RESEARCH PAPER

Making Visible a Teacher's Pedagogical Reasoning and Actions Through the Use of Pedagogical Documentation

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Mathematics education research has focused on developing teachers' knowledge or other visible aspects of the teaching practice. This paper contributes to conversations around making a teacher's thinking visible and enhancing a teacher's pedagogical reasoning by exploring the use of pedagogical documentation. In this paper, we describe how a teacher's pedagogical reasoning was made visible and highlight aspects of his thinking in relation to his instructional decisions during a series of lessons on division. Implications for professional learning are discussed.

Teaching students for *relational understanding* (Skemp, 1978) in mathematics is challenging. Doing this requires teachers to choose, adapt, or design tasks that promote reasoning (Sullivan et al., 2014); orchestrate interactions and discussions amongst students around these tasks to focus on the key mathematical ideas while catering to students with different levels of readiness and interest (Lampert, 1985); and making in-the-moment instructional decisions by considering students' responses to these learning tasks (Jacobs et al., 2011). Much of the current research in mathematics education has focused on developing teachers' mathematical knowledge for teaching or what Kennedy (2015) described as the more "visible behaviours of teaching" (p. 6) to do the ambitious work of teaching. As argued by Kavanagh et al. (2020), it is critical that teacher education aimed at ambitious teaching "must involve unfolding the invisible professional thinking behind discrete elements of teaching practice" (p. 3). This paper contributes a new perspective to this ongoing conversation about enhancing teachers' pedagogical reasoning (Kavanagh et al., 2020; Loughran et al., 2016; Pella, 2015), an invisible aspect of ambitious teaching, by considering how a primary school mathematics teacher's pedagogical reasoning and action can be made visible in the context of day-to-day teaching activities. We explored how the idea of pedagogical documentation (Alcock, 2000; de Sousa, 2019; Lee-Hammond & Bjervås, 2021) can be used to make visible a teacher's pedagogical reasoning and actions about the teaching of division. We frame our discussion in this paper around the following research question: What insights can we gain into a teacher's pedagogical reasoning and action when his thinking during a series of lessons on division is made visible?

Theoretical Considerations

Our study was motivated by our interest in supporting teachers to do this ambitious teaching by providing meaningful opportunities for teachers to reflect and learn from their teaching practices. However, our premise differs from current perspectives about practice-based professional development (Chapman, 2014; Timperley et al., 2007) that also embed learning into teachers' work (e.g., lesson studies) in two ways. First, we want to go beyond the confines of professional development activities and explore how individual teachers can learn from their own teaching as well as their colleagues' teaching experiences. Next, we see a need to maximise learning opportunities for teachers by empowering them to learn from these opportunities as part of their teaching activities. Hence, our vision is to see professional learning as part of a teacher's daily teaching activities beyond a teacher's participation in

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professional learning communities. To introduce our ideas, we will revisit three aspects of this *invisible work* of thinking about one's practice (Kavanagh et al., 2020).

We begin by elaborating on the ideas of pedagogical reasoning and action as proposed by Shulman (1987), who perceived teaching as an iterative "act of reason" and an ongoing "process of reasoning" that culminates in a series of pedagogical actions (p. 13). This process involves taking what one understands about content and "making it ready for effective instruction" (Shulman, 1987, p. 14), through a cycle of activities involving comprehension, transformation, instruction, evaluation, and reflection, leading to new comprehension. Briefly, teaching first involves *comprehending* the content and its purposes in different ways and relate them to other ideas within and beyond the subject to be taught. The level of comprehension then influences how a teacher *transforms* his or her content knowledge into "forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (Shulman, 1987, p. 15). Transforming this knowledge involves preparation, representation, instructional selections, adaptations of these representations and tailoring the representations to specific students' profiles. While we may argue that comprehension and transformation can occur at any time during teaching, Shulman (1987, p. 18) sees these two processes as "prospective", occurring before *instruction*—an "enactive" performance in the classrooms. Shulman highlights *evaluation* and *reflection* as retrospective processes by which a teacher looks back at students' responses to instruction to learn from his or her experiences. This learning is encapsulated in the notion of *new comprehension* where teachers have a better understanding of teaching and learning. Shulman and Shulman (2004) position this kind of learning from experience through reflection as the capacity for enacting purposeful change. But *reflection does not necessarily lead to learning from experience*.

