GRADED ARITHMETICS
# BOOK SIX-GRADE VII.
CHANCELLOR

ELEMENTARY SCHOOL MATHEMATICS BY GRADES

GLOBE SERIES

SIXTH BOOK

COMMERCIAL AFFAIRS

BY

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SUPERINTENDENT OF SCHOOLS, BLOOMFIELD, N.J.
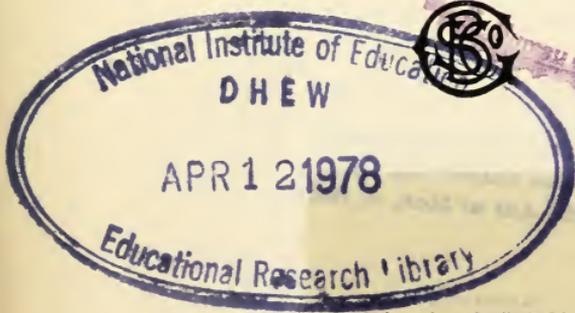
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NEW YORK AND CHICAGO

" Geometry has many and very important applications in
the arts and trades. Its applications are as important as
those of arithmetic, if we except the very simplest and com-
monest arithmetical operations. That the great mass of
American children should leave school without ever having
touched this subject — except perhaps in arithmetic under
the head of mensuration — is a grave public misfortune."

President Charles William Eliot, LL.D.,
Harvard University.

From "Educational Reform."

" The course . . . that I would suggest for the American
school . . . involves beginning algebra and plane geometry
shortly after . . . ten or eleven years of age. . . . In order
to prevent the pupil's development from being arrested and
his capacity for education from being brought to an end,
he must constantly be led on to new heights."

President Nicholas Murray Butler, LL.D.,
Columbia University.

From "The Meaning of Education."

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M. P. 1

PREFACE

Boys and girls in the upper grammar grades of the free common schools of American towns and cities are pursuing courses of study by no means uniform. There is perhaps no general agreement in educational practice beyond the fact that it is considered desirable that boys and girls should advance from elementary to secondary schools at about fourteen years of age. There are, however, marked tendencies to agree upon several other points.

There is increasing agreement to the effect that pupils in the upper grammar grades should study what will most benefit them in discipline and in knowledge. We have come to see in the light of the new knowledge of the facts and principles of mental and moral growth, presented by genetic psychology, that what a child enjoys learning he profits by, and that what he profits by develops in him the normal life of the child, which is the guarantee of an efficient life as an adult. There is a growing tendency to decrease the range of arithmetical instruction and to introduce much more geometrical instruction. There is also a distinct tendency to rely more and more upon the various forms of "manual training" in the education of boys and girls; and this development along the lines of the industrial arts, which afford the materials of manual-mental discipline, lends itself notably to the encouragement of the study of geometry early in life.

In the primary and first grammar grades children may easily learn how to add, subtract, multiply, and divide accurately and rapidly, and common and decimal fractions, factoring, cancelling, finding least common multiples and greatest common divisors, and all the facts and operations of denominate numbers. They ought to learn to image correctly the facts involved

in ratios, percentages, and measurements, and to understand the elements of simple proportion.

It appears from the investigations of child-students and of psychologists that unless a boy learns before the age of ten or twelve how to perform the fundamental operations both correctly and quickly he seldom becomes proficient later. Early proficiency, however, can be maintained only by constant exercise. For boys and girls who are prepared in the essential elements of arithmetic this later book proposes extended instruction in percentage and commercial transactions, and in the elements of geometry. It is one of a series of handbooks for pupils, and consists of progressive lessons, arranged mainly in topics, either reviewing the treatment in earlier books, or completing the subject, or preparing for the next book.

In mathematics we have our traditions as to what ought and what ought not to be taught in the different grades. These traditions had their origins long before either courses of study were scientifically ordered or men questioned themselves as to the stages in the growth of the mind. In consequence there are many easy processes in mathematics which are postponed until after much more difficult processes have been mastered, at needlessly great costs in time and energy. It too often happens that the attack upon these more difficult processes results in such discouragement that the student never completes even the elementary school courses. It is not the purpose of this Series to overturn the accepted order of mathematical topics ; but certain changes have been made in the direction suggested. The utilitarian value of the simplest geometrical exercises is not ·less than that of many arithmetical exercises ; and their cultural value is greater because they fit more closely the powers and needs of the minds of boys and girls. It is unquestionably good pedagogy and sound common sense to develop for boys and girls fundamental geometric principles, of angles and areas, of forms and of volumes, even at the expense of an encyclopedic knowledge of the rules and methods of interest,

discount, partial payments, and cube root at an age when the student is still living the natural life of the boy or girl and has years yet ahead before needing or caring to know all the conventions of the world of finance. We are not all destined for bank clerkships. There are more mechanics than merchants in the world. We cannot use tools or examine the construction of things made with tools unless we know something of geometry.

A great amount of material has been presented in this book so as to give the teacher an unusually large and free range of selection. No class in one year is expected even to try to solve every problem. Classes in the same grade vary radically in power. The first principle of each book, that it is a text-book for the students in their personal study rather than a handbook for teachers, necessitates the introduction of a considerable amount of explanatory instruction. Too much dependence upon oral teaching makes the pupil weak. To develop the self-activity of the boys and girls is the most important aim of education; and to secure such original effort is to establish the foundation of self-reliance, which is the substance of true character.

Author and publishers desire to acknowledge the helpful and valuable suggestions of Mr. G. I. Aldrich, Superintendent of Schools, Brookline, Massachusetts, in revising the proofs of this book. We are indebted also for criticisms of methods and problems to several business men and teachers, among whom Miss Grace E. Jones, Center Grammar School, Bloomfield, has given important assistance. No effort has been spared to make the text at once modern and practical.

W. E. C.

Bloomfield, N. J.,
    May 24, 1902.

SUGGESTIONS TO TEACHERS

1. The preface explains the general purpose of this book.

2. Read also the prefaces and suggestions to teachers in each of the earlier books of this Series. It may be desirable to review some of their exercises before taking up this book systematically. The value of these exercises in awakening the pupils' interest and activity is speedily evident upon trial.

3. Read this book itself. The purposes of certain features appear only when considered in relation to other features.

4. This book deals with eminently practical matters. When discussing any special topic and at any time after having discussed it, welcome suggestions and information from the pupils regarding the way business men, artisans, mechanics deal with the same subject. Encourage the boys and girls to get into touch with the world of affairs. If the time of the recitation is being unduly encroached upon, postpone lengthy discussions to private talks, or, if the matter is important to all, to some suitable time " between periods," or at the beginning or end of the session. It is worth very much to boys and girls, especially to those who will not continue in school long, to be encouraged to observe and to think for themselves.

5. Remember that in our American schools, during or just after each of the fourth, fifth, sixth, seventh, eighth, and ninth years in school, from ten to twenty-five per cent of a class drop out of school. In a sixth or seventh year class of forty-five boys and girls using this book, a half dozen, more or less, will remember throughout life this instruction as the highest stage of their formal education. Some of these may be among the most promising students, sifted out from their

class by economic or social forces. For these the cultural quality of their instruction and association in school is even more important than the utilitarian. Even more than the other students those who drop out early need to know not only the processes and the methods of arithmetic, but the reasons involved. We are too apt to judge the ability of students in comparison with our own experienced skill or in comparison with the rapid work of the most forward students, who are by no means always the most thorough, the most retentive, and the most accurate. There is danger in teaching too rapidly just as there is in overdeveloping a lesson.

6. The nature of the human mind is such that when in a student's effort upon a problem he shows that he is radically deficient in the fundamental operations, it becomes the teacher's duty to give to him individual exercises. The dropped stitch in knitting is a trifle compared with an omitted process in an art. And further, if anything has been thoroughly demonstrated by the study of the psychology of children and youth, it is this, that we become proficient in addition, subtraction, multiplication, and division most easily when from eight to twelve years of age. To postpone to later years the boy's acquirement of rapidity and of accuracy is to make that acquirement yet harder for him. When we find our students compelled to add columns over and over again, because of getting different results, we know that the time when they could learn addition most quickly and surely has already passed. With every later added year the difficulty becomes greater. This book, however, does not devote very much space to the fundamental operations. Individuals who need special drill in the elements may be trained in the earlier books of this Series.

7. As all measurements involve ratio, and as measurement is the chief topic of this book, it is desirable to cultivate the habit of observation in the children. Various passages in the text suggest the sort of questions we may ask from time to time in order to lead the student to make comparisons. The

habit of noticing sizes and weights is as valuable as that of noticing forms, colors, and textures, which is developed by drawing and manual training.

8. All arithmetic must be mental; but oral recitations, passing immediately from one kind of problem to another and using small numbers, insure the student's reasoning upon the problems. Reasoning is the soul of arithmetic.

9. Neatness tends to accuracy in all the written work. But it is easy to cause much unprofitable time to be spent in copying correct solutions, carelessly written. As far as possible our pupils should be induced to write neatly the first time. Even permission to copy encourages in some natures the habits of carelessness and of slovenliness. It becomes extremely important for this reason, as well as for others even greater, to study and to know the characters, needs, and powers of each individual in a class.

10. From a half to a whole page will be found usually a sufficient lesson. One hundred pages of problems make a reasonable year's work. Topics and problems are offered here in sufficient variety to permit a considerable range of choice in planning a grade's assignment. Whether problems should be given out for home-study is a question not entirely settled; but the reasons for having all problems done in school hours by pupils of this grade rather outweigh the convenience and the apparent saving of time in school resultant from giving problems to be done at home.

11. Never refuse to accept a correct solution of a problem, which can be explained by the student, even if the solution is extremely indirect and inconvenient. But if there is a better method, make its excellence plain.

12. Many problems at first to be solved only in writing may later be solved orally. In reviews of problems on earlier pages oral explanations should be encouraged.

13. Use concrete materials and illustrative drawings as much as time permits. Arithmetic cannot be too clear.

14. One good mode of solution well understood is worth any number of solutions but partly comprehended. On the other hand, one method of solution often throws much light upon another method.

15. Pupils who have thoroughly mastered the fundamental operations need not perform the work of all problems; let them rather indicate what must be done, giving reasons. It is well to remember that the great error of arithmetical teaching has been too much drill on problems to the neglect of reasoning and of logic, and that the great error of geometrical teaching has been too much drill in reasoning and in logic to the neglect of problems.

16. The five principles of the recitation carried out systematically insure success in the arithmetic lesson. Let the *preparation* of the class for recitation be oral, with easy review exercises and questions. Make the *presentation* definite, with the written test-exercise after it for the *generalization*. Question here closely. Secure brief oral explanations of problems for *recapitulation*. And by further questioning bring the *application* home to the children's lives.

17. The fact that a problem is hard is not necessarily a reason for not requiring its solution. Effort is the mountain air of the soul. Difficulty lends interest. Arithmetic is the main reliance of modern education to develop carefulness of mind and power of attack and persistence. Only those boys and girls know the meaning and get the benefit of arithmetic who study and master its difficulties.

