RESEARCH PAPER

Exploring Visual Representations of Multiplication and Division in Early Years South African Mathematics Textbooks

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Early years mathematics textbooks are support material for teachers and learners as they contain visual representations which communicate and clarify mathematical concepts. In this paper, we report on the types and functions of visual representations of multiplication and division in South African early years mathematics textbooks. There seems to be a reliance on textbooks amongst teachers in South Africa as they assist in providing pedagogic content knowledge. The research from which this paper emerged was a document analysis of multiplication and division visual representations in nine textbooks (Grade 1–3). A Visual Representation Framework was used to analyse the textbooks. The findings indicated that the most dominant type of visual representations across all three textbook series and across the three grades were equal groups and the majority of visual representations had an exemplifying function.

Spaull and Taylor (2015) maintain that only 58.6% of South African Grade 6 learners are functionally numerate. The Southern and Eastern Africa Consortium for Monitoring Education Quality IV (SACMEQ) results attested to this and highlighted that South African learners had difficulties with multiplication and division (DBE, 2011). In response to the need to improve the South African learners' performance in mathematics, the Department of Basic Education (DBE) introduced a series of mathematics textbooks for Grades R to 7 learners in public schools. The textbooks were intended to standardise the content learning covered in each grade. This paper focuses on textbook research and asks the question: *What are the types and functions of visual representations of multiplication and division in early years (Grades 1–3) mathematics textbooks in South Africa?* Although the research was set within South Africa, textbooks are used internationally. We thus maintain that this research is of relevance to many countries worldwide and illustrates the application of the visual representation framework as an analytical tool.

Literature review

Ben-Peretz (1990) asserted that textbooks were the leading pedagogic tool used by teachers in trying to understand the intended curriculum. In many countries this is still the case. Textbooks have been used predominantly as a monitoring tool to inform teachers on what to teach, when to teach and how to teach (Nicol & Crespo, 2006). Textbooks provide a framework that informs teachers about possible ways to teach the required content (Nicol & Crespo, 2006) and assist teachers with learner assessment.

During the COVID-19 pandemic and ensuing lockdown, textbooks became an essential resource for learners during the closure of schools. In contexts such as South Africa, where most learners do not have access to modern technologies and data, the use of textbooks provided the only opportunity for learners to develop an understanding of multiplication and division concepts. This issued was raised more than 20 years ago by Harries and Spooner (2000).

Mathematics is a language rich in visual representations (Mudaly & Rampersad, 2010). Visual representations assist learners in developing an understanding of mathematics as they support and clarify mathematical concepts (Presmeg, 1986). The visual representations in a textbook may assist non-expert readers in understanding a mathematics concept (Fotakoupoulou & Spiliotopoulou, 2008; Mazumder et al., 2020). Presmeg (2006) asserted, a

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visual representation can also assist learners in making connections between and across mathematics concepts.

There are two broad views on the order in which to teach multiplication and division. First, Harries and Barmby (2007) asserted that mathematics is hierarchical and that the number operations should be taught sequentially. They proposed that number operations be taught in the following order: addition, subtraction, multiplication and division. In other words, they suggested that multiplication be taught before division. Second, Nunes and Bryant (1996) asserted that multiplication and division should be introduced to learners simultaneously to understand the inverse relationship between the two operations (multiplication and division). Although it is some time since these recommendations were published, the practices have endured in South African classrooms.

The South African Curriculum and Assessment Policy Statement (CAPS) for Mathematics in the early years (Grades R–3) introduces repeated addition leading to multiplication before repeated subtraction leading to division, grouping objects into groups and sharing a set of objects into groups. In the CAPS document, division is viewed as separate from multiplication and multiplication is taught first (Askew et al., 2019; DBE, 2011).

Theoretical Framework

This paper is underpinned by constructivism. Constructivism involves an active and meaningful learning process in the classroom (Kukla, 2000). Learners use *signs* and *tool*s to assist in making meaning and mediating the learning process (Daniels, 2008). Signs are defined as an internal psychological activity that involves mastering the tool used (Vygotsky, 1978). Veraksa (2013) suggested that a sign mediates the transitions between speech, actions, and concept formation. Tools are cultural artifacts that control behaviour from the outside (Daniels, 2008) and assist learners in accomplishing an activity. Signs and tools are mediated by actions (Werstch, 1997). In this study, the tools are the textbooks that learners use in the classroom.