Second, we explain why some teachers learn from their teaching experiences, while others do not, by referring to Schoenfeld's (2011) conceptualisation of teaching as an goal-directed activity, which rests on a set of resources—a teacher's knowledge—and is driven by a teacher's orientations—one's beliefs and preferences. A teacher's existing knowledge provides the basis for an initial comprehension of content. The teacher then transforms the initial comprehended ideas into a form suitable for teaching the students, subjected to the teacher's cluster of goals and orientations. The orientations aspect is important because beliefs have high inertia (Schoenfeld, 2011), and this explains why it is sometimes difficult to change teachers' practices even after professional development activities. The iterative and cyclical processes of transformation, actual instruction, and evaluation feed forward to the reflection of the teacher (to different extents for different teachers). This process leads to some new comprehension, which may lead to an *expanded* set of resources, orientations, and goals (ROGs), corresponding to the idea of learning from experience, and the cycle repeats.

But what enables this learning from experience for an expansion of resources, orientations, and goals to take place? Here, we refer to Choy's (2016) ideas of productive noticing and argue that learning from experience occurs when a teacher notices critical aspects about the content, aspects of student learning, and appropriateness of teaching actions. Simply stating, a teacher's productive noticing is likely to raise an awareness of new possibilities (new comprehension), leading to an expansion of one's current cluster of ROGs (Choy, 2016). This expanded cluster of ROGs thereby becomes the base from which the teacher makes sense of instruction. Moreover, as Choy (2016) highlighted, productive noticing can take place during planning, instruction, and reviewing of lessons. Consequently, we argue that new comprehension leading to learning from experience can occur during any of the activities of Shulman's model. Putting these three ideas together, we conceptualise the invisible work of thinking about one's practice in terms of an adaptation of Shulman's model of pedagogical reasoning and action. Referring to Figure 1, we see how a teacher's prior set of ROGs may influence the initial comprehension of the content to be taught. This initial comprehension leads to what we perceive as an iterative cycle of transformation, instruction, and assessment. Note that we replace the term "evaluation" with "assessment" to denote a more comprehensive notion of assessing for learning and assessment of learning. By reflecting on the cycle of transformation, instruction, and assessment, a teacher may gain new comprehension, which may lead to an expanded set of ROGs, resulting in learning from one's experiences. The key to learning from the processes of reflection on experiences during planning, teaching, and reviewing of a lesson is to productively notice aspects of teaching and learning that promote student understanding of mathematics (Choy, 2016).

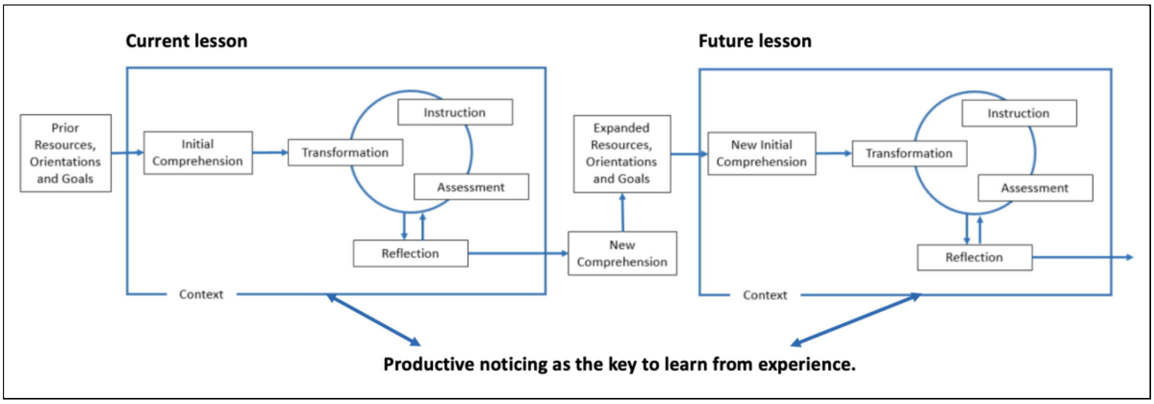

Figure 1. Adapted model of pedagogical reasoning and action.