TABLE OF CONTENTS

INTRODUCTORY REVIEW — ORAL

1. A dealer sold a piano for $375, at a profit of 25%. What was its cost to him?

2. His expenses of sale and delivery were 10% of the price that he paid for the piano. What was his net profit?

3. What is the cost of $8\frac{3}{8}$ yd. of silk at $5 per yard?

4. How many pecks make $7\frac{1}{2}$ bu.?

5. What is left from six hundred thousand tons of coal after 60,897 tons are sold?

6. What will 15 lb. of bacon cost at $9\frac{1}{2}$ ¢ a pound?

7. $20 - \frac{3}{5} = ?$ $168 - 30\frac{3}{5} = ?$ $729 - 125\frac{33}{40} = ?$

8. What is the radius of a wheel 28 in. in diameter? What is the length of its circumference?

9. What will $7\frac{3}{4}$ yd. of lining cost at 12¢ a yard?

10. Change 10,000 pt. to gallons.

11. How many cubic feet are there in a wall 20 ft. long, 3 ft. high, and 3 ft. thick?

12. A garden is 45 ft. by 25 ft. in size. How many square feet has the gardener to tend?

13. What will $6\frac{7}{8}$ yd. of gingham cost at 12¢ a yard?

14. What time elapsed from October 12, 1492, to April 19, 1775?

11

15. Add by columns and rows:

2	5	7	9	6	4	8	1	3	6
4	7	3	8	9	1	7	5	2	9
7	2	1	6	5	3	4	8	9	5
1	6	8	2	3	5	9	7	4	4
8	1	4	5	2	6	3	9	7	8
3	8	2	7	4	9	5	6	1	3
5	9	6	4	7	2	1	3	8	7

16. Add by columns and rows :

2	4	2	8	7	9	9	9	8	7
2	5	3	7	5	1	8	8	8	7
2	6	2	6	9	1	5	6	8	7
5	5	5	9	6	1	7	6	6	6
5	6	4	5	5	2	7	6	5	8
8	7	5	5	7	3	7	3	3	3
8	8	4	6	3	2	4	4	4	2

17. When 96¢ is the price of a bushel of wheat, what do 1½ bu. cost ?

18. Divide the number 35 into two parts, whose ratio is as 6 to 1.

19. When 1 yd. of muslin is worth 20¢, what is the cost of 16½ yd. ?   12¼ yd. ?

20. A rectangular city lot, 150 ft. by 100 ft., is worth $2 per square foot.   What is its total value ?

21. Give the Multiplication Tables in the following orders :

    (a) 5, 10, 3, 6, 12, 2, 4, 8, 7, 9, 11.

    (b) 2, 3, 6, 4, 8, 5, 10, 12, 11, 9, 7.

    (c) 12, 9, 6, 3, 8, 4, 2, 10, 5, 11, 7.

22. What are the prime factors of 100 ? 96 ? 144 ?

23. At 3¢ per foot, what is the cost of picture molding for a room 14 ft. × 12 ft. ?

24. Mr. S took his four boys with him to the city. He bought 3 full-fare " return " tickets at 60¢ each and 2 half-fare tickets. His other expenses averaged 35¢ for each person in the party. How much more or less than $5 did he spend?

25. The conductor on a street car collected 85 5¢ fares and 6 3¢ fares in one trip. How much was this in all ?

26. A commercial traveler was away on a trip for 38 da. at an average expense of $8 a day for railroad, hotel, and all other costs. What were his total expenses?

27. A man and his wife traveled and lived in Europe for 42 wk. at a total cost of $840. What was the average cost per week for each person ?

28. A roast of beef at 18¢ a pound cost $1.53. How many pounds did the meat weigh?

29. A boy wore out in one year 3 pr. of shoes, costing $1.75, $2.25, and $1.50 per pair. Repairs cost $1.70 during the year. What was the average cost per month of shoe leather for this boy ?

30. Mr. Z furnished a house at a total cost of $1080. He spent $200 on the parlor furniture, $100 on that in the sitting room, $250 for the dining room and kitchen, $30 for the hall, and the balance for the rooms upstairs, of which there were six. What was his average expenditure per room upstairs ?

31. A man earning $3.50 a day at his trade worked 200 da. in 1 yr. He earned also $30 by work outside of his trade. What was the average amount that he had to spend per day for the year ?

32. Write in Roman notation : 1776; 1837; 1949.

33. A boy with 36 chickens had 3 worth $2.50 each, 6 worth $1.50 each, and the rest worth on the average 50¢ each. How much were all the chickens worth?

34. Which is more, 25×12 or $33\frac{1}{3} \times 9$?

35. A train going at the average rate, including stops, of 35 mi. per hour requires how many hours to go 1050 mi.?

36. Mr. Q bought a 12-room house for $6000 and paid $800 for repairs and $400 for additions. Mr. R bought a plot of ground for $1200, built a 10-room house costing $5000, and paid $400 for interior decorations.

a. What was the cost of each house?

b. What was the difference in cost?

c. Considering Mr. Q's land worth $1200, what did his house cost per room?

d. What did Mr. R's house cost per room completed?

37. In a town with a 2% tax rate A paid $50 tax on his house, B $120, and C $200. What was the assessed valuation of each man's house?

38. William ran a half mile in 4 min., and John two miles in 18 min. Which ran faster? At the same rate, how long would it take William to run two miles?

39. John with a 10-yd. start was caught by William at the end of 100 yd. How much faster was William's rate than John's?

40. What is the cost of 6 lb. of butter at 22¢ per pound?

41. What is the cost of a carpet at 80¢ per yard for a room 15 ft. × 15 ft.?

42. Subtract 5^3 from $10 \times 10 \div 4 \times 6$.

43. Multiply 3^4 by 2^5.

44. What was the interest on a promissory note for 90 da. for $500 at 6% per annum?

45. Review the Tables of Denominate Numbers.

INTRODUCTORY REVIEW — WRITTEN

1. Find the number of inches in a mile.

2. Find the number of square inches in an acre.

3. Find the number of cubic inches in a cubic yard.

4. Find the number of gills in a barrel.

5. What is the number of pints in a bushel?

6. What is the number of ounces in a ton?

7. Find the number of seconds, circular or angular measure, in a circumference.

8. What is the number of sheets in a bale?

9. Find the number of seconds in a day.

10. A township is 12 miles square. How many acres of land does it contain?

11. A railroad tunnel is 400 ft. long, 21 ft. high, and 1 rd. wide. How many cu. yd. of earth and rock were excavated?

12. How many days were there in the life of a man who died, aged 96 yr. 3 mo. 2 da., if Feb. 29, 1804, was his birthday?

13. Draw to the scale of $\frac{1}{8}$ in. to 1 in. each of the following figures :

   a. parallelogram, 9 in. × 8 in.

   b. isosceles triangle, 6 in. × 12 in.

   c. rhombus, 12 in. × ? in.

   d. rhomboid, 15 in. × 10 in.

   e. isosceles trapezoid, 12 in. × 6 in. × 8 in. × 6 in.

   f. right angle triangle, 9 in. × 10 in.

   g. trapezium, with any measurements.

   Use compasses and straight-edge ruled to $\frac{1}{8}$ inches.

14. What is the cost to the buyer of 817 suits at $100 per doz. less 2% for cash?

15. Divide :

a. 1496.04 by 10.   e. .0992 by .32.

b. 1596.04 by 1000.   f. 17.6 by 44.

c. .00207 by .009.   g. .00021 by .0007.

d. 307.14646 by 12.17.   h. .56 by 1.12.

16. Add :

a. 447147.909 + 365.18698 + 909627.83 + 5987.3501 + 967390.293 + 10935.6 + 29627.03.

b. 5582580.14 + 72102.162 + 739.7918 + 3547600.7 + 421654713 + 578075241 + 32194320 + 864127659.

17. Multiply :

a. 36 by $\frac{7}{99}$.   b. 144 by $\frac{11}{18}$.   c. 375 by $\frac{13}{15}$.

d. 2277 by $\frac{70}{99}$.   e. 3228 by $\frac{15}{64}$.   f. 1000 by $\frac{4}{75}$.

18. Add :

yr.	da.	hr.	min.
50	127	7	50
	210	9	44
76	121	11	44
6	47	3	41
8	9	11	17

19. Add :

yds.	ft.	in.
16	2	9
17	1	11
18	0	55
17	2	7
12	1	8

20. Change 4,178,340 pt. to quarts and gallons.

21. At 3¢ a pint how many gallons of milk can one buy for $1.92?

22. If it takes 6 weeks for 5 men to earn $405, how long will it take 4 men to earn $540?

23. A, B, and C rent a pasture for $192. A puts in 8 cows; B, 9 cows, and C, 11 cows. How much money should each man pay?

24. From a tank holding 3465 gallons there were drawn out 75¼ barrels, of 31.5 gallons each. How many gallons were left in the tank?

25. Add :

rd.	yd.	ft.	in.
10	4	2	8
1	3	0	5
8	2	1	6
1	1	0	4

26. Add :

sq. yd.	sq. ft.	sq. in.
100	8	130
50	0	100
0	8	143
13	2	8

27. $1414103697906795 \div 467985 = ?$

28. $985903616419668 \div 589476 = ?$

29. Four partners divided $336 as follows : the first partner took one-sixth of the whole, the second one-fourth of what was left, the third one-half of what was then left ; the fourth had the remainder. How many dollars did each partner get ?

30. What is the interest at 6% on a note of 90 days for $1250?

31. Perform the operation indicated :

a. $25.001 - .608975$. *d.* $3.1 - .76805$. *g.* $4.0125 - .90867$.

b $.0021 - .000685$.   *e.* $201 - .98765$. *h.* $3.215 - .96875$.

c. $.001 - .0006875$. *f.* $31 - .316095$. *i.* $4 - .487595$.

32. Find the cost of 40 trunks at $$8.37\frac{1}{2}$ apiece.

33. A merchant bought 1845 pounds of cheese at 9 ¢ a pound. He sold 528 pounds at 9 ¢ a pound, and 856 pounds at 8 ¢ a pound. How many pounds had he left ? What was their net cost to him ?

34. Add : six hundred nineteen, eight thousand ninety-eight, nine, nine hundred thousand seven hundred seventeen, one billion eighty-three million seven thousand eighty-six, sixty-five thousand four hundred eighty-four, nine thousand ninety, nineteen thousand nine, seventy-seven thousand seventy, sixteen million eighty-five thousand eight hundred, nine hundred seventy-four, eighteen.

35. $7 : \$30 :: 9 : \$x$.   **36.** $16 : 27$ yd. $:: x : 84$ yd.

37. What decimal part of 81 is 200 ?

38. A farmer gave $46 for some calves, 5 of which he sold for $5⅗ each. He traded the rest for 5½ barrels of flour at $4 a barrel. How much did he gain ?

39. Subtract :

a. 10602	*b.* 14002	*c.* 20601	*d.* 13002
9087	8607	4908	6709

40. In 1896 the Republican vote for Presidential electors was 7,106,779, and the Democratic 6,502,925. *a.* What was the sum of the votes? What was the difference?

In 1900 the Republican vote was 7,208,244, and the Democratic 6,358,789. *b.* What was the sum of the votes? What was the difference between the votes?

c. What was the difference between the Republican votes of 1896 and 1900? the Democratic votes?

d. What was the difference between the Republican pluralities in the two elections?

41. A can do a piece of work alone in 10 days, and B in 13 days. In what time can they do it when they work together?

42. How many bricks, 8 inches long and 4 inches wide, will pave a yard that is 100 feet by 50 feet ?

43. What is the value of 10 bu. 3 pk. 5 qt. of corn, at 62½ ¢ a bushel ?

44. Mr. Jones employs 28 of Mr. Smith's workmen for 15 days. For how many days must 21 of Mr. Jones's workmen work for Mr. Smith in order to settle the account ?

45. Divide :

 a. 18 × 36 × 72 × 144 by 6 × 6 × 8 × 9 × 12 × 8.

 b. 4 × 6 × 3 × 5 by 5 × 9 × 12 × 16.

46. Find L. C. D. and add :

    a. $\frac{5}{7}, \frac{11}{12}, \frac{2}{15}, \frac{8}{27}, \frac{9}{35}, \frac{17}{40}.$

    b. $\frac{14}{15}, \frac{7}{8}, \frac{4}{3}, \frac{11}{12}, \frac{6}{11}, \frac{19}{20}, \frac{6}{7}, \frac{29}{35}.$

47. Multiply :

  a. 39948123 by 6007.   *b.* 534.28 by 84.629.