Vygotsky (1978) argued that cognitive processing is still necessary for learning to occur and that signs and tools are auxiliary means of problem-solving. Learners construct meaning by using visual representations (Yackel, 2001). Thus, the visual representations used in textbooks are essential in supporting understanding of mathematics concepts as they are the signs and tools used to mediate the learning process of multiplication and division in the early years (Nghifimule, 2016).

Visual Representation Framework

A Visual Representation Framework developed by Fotakopoulou and Spiliotopoulou (2008) was used to analyse the nature of visual representation in textbooks. Fotakopoulou and Spiliotopoulou's (2008) framework examines the type of visual representation, its relation to content and reality, and the function of the visual representation. This paper focuses specifically on the type and function of the visual representation in the textbooks.

The visual representations *type* refers to the nature of the visual representations evident in the textbooks, such as equal groups, arrays, tables, function diagrams and number lines. The *function* of a visual representation refers to the purpose of the visual image and its relation to what is taught. The different functions include a decorative function, explanatory function, exemplifying function, complementary function, and organising function. A decorative function is a visual representation that serves aesthetic purposes. An explanatory function assists in expanding the concept to assist learners in completing the exercise. An exemplifying function provides an example of how to solve the problem. A complementary function is a visual representation accompanying the exercise to be completed. It provides the learner with additional information that might not be required to solve the problem. An organising function assists learners in structuring their thought processes (Fotakopoulou & Spiliotopoulou, 2008).

Methodology

An interpretivist orientation underpinned this research as the focus was meaning-making, interpreting, and understanding (Cohen et al., 2018) the types and functions of visual representations in textbooks. This research study has a subjective view of reality based on the researchers' interpretations of the textbooks analysed.

The research included both qualitative and quantitative data. Qualitative research allows for a thick description of the data, while quantitative research focuses on basic statistical methods (Apuke, 2017). A mixed-methods approach allows different areas of the research process to blend, for example, the research questions, orientations and methodologies (Cohen et al., 2018). By combining qualitative and quantitative data, the researcher can develop a more comprehensive understanding of the research question (Creswell, 2014). A mixed-method approach allows greater insights into the type and function of visual representations found in South African textbooks. The benefit of a mixed-method study is that it increases the credibility of the research results. Document analysis was used to generate the data. The sample included nine South African early years (Grades 1 to 3) textbooks. Three of the textbooks were the Department of Basic Education (DBE) workbooks and six of the textbooks were from two textbook series widely used in South Africa.

The textbooks were analysed using the Fotakoupoulou and Spiliotopoulou (2008) Visual Representation Framework. Initially, we analysed a Grade 4 textbook separately to ensure inter-rater reliability. After that, the first author focused on the Grades 1 to 3 textbooks. This involved identifying the types and functions of visual representations of the multiplication and division content present in the textbooks. The frequency of the different categories (i.e., the types of images and the function of each visual representation across the nine textbooks) was tallied using an excel spreadsheet.

Data Analysis and Findings

Types of Visual Representation

This section presents some of the Grade 1, 2 and 3 textbook analysis findings on the visual representations used to support learners' understanding of multiplication and division. The findings focus on the types and function of visual representations in the nine Grades 1 to 3 South African textbooks.

Across the three textbooks, Textbook A has the most visual images (n =282), followed by Textbook C (n = 76) and then Textbook B (n = 55). From Table 1 it is evident that the textbooks had more than double the number of multiplication visual representations (n = 209) compared to division visual representations (n = 83). There were 47 examples that consisted of both multiplication and division visual representations and 74 that were categorised as "other". This category included problems focused primarily on doubling and halving. This category was more evident in Textbook A.

There are a total of 122 images in the Grade 1 textbooks (A, B and C). As noted in Table1, Textbook A has the most visual images. Textbook A has 80 more visual representations than Textbook B.