Methods

The data presented in this paper were collected as part of a larger project, involving a total of 39 mathematics teachers from three different primary schools, which aimed to develop the proof of concept for a new professional learning model. Drawing on current theoretical perspectives of teacher noticing (Fernandez & Choy, 2019), we conceptualised professional learning sessions where teachers would have opportunities, in the context of a community of inquiry, to work and co-learn with us by engaging in activities related to our adapted model of pedagogical reasoning and action (see Figure 1). For this paper, we focus on a series of sessions and activities we had conducted with teachers from Sandy Shore Primary School (pseudonym).

First, we engaged all the teachers in two online professional learning sessions conducted over the Zoom platform. The sessions were conducted online because of prevailing COVID-19 restrictions at that time, which prevented us from holding in-person meetings. During the sessions, which were facilitated by one of the teachers, we took on the role of a knowledgeable other to share new ideas for teaching or critique existing ideas. The teachers from Sandy Shore Primary School decided to work on teaching of division for Primary Three pupils. Although we provided teachers access to relevant research and practice-based articles when requested, we did not insist that the teachers must adopt our ideas. Instead, we left all the instructional decisions to them because we wanted to investigate the thinking behind their instructional decisions. After the second session, we followed Shawn (pseudonym), one of the teachers in the discussion, as our focus teacher and observed his teaching of the unit on division. An accountant by training, Shawn had worked in the finance sector and had some part-time teaching experiences before he went on to be trained as a teacher in 2018. At the time of this study, Shawn had over three years of teaching experience at Sandy Shore Primary School. After Shawn's last lesson on division, we conducted the third and final professional learning session to collectively reflect on our learning. Data collected include voice and video recordings of the discussion during the sessions, photographs of lesson artifacts such as lesson plans, discussion notes, and when available, samples of students' work.

To make Shawn's pedagogical reasoning more visible, we turned to the idea of pedagogical documentation (Alcock, 2000; Lee-Hammond & Bjervås, 2021), which is widely practised in early childhood education settings. The practice of pedagogical documentation involves teachers in collecting written notes, audio and video recordings, photographs, or students' learning artifacts for describing what and how students learn, which then serve as a basis for reflection and making instructional decisions (Lee-Hammond & Bjervås, 2021). In this way, the documentation serves as both a product and a process to support teachers in their professional learning. For this study, we used Padlet (https://padlet.com/), a digital notice board, as a platform for Shawn to curate his pedagogical documentation. We did not impose any number for the reflections—instead, we asked him to post his reflections, photos, videos, or documents related to any incident that he had found interesting on the Padlet—and we left all instructional decisions to Shawn. Our role was to observe what he had learned from our sessions, his selection of tasks and the instructional decisions made during his lessons.

Findings were developed through analyses of Shawn's Padlet entries, his teaching materials, and his teaching videos. We identified parts of his reflections, artifacts, and actions that were related to aspects of division concepts (Brown et al., 2011; Takker & Subramaniam, 2018), students' confusion about division concepts and algorithm (Holland, 1942), as well as Shawn's knowledge and instructional decisions about the teaching of division (Simon, 1993; Takker & Subramaniam, 2018). By relating these selected artifacts to our adaptation of Shulman's (1987) model, we made inferences about Shawn's comprehension of division, his transformation of understanding into lesson materials, his instruction, assessment, and reflections. We also identified *critical incidents* (Goodell, 2006), which are "everyday" events encountered by teachers that made them question their instructional decisions, thus providing "an entry to improving teaching" (p. 224), that occurred during Shawn's lesson and reflection. More specifically, we wanted to examine if Shawn had *noticed* critical aspects of content, student thinking, and teaching approaches from these incidents (Choy, 2016).

Findings

Here, we present snapshots of Shawn's pedagogical reasoning as seen from his pedagogical documentation in the Padlet, supported by evidence from the lesson materials he used, relevant snippets from his teaching, and points of discussion raised during our professional learning sessions. It is important to note that we are *not criticising* his way of teaching nor evaluating his practices. Rather, we are *making visible* his pedagogical reasoning to gain insights into what he noticed about the content, student thinking, and the teaching approach adopted.