48.   COLUMBUS, O., July 12, 1900.

J. E. KNOX,

               To THE WASHINGTON SUPPLIES CO., *Dr.*

To 1 Harvester,	$170.00
1 Boston Mower,	200.00
1 Gem Thresher,	625.00
2 Saratoga Corn Mills,	30.00
3 A1 Hay Rakes,	60.00
2 Wheelbarrows,	3.25
6 Shovels,	1.15

             Paid, July 15, 1900,

                 THE WASHINGTON SUPPLIES CO.,

                        Per W. H. M.

  Complete the entries and find the total.

49. Subtract :

	yd.	ft.	in.			sq. yd.	sq. ft.	sq. in.
a.	15	1	5		*b.*	12	4	72
	13	2	7			10	7	99

	cu. yd.	cu. ft.	cu. in.			yr.	da.	hr.	min.	sec
c.	37	18	857		*d.*	7	129	13	26	17
	35	24	1280			3	273	18	34	29

50. A lot is 20 rd. wide. How long must it be to contain 10 A.?

51. How many square feet are there in a floor 40 ft. long and $13\frac{1}{2}$ ft. wide ?

52. How long must a street be made to contain 45 A. of surface when it is 4 rd. wide?

53. How many yards of carpeting, 27 in. wide, with its design repeated at every yard, are required to cover a room 15 ft. 9 in. by 16 ft. 7 in.?

54. How many square feet are there in 12 boards, averaging 12 ft. 8 in. long by 1 ft. 9 in. wide each?

55. Since the moon's average daily motion is 13° 10′ 35″, how much of her orbit does she go through in 14 da.?

56. When a man averages daily 7 hr. 44 min. 52 sec. of sleep, how much time does he spend in sleep in 100 days?

57. X purchased a hogshead of molasses containing 76 gal. 3 qt. 1 pt., for $58.25; lost 9 gal. 1 qt. 1 pt. by leakage; and sold the rest for 88 ¢ a gallon. What was his gain?

58. From a tract of land 80 rd. square a field 40 rd. square was sold at $75 per acre. On the rest was raised wheat, which at $1 a bushel brought $600. What was the yield per acre, and what was the total of money received for land and grain?

59. What is the difference of time between Aug. 5, 1896, and Mar. 14, 1899?

60. What time elapsed between Sept. 22, 1895, and July 1, 1896?

61. Subtract 161° 34′ 11.8″ from 180°.

62. *a.* Subtract 4 mills from $40. *b.* 1 ¢ from $1 and 1 m. *c.* .08 of a dime from 1 ¢ 8 m.

63. *a.* Divide 19 hr. 53.2 sec. by 7. *b.* 152° 46′ 2″ by 9. *c.* 3 yr. 7 mo. 2 wk. by 12.

64. How many steps, each 2 ft. 9 in., does a man take in going $3\frac{1}{4}$ mi.?

65. The wheels of a locomotive are 15 ft. 5 in. in circumference, and make 8 revolutions a second. How long does it take to run 100 mi. ?

66. When at one load a span of horses can draw 1 cd. 112 cu. ft., how many cords can they draw in 10 loads ?

67. When a thousand soldiers consume 12 gal. 3 qt. 1 pt. of molasses in 1 week, at the same rate how much will they consume in 1 year ?

68. When a man walks 4 mi. 145 rd. in an hour, how far at the same rate can he walk in 2 days of 10 hours each ?

69. The difference between three times a certain number and twice the number is 15. What is the number?

70. A boy sold a certain number of papers. If he had sold four times as many, he would have sold in all 196. How many did he sell ?

71. A speculator bought a lot 25 rd. long and 16 rd. wide for $100,000, and sold it for $1.25 a square foot. How much did he gain ?

72. A man walked in 1 day 23 mi. 264 rd. How many paces did he take, allowing 3 ft. for each pace ?

73. How many acres are there in the desert of Sahara, which is 3000 mi. long and 1000 mi. wide ?

74. How many yards of plain carpeting, 1 yard wide, will carpet the floor of a room 15 ft. long and 12 ft. wide, deducting a tiled space 6 ft. long and 18 in. wide ?

75. A bushel of wheat is produced on a piece of land 2 rd. square. What is the ratio of the yield of this piece to the yield of an acre ?

76. A carpenter received a bill for 25 planks, each 18 ft. long by 16 in. wide by 2½ in. thick, at $19 per M. What was the amount of his bill ?

77. Marble weighs 170 lb. a cubic foot. What is the weight of a marble slab 6 ft. × 3 ft. × 4 in. ?

78. A cubic foot of turpentine weighs 870 oz. avoirdupois. What is the total weight of 48 bbl. of turpentine, each barrel containing 36 gal., allowing 60 lb. for the weight of a barrel?

79. What do 3 bu. 3 pk. 5 qt. of clover seed cost at 7 ¢ a quart?

80. The Declaration of Independence was adopted July 4, 1776. How long was it from that date to the declaration of war against England, June 18, 1812?

81. A man borrowed some money Aug. 15, 1895, and paid it April 7, 1899; how long did he keep it?

82. How many cords of wood are there in 10 piles, each pile containing 112 cu. ft. 864 cu. in.?

83. *a.* $6 \times (5 + 3) - (15 - 12 + 4) + 5(6 + 7 - 5) = ?$

b. $9(8.12 - 3.19) + .3(8 - 7.2138) - (8 - .8) = ?$

c. $\dfrac{16 \times 15 + 3 \times 4 \times 18 - 2 \times 6 + 3 \times 5}{3 \times 2} = ?$

84. Draw to scale various parallelograms each with an area of 24 sq. in.

85. Divide a farm of 100 acres 168 rd. of land between 4 persons, A, B, C, and D, so that A shall have one-sixth of the entire farm, B one-fourth of the remainder, C one-third of what then remains, and D the rest. How much does each one have?

86. 25 men together spend on a piece of work enough time to amount to 37 yr. 11 mo. What was the average time spent by each man, and what amount of money on the average did each earn at the rate of $2.25 a day?

87. When a steamer goes 3000 mi. in 10 da. 13 hr., what is the average hourly rate?

88. The moon moves through an entire circumference of 360° in 27 da. 7 hr. 43 min. 11.5 sec. How far in circular measure does it move each day ?

89. Estimating one degree of the earth's equator at $69\frac{1}{6}$ mi., find what is the length of an equatorial arc of 14° 18 ′.

90. Since the circumference of the earth at the equator is 24,902 mi., through what distance is a point of the equator carried each hour by the earth's daily revolution ?

91. A man was employed during the first week of winter $\frac{5}{6}$ wk., counting 6 da. to the week ; during the second week, $\frac{7}{10}$ da., counting 10 hr. to the day ; during the third week, $1\frac{3}{10}$ da. ; during the fourth week, $6\frac{1}{2}$ hr. ; during the fifth week, $\frac{3}{4}$ hr. How much did he earn at the rate of $12\frac{1}{2}$ ¢ per hour ?

92. From $\frac{1}{9}$ of a hogshead take $\frac{2}{3}$ of a quart.

93. A man bought 40 boxes of oranges at $8.75 a box, and sold them for 9.12\frac{1}{2}$ a box. How much did he gain ?

94. When a man consumes $\frac{9}{24}$ of every day in sleep, $\frac{3}{24}$ of every day in reading, $\frac{2}{3}$ hr. each day in amusement, $\frac{3}{4}$ hr. each day in idleness, how many hours has he left for work in the course of the year ?

95. A merchant purchased 36 bolts of silk, each bolt containing 44 yards, at $4 a yard. How much did the silk cost him ?

96. Add $\frac{1}{2}$ of a week, $\frac{1}{3}$ of a day, and $\frac{1}{4}$ of a minute.

97. At 6%, what is the interest on a note at 60 days for $950 ?

98. London, England, is 0° 5 ′ 38 ″ W., and Sydney, Australia, is 151° 11 ′ 0 ″ E. What is their difference in longitude ?

99. The shortest distance from X to Y is 157,132 rd. How many miles apart are the two places ?

100. What is the cost of 13 gross of pencils, at $2\frac{1}{2}$ ¢ a pencil?

101. A man bought 24 reams of paper at 20 ¢ per quire. How much did the paper cost?

102. A man wishes to put 2 hhd. of oil into bottles that contain 3 qt. each. How many bottles are needed?

103. A man purchased 6118 lb. of cheese at 12 ¢ per pound, and sold it at 5% loss. How much did he lose?

104. A railroad was graded at $5.75 a rod; what was the cost of grading the road, its length being 37 mi. 309 rd.?

105. When it costs $17.29 a rod to grade a street, what is the expense of grading a street 15 mi. 277 rd. long?

106. What is the value of a lot containing 40 sq. rd. 200 sq. ft., at $1.50 a square foot?

107. How many yards of matting, one yard wide, are required to cover the floor of a room 18 ft. long and 15 ft. wide?

108. In a field 30 rd. wide and 50 rd. long, how many acres are there?

109. A man bought 9 lots of cloth, each lot containing 15 bolts, each bolt 23 yd., at 8 ¢ a yard; what was the cost of the cloth?

110. A township is 6 mi. long and $4\frac{1}{2}$ mi. wide. How many farms of 90 acres each might be made out of it?

111. How many shingles does it take to cover two sides of a roof, each side 36 ft. long, with rafters 16 ft. in length, when one shingle covers 27 sq. in.?

112. Light travels at the rate of 186,000 mi. per second: sound travels at the rate of 1080 ft. per second. What is the ratio of the speed of light to that of sound?

113. Write in words:

296582367412879562.843968317+.

114. How many square yards are there in a floor which is 48 ft. 6 in. long, and 36 ft. 10 in. wide?

115. What will the plastering of a room cost, at 18 ¢ a square yard, with no allowance for windows or doors, when the length of the room is 30 ft. 10 in., the width 24 ft. 6 in., and the height of the walls 8 ft. 4 in.?

116. At 12½ ¢ per square yard, what does it cost to plaster a room 20 ft. 6 in. long, 18 ft. wide, and 10 ft. high?

117. How many square feet are there in a board 15 ft. 7 in. long and 1 ft. 10 in. wide?

118. In a floor 16¼ ft. long and 12⅓ ft. wide, how many square feet are there?

119. How many square yards of carpeting will cover two rooms, one 18 ft. long and 12 ft. wide, the other 21 ft. long and 15 ft. wide?

120. How many square yards are there in a ceiling 18½ ft. long and 14¾ ft. wide?

121. In a field 35 rd. long and 32 rd. wide are how many A.?

122. A floor containing 132 sq. ft. is 11 ft. wide. What is its length? Draw this to scale.

123. A floor is 18 ft. long and contains 30 sq. yd. What is its width? Draw this to scale.

124. A field containing 9 acres is 45 rd. in length. What is its width?

125. A field 35 rd. wide contains 21 acres. What is its length?

126. What does it cost to paper the walls of a room 21 ft. long, 18 ft. wide, and 10 ft. high, at $1.25 a roll, when each roll is 8 yd. long and 15 in. wide?

127. Insurance for $30,000 was placed on a schoolhouse for 3 yr. at 17 ¢ per $100. What was the average cost per year for the protection?

128. A man owns real estate, valued by the assessors at $130,000, and admits the ownership of $65,000 personal property not exempt from taxation. In a city where the tax rate is $17 per $1000, what tax does he pay?

129. 4 men, A, B, C, and D, owning 184 bu. 2 pk. of wheat, agree to divide it as follows: A is to have one-half of the whole; B is to have one-third of the rest; C is to have one-fourth of what then remains; and D is to have what is left. What is the portion of each?