The Grade 2 textbooks had a total of 128 images. Textbook A had 78 and 82 more images than Textbooks B and C, respectively (Table 1). There were 163 visual representations across the three Grade 3 books. Textbook A had 93 visual representations, Textbook B had 24 and

Textbook C had 46 visual representations (Table 1). Across all three grades, Textbook A has the most visual representations.

Table 1
Number of Visual Representations in Each Grade of Three Different Textbooks

Visual Representations of multiplication and division problems	Textbook A			Textbook B			Textbook C			Total
	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	
Number of visual representations	93	96	93	13	18	24	16	14	46	413
Multiplication	60	54	50	4	13	9	4	8	7	209
Division	12	17	11	6	5	8	0	1	23	83
Both Multiplication & Division	2	0	22	0	0	7	0	0	16	47
Other (Doubling, halving.)	19	25	10	3	0	0	12	5	0	74

A single exercise in a textbook, might consist of more than one type of visual representation (Figure 1). As noted in Table 2, the most prominent visual representations across the three textbooks are equal groups (256) followed by arrays (67). In Grade 1, Textbook B only has examples of equal groups and arrays while Textbook A includes number lines and Textbook C, a function diagram (Table 2). The Grade 2 textbooks have a wider variety of visual representation than Grade 1 (Table 2). Textbook A has more types of visual representations than Textbooks B and C. Textbook A is the only textbook that has multiplication grids, tables and unifix blocks in Grade 2 (Table 2). Textbook A and C have all the different types of visual representations listed in Table 2 except for multiplication grids and unifix cubes. In Grade 3, Textbooks A and C have number lines, equal groups, arrays, tables and function diagrams, Textbook B has no tables.

Table 2
Types of Visual Representations in Textbooks

Type of visual representations	Textbook A			Textbook B			Textbook C			Total
	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	
Number lines	4	12	10	0	3	3	0	1	9	42
Multiplication gird	0	2	0	0	0	0	0	0	0	2
Equal groups	105	57	26	9	7	10	10	10	22	256
Array representation	18	9	13	5	5	2	4	0	11	67
Tables	0	3	12	0	0	0	0	0	6	21
Function diagram	0	0	21	0	0	6	1	2	1	31
Unifix cubes	0	8	0	0	0	0	0	0	0	8

Figure 1 is an example of three different types of visual representations, that is, equal groups, arrays and number lines. This example was taken from Textbook A, Grade 2.

