RESEARCH PAPER

George Preferred Learning Fraction Concepts with Physical Rather than Virtual Manipulatives

Seyum Getenet
University of Southern Queensland
Seyum.Getenet@usq.edu.au

This case study aims to describe the learning characteristics of a child and evaluate his preferences for using physical manipulatives (PM) and virtual manipulatives (VM) to solve fraction problems. The participant in this study was a fourth-grade child. The child was given similar problems to solve using PM and VM. Data sources were observations and interviews conducted with the child during and after the tasks were completed. The results showed that the child engaged and preferred solving fraction problems using PM more than VM. The child stated that PM helped him quickly understand the relationship between various representations of fractions and model them using manipulatives. He reported the VM did not help him solve the problems.

The use of manipulatives in teaching mathematics is explicitly encouraged in various studies to support students to understand mathematics concepts easily (e.g., Getenet & Callingham, 2021; Golafshani, 2013). These studies reported that students who use manipulatives in their mathematics classes outperform those who do not. They suggested that manipulatives could support students' mathematics learning through a wide range of visual representations, reducing anxiety, increasing engagement, and improving problem-solving skills. Recent studies by Donovan and Alibali (2021) and Basargekar and Lillard (2021), suggested that using perceptually rich manipulatives improved students' problem-solving skills and retention of information. As a result, teachers were encouraged to use manipulatives and technologies to help children understand complex mathematical concepts in the primary context (Reys et al., 2018). There seems to be a consensus that manipulatives help teachers enact effective pedagogy that makes abstract mathematical concepts concrete and relevant to children's lives. Naiser et al. (2003) found that the use of manipulatives is one way that teachers can make the lessons more engaging by creating a concrete experience and providing an effective way for children to represent their thinking. A study conducted in a New Zealand classroom by Getenet and Callingham (2021) showed that manipulatives helped a teacher transform her pedagogical practice by encouraging children to concretely demonstrate fraction concepts.

Technological innovations now allow teachers to use virtual manipulatives (VM) for teaching mathematics in place of physical manipulatives (PM). VM are computer-based versions of physical mathematics manipulatives. These digital substitutions have become popular over the past few years for supporting children's learning of mathematics concepts. There are various studies on VM and PM use in teaching mathematics in general (e.g., Day & Hurrell, 2017; Hunt et al., 2011; Wong, 2010). However, there are limited studies on children's preference between PM and VM to learn a specific mathematics concepts. The purpose of this study was to explore the preference of a year four child, George, in using various PM while engaging in activities such as paper folding, shading paper strips, and manipulating wooden area models or substituted VM for solving fraction problems. The study answers the research question, "How does a child engage with and prefer between VP and PM while learning fractions?" The author reports the observation and interview results from the fourth-year child in the Australian curriculum context. The importance of this paper is its implication for mathematics teachers on the use and preference of using various forms of manipulatives to

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 234–241. Launceston: MERGA.

engage and support children to learn fractions. Research on the use of various manipulatives will eventually help to discern the most effective uses of these manipulatives.

Literature Review

As highlighted above, mathematical manipulatives can build foundational knowledge for children to understand various mathematical concepts, which can support children in solving abstract mathematical concepts.

Teaching Fraction Concepts

Fractions are difficult to learn for children and create pedagogical challenges for mathematics teachers (e.g., Hackenberg & Lee, 2015; Siemon et al., 2015). These difficulties are observed across all year levels (e.g., Gupta & Wilkerson, 2015). Different reasons have been identified for these difficulties, particularly in the primary school context, including the complex nature of the concept itself and teachers' pedagogical approaches.

Hackenberg and Lee (2015) showed that limited understanding of particular aspects of the different meanings of fractions affected children's ability to generalise and work with fraction concepts. Similarly, Siemon et al. (2015) indicated that learning fraction concepts were difficult because they were commonly used to represent a relationship between numbers rather than an absolute quantity. Other studies (e.g., Blömeke et al., 2011; Getenet & Callingham, 2021), however, have shown that teachers' professional competencies, including pedagogical knowledge, are essential to the learning and teaching of mathematics concepts. For example, equivalent fractions are introduced in the fourth grade and the subsequent grades in the Australian curriculum (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2018). However, a majority of children do not thoroughly understand equivalent fractions due to ineffective teaching of equivalent fractions (e.g., Reys et al., 2018; Wong, 2010). Studies showed that children could develop the necessary conceptual understandings of fractions with teaching approaches that emphasised many representations—manipulatives, pictorial, real-world and symbolic—over more traditional didactic and procedural approaches (e.g., Bouck et al., 2020, Way, 2011; Wong, 2010). For example, Bouck et al.'s. (2020) study results showed that using virtual manipulatives to teach fractions to third grade increased significantly their test results and acquired knowledge of fraction concepts. This was because the children's understanding of fractions was influenced not only by how their knowledge was structured but also more profoundly, by how the concept was taught and structured by the classroom teachers, reported similarly by Getenet and Callingham (2021).