Shawn's Comprehension of Division: Division as Equal Sharing?

In our first two professional learning sessions, we elicited the teachers' prior understanding of division by getting them to articulate ideas about division and how they would explain 48 ÷ 4 and 48 ÷ 3 using different ways. From his Padlet entries, we see that Shawn was familiar with the idea of division as equal sharing or partitive division (Simon, 1993), or the idea of finding the number of items in each equal group. He wrote "For division, the idea is on equal sharing. For chapter 4, students need to understand that there is an algorithm to learn to solve 2 or 3-digit division." In Figure 2, we see his "demonstration" of the two division problems. He demonstrated his understanding of partitive division, his emphasis on place values (Holland, 1942), and his coordination between the pictorial representation and the symbolic representation of the long division algorithm. For instance, in 48 ÷ 4, he "shared" the 4 tens into the four equal groups ("1 ten in each group") and the 8 ones into four equal groups ("2 ones in each group") and showed how he would explain the algorithm by referring to the place values of the digits (e.g., the digit 4 is "4 tens", which came from 1 ten × 4).

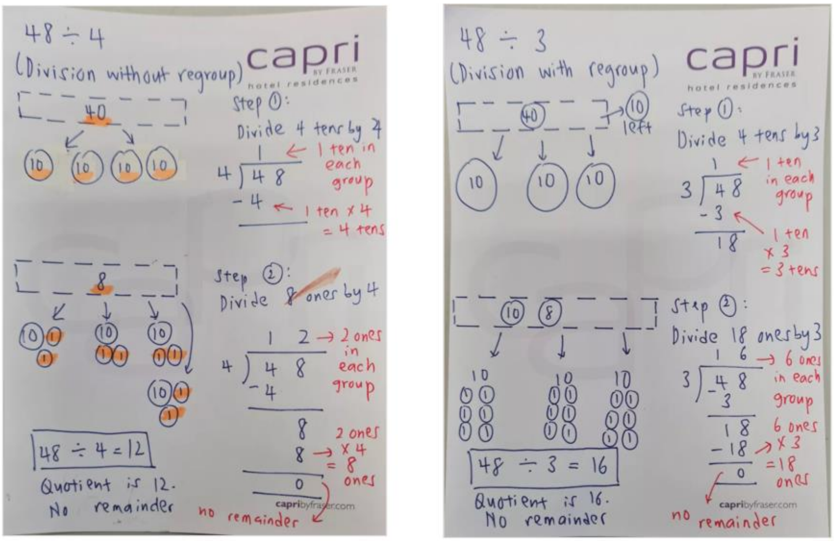

Figure 2. Shawn's demonstration of the two division problems.

Shawn's Transformation, Instruction, and Assessment: Emphasis on Equal Sharing and the "DMSCB" Algorithm

At this point, it appears that Shawn seemed to see division only as equal sharing. However, from the slides he used, it was clear that he was aware of division as equal grouping (see Figure 3, left). Nevertheless, an analysis of his explanations from the video recordings and his use of the lesson materials suggests a stronger emphasis on division as equal sharing, as evidenced from his transformation made visible by his lesson materials (see Figure 3, right).

We noted three interesting insights about his pedagogical reasoning from his materials and instruction. Firstly, we see that Shawn was aware of division as equal grouping or quotitive division (Simon, 1993) even though he did not mention the idea during the professional learning sessions. Besides the slide on equal grouping, he also selected the storybook "Divide and Ride" by Stuart Murphy, which depicted equal grouping scenarios (such as "11 divided by 2 = 5 full seats with 1 friend left over."). Next, Shawn incorporated a mnemonic "GET" to support his students in remembering the relationship between the number of equal groups (G), the number of items in each equal group (E), and the total number of items (T). As seen from Figure 3 (left), he used the equation "Total ÷ Each = Group" to remind students about the idea of equal grouping. Likewise, he had a similar equation for equal sharing. His thinking behind this move was captured in one of his reflections on Padlet, dated 30 March, as follows:

> Since P2, they have learnt that G × E = T, Group × Each = Total. In P3, they have to interpret this equation and understand that Each = Total ÷ Group and Group = Total ÷ Each.

