şengül (2013) also sought to understand the strategies used by PSTs in solving problems. In that study, 133 PSTs were given a "Number Sense Test" and it was also found that the participants "preferred using rule-based methods instead of number sense" (p. 1965). Similarly, Lemonidis et al. (2004) investigated the strategies used by PSTs when solving two-digit multiplication problems and their flexibility in applying these strategies. Their findings showed that the participants were "not flexible in two-digit multiplications and they mostly used the written algorithms mentally in order to calculate" (p. 110).

Teaching a Variety of Strategies

There were seven articles that focused on interventions: Baranyai et al. (2009); Ineson (2008); Mutawah (2016); Son et al. (2019); Whitacre (2015, 2018); and Whitacre & Rumsey, (2018). Three of the articles were based on research that had a pre-test, intervention, post-test design (Baranyai et al., 2009; Son et al., 2019; Whitacre, 2015) and one conducted a post-test after an intervention (Ineson, 2008).

Ineson's (2008) research sought to assess the development of PSTs' connected and relational knowledge (i.e., "use of multiple strategies, estimation and justification of solutions" (p. 51)). The intervention ran for 33 weeks over 2 years. One hundred and seventy PST were explicitly taught different non-standard strategies. In the first year, the pre-service teachers were introduced to addition, subtraction, multiplication, and division strategies sequentially using mental calculations prior to written calculations. The results of the post-test at the end of the first year suggested that the PSTs showed "little evidence of relational or connected thinking" (p. 47). In second year the emphasis was on the PSTs' strategy choices and the justifications for their choices. The results demonstrated that most students in the second year of the intervention used "a variety of informal checking strategies and recognised that justifying results mathematically was a crucial part of the process" (p. 51), thus demonstrating evidence of connected and relational thinking.

Baranyai et al.'s (2009) research examined the efficacy of three different types of games (didactical games, board games and mobile games) on the development of 85 preschool and primary school PSTs. The PSTs were divided into three experimental groups, with each group playing a different type of game. The intervention consisted of eight 20–25-minute sessions conducted over a period of 8 weeks. The results of the research showed that the didactic games, where calculations were asked orally, were most beneficial in developing the PSTs' mental mathematics. The researchers made no comments as to the implications of their research for teacher education.

Fifty-eight PSTs participated in the research of Son et al. (2019). The PSTs wrote pre-and post-tests that consisted of an estimation task, a computational test and a belief survey. The intervention was designed to develop PSTs' use of various strategies for computational estimation. The intervention concentrated on understanding the value of computational estimation and developing and practising computational estimation strategies. The results of the study showed that PSTs performed better on the computation test than the estimation test, and PSTs with a positive perception of their mathematics abilities achieved higher scores in mental estimation.

Three of the articles in this review were either authored or co-authored by Whitacre (Whitacre, 2015; 2018; Whitacre & Rumsey, 2018). All three articles were based on data from a single study in which 39 undergraduate PSTs participated. The learning goal for the research was for PSTs "to move from dependence on standard algorithms to reasoning flexibly about numbers and operations" through participation in collaborative activities and engaging in meaningful mathematical discussions. The intervention that sought to achieve these learning goals was based on five classroom mathematics practices presented sequentially: (1) understanding the standard algorithm, (2) making sense of place value, (3) applying knowledge of place value to make sense of the standard algorithm and transitional strategies (e.g., calculating from right to left), (4) reasoning flexibly about addition calculations, and (5) reasoning flexibly about subtraction algorithms. The intervention aimed to shift the PSTs from a dependence on the *Mental Analog of the Standard Algorithm* [MASA] to working more flexibly with a variety of non-standard strategies. The strategy framework of Markovits and Sowder (1994) and flexible mental computation framework of Heirdsfield and Cooper (2004) guided the analysis across the three articles. Markovits and Sowder's (1994) strategy framework is a continuum that was used by Whitacre (2015, 2018) to identify PST strategy ranges and the changes that occurred during the intervention. The continuum includes *MASA, transition* (the PST is still dependent on the standard algorithm but gives attention to the numbers in the calculation rather than simply performing the procedure), *non-standard with no reformulation* (calculating from left-to-right) and *non-standard with reformulation* (non-standard strategies).