130. 24 men agree to build a bridge 168 ft. long. After completing $\frac{1}{6}$ of it, they employ 8 more men. What average distance does each man construct before the 8 men are employed? after?

131. A farmer has 1000 bu. of potatoes, which he puts into 350 bbl. How many bushels does each barrel hold?

132. An estate worth $2570 is divided as follows: the widow has one-third; the remainder is divided equally among 7 children. How much did the widow receive, and how much did each child get?

133. An estate of £2173 15s. 10d. is divided among three children: the first has £927 14s. 9d.; and the second £645 12s. 10d. What does the third receive?

134. When a locomotive moves 1 mi. 40 rd. in 1 min., how far does it move in 1 hr.? in 6 hr.?

135. When a horse travels 17 mi. 26 rd. in $1\frac{1}{2}$ hr., how far does he travel in 1 hr.?

PERCENTAGE

In problems involving percentage we have always the **base**, the **rate**, and the **percentage** ; and sometimes the **amount** or the **difference**.

Discuss these definitions :

1. *Rate per cent*, or *rate*, is the number which denotes how many hundredths of the base are to be taken.

2. The *base* is the number on which the percentage is computed.

3. The *percentage* is the result obtained by taking a certain per cent of the base.

4. The *amount* is the sum of the percentage and the base.

5. The *difference* is the remainder obtained by subtracting the percentage from the base.

USES OF PERCENTAGE

In every problem in percentage or in any topic growing out of percentage we have the number upon which the per cent is calculated, called in *percentage* the **base**, in *interest* the **principal**, in *taxes* the **assessment**, in *customs* the **appraised value**, in *commission* the **consignment**, in *commercial discount* the **list price**, in *insurance* the **policy amount**, in *bank discount* the **face** of the note, in *stocks* and *bonds* the **market value**, or **par value**, in *exchange* the **bill**, **draft**, or **check**, and in *partnership* the **interest** or **share**.

In every problem in any of these percentage topics the first thing to do is to decide upon what the rate per cent is being calculated.

Tell what is the *rate*, the *base*, and the *percentage* in each of the following :

1. A boy sold 15 chickens for $5 at a gain of 20%. How much did he gain?

2. A note at 6% for 6 mo. was paid with $203. What was the face of the note?

3. The tax rate was .017 and the tax paid was $3400. What was the assessment?

4. The rate of duty was 20%. The value of the goods was $1900. What was the duty?

5. A farmer shipped goods. The merchant sent him $97½, after deducting a commission of 2½% and expenses amounting to $15.75. What was the value of the goods?

6. The discounts were: 1st, 20% off, and 2d, 5% from that. The list price was $320. Find the net amount.

7. A man paid a premium of $31 per $1000 on a policy for $4000. Find the amount of his premium.

8. A note for 60 days at 6% was discounted, and the proceeds were $297. What was the face of the note?

9. A railroad director added to his property 1000 shares, of $100 each, at 97½%. What was his investment?

10. A draft was sent to London for £2000 at $4.85¼. What was the cost of the bill?

<center>FIRST CASE</center>

The base and rate given, to find the percentage.

A man who earned $3500 a year saved 25% of his earnings. How much money did he save?

The *base* is $3500.	$3500 = year's earnings.
The *rate* is 25%, or .25.	.25
	17500
	7000
The *percentage* is $875.	$875.00 = the amount he saved.

To find the percentage when the base and the rate are given, we multiply the base by the rate expressed decimally.

Base × Rate = Percentage $B \times R = P$

1. A cavalry regiment took 960 horses into a battle. $16\frac{2}{3}\%$ of them were killed, and 25% of them wounded. How many were left for service?

2. A workman who earned $2.25 a day had his wages reduced 15%. How much less did he receive at the end of a year of 305 working days?

3. A bought a house for $4000 and sold it to B at a gain of 5%. B afterward sold it to C at a loss of 5%. How did B's loss compare with A's gain?

4. A man bought 8 piles of wood, each 18 ft. long, 4 ft. wide, and 6 ft. high, at $3.50 a cord. He sold the wood at a gain of 8%. How much did he gain?

5. A dealer bought 23 firkins of butter, each containing 56 lb., at $37\frac{1}{2}$ ¢ a pound, and sold it so as to gain 5%. What was his selling price?

6. A farmer raised 325 bu. of wheat, 215 bu. of corn, and 400 bu. of oats. He sold 75% of the wheat at $.90 a bushel, 80% of the oats at 35¢ a bushel, and $62\frac{1}{2}\%$ of the corn at 53¢ a bushel. How much did he receive for all?

7. A man who traveled 5649 mi., traveled 25% of the whole distance on his wheel. How many miles did he wheel?

8. *a.* A firm of three men had $750,000 in a business. A put in $66\frac{2}{3}\%$ of the capital; B, $20\frac{5}{6}\%$; and C, $12\frac{1}{2}\%$. *b.* The business paid one year $120,000. What was each man's share of the gain?

9. A boy with $24 invested in chickens made money at the rate of 75%. What was his gain?

SECOND CASE

Percentage and rate given, to find the base.

A man spent $280, which was 20% of his money. How much money had he?

FIRST METHOD

(a) Since $280 was 20%, or $\frac{20}{100}$, of his money, 1%, or $\frac{1}{100}$, was $\frac{1}{20}$ of $280, or $14, and 100%, or the amount he had, was $14 × 100, or $1400.

SECOND METHOD

(b) $280 was 20%, or $\frac{1}{5}$, of his money; $\frac{5}{5}$, or the whole, was $280 × 5, or $1400.

ORAL PROBLEMS

1. A boy bought a book for 30¢, which was 50% of his money. How much money had he?

2. During one month there were 45 pupils of a certain school tardy. The number tardy was 50% of the whole number of pupils. How many pupils in the school?

3. Find the number of which 50 is 25%.

4. I am thinking of a certain number of which 80 is $66\frac{2}{3}$%. Of what number am I thinking?

5. 7% of a man's money is $630. How much has he?

6. Ten years is 20% of a man's age. How old is he?

7. By selling my house at a loss of $1000 I lost $12\frac{1}{2}$%. What was the value of the house?

8. When a train had gone a distance of 300 miles, it had traveled $66\frac{2}{3}$% of the length of the road. How long was the road?

9. 15% of a man's wealth is $1200. What is his wealth?

10. If $12\frac{1}{2}\%$ of the cost of a yard of cloth is 20 cents, what is the cost of two yards?

11. Roy has 26 marbles, which is $16\frac{2}{3}\%$ of the number Fred has. How many has Fred?

12. A dealer made a profit one day of $108 at an average gain of $16\frac{2}{3}\%$. What was the cost to him of the merchandise?

WRITTEN PROBLEMS

John has $90, which is 45% of the amount James has. How much has James?

PROCESS

(a) $45\overline{)\$90} = 45\%$, or $\frac{45}{100}$ of the amount James has.

$\$2 = 1\%$, or $\frac{1}{100}$ of the amount James has.

$\$\dfrac{100}{200} = 100\%$, or $\frac{100}{100}$ — the amount James has.

(b) But dividing by 45 and multiplying by 100 is really the same as dividing by .45, which is the rate.

Rate .45$\overline{)\$90.00}$ Percentage
$\$200 = $ Base

When the percentage and the rate are given, to find the base, we divide the percentage by the rate.

Percentage ÷ Rate = Base $P \div R = B$

1. By selling some corn at an advance of 12% I gained $102. What was the value of the corn?

2. A man who spends 90% of his salary spends $981. What is his salary?

3. When a grocer had paid $86.50, he found he had paid $16\frac{2}{3}\%$ of the amount he owed. How much did he owe?

4. After wheeling $12\frac{1}{2}$ miles a boy found he had traveled $83\frac{1}{3}\%$ of the distance he had intended to go. How long a ride did he expect to take?

5. 20% of the population of a certain place is 8250 inhabitants. What is the population of a place in which there are three times as many people?

6. A man gained $9\frac{1}{2}\%$ by selling a house for $228 more than it cost him. What was the cost of the house?

7. Mr. Wilde sold some land for $36,000, which was $85\frac{5}{7}\%$ of its cost. What was the cost of the land?

8. Peter sold to Robert 15 hens. This number was $37\frac{1}{2}\%$ of the number of hens Peter owned in the first place, and $\frac{5}{8}$ of the number Robert had before the sale. How many had each boy after the sale?

9. A boy gave to his brother 20% of his marbles, and lost .05 of the remainder. He then had 152 marbles. How many had he at first?

10. A girl having spent 88% of her money for Christmas presents had left 54 cents. How much did she have when she started?

<div align="center">THIRD CASE</div>

The percentage and base given, to find the rate.

A boy who owned 80 chickens sold 20 of them. What per cent did he sell?

<div align="center">EXPLANATION</div>

If he owned 80 chickens, one chicken was $\frac{1}{80}$ of what he owned, and 20, or the number of chickens he sold, made $\frac{1}{80} \times 20$, which is $\frac{20}{80}$, or $\frac{1}{4}$, of the number he owned. But $\frac{1}{4}$ is equal to $\frac{25}{100}$, or 25%. Therefore he sold 25% of his chickens.

Oral Problems

1. A yard of ribbon was bought for 24¢, and sold at a loss of 6¢. What was the rate of loss?

2. What per cent of 3 gal. are 3 pt.?

3. Joseph earned $15, and James $30. What per cent of the whole amount did each earn?

4. A farmer who raised 300 bu. of potatoes sold 150 bu. The amount he sold was what per cent of the amount he raised?

5. Mr. Smith owed the grocer $25, and paid all but $5. What per cent of his bill did he pay? What per cent of the whole amount does he still owe?

6. What is the rate of gain from selling a $50 bicycle for $2½ more than it cost?

7. A boy misspelled 15 words out of the 75 dictated. What per cent of the whole number did he spell incorrectly?

8. 16 sq. in. are what per cent of a square foot?

9. From a drove of 250 sheep 50 were killed. What per cent were killed?

10. A bicycle that cost $40 was sold for $32. What was the rate of loss?

11. A sofa that cost $64 to make was sold for $80. What was the rate of gain?

12. A piano costing $175 was sold for $140. What was the per cent of loss?

13. A regiment of 880 men lost in battle 160. What was the rate of loss?

Written Problems

1. A farmer who raised 128 bu. of potatoes kept 32 bu. What per cent did he keep ?

Process

1 bu. $= \frac{1}{128}$ of the amount raised.

32 bu. $= \frac{32}{128}$ of the amount raised.

But $\frac{32}{128} = 32 \div 128$.

$$\begin{array}{r} \text{Percentage} \\ \text{Base } 128)\overline{32.00}(.25 \quad \text{Rate} \\ \underline{25\ 6} \\ 6\ 40 \\ \underline{6\ 40} \end{array}$$

Therefore, 32 bu., the amount he kept, is .25, or 25%, of the amount he raised.

When the base and percentage are given, to find the rate, we divide the percentage by the base.

Percentage $\div$ Base $=$ Rate $\qquad P \div B = R$

1. An automobile that cost \$560 was sold for \$700. What was the rate per cent of gain ?

2. Find the rate of profit or of loss :

	Cost	Selling Price		Cost	Selling Price
a.	\$12	\$15	d.	\$450	\$512.50
b.	\$150	\$180	e.	\$3.75	\$2.00
c.	\$225	\$198	f.	\$500	\$500.50

3. From a bin containing 450 bu. of oats, 150 bu. were sold. What per cent was sold?

4. A man who was worth $186,000, left $\frac{1}{2}$ of it to his wife, $\frac{1}{3}$ to his son, and the remainder to his daughter. What per cent of the whole amount did the daughter receive?   ·

5. Mr. White bought goods to the amount of $2550, and gave in payment a check for $2000. What per cent of the bill was still due?