Although we may question the use of such mnemonic in promoting relational understanding, it is clear that Shawn wanted to build on what students had already been taught in Primary Two to bring across the relationship between multiplication and division, as well as the two different notions of division—equal grouping and equal sharing.

Thirdly, despite his awareness of equal grouping, his transformation and instruction strongly suggested a preference towards the use of equal sharing. In Figure 3 (right), we see how Shawn used the idea of equal sharing to distribute the 3 hundreds, 6 tens, and 9 ones into three groups and coordinated the use of the division algorithm or what he called "DMSCB" algorithm ("Divide, Multiply, Subtract, Check, and Bring Down") with the different representations (number discs and symbolic).

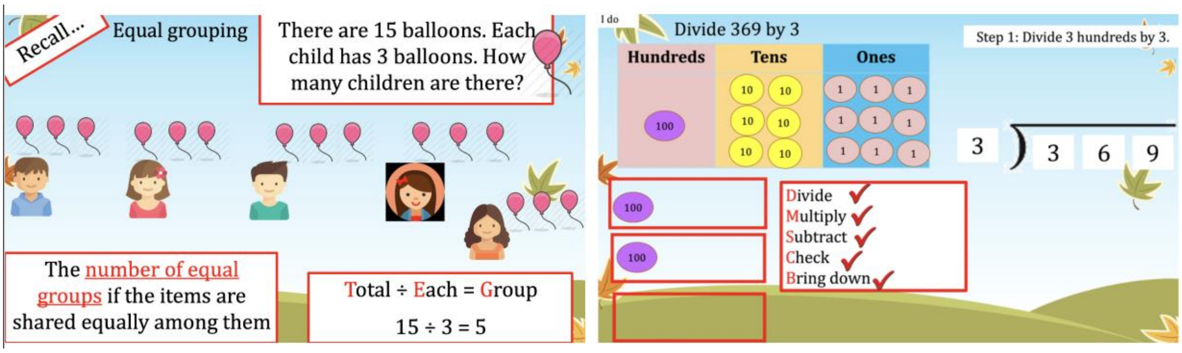

Figure 3. Snapshots from Shawn's teaching materials.

His instruction and assessment clearly reflected a strong emphasis on teaching the "DMSCB" algorithm. For example, he noted, in his reflection on Padlet, that "students had issue placing the wrong number at the wrong place value" and he "reinforced" the idea that students "have to clear the 'particular lane' (referring to place value) by doing the DMSCB first, before they proceed to the next lane" (Shawn's reflections, 5 April 2021). Again, he reflected that while "some students could follow the 5-step algorithm to derive the answer" to 682 ÷ 2, a "handful of them did not apply the 5 steps" (Shawn's reflections, 6 April 2021). He noticed that students might not have applied the 5-step algorithm because "the numbers 6, 8, and 2 are in the 2 times table and got the answer immediately by dividing". Interestingly, he acknowledged that he "should not dismiss that the method is wrong" even though he had actually insisted his students to use the full DMSCB algorithm during the lesson. While Shawn was cognisant of the idea that the long division algorithm "is a way of keeping track of the numbers", he seemed to prefer his students to follow the DMSCB method. This can also be seen from his proposed use of questions such as "672 ÷ 4" so that his students "have to rely on the long division method to keep track of the numbers" (Shawn's reflections, 6 April 2021).

Shawn's Reflection of a "Critical Incident"

This dominant use of the DMSCB algorithm was questioned by Shawn himself after the lesson on "long division with regrouping" when he realised that "students were initially confused by multiplication and division" (Shawn's reflections, 7 April 2021). In that reflection, he also wrote: "It suddenly occurred to me that there might be a simpler way of long division without going through the whole motion of DMSCB". We had also pointed out to Shawn that he "might have relied too heavily on the DMSCB method" (Shawn's reflections, 8 April 2021). Consequently, it appeared that he considered another approach that resembled a chunking strategy (see Figure 4), or what was commonly referred to as the partial quotient strategy (Takker & Subramaniam, 2018). In his reflection (dated 8 April), he wrote:

> In the question 72 ÷ 4, I solved it with the use of number disc, where I can share 1 group of 10 each equally into the 4 groups, then students recall that they can use 32 ÷ 4 = 8, all they have to do is to use 10 + 8 = 18 to get the answer. However, one drawback about this method is that it requires the use of number disc to solve the question.