Whitacre's 2015 article was based on data generated through pre- and post-instruction interviews of whole number mental calculation tasks with seven PSTs. The purpose of the interviews was to elicit the strategy ranges of the PSTs before and after the intervention to identify if there were any changes to the choice of strategies used. The participants were presented with several word problems and were asked to give the answer and explain the strategy they used to obtain the answer.

The data were coded using Markovits and Sowder's (1994) strategy framework and the flexible mental computation framework of Heirdsfield and Cooper (2004). The data revealed a greater variety of categories than the Markovits and Sowder (1994) strategy framework, which reflected changes in PSTs' strategy ranges. These included (1) MASA-bound strategies, (2) Polarised strategies (a combination of MASA and non-standard strategies), (3) Transitional strategies (MASA and versions thereof, e.g., calculating from right to left), (4) Spread (MASA and two non-standard strategies), (5) Transition strategies (transitional or non-standard strategies) and (6) Independent strategies (non-standard strategies). The profile of strategy ranges assisted in identifying the flexibility of the PSTs' reasoning in both the pre-and post-interviews. The results provided evidence of increased flexibility and a shift to non-standard strategies. Whitacre's research generated an analytic framework that views the progression of flexible reasoning as a process of development from MASA-bound strategies to non-standard strategies.

The articles written by Whitacre (2018) and Whitacre and Rumsey (2018) focused on the intervention. Whitacre (2018) "presents a viable learning trajectory … with a focus on whole-number place value, addition and subtraction" (p. 56). Drawing on the five classroom mathematics practices (mentioned above), Whitacre (2018) showed the shift in PST flexibility in thinking and a change to using more non-standard strategies for mental calculations. Like Rasmussen and Stephan (2008, as cited in Whitacre, 2018), he drew on an anatomy of argument framework to show this shift by focusing on the *claims* made by the PSTs, the *warrants* (i.e., the evidence to support the claim), and *backing* (i.e., justification of the warrant) as the PST engaged in collaborative activities. The results showed: (1) number sense development as a "cumulative process" (p. 76) that involved a shift from MASA to the use of more non-standard strategies for calculating; (2) The development of PSTs' number sense should start with their knowledge of the standard algorithm. Unlike learners whose number sense development starts with the use of informal strategies, PSTs "approach their learning from a fundamentally different starting place because they have long since learned the standard algorithm and often grown dependent on it" (p. 77); (3) The use of accountable arguments as PSTs engaged in collaborative, meaningful, problem-solving activities and discussions that highlighted that

> taken-as-shared ideas do not have to always be viewed as an element of only one classroom mathematical practice—they may contribute to the emergence of other practices and form a network of practices instead of a sequential chain of practices with distinct taken-as-shared ideas. (Stephan & Rasmussen, 2002, as cited in Whitacre, 2018, p. 62)

Put differently, the classroom mathematics practices in the intervention enabled "as-if shared ideas" (e.g., the standard algorithm) to produce more "as-if shared ideas" (non-standard strategies).

Whitacre and Rumsey's (2018) article focused on a single participant (Brandy) and demonstrated how the scaffolding of the strategy ranges framework from standard to non-standard (Whitacre, 2015) influenced the development of an understanding of non-standard strategies and the ability to reason more flexibly when calculating. Specifically, the research examined how the socio-mathematical norms established during collaborative activity supported Brandy in her transition from her initial reliance on the standard algorithm to calculating in more flexible ways. The authors coded the argument log that formed part of the previous paper (Whitacre, 2018) according to type and frequency of mental calculation strategies used over time. The argument logs were also coded for socio-mathematical norms. The researchers drew on Fukawa-Connelly (2012, as cited in Whitacre, 2018) to identify the socio-mathematical norms. Three socio-mathematical norms were identified relating to discussions of addition and subtraction strategies: *desirable characteristics of strategies*, *distinguishing and communicating the details of strategies* and *strategy naming conventions* (p. 342). Brandy's strategy range showed improvement between the pre- and post-interviews. As Brandy participated in the intervention, the researchers noted that her unscaffolded strategy range (i.e., the strategy range she used during pre-interview) differed from her scaffolded strategy range (i.e., her strategy range when asked to use an alternative strategy for calculating mentally). In the first interview, Brandy was bound to the standard algorithm. However, in the scaffolded activity tasks she showed some indication of semi-flexibility in her reasoning. In the second interview, Brandy showed more flexibility in her thinking when presented with unscaffolded tasks. While not proficient in some of the non-standard strategies, she demonstrated she was no longer bound to MASA and chose rather to use transition and non-standard strategies. She was able to reason and justify her strategies, a change consistent with the three socio-mathematical norms. The results of Whitacre and Rumsey's (2018) research suggest that PSTs may be more flexible in their use of strategies when presented with scaffolded tasks and that the development of socio-mathematical norms as the PSTs engaged in collective activities possibly lead to greater flexibility in the choice of non-standard strategies used. The latter is an important finding in that it challenges the dichotomous view that PSTs use either the standard algorithm or non-standard strategies (viewed as number sense) when calculating mentally. Rather, the research suggests that the process of engaging collectively in classroom mathematical practices that require PSTs to attend more closely to different non-standard strategies promotes greater flexibility. With the development of flexible thinking, Brandy's initial scaffolded strategy ranges became her unscaffolded strategy range.