6. A tourist started on a trip of 1275 mi. He traveled $\frac{2}{5}$ of the distance by rail, 435 mi. by water, and the remainder of the distance on his wheel. What per cent of the distance did he travel by wheel?

7. If a fire destroyed $1546 worth of stock valued at $4626, what was the rate of loss?

8. From a rope which measures 6 rd. 3 yd. 2 ft., 22 ft. were cut off. What per cent of the rope was cut?

FOURTH CASE

The amount and rate given, to find the base.

There are 702 pupils in School No. 1 of a certain town, which is 17% more than there are in School No. 2. How many pupils in School No. 2?

PROCESS

100% represents the number of pupils in No. 2.

117% represents the number of pupils in No. 1.

117% = 702 pupils.

 1% = $\frac{1}{117}$ of 702 pupils, or 6 pupils.

100% = 6 pupils × 100, or 600 pupils.

Oral Problems

1. A boy who sold a book so as to gain 20% received 60 ₵ for it. What was the cost of the book?

2. A number increased by 25% of itself is 50. What is the number?

3. Tom collected 92 stamps, which was $33\frac{1}{3}$% more than his brother had. How many had the brother?

4. A boy rode 77 miles on Monday, which was $37\frac{1}{2}$% farther than he went Tuesday. How far did he travel on Tuesday?

5. David lives 198 rd. from the schoolhouse, which is $12\frac{1}{2}$% farther than Sarah lives. How far from the school is Sarah's home?

6. A girl who weighs 56 lb. weighs $16\frac{2}{3}$% more than her little sister. What is the weight of the sister?

7. A dry-goods man sold some ribbon at $1.25 a yard, thus gaining 25%. What did he pay for 6 yd. of the ribbon?

8. After a clerk's salary was increased 10% he received $77 a month. What were his monthly wages before this increase?

Written Problems

A man's house rent was $530 a year, which was 6% more than he had paid the year before. What did he pay the preceding year?

Process

106% of former rent = $530.

1% of former rent = $\frac{1}{106}$ of $530, or $5.

100% of former rent = $5 × 100, or $500.

But dividing by 106 and then multiplying the quotient by 100 gives the same result as dividing by 1.06.

Base = Amount ÷ (Base + Rate) = 1 ÷ (1 + .06) = 1.06.
$$\$530 \div 1.06 = \$500.$$

To find the base when the amount and the rate are given, we divide the amount by 1 plus the rate.

$$\left.\begin{array}{l}\text{Amount divided}\\ \text{ by 1 plus Rate}\end{array}\right\} = \text{Base} \qquad A \div (1 + R) = B$$

1. The population of a certain city is 51,750, which is 15% more than it was last year. What was the population last year?

2. A man sold a farm for $14,804, which was $17\frac{1}{2}$% more than he paid for it. How much did the farm cost him?

3. A merchant selling goods for $6835.50 thereby gained 26%. What was the cost of the goods?

4. A man bought a house for a certain amount and then spent 11% of the cost of the house for improvements. He found the cost of both to be $8214. What did he pay for the house?

5. A grocer sold 275 bbl. of flour for $1375 and in so doing gained $37\frac{1}{2}$%. What was the cost of a single barrel of flour?

6. A vessel sailed for New Orleans with 1600 tons of ice, which was $2\frac{1}{2}$% more than actually arrived there. How much ice reached New Orleans?

7. Some boys studying botany together collected 648 specimens, which was 62% more than they were required to have. How many were they required to collect?

The difference and the rate given, to find the base.

A chair was sold for $64, the seller thereby losing 20%. What was the cost of the chair?

First Process

(a)   100% represents the cost price of anything.
    20% was the loss.
    80% was the selling price.
    80% = $64.

    $1\% = \frac{1}{80}$ of $64, or 80¢.
    $100\% = 80¢ \times 100$, or $80.

Second Process

(b) $100\% = \frac{100}{100}$, or $\frac{5}{5}$, the cost price of the chair.
    $20\% = \frac{20}{100}$, or $\frac{1}{5}$, = the loss.
    $\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$, or the selling price.

    $\frac{4}{5} = 64, the selling price.
    $\frac{1}{5} = \frac{1}{4}$ of $64, or $16.
    $\frac{5}{5} = 16×5, or $80, cost price

Oral Problems

1. Mr. Black sold 20% of his flock, and then found he had 84 sheep. How many had he at first?

2. A girl spent 75% of her money, and had left $14. How much had she at first?

3. Harry lost $12\frac{1}{2}\%$ of his marbles, and still had 28 marbles. How many had he before he lost any?

4. $16\frac{2}{3}\%$ of the length of a string is cut off. If the string still measures 100 ft., what was its original length?

5. A man who sold his goods for $12\frac{1}{2}\%$ less than they cost, received for them $42. What was the cost of the goods?

6. A clerk spent $66\frac{2}{3}\%$ of his salary, and then had $150. What was the amount of his salary?

7. A wheelman traveling from A to B found when he had gone $37\frac{1}{2}\%$ of the distance that he still had 55 miles to wheel. What was the distance from A to B?

8. A number diminished by $11\frac{1}{9}\%$ of itself is 200. What is the number?

9. A boy earned $1.70 shoveling snow, which was 15% less than the amount he hoped to make. How much did he expect to earn?

Written Problems

A man was forced to sell his farm for $8250, which was 25% less than it was worth. What was the value of the farm?

$$100\% = \text{value of the farm.}$$
$$25\% = \text{loss.}$$
$$75\% = \text{selling price.}$$
$$75\% \text{ of value} = \$8250, \text{ or selling price.}$$
$$1\% \text{ of value} = \tfrac{1}{75} \text{ of } \$8250, \text{ or } \$110.$$
$$100\% \text{ of value} = \$110 \times 100, \text{ or } \$11,000, \text{ cost price.}$$

Or,

$$\text{Base} = \text{Difference} \div (\text{Base} - \text{Rate}) = 1 \div (1 - .25) = .75.$$
$$\$8250 \div .75 = \$11,000.$$

When we know the rate and the difference, we divide the difference by 1 minus the rate to find the base.

Difference divided by 1 less the Rate $\Big\}$ = Base $D \div (1 - R) = B$

1. After 18% of a man's property had been destroyed by fire, he still had real estate valued at $61,500. How much was he worth before the fire?

2. A farmer sold 46% of his potatoes, and had left 513 bushels. How many bushels did he raise?

3. In a town in which $37\frac{1}{2}$% of the people are foreigners there are 31,250 Americans. What is the population of the town?

4. A house was rented in December for $35.50 a month, which was 20% less than the amount it brought the following May. What was the rent for a year at the latter rate?

5. A man who deposited 85% of his money in the bank still had $9000, which he invested in lots. How much was he worth altogether?

6. After shrinking 5%, some cloth was found to measure 380 yards. What was the length of the cloth before shrinking?

7. $12\frac{1}{2}$% of the length of a pole was broken off. The part remaining was 13 ft. 10 in. long. How many inches in length was the pole before it was broken?

Miscellaneous Oral Problems

1. How many bushels in 20% of 200 pecks?

2. A farmer bought 120 cows and sold 5% of them. How many cows had he then?

3. After spending 50% and 25% of his money, a man had left $50. How much had he at first?

4. $12\frac{1}{2}$% of a cargo of 1600 tons of ice melted. How much ice remained?

5. A boy sold his wheel for $30, thus losing 50%. What was the cost of the wheel?

6. Edgar has 30 rabbits, which is 25% more than Willis owns. How many has Willis?

7. A ticket from A to C costs $1.60, or $33\frac{1}{3}$% more than from A to B. What is the fare from A to B.

8. A merchant bought goods for $3000, and sold them at a gain of 10%. What was the gain?

9. A horse was bought for $75 and sold for $100. What was the gain per cent?

10. On Friday Mr. Storm's sales amounted to $36. On Saturday they were increased 25%. What was the value of the goods sold on Saturday?

11. Express the following as common fractions in their lowest terms:

$16\frac{2}{3}$%,   75%,   150%, $62\frac{1}{2}$%,   85%.

12. 30 is what per cent of 120?

13. In January a man earned $150. In February his wages were increased 2%. How much did he earn in February?

14. Of a school of 500 pupils 20% were neither absent nor tardy. How many pupils were neither absent nor tardy?

15. A man having $25,000 spent 3% for a lot. What did the lot cost him?

16. A boy who wrote 120 words misspelled 5% of them. How many did he spell correctly?

17. A piece of ribbon measured a yard and a foot before 25% of its length was cut off. How many inches in the piece cut?

18. A girl's carfare for a month is usually $2.50, which is 50% of her monthly allowance. How much is her allowance?

19. A field 140 rods long is 70% of the length of another field. How long is the second field?

20. A certain number increased by 20% of itself equals ½ of 120. What is the number?

21. An agent receives 4% commission for selling goods to the amount of $1500. What is his commission?

22. What is the interest on $500 for 2 years at 6%?

23. A boy bought 25 marbles, which was 5% of what he already had. How many had he after buying?

24. A man owned 30% of a vessel valued at $25,000. What was the value of his share?

25. Mr. Case bought a sofa for $140, and sold it at a loss of 20%. What was the selling price?

26. John earned in a week $2.60, which was 12½% of what his father earned. What did the father earn?

27. $3.90 is 75% of how much money?

28. Last fall Harry gathered 6 pecks of nuts, Edward 33⅓% as many. How many quarts did both boys gather?

29. What is the interest on $17.50 for 2 yr. at 10%?

30. A man received $8.50 as commission for selling goods to the amount of $850. What was the rate of commission?

31. A merchant received a discount of 12% on goods worth $500. What did he pay?

32. By selling a horse for $210 a man gained 50%. What was the cost of the horse?

33. The yearly rent of a house is $282, or 6% of its value. What is the value of the house?

34. 7500 ft. of lumber were found to be 25% less than the required number of feet. How many feet were needed?

35. A man who earns $1320 in a year has his wages increased $16\frac{2}{3}\%$. What was the yearly increase?

36. A farmer raised 400 bu. of grain. 50% was wheat, 20% oats, and the rest corn. How much corn did he raise?

37. $33\frac{1}{3}\%$ of a pole is in the mud, 25% in the water, and 25 ft. in the air. What is the length of the pole?

38. If $66\frac{2}{3}\%$ of my money is $60, how much is $\frac{1}{9}$ of it?

39. A boy who earned $30 gave $33\frac{1}{3}\%$ of it to his mother, and spent 50% of the remainder for clothes. How much had he left?

40. A lady who had $300 spent $50 for coal and $25 for a month's rent. What per cent of the money did she spend?

41. A man is 40 years old. 50% of his age is 8 years more than his son's age. How many years old is the son?

42. Elmer paid $330 for a horse and cow. The horse cost 75% more than the cow. What was the cost of each?

43. 15 is 5% of what number?

44. What is 8% of $512?

45. 50 is what % of 500?

46. A paid $1000 for a lot which he sold to B at a gain of 10%. B sold it to C at a gain of 10%. What did C give for the lot?

47. The distance from X to Z is $\frac{3}{4}$ mi. A walked $\frac{1}{3}$ of the distance. What per cent of a mile had he then to go?

48. A boy sold for his father 3 bbl. of apples, worth $3.75, at 20% gain. What did he receive?

49. Goods that cost $12 were sold for $10.50. What was the rate of loss?

50. A lot of goods was sold for a gain of $100 at the rate of 15%. What was the amount of the sale?

51. A flock of sheep was sold at a loss of $27, at the rate of 3%. What was their selling price?

Miscellaneous Written Problems

1. A room 24 ft. long by 20 ft. wide was carpeted with carpet a yard wide, the strips running lengthwise. If the value of the carpet was 95¢ a yard, and the cost to lay it $2\frac{1}{2}$% of the value of the carpet, what was the entire expense of carpeting the room?