Using Manipulative for Teaching Mathematics

The use of manipulatives can be traced back to Piaget's (1952) suggestion that children cannot comprehend abstract mathematics through explanations and lectures; therefore, they need models and instruments to grasp the mathematical concepts. Piaget's ideas of using manipulatives are well received in today's mathematics classroom. Teachers are encouraged to start with manipulative materials to teach for understanding, then transfer to representational models like pictures or diagrams, leading and bridging learning to the abstract level of understanding symbols and operation signs (Reys et al., 2018). Getenet and Callingham (2021) suggested that children could be supported to understand the links between ratio and measurement concepts of fractions using manipulatives such as a thin strip of paper. Strip paper can be folded into halves, quarters and so on and later, children can use length partitioning to represent fractions as points on a number line. Jordan et al. (1999) compared the teaching of fraction concepts to fourth-grade children using PM and a traditional textbook approach. The finding showed that children who used manipulatives showed more significant gains in acquiring fraction concepts and skills than children receiving traditional instruction. Similarly, Strom (2009) reported that children who use manipulatives in their mathematics classes outperform those who do not. They suggested that manipulatives could support children's mathematics learning through a wide range of visual representations, reducing their mathematics anxiety and increasing their engagement.

The increased access to computers, software and internet access has brought VM into the majority of classrooms (e.g., Day & Hurrell, 2017; Dewi & Verawati, 2022; Hunt et al., 2011). It is assumed that VM can offer a visual image or a pictorial model, and they can be manipulated like a physical model. Furthermore, they can allow differentiation for the varied ability levels of the learners. Moyer et al. (2002) showed that VM supported children learning to work at their own pace. In addition, they argued that VM were great resources for classroom use because of their unique features to record and store user movements online and their potential for alterations, such as size and colouring.

Similar to their international peers, teachers in Australian schools are encouraged to use manipulatives such as fraction bars and pattern blocks or VM such as fraction circles. These resources can help children develop concepts about fractions (Wong, 2010). Some of the most effective materials, such as paper strips, fraction bars and counters, are readily available and used in most Australian school classrooms. Children can make concrete models (e.g., fraction bars) and then use them to find equivalent fractions. They can be further used to order fractions and connect the concrete device to the symbolic representation (Wong, 2010; Wu, 2013).

However, there are a few discussions and studies on students' preference for using either physical, virtual or both VM and PM and their effectiveness when teaching mathematics in general (e.g., Day & Hurrell, 2017; Hunt et al., 2011; Moyer et al., 2002). Day and Hurrell (2017) showed that VM do not necessarily provide the same experience as concrete materials. They can provide a bridge between the concrete materials and other representations. Additionally, Hunt et al. (201l) and Moyer et al. (2002) recommended VM to record and store users' movements, online and constant availability and their potential for flexible learning. However, a recent study by Dokić et al. (2022) showed that there was no difference in 4th grade (10–11 years old) students' 3D geometry achievement regardless of the learning support through either VM or PM. Few studies have explored children's preferences and experiences using the two forms of manipulatives to learn a specific mathematics concept, such as fractions.

Method

Purpose of the Study

The *Australian Curriculum: Mathematics* (ACARA, 2018) states that children in Year 4 should be confident to solve fraction problems using the concept and skills associated with equivalent fractions in various contexts (ACMNA077). For example, children are expected to explore the relationship between families of fractions (halves, quarters and eighths or thirds and sixths) by folding a series of paper strips. The purpose of this study, therefore, was to evaluate a child's preferences for using the two forms of manipulatives while learning fractions. In this case study, the author made the two forms of manipulatives available to a child, George, to solve fraction problems. The VM were from the National Library of Virtual Manipulatives (NLVM), and PM examples are shown in Figure 1.

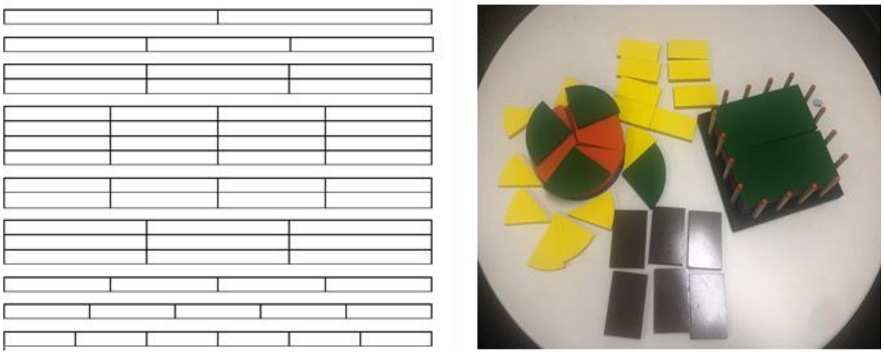

Figure 1: Examples of the physical manipulatives used by the child.