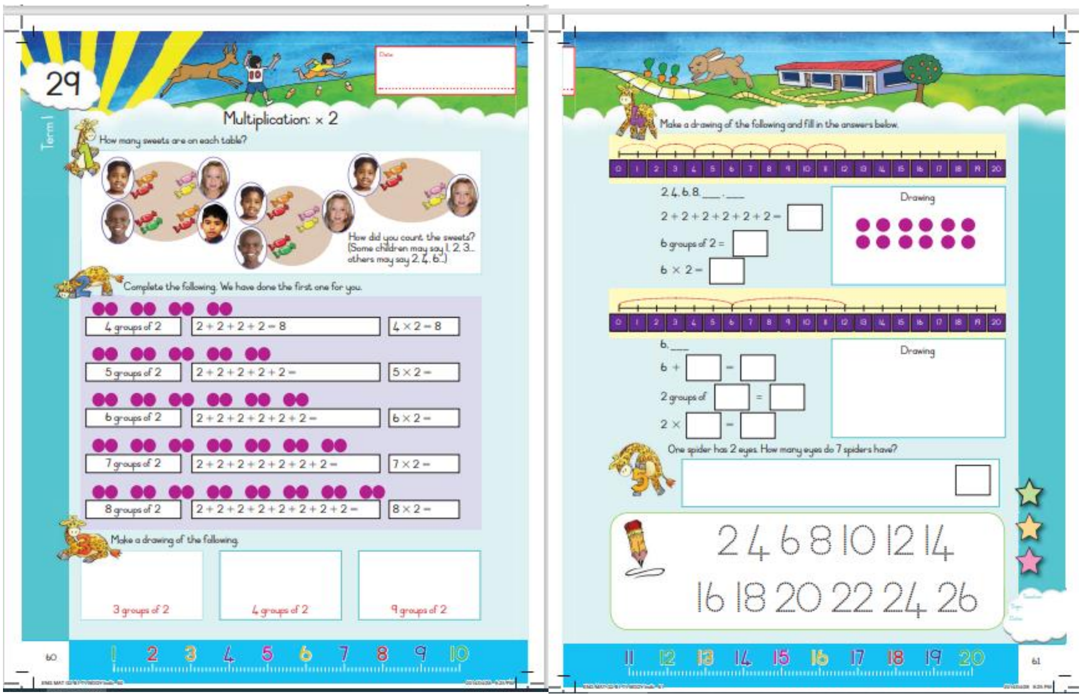

Figure 1. Equal groups and the number line.

In the exercise numbered 2 on page 60 of Figure 1, learners are expected to form a link between 4 groups of 2 as 2 + 2 + 2 + 2 (repeated addition) and 4 x 2 (multiplication). In the exercise numbered 4 on page 61 of Figure 1, the learners are required to make the link between skip counting, repeated addition, groups of and multiplication using a number line and drawing. The drawing is given in array form.

Function diagrams (n = 31) appear in all three textbooks but are most prominent in Textbook A (Figure 2). However, in Grade 1, Textbook C is the only textbook that exposes Grades 1 and 2 learners to function diagrams. A function diagram defines a relationship between the input number and the output number using a rule. For example, the function diagram provided in Figure 2 shows the relationship between the input (n = 11) and output (n = 55) by implementing the rule (x 5). After identifying the types of visual representations in the textbooks, analysis of the function (purpose) of each visual representation was undertaken.

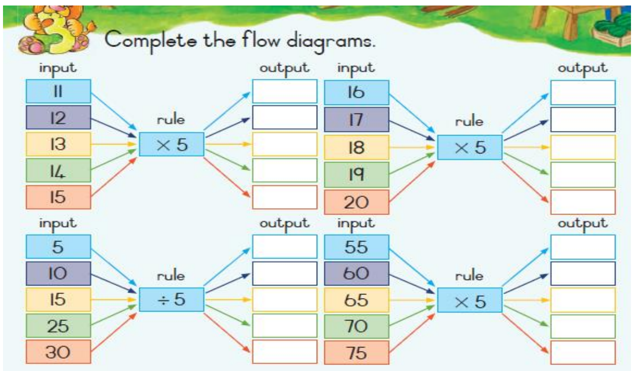

Figure 2. A function diagram.

The Function of the Visual Representations

Table 3 presents the functions of the multiplication and division visual representations across all Grade 3 textbooks. The most prominent visual representations had an exemplifying (n = 353) function followed by a complementary (n = 40) and explanatory function (n = 21). A visual representation with a decorative function (n = 2) was only evident in Textbook A.

Table 3
The function of visual representations in textbooks

Function of visual representations	Textbook A			Textbook B			Textbook C			Total
	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	
Decorative	0	0	2	0	0	0	0	0	0	2
Explanatory	4	3	3	0	0	4	0	0	4	21
Exemplifying	88	91	68	12	11	17	15	12	39	353
Complementary	0	2	22	0	7	3	1	2	3	40
Organising	0	0	0	0	0	0	0	0	0	0

Figure 3 is an example of a visual representation with an exemplifying function. The example shows that 5 x 4 is represented visually as 5 groups of 4 and 4 x 5 is represented visually as 4 groups of 5.