2. A man bought 3356 ft. of board pine at $19.50 per M., and was obliged to sell at a loss of 5%. What was the selling price?

3. Mr. Long sold 1240 bu. of corn, which was 31% of his entire crop. How many bushels of corn did he raise?

4. The area of a rectangle 12 ft. long and 10 ft. wide is 15% of the area of another rectangle. How many square yards in the second rectangle?

5. A man bought 10 yd. of velvet at $3.90 a yard, 6 yd. of lace at $2.50 a yard, and 8 yd. of silk at $1.75 a yard. If what he paid for the goods was 20% more than they were worth, what was their real value?

6. Six miles of a country road are macadamized. When that distance is 32% of the length of the road, how much has still to be macadamized?

7. A man bought a horse for $225 and sold it at a loss of 15%. What did he receive for the horse?

8. What will it cost to plaster the walls and ceiling of a room 27 ft. long, 24 ft. wide, and 12 ft. high, at 36¢ a square yard, allowing 5% for openings?

9. A house was rented for $48 a month. The yearly rent was 8% of the value of the house. What was its value?

10. Two houses were sold for $6000 apiece. One was sold at a loss of 25% and the other at a gain of 25%. What was the cost price of each house?

11. It required 1200 M. bricks to build a dwelling house, which is 22% of the number needed for a church. What would be the value of bricks needed for two such houses and for such a church, at $8.50 per M.?

12. A borrowed $6450 and paid back $3740. What per cent of the original debt does he still owe?

13. In an army were 7569 men. 338 were killed, 492 were wounded, and 263 deserted. What per cent of the army remained for duty?

14. In 1899 our sales of manufactured articles to foreign customers amounted to $339,000,000, and in 1900 to $434,000,000. What was the rate of increase?

15. A merchant sold on Monday goods amounting to $5000, which was 80% of the amount he sold on Tuesday. Tuesday's sales proved to be 25% more than Wednesday's. What was the value of the three days' sales?

16. A's property is assessed at $13,518, B's at $12,650. What tax does each pay at the rate of $8\frac{1}{5}$ mills on the dollar?

TRADE DISCOUNT

When merchandise is sold at wholesale, there is usually a **list price**. Competition and improved processes reducing the costs of production cause reductions of price by discounts.

A **trade discount** is a percentage taken from the list price.

When prices are raised above the list price, a new price is made. When they are permanently lowered, usually a new list price is made.

Discounts are not common in retail trade.

1. A jobber offered a line of school supplies, list price $575, at 40% off to meet the market, 10% special, and 2% for cash. Cash was paid. What was the bill?

It makes no difference in what order the discounts are taken.

$$\$575 \times .40 = \$230.00 \qquad \$575 - \$230 = \$345$$
$$\$345 \times .10 = \$34.50 \qquad \$345 - \$34.50 = \$310.50$$
$$\$310.50 \times .02 = \$6.2100 \qquad \$310.50 - \$6.21 = \$304.29$$
$$\text{Or} \quad \$575 \times .02 = \$11.50 \qquad \$575 - \$11.50 = \$563.50$$
$$\$563.50 \times .10 = \$56.35 \qquad \$563.50 - \$56.35 = \$507.15$$
$$\$507.15 \times .40 = \$202.860 \qquad \$507.15 - \$202.86 = \$304.29$$

2. Find the discounts and amounts of these bills:

	List	1st Rate of Discount	2d Rate	3d Rate
a.	$500	10%	8%	2%
b.	$1175.60	$7\frac{1}{2}$%	5%	$1\frac{1}{2}$%
c.	$839.27	$33\frac{1}{3}$%	10%	3%
d.	$10000	2%	1%	1%
e.	$29.75	8%	3%	2%
f.	$1000	95%	10%	3%

Discounts like this last have often taken place from the substitution of machine for hand methods, as in the case of wire nails. There may be any number of successive discounts.

3. Mr. Smith bought of the Newark Glass Co. the following bill of glass: 3 boxes 10 in. by 14 in. listed at $32 a box, 5 boxes 14 by 20 at $33.50 a box, and 10 boxes

$24'' \times 30''$ at $38.75 a box.   He received discounts of 70, 10, and 5%, with an additional discount of 2% for cash, payment within 10 days. What was the amount of his bill?

4. A bookseller ordered a bill of books amounting to $262.50, upon which there was an agent's discount of $33\frac{1}{3}$%, and a cash discount of 4%. What did he pay for the books?

5. I paid $351.86 for goods after a discount of 27% had been made from the list price. What was that price?

6. A druggist bought medicines listed at $1900 for 27 and 13% off. Had he waited a week he would have been able to get a trade discount of 40%. How much did he lose by not waiting?

7. When a man pays $1026 for goods on which discounts of 10 and 5% have been made, what is the list price?

8. What is the net amount of a bill of $560, the trade discounts being 25, 15, and 10%, and an extra discount of $2\frac{1}{2}$% for cash?

9. A dry-goods man paid $5000 for goods after he had received discounts of 25, 10, and 5% off. Did he pay more or less than he would have paid had a discount of 45% been made?

10. Captain Strong of the ship Jersey bought of Gray & Co., ship chandlers, 4 bolts $22''$ #8 Sail Duck 102 yd. each at 25¢ a yd., 2 bolts $24''$ #6 Sail Duck 98 yd. each at 29¢ a yd. He received trade discounts of $27\frac{1}{2}$%, 5%, and 2% off for cash.   What was the amount of his bill?

In some lines of business such terms as the following are given. A man buys $100 worth of merchandise, payable in 60 days, but by paying cash can obtain a cash discount amounting to 2%.

He may have his bill dated ahead 60 days, which serves to postpone the maturing of the bill that length of time, so that a bill dated March 1 starts to mature May 1, just as if the bill were bought May 1.

This dating can be discounted for prepayment at the rate of 6 % per annum. Hence a bill bought March 1 and paid for at once can have a discount of 2 % and 1 % taken from it. If paid May 1, 2 % discount can be taken.

A. B. Van Dyke & Co. buy from the L. S. Valentine Company:

Bill dated March 1, 1902.

12 pcs. 10/4 sheeting, 463 yards, at 18 ¢ =			$ 83.34
less 10 and 5 and 2½ %	10 %		8.33
net 60 days			75.01
or less 1 % 30 days, or less 2 % cash	5 %		3.75
60 days postdating			71.26
	2½ %		1.78
			$ 69.48

Dating discountable for prepayment @ 6 %.

Amount payable at end of term, 120 days, $ 69.48. Postdating starts to mature first. If paid at once, the postdating can be discounted at 6 % per annum, or 1 % for 60 da. 69
and the regular discount of 2 % for cash can also be 68.79
deducted 1.37
$ 67.42

If paid at the expiration of 60 days, the postdating having $ 69.48
expired only, the regular cash discount is allowable. 2 % 1.38
$ 68.10

If paid at the expiration of 90 days, the postdating and 30 days of the regular time having expired only, the 30-day $ 69.48
discount is allowable . . . . 1 % .69
$ 68.79

If paid at the expiration of 120 days, the postdating and regular time having expired, the total amount of the bill is due and payable . . . . $ 69.48

Any further delay in payment makes the amount due liable to a debit interest charge at the rate of 6 % per annum.

11. Mr. Benedict, proprietor of "Hotel Sussex," bought from Ketcham & Co., dry-goods dealers for sheets:

1 case 10/4 (2½ yd. wide) Sheeting, 594 yd. at 22½ ¢ a yd. The trade discounts were 12 + 6 + 2½ % — cash discount 2 %. He also had given him a dating of 60 days, which was discountable for cash at the rate of 6 % per annum. What was the amount of his bill?

PROFIT AND LOSS

Profit and **loss** are commercial terms used to express the gain or loss in business transactions, the profit or loss being reckoned at a certain per cent on the cost.

The *cost* is the **base** and is always 100%.

The *per cent of profit or loss* is the **rate**.

The *amount of profit or loss* is the **percentage**.

Since $\quad B \times R = P, \quad B = \dfrac{P}{R}, \quad$ and $\quad R = \dfrac{P}{B},$

then $\qquad$ Cost $\times$ Rate = Profit or Loss,

$$\text{Rate} = \frac{\text{Profit or Loss}}{\text{Cost}}, \quad \text{and} \quad \text{Cost} = \frac{\text{Profit or Loss}}{\text{Rate}}.$$

When the cost and the selling price are given, their difference will be the profit or loss ; profit if the selling price is greater than the cost, loss if the cost is greater than the selling price.

Oral Problems

1. A man by selling silk at $\$1.25$ per yard gained 20%. What was the cost of a yard of the silk ?

2. A piece of furniture was sold at a loss of $37\frac{1}{2}\%$. If the loss was $\$12$, what was the cost price of the article ?

3. How much does a man lose when he sells goods worth $\$4800$ at a loss of $16\frac{2}{3}\%$?

4. Bought 1000 lb. of sugar at $4\frac{1}{2}\cancel{c}$ a pound, and sold it at a gain of 10%. What was my gain ?

5. Eggs that cost $24\cancel{c}$ a dozen were sold for $30\cancel{c}$. What was the gain per cent ?

6. The cost is $\$.90$, the selling price of $\$.45$. What is the rate of loss ?

7. A pair of shoes was bought for $2, and sold at a gain of 20%. What was the gain?

8. An acre of land that cost $125 was sold for $137½. What was the rate of gain?

9. What is the gain on a yard of cloth bought for $1.60, and sold at a gain of 37½%?

10. A grocer sold a pound of coffee for 30¢ and made 5¢. What per cent of the cost did he gain?

11. When a man buys a yacht for $2180, and sells it at 10% advance, how much money does he make?

12. A merchant bought goods amounting to $3400, and retailed them at 20% net profit. What sum did he gain?

13. A merchant bought a piece of cloth for $40, and sold it at 10% net profit. How much did he gain?

14. Goods that cost $150 were sold at 10% gain. Find the amount of the gain.

15. Mr. K. bought goods for $500, and sold them at 12% profit. For what sum did they sell?

Written Problems

1. For what sum must a house which cost $2900 be sold so as to gain 15%?

A gain of 15% means 100% + 15% as the rate for the price of sale.

For every $1 of cost $1.15 is to be received.

The selling price is $2900 × 1.15 = $3335.

2. A person bought a horse for $230, and sold it so that he lost 11%. How much did he receive for it?

For every $1 he paid for the horse he received only $0.89 (since he lost 11%).

The selling price is $230 × .89 = $204.70.

3. A man bought a quantity of flour for $1793.80.

a. For how much must he sell it in order to gain 8%? *b.* For how much, to gain 15%? *c.* What was the rate per cent of a total gain of $89.69?

4. If a dealer bought 908 tons of coal at $5.22 per ton, and sold at a loss of $1\frac{1}{2}$%, what did he get for the coal?

5. What is the rate of profit or loss in the following cases, taking all the percentages upon each of the bases?

	Cost Price		Gain or Loss
I.	$100.	(*a*)	$20 gain.
II.	$200.	(*b*)	$20 loss.
III.	$400.	(*c*)	$40 gain.
IV.	$600.	(*d*)	$60 loss.
V.	$750.	(*e*)	$ 5 gain.
VI.	$900.	(*f*)	$15 loss.