The Procedure of the Study

George was provided with two separate but similar fraction problems to solve using various manipulatives. First, George was asked to represent a list of fractions using various PM and later using a VM (see Figure 2).

Poblem 1. Model the following fractions by

1. Shading the provided paper strip
2. Folding the provided paper strip
3. Using the provided wooden resources
4. Using the virtual manipulative in your computer

The list of fractions

a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ b) $\frac{1}{3}, \frac{1}{5}, \frac{1}{6}$

Poblem 2. First, model the following fractions by

- Shading the provided paper strip
- Folding the provided paper strip
- Using the provided wooden resources
- Using the virtual manipulative in your computer

Second, group equivalent fractions togather

The list of fractions

a) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{4}, \frac{2}{6}, \frac{2}{8}, \frac{4}{8}, \frac{3}{9}, \frac{4}{16}, \frac{3}{8}$

Figure 2. Fraction problems provided to George.

Next, George was provided with a list of fractions and asked to model the fractions using VM. Screen captures of the front page of the VM the child used is shown in Figure 3. The problems further asked George to group the equivalent fractions. Using these VM allowed the child to compare fractions and determine if two fractions are equivalent. The VM George used were sourced from the NLVM website. In addition, George was provided with various forms of PM, such as paper strips and wooden manipulatives (Figure1) to solve the same problems.

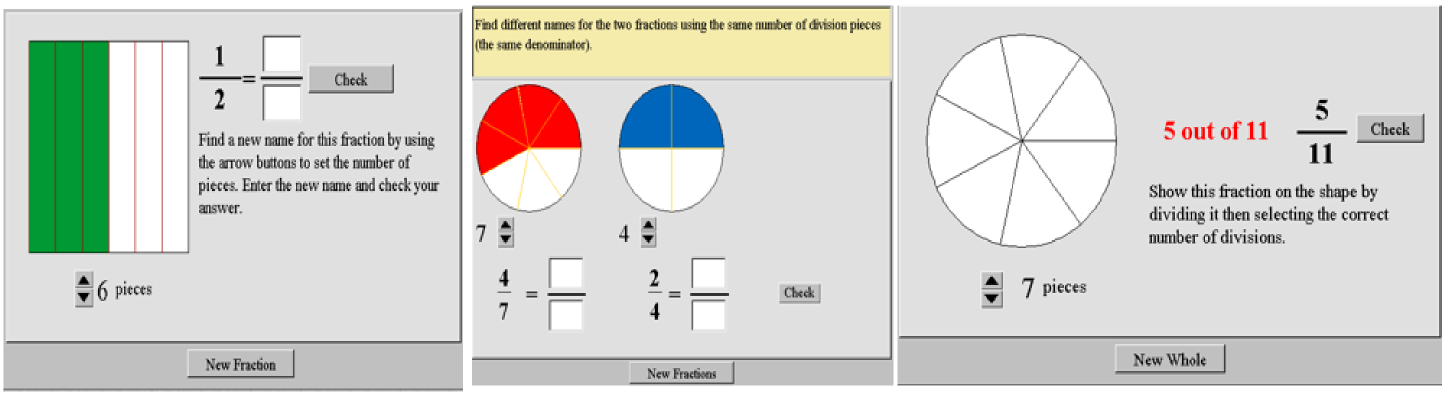

Figure 3. The screen capture of various sections of the NLVM apps.

Data Source and Analysis

The data were collected using interview and observation techniques. The author observed and video recorded while George solved the problems. At the conclusion of each activity, George was asked four questions to reflect on this process, which included, “Which resources do you think helped you to solve the problems” and “Which resources do you like or dislike and why?”

The author transcribed and analysed the interview and observation data (watching the recorded video, which showed George’s actions while using various manipulatives). Consistent with Barron and Engle’s (2007) advice, the analysis emphasised the characteristics of the child’s learning using the manipulatives, such as how the child interacted with the resources and how he worked to make sense of particularly equivalent fractions. The analysis involved viewing the recorded video and interview data by iteratively revising until the transcripts eventually provide a reliable record of what the researcher views as the most relevant aspects of the research question (Barron & Engle, 2007).

Results and Discussions

The results are presented in two sections - the first section describes the child’s experience of using the PM, and the second section presents the child’s experiences of using VM.

Using Physical Manipulatives

As described, George was asked to group sets of fractions using the provided PM—see Figure 1. He used the manipulatives for diverse purposes, as highlighted in Table 1.