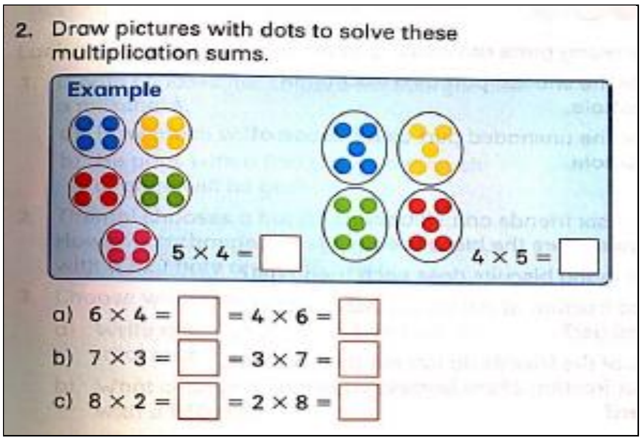

Figure 3. A visual representation with an exemplifying function (Textbook B).

Visual representations with a complementary function (n = 40) were evident across all the textbooks except for Grades 1 in Textbook A and B. Figure 4 is an example of a complementary function. The visual representation includes a stack of four books. Without the visual representation, the learners would still be able to solve the problem.

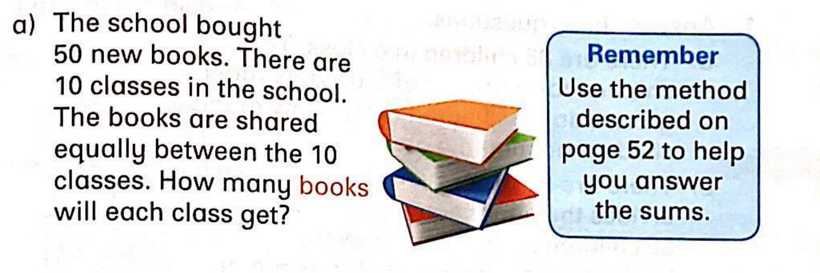

Figure 4. A visual representation with a complementary function (Textbook B).

Textbooks A, B and C all had examples, albeit very few, of visual representations with an explanatory function (Table 2). An example of a visual representation with an explanatory function is included in Figure 5. The speech bubbles show how the two learners solved each word problem.

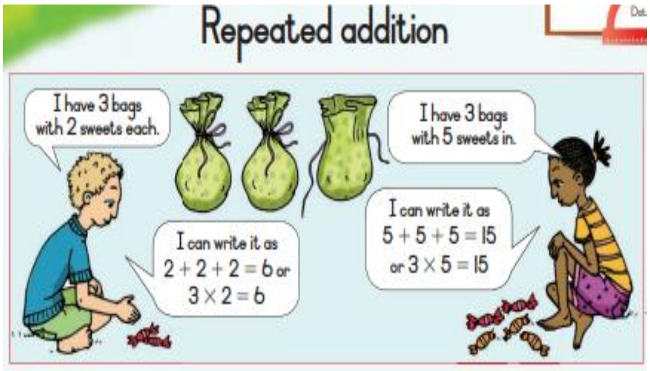

Figure 5. A visual representation with an explanatory function (Textbook A).

Discussion and Conclusion

Visual representations assist learners in understanding mathematical concepts (Kosko, 2019). In this research, equal groups were the most common visual representation type across all three textbooks. This is not surprising given the emphasis on multiplication and division as grouping in the curriculum document. CAPS (DBE, 2011) emphasised the different types of visual representations, namely equal groups, arrays, multiplication grids and number lines. However, there is a misalignment between the curriculum and the textbooks as the types and frequency of the visual representations differed. Textbook A in Grade 2 was the only textbook with a multiplication grid. Learners exposed to Textbook B and C were not exposed, through use of the textbook, to this type of visual representation as CAPS suggested.

Textbook B and C had fewer visual representations than Textbook A. This is concerning as learners use visual representations to make sense of the problem. Greeno and Hall (1997) ascertained that learners needed to be able to draw on multiple representations of a problem in order to develop their mathematical thinking fully.

The most common function of the visual representations across the grades in the three textbooks was an exemplifying one. An exemplifying function is crucial as it provides the learner with support in how to solve the problem. The second most prominent function, an explanatory function, was found in all grades in Textbook A, but only in Grade 3 in Textbooks B and C. An explanatory function elaborates on the thinking process necessary to solve the problem.