6. A man bought apples at $1.60 per barrel, and desires to sell them so as to gain 14%; what must he charge per barrel?

7. For what must a quantity of lumber that cost $7437.80 be sold so as to gain 16%?

8. A man bought a quantity of flour for $7190 and sold it at a loss of 6%; what did he get for the flour?

9. Mr. X bought a quantity of tea for $293; for what must he sell it in order to gain 10%?

10. For what must a quantity of leather that cost $990 be sold so as to gain 17%?

11. When a man buys 630 barrels of potatoes at $1.23 per bushel, and sells at a loss of 8%, what does he receive for the potatoes?

12. When one buys 411 barrels of flour at $5.22 per barrel, for what must one sell the whole amount in order to gain $12\frac{1}{2}$% ?

13. A man bought 950 posts at 44¢ each; for what must he sell them in order to gain 33% ?

14. When a man buys 512 dozen brooms at $2.80 per dozen, and sells at a loss of 15%, what does he receive for the lot ?

15. A wholesale grocer bought a lot of sugar for $6235, and sold it at 15% less than cost. How much money did he lose ?

16. A speculator bought a quantity of cotton for $24,850, and sold it at $5\frac{1}{2}$% advance. How much money did he gain ?

17. A man bought real estate for $582,460, and sold it at 118% advance. Find the amount of his gain.

18. I bought a bale of cotton for $80, which I sold at 8% loss. What sum did I lose ?

19. Mr. Wise owned $\frac{3}{5}$ of a vessel valued at $85,000. He sold .75 of his share at a gain of 5%. What did he receive for the part sold ?

20. An engineer, after saving up $4000, went into the grocery business. For the second year his total net profits were $1000. His wages for running a locomotive had been $100 a month, and his savings had been invested in mortgages at 5%. If he considered these two items, was the business advantageous to him ?

21. A man bought two lots at $300.50 apiece. He sold the first at a gain of 40% and the second at the loss of 20%. What was his total profit ?

22. A merchant bought goods for $1045.50 and sold them for $1254.60. What was his gain per cent ?

23. A boy bought 2 bu. 3 pk. 5 qt. 1 pt. of peanuts at 4 ¢ a pint and sold them at 10 ¢ a quart. What was his gain per cent?

24. A dry-goods man gained $1872.50 in three months. The rate of profit was 25%. What was the cost of the goods sold?

25. A farmer sold his land for $7762.50, thus gaining $12\frac{1}{2}$% on the cost. A year before, he might have sold it at a gain of $1000 but refused to sell. How much did he lose by waiting?

26. A merchant bought 30 casks of molasses, each containing 63 gallons, at 40 ¢ a gallon. For what must he sell the entire quantity in order to make a profit of 15%?

27. A vessel was sold for $20,000, the seller thereby gaining $2000. If he had sold it for $16,000, what would have been his loss per cent?

28. Mifflin & Co. bought goods for $12,500. Half of them were sold at a gain of 20%, one-fifth at cost, and the remainder at a loss of 10%. Did they gain or lose on the goods, and how much?

29. A man recovered $3150 for 15 carriages sold at a loss of 16%. What was the cost of a single carriage?

30. A milkman sold 12 gal. of milk at 32 ¢ a gallon, gaining $33\frac{1}{3}$%. What did a quart of the milk cost him?

31. Find illustrations in business of gross and net profits. Is it possible to make a gross profit and still to lose money?

In actual business there are two, and even three, kinds of profit: (1) gross profit, that is the difference between buying price and selling price; (2) net profit, the difference between buying price, plus all expenses of sale, such as clerk hire, delivery, waste, etc., and selling price; and sometimes (3) profit of the business, that is, annual total net profits, less interest on capital, an allowance of wages or salary to owner for his services, etc.

INVENTIONAL GEOMETRY

A **right** angle has an arc of 90°.

It is made by perpendicular lines.

An **acute** angle is less than 90°.

An **obtuse** angle is more than 90°, but less than 180°.

A **straight** angle is 180°.

It is made by the diameter of a circle.

A **reflex** angle has an arc of more than 180°.

a is what kind of an angle?

b is what kind of an angle? *c*? *e*?

d is what kind of an angle?

What combination of the angles here would make a reflex angle?

With a protractor measure accurately each of these angles.

Chords are straight lines connecting the ends of arcs.

In any circle, angles, arcs, and chords are always in proportion to each other. Compare these in the figure.

A **secant** is any straight line that cuts the circumference of a circle. A *secant* is a part of a *chord*.

The part of the circumference defined by the *chord* is called an **arc**.

A **tangent** is any straight line that touches the circumference of a circle without cutting it.

A *tangent* has one, and only one, *point of contact* with the circumference.
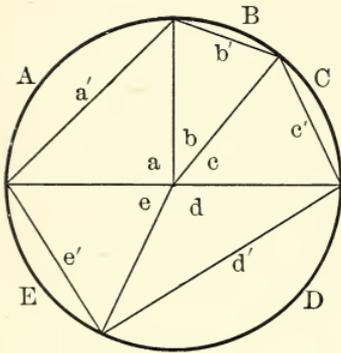
To draw a tangent to a circle at any point

From X, any point, in the circumference XY, draw the radius XC. To this radius at X draw the perpendicular AB.

AB is tangent to the circle.
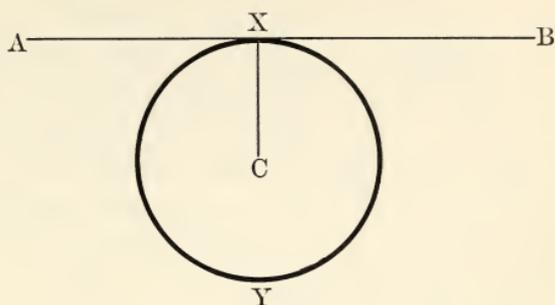
A **sector** is the part of a circle included between an arc and two radii drawn to the circumference.

A **segment** is the part of a circle between a chord and its arc.

Point out : *arcs, sectors, radii,* a *diameter, tangents, secants, segments.*
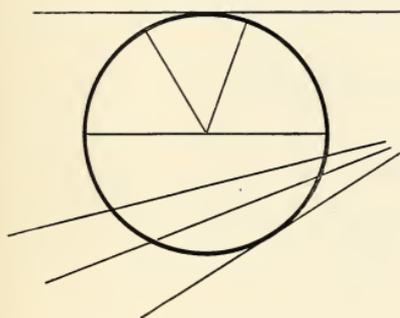
SUGGESTED DESIGNS
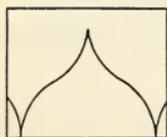
Discover by the dividers the centers from which each of these arcs is drawn, and reproduce the figure upon a larger scale.
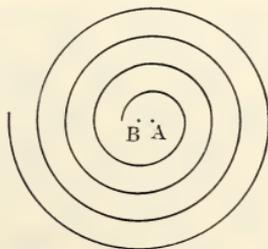
Draw semicircles from A and B alternately.

To draw an equiangular triangle
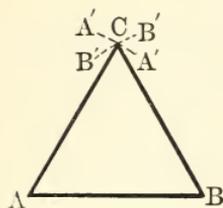
From the end A of the given side AB
describe with a radius AB the arc AA'.
From the end B of the given side de-
scribe with the same radius the arc $B'B'$.

The point C of intersection of the
arcs is equally distant from A and from
B. But if we connect it with A and B by straight lines,
these lines AC and BC equal AB.

The figure ABC is an equilateral triangle. An equi-
lateral triangle is a figure with three equal sides.

With the protractor measure the three interior angles
of the triangle.

Is this equilateral triangle equiangular?

To draw an equilateral triangle

From the end A of one side AB of
the desired equiangular triangle de-
scribe with any radius the arc $A'A''$.
Upon this arc with the protractor
measure 60°. From the point C of
60° draw a line to A. The angle
CAB is one of 60° because its arc is
60°. From the same point C describe
the arc $C'C''$. Measure from A with the protractor an arc
of 60°. Connect the 60° point B with C. The angle ABC
is 60°. Draw a line from B to C.
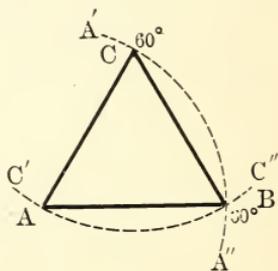
$$60° + 60° = 120°. \quad 180° - 60° = 120°.$$

The three angles of a triangle always equal 180°.
The remaining angle is one of 60°.
The triangle ABC is equiangular.

To draw a line parallel with another line

At any point A in the line XY erect a perpendicular AB.

At any point C in the perpendicular AB erect a perpendicular CD. This perpendicular extended in both directions at the line $X'Y'$ is parallel with XY.
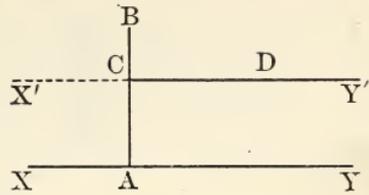
Various Methods

The two methods of drawing parallel lines, shown on this page and page 119, and the two methods of erecting perpendiculars from points in lines, page 117, illustrate the variety of ways to do things geometrically.

Carpenters employ both of the methods given in this book for drawing parallel lines. Their "squares" enable them to correct their work by using one method to prove the other.

Suggested Designs

Find parallels in these different designs.
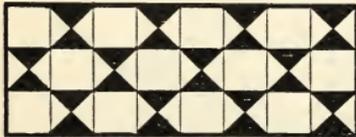
SQUARES AND RIGHT ANGLE TRIANGLES

EQUILATERAL TRIANGLES
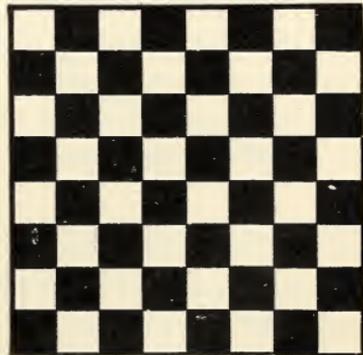
CHECKERBOARD DESIGN
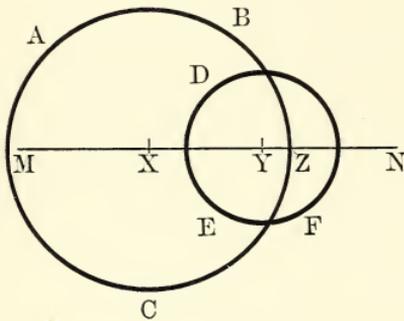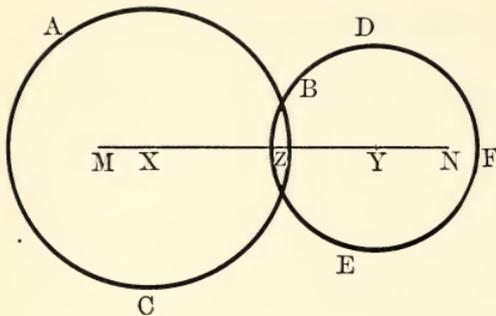
To draw intersecting circles

Take any two points, X and Y, in the line MN. From X, with any radius less than XY, describe the circumference ABC. Z is the point where the circumference ABC cuts the line MN. From Y, with any radius greater than YZ, draw the circumference.

The circles ABC and DEF intersect.

To draw an angle equal to another angle

Draw any line, MN, cutting the sides XY and ZY of the angle XYZ. Upon the line AB take the distance AN' equal to YN. From N' as a center, with a radius equal
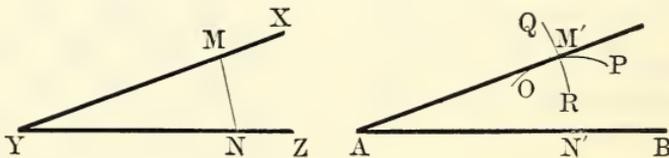
to MN, describe the arc OP. From A as a center, with a radius equal to YM, describe the arc QR. Through the point M' of intersection of the two arcs, draw a line to A.