As seen in Figure 4 (left), Shawn continued in his use of "equal sharing" to explain the chunking strategy in his reflection. It was interesting to note that Shawn did not think that this method could be an alternative to the DMSCB algorithm. He also highlighted that the method "requires the use of number discs to solve the question" and resisted teaching this method to his students even though he opined that his students might understand this method better. In addition, Shawn did not consider how an equal grouping notion of division might offer an alternative explanation for the division algorithm (Simon, 1993), and hence the chunking strategy. It appeared that Shawn did not consider the use of the chunking strategy because he did not notice the connection between the chunking strategy and the standard algorithm (Choy, 2016), as highlighted by us in Figure 4 (right, parts (a) to (c)).

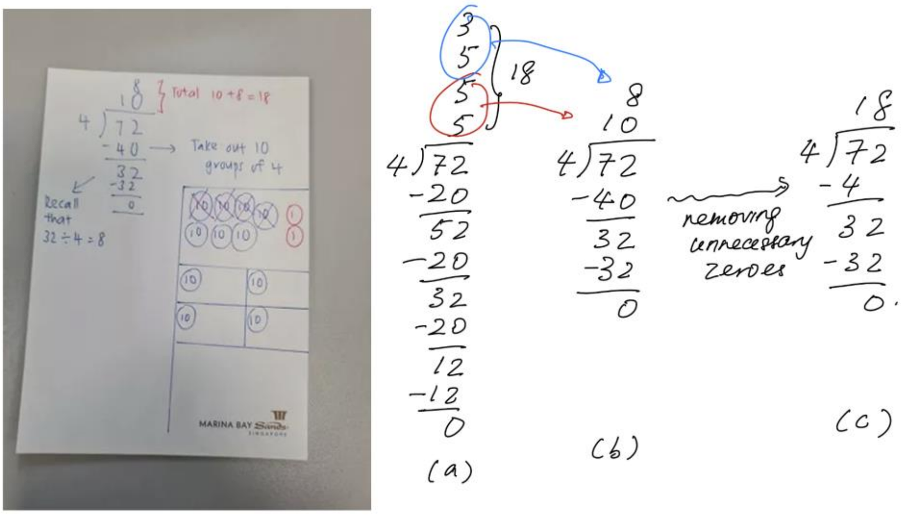

Figure 4. Shawn's idea of chunking (left); relationship between chunking and standard algorithm (right).

Concluding Remarks

Our analysis of Shawn's pedagogical reasoning and action made visible by his pedagogical documentation during his series of lessons on teaching division reveals *unseen* aspects of his visible practice (e.g., his understanding of division as equal sharing and equal grouping, the use of "GET" mnemonic and his preference for the DMSCB algorithm). More importantly, we uncover the complexity of learning from practice by highlighting that while Shawn might have learned some new ideas (chunking strategy), he might not have *productively* noticed (Choy, 2016) about the connections between this strategy and the standard algorithm to impact his practice. This paper also illustrates the affordances of pedagogical documentation to make visible a teacher's thinking, beyond its usual use to reveal students' understanding (Alcock, 2000; de Sousa, 2019). By making one's pedagogical reasoning visible, we can make connections between a teacher's visible actions and the corresponding invisible aspects of practice. This is critical for supporting teachers' efforts to develop their adaptive expertise in teaching in three ways. Firstly, as a form of reflection, the teacher's pedagogical reasoning made visible by the use of pedagogical documentation serves as a means for teachers to learn from their day-to-day teaching experiences (Shulman & Shulman, 2004). Secondly, snapshots of how Shawn learned from his practice reaffirms that learning takes place over a sequence of lessons. Hence, we need to follow a teacher's pedagogical reasoning through several lessons to uncover the invisible aspects of practice. Thirdly, by making visible a teacher's thinking about instruction, we can better pinpoint the components of pedagogical reasoning that needs attention and the support that teachers need. Doing this can potentially transform the way we approach professional learning and development. However, documentation is hard work even though it can unpack the invisible aspects of one's practice. How this can be done sustainably in the context of a teacher's busy schedule will be an important area for future research.

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