Table 1
The PM Used to Solve the Fraction Problems

Manipulatives	Used in the challenge
Folding papers	To show different sizes of fractions
Shading	Compare fractions and group equivalent fractions
Wooden manipulatives	Identify different sizes of fraction and group equivalent fractions

George stated that he believed the PM helped him to be more successful when solving the problems. He used manipulatives to represent various fractions and later identified the equivalent fractions. This was demonstrated in his response to the interview question. “Which resources do you think helped you to solve the problems?”

“Blocks, you can tell equivalent fraction by putting on one on the top of the other.”

Figure 4 shows George demonstrating equivalent fractions putting one on the top of the other (e.g., two halves on the top of the whole).

Figure 4. George demonstrating equivalent fractions.

A similar study by Jordan et al. (1999) showed that manipulatives helped children represent fractions and build a foundational knowledge for various mathematical concepts, which can then lead to understanding abstract mathematical concepts. Before moving to operations with fractions, you must make sure that children have a clear understanding of equivalence (Reys et al., 2018). In addition, the child enjoyed working with folding papers. He said:

"Folding papers, you have to think ways to fold to the right fraction. It gives you more exercise to your brain."

The observation results and George's responses highlighted the importance of teaching and learning using manipulatives to increase student engagement when learning mathematics. This approach is supported by previous studies such as Hunt et al. (2011) and Strom (2009). Hunt et al. (2011) showed that manipulatives could help children learn mathematics through visual representations by increasing engagement. However, George mentioned the wooden area model manipulatives reduced his engagement in solving the problems. This was reflected in one of his responses to the interview questions:

"The fraction is already done for you. Your task is to take it out."

This finding resonates with Wong (2010), who showed that children could develop the necessary conceptual understandings of fractions with teaching approaches that use the right manipulative for each concept. In addition, the literature reports teachers' effective pedagogical approaches were strained in this area (e.g., Blömeke et al., 2011; Getenet & Callingham, 2021).

Using Virtual Manipulatives

George loved and enjoyed being around technologies. He spent holidays playing with computers, iPads and Nintendo switches. However, George inclined more towards using PM than the VM in solving the fraction problems. He had the opportunity to use the VM for various purposes, such as grouping equivalent fractions and shading fractions sizes as part of a whole. During the interview, George mentioned that he had no positive experiences in using the VM. He was not as engaged with VM compared to the PM to solve the problems. His responses from the interview and the author's observation supported this conclusion. He said:

"Using the computer was not fun, it is only answering questions, but folding the paper is more fun and helps me to think."

This result supports the argument made by Day and Hurrell (2017). They showed that VM do not necessarily provide the same experience as concrete materials but still bridge the concrete materials to other representations.

George identified a few advantages and disadvantages of using the two forms of manipulatives (Table 2). He mentioned that one advantage of VM was for checking answers. Similar studies by Hunt et al. (201l) and Moyer et al. (2002) showed that using VM helped teachers to record and store users' online responses and movements, which makes the resources suitable for flexible learning.

Table 2
Likes and Dislikes Reasons for the Two Forms of Manipulative

Manipulatives	Likes reasons	Dislikes reasons
Physical	It helps to solve the fraction problem through thinking. The blocks are great to compare fractions. The blocks are great to find the equivalent fractions. You can do this by putting one on the top of the other. Folding papers is more fun and helps to think.	The blocks do not help to think when identifying fraction sizes. The parts are already made for you.
Virtual	You can check your answer. Different responses to a question.	Some of the questions are confusing and were not fun.

Conclusion

Various studies and literature stated that using a variety of manipulatives in mathematics classrooms is beneficial for children to understand mathematics concepts (e.g., Day & Hurrell, 2017; Reys et al., 2018). However, there have been a few studies on whether children equally prefer VM and PM to learn mathematics (Day & Hurrell, 2017; Dokić et al., 2022). On the one hand, various studies (e.g., Hunt et al., 2011; Moyer et al., 2002) showed VM were innovative and useful ways to enhance mathematics teaching. On the other hand, there is a line of thought that states VM should follow, not precede, the use of concrete manipulatives (e.g., Wu, 2013). In this study, George preferred and was more engaged when solving the fraction problems using PM than when using the VM. This finding highlights the need for further studies on VM use in teaching mathematics. It is worthy determining VM impact on students' learning and understanding of mathematical concepts.

The results of this case study contribute to the body of literature in several ways. First, they replicate previous findings in the positive effect of using manipulative in learning mathematics concepts (e.g., Day & Hurrell, 2017; Hunt et al., 2011). Second, the present findings suggest teachers should be selective when using PM and VM in their classrooms. Teachers should consider children's preferences and the resources pedagogical advantage when using these forms of manipulatives. It appears, in a well-planned teaching setting, both physical and virtual manipulatives can encourage students to make their knowledge explicit and help to build concrete mathematical knowledge. Finally, the study provides insight into using PM over VM to teach fraction concepts and skills. Perhaps some concepts are supported better by one or the other form of the two manipulatives.

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