The angle $M'AN'$ is equal to the angle XYZ.

POST–OFFICE MONEY AFFAIRS

For orders not exceeding $2.50, three cents; over $2.50 and not exceeding $5, five cents; over $5 and not exceeding $10, eight cents; over $10 and not exceeding $20, ten cents; over $20 and not exceeding $30, twelve cents; over $30 and not exceeding $40, fifteen cents; over $40 and not exceeding $50, eighteen cents; over $50 and not exceeding $60, twenty cents; over $60 and not exceeding $75, twenty-five cents; over $75 and not exceeding $100, thirty cents.

POSTAL RATES

	Domestic	* Foreign (Postal Union)
Postal Card,	1 ¢.	2 ¢.
1st Class, Letter,	2 ¢ per oz.	5 ¢ per ½ oz.
2d Class, Periodicals,	1 ¢ per 4 oz.	1 ¢ per 2 oz.
3d Class, Print,	1 ¢ per 2 oz.	
4th Class, Merchandise,	1 ¢ per oz.	

* These rates include Europe, Turkey in Asia, Egypt, India, Japan.

1. A boy had $8.75 to send by mail. What was the cost of the money order? What per cent was this on the money sent?

2. A bank draft for $60 from one city to another cost 15 ¢. Was this less expensive than a money order?

3. What are the advantages of money orders?

4. A letter weighed $3\frac{1}{2}$ oz. Find the cost of postage.

5. A letter containing 2 two-dollar bills was never delivered. What would a money order have cost? What per cent was this?

6. When a man's time is worth $2 an hour, and it would take 15 minutes to go to the post office, is he wise to send a one-dollar bill by ordinary mail? Consider the receiver, also his time, the temptations to carrier, and the value of the habit of carefulness.

7. What is the cost of a P. O. Money Order for $27.50? for $35? for $18.65? for $91? for $8.89?

8. Why do we write foreign letters on very thin paper?

9. Why is it good public policy to have cheap mails?

THE DATE LINE

Difference in longitude causes a difference in time. Each 15° of longitude make a difference of an hour's time.

Places *east* have *later* time.

Places *west* have *earlier* time.

A traveler starts westward from New York to go around the world, taking a two-faced watch with him. One face marks New York time throughout the trip. The other he sets according to the time of the place at which he stops.

When he has traveled a distance of 15° of longitude, he notices a difference of an hour's time. Since he has been traveling toward the *west*, it is an hour earlier than at New York. If it is 12 o'clock noon by New York time, he must set the other face at 11.

When he has traveled 90° of longitude, he finds a difference of 6 hours in time. By New York time it is 12 o'clock noon, but the local time is 6 hours earlier than noon, or 6 A.M.

When he has traveled 180° the difference in time is 12 hours; that is, it is 12 hours earlier where he now is than at New York. By looking at the face of the watch that is still keeping New York time he sees it must be 12 o'clock noon (Friday) there, while the clocks around him register midnight of Thursday. He has traveled halfway around the globe and has lost half a day, or 12 hours. At that rate, if he continues to travel in the same direction, he will continue to lose 1 hour for each 15° of longitude until he reaches his starting place, when he will have lost 24 hours, or a day. According to the face of watch which he has set from time to time during his journey, it is Thursday noon, while in New York it is Friday noon. He has apparently lost a day.

Now let E start on a similar journey, traveling eastward. At the end of 15° he too finds a difference of an hour's time, but since he is going toward the *east*, it is an hour *later*. It is 12 o'clock noon in New York; he finds it is 1 o'clock at a place 15° east of New York. When he has traveled 180°, he finds that he has in this way gained 12 hours, or half a day, and on returning finally to the place from which he started he has apparently gained a day, for it is Saturday noon to him, while it is only Friday noon in New York.

In order to prevent such a disagreement, a date line, or change-of-day line, has been established.

The meridian of Greenwich, England, is standard for longitude. If a man could go all around the world in 24 hours with the sun, he would lose the day going westward from Greenwich, but gain it if he traveled eastward. At about 180° a **date line** is established. Sailors crossing it in going eastward count the same day twice; those going westward omit a day.

The date line does not closely follow the 180° meridian. It is really an irregular line in the Pacific Ocean, the general direction being north and south. The date line passes through Bering Sea, west of the Aleutian Islands, east of Japan, southeast of Formosa, and enters the China Sea east of Hong Kong. It next passes just west of the Philippines, southeast, then parallel with the equator to longitude 165° E. From that point it passes northeast of the Samoan Islands, east of the Friendly Islands, through Chathams Island, then southward to the south pole.

1. Saturday in San Francisco or New York is Sunday at Hong Kong or Yokohama. What day is it at Manila?

2. A sea captain made a voyage from Valparaiso to Liverpool in 49 solar days. If he started July 9, what day did he arrive?

3. Two men raced each other around the world. One made the trip in 64 solar days, going westward, the other required 65 solar days, going eastward. Which won?

4. Find the *date line* in the geography. In what ocean is it? Near what islands? Does it touch Alaska? Hawaii?

5. We say that the battle of Manila Bay was won by Admiral Dewey before noon, May 1. Was that the date by our time? Manila is 120° E.

6. A traveler crossing the Pacific from San Francisco to Hong Kong reaches the Philippines Wednesday, June 5, at 6 a.m. A day later he entered Hong Kong. What was the date?

7. An east-bound vessel leaves Pekin, China, on Jan. 5, and two days later arrives at a place in the Pacific 150° E. longitude. What was the date of its arrival at this point?

8. It took 3 days for a ship to sail from the Society Islands to New Caledonia. If the ship started Monday, March 20, what day did it reach New Caledonia?

9. A vessel crossed the Pacific from North America to Asia, keeping as nearly as possible 15° north latitude. Where was the change of day made? What was the change?

VALUES OF TIME

1. Time Sheet in Factory. Room 142

Name	Rate per Hr.	Hours of Work						Total	Due	Note
		Mon.	Tues.	Wed.	Thu.	Fri.	Sat.			
John Mason,	.55	12	10	11	10	12	6			
T. Benman,	.35	10	10	10½	10	12	10			
G. Downes,	.25	9	10	10½	10	9	10			
N. Howard,	.17	8½	8½	10	10	9	10			
K. Smith,	.17	10	10*	10	10	9	10			* $1 fine.
R. Williams,	.17	10	9	10	10	9	10			
J. Cowper,	.09	12	10	10	10	12	6			
Totals.										

Complete the entries.

2. Frank invested $4 in pine wood of suitable dimensions, $2.50 in tools that he needed, and $1.75 in paint and brushes, and made a bookcase. The work, including the drawing of the plan and the errands to the stores and

to the lumber yard, required in all 144 hr. When the bookcase was done his father gave him for it a new bicycle worth $22.50. What was the money value of Frank's time per hour?

3. His brother invested $7 in fowls, $3 for their food, and $10 for lumber and nails for a house for them. In six months he had received $8 for eggs, $3 for fowls, and had then fowls worth $22. He had spent, in care of them, an average of 1½ hr. per day. If the house was worth $5 to sell, what was the money value of his time per hour?

4. His sister spent $4.50 in cloth, and $2 for other materials, and made herself a dress in 48 hr. The dress would have cost $12 if made by others. What was the money value of her time per hour?

5. One man earns $3000 a year, working 8 hr. a day, with 4 wk. vacation; another earns $3.50 a day for 9 hr. work; and a third has a salary of $18,000 for duties sometimes 15 hr. a day, often 6 hr., so that, counting several vacations a year, he averages 8½ hr. a day for 40 wk. a year. Compare the money value per hour of the time of each of these men. In what lines of work do men receive these or similar amounts?

6. What are the ratios in day's pay of $16 a month (30 da.), $5.50 a week, and $300 a year? In what lines of work do persons receive these or similar amounts?

7. A girl hired a typewriter for a month for $3. She paid $2.50 for paper and 50¢ for carbon paper. During the month she worked 35 hours and received $25. What was the money value of her time per hour?

8. Two girls decided to earn money by typewriting. The first wrote 4 pages an hour and the second 6 pages an hour. Both were paid 10¢ a page and worked until they had earned $15. How many more hours did the first girl work than the second? How much less was the money value of her time per hour?

9. A schoolboy wished to earn money by selling peanuts. Each Saturday he bought 15 qt. at 7¢ a quart. As he had no wheel, he had to spend 10¢ for car fare each time he went for them. He sold his peanuts at 5¢ a pint, but it took him from 9 to 12 A.M. and then again from 1.30 until 5.30 to deliver the nuts. What was the money value of his time per hour?

REVIEW OF PROFIT AND LOSS

1. Mr. Munn sold a quantity of coal for $719 and lost 7% by the sale. What was the cost of the coal?

SUGGESTION. 100% = cost price.
  7% = loss.
  93% = $719.

2. How much did X pay per yard for cloth which he sold for $4 a yard, and thereby gained 10%?

SUGGESTION. 100% = cost price.
  10% = gain.
  110% = $4.

3. A man bought a lot for $4325 and sold it for $6216. What per cent did he make?

4. A speculator invested $75,000 in stocks which he sold for $77,225. What was his gain per cent?

5. A man paid $75 for a wagon and sold it for $105. What per cent did he gain?

6. How much did X pay for sugar which he sold for $21.60, thereby losing 4%?

7. A library was sold for $2880. This sum was 10% less than cost. What was paid for the library?

8. A man sold a farm for $7881, gaining 11% on the cost. How much did he give for the farm?

9. An importer sold a quantity of silk velvet for $7578.09 at a gain of 17%. What was the buying price?

10. A herd of oxen was sold for $2740.50, a sum 13% less than cost. What was the cost price?

11. What did Mr. James pay per bushel for wheat which he sold for $171.10 at a gain of 18%?

12. What did C pay for a quantity of shingles which he sold for $1296.45, gaining 29% by the sale?

13. X sold 350 bu. of clover seed for $1780, losing thereby 11%. What did the seed cost per bushel?

14. A company listed a sewing machine at such a price that, after paying the agent's commission of 30% for sale and delivery, it received $35 for each machine. What was the list price?

15. Z bought 30 books for $10. At what price must he sell each to gain $2 on his outlay?

16. A farmer sent $250 worth of produce to a commission merchant, who remitted to him that amount less $12\frac{1}{2}$% commission and expenses. How much was remitted?

17. A book agent bought books at 60% of the list price. He sold 6 at a gain of $12. What was the list price?

GENERAL ORAL REVIEW

1. Perform the operations indicated :

$a.$ $\frac{1}{2} - \frac{1}{3}$. $b.$ $\frac{3}{4} - \frac{2}{3}$. $c.$ $\frac{5}{6} - \frac{4}{9}$. $g.$ $\frac{3}{20} - \frac{1}{40}$.

2. What per cent of a bushel is a half peck ?

3. From a farm of 760 acres 100 acres were sold at one time, and 90 acres at another. What per cent of the whole farm was sold ?

4. A man had $600, but lost $\frac{1}{4}$ of it at one time and $\frac{1}{3}$ of the remainder at another time. What per cent of the $600 did he have then ?

5. What premium must X pay for insuring a cargo of flour worth $36,000, from Boston to Havre, at 3 % ?

Multiply :

6. 5 by $\frac{1}{5}$.   **7.** 12 by $\frac{3}{4}$.   **8.** 16 by $\frac{1}{2}$.

9. Out of a cask containing 300 gal., 60 gal. were drawn. What per cent was drawn ?

10. Goods bought for $5 were sold for $6. What was the rate of gain ?

11. $a.$ 90 bu. is what per cent of 1800 bu. ?

    $b.$ What per cent