Discussion and Conclusion

All of the papers advocated that PSTs require training in order to develop a broad repertoire of mental calculation strategies. The five non-intervention studies provided no indication of what the nature of those potential interventions should entail. There exists a wealth of research that reports PSTs tend to rely on the standard algorithms for mental calculations (MASA), and seemingly lack flexibility in their mathematical thinking when calculating mentally. Five of the studies in this review appear to confirm what is already known. These five articles thus promote a deficit view of PSTs ability to reason and calculate in flexible ways.

All seven articles on interventionist studies were based on the view that interventions are required to develop PSTs' knowledge of non-standard strategies for calculating mentally. Ineson (2008), Son et al. (2019), Whitacre (2015, 2018) and Whitacre and Rumsey (2018) commented on the types of the non-standard strategies that PSTs develop as they engage in various interventions.

Three of the authors whose research focused on intervention studies prioritised the teaching of non-standard methods only, ignoring what PSTs are already familiar with, namely the standard algorithm (Ineson, 2008; Mutawah, 2016; Son et al., 2019). By contrast, Whitacre (2018) proposed that a learning trajectory that starts with what is already known to the PST is required prior to introducing non-standard strategies. His use of an adaptation of the strategy range framework developed by Markovits and Sowder (1994), challenges the dichotomous view that PST either use standard algorithms or non-standard strategies.

Whitacre (2018) argued that interventions should be based on classroom practices that move PSTs from the known to unknown. He advocated that developing an understanding of the standard algorithm, place-value and transitional strategies are ultimately necessary to enable PSTs to reason flexibly with a variety of non-standard strategies. Whitacre and Rumsey (2018) extended this argument suggesting that classroom practices that encourage collaboration and shared sense-making develop social-mathematical norms that encourage PSTs to identify desirable characteristics of strategies, distinguishing and communicating the details of strategies and strategy naming conventions (p. 342). These socio-mathematical norms also promote greater flexibility in using a wide repertoire of mental calculation strategies.

There is little doubt that PSTs need to be familiar with a range of strategies in order to support the development of learners' number sense. As Shulman (1987) noted "to teach is first to understand" (p. 14). Given the importance of developing learners' number sense, it is surprising that there are seemingly few studies that have been published in peer-reviewed journals on how to develop PSTs' knowledge and promote use of various mental calculation strategies.

The research reviewed in this paper comprises non-interventionist studies that focused on identifying PST strategy ranges through pre-test and post-test, interventionist studies that started with the development of a wide repertoire of non-standard strategies, and interventions that began with developing an understanding of the standard algorithm and place value prior to supporting PSTs development of a range of strategies. The implication of this is that there is no consensus on how best to enable pre-service teachers to develop learners' number sense. We suggest that research that does not take cognisance of PSTs' use of the standard algorithm for mental computation assumes that PSTs understand the standard algorithm. This is not our experience as teachers of PSTs. Interventions that ignore the standard algorithm assume that it is not a strategy that should be regarded as one of various strategies in the development of number sense, particularly when calculating in writing. Notably, absent from the research reviewed is the transition from the lecture room to the classroom and the extent to which PSTs are able to translate their knowledge and understanding of non-standard strategies during their teaching practice. We suggest that transferability and sustainability of practice may be worth researching in the future.

The review presented in this paper provides indicators of practice that could be explored in more depth in a more comprehensive review. Future reviews should include a broadening of the search terms. For example, inclusion of "number sense", "number talks", "number flexibility", and variations of the terminology associated with mental mathematics, such as "mental math" and "mental maths".

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