This study found that textbooks for early years contained written explanations for young learners who may or may not be proficient in reading and comprehending the explanation. The learners may depend on the teacher to explain the explanatory visual representations. The second most prominent function in Textbooks A, B and C in Grade 2 and 3 was a complementary function.

The government-provided textbook (Textbook A) had the most visual representations and reflected a greater variety of visual representations. Textbook B and C were the most popular textbooks in South Africa, yet they had very few visual representations to support learners. An implication of this study is the importance of quality textbooks with a variety of visual representations and their alignment with the intended curriculum.

References

Apuke, O. (2017). Quantitative research methods: A synopsis approach. *Kuwait Chapter of Arabian Journal of Business and Management*. https://doi/10.12816/0040336

Askew, M., Venkat, H., Mathews, C., Ramsingh, V., Takane, T., & Roberts, N. (2019). Multiplicative reasoning: An intervention's impact on foundation phase learners' understanding. *South African Journal of Childhood Education*, *9*(1), 1–10. https://doi.org/10.4102/sajce.v9i1.622

Ben-Peretz, M. (1990). *The teacher-curriculum encounter: Freeing teachers from the tyranny of text*. State University of New York Press.

Cohen, L. Manion, L., & Morrison, K. (2018). *Research methods in education* (8th ed.). Routledge

Creswell, J. 2014. *A concise introduction to mixed methods research*. SAGE Publications.

Daniels, H. (2008). *Vygotsky and research*. Routledge.

Department of Basic Educations (DBE). (2011). *Curriculum and assessment policy statement (CAPS): Foundation phase mathematics grade R–3.* Pretoria.

Fotakopoulou, D., & Spiliotopoulou, V. (2008). Visual representations in secondary school textbooks of economics*. Paper presented at the Conference on Applied Economics: ICOAE School of Pedagogical and Technological Education (A.S.PE.T.E.),* Division of Patras: Patras

Greeno, J. G. & Hall, R. P. (1997). Practising representation: learning with and about representational forms. The Phi Delta Kappan,*78*(5), 361–367.

Harries, T., & Barmby, P. (2007). Representing and understanding multiplication. *Research in Mathematics Education*, *9*(1), 33–45.

Harries, T., & Spooner, M. (2000). *Mental mathematics for the numeracy hour.* Routledge

Kukla, A. (2000). *Social constructivism and philosophy of science*. Routledge.

Mazumder, S., Latulipe, C., & Perez-Quinones, M. (2020). Are variable, array and object diagrams in java textbooks explanative. *Proceedings of Innovation and Technology in Computer Science Education* (Vol 1, pp. 425–431). ITiCSE.

Mudaly, V. & Rampersad, R. (2010). The role of visualisation in learners' conceptual understanding of graphical functional relationships. *African Journal of Research in Mathematics, Science and technology Education, 1*(14), 36–48.

Nghifimule, S. (2016). *An analysis of the nature of visualisation objects in three Namibian grade 9 mathematics textbooks: A case study in Namibia.* [Master's thesis, Rhodes University].

Nicol, C. C., & Crespo, S. M. (2006). Learning to teach with mathematics textbooks: How preservice teachers interpret and use curriculum materials. *Educational Studies in Mathematics*, *62*(3), 331–355. https://doi.org/10.1007/s10649-006-5423-y

Nunes, T., & Bryant, P. (1996). *Children doing mathematics*. Blackwell.

Presmeg, N. C. (1986). Visualisation in high school mathematics. *For the Learning of Mathematics*, *6*(3), 42–46.

Presmeg, N. (2006). Research on Visualization in Learning and Teaching Mathematics. *Handbook of Research on the Psychology of Mathematics Education*. https://doi.org/10.1163/9789087901127_009

Spaull, N., & Taylor, S. (2015). Access to what? Creating a composite measure of educational quantity and educational quality for 11 African countries. *Comparative Education Review, 59*(1), 133–165.

Veraksa, A. N. (2013). Symbol as a cognitive tool. *Psychology in Russia: state of the art*, *6*(1), 57-65.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Harvard University Press.

Yackel, E. (2001). Explanation, justification and argumentation in mathematical classrooms. *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 9–24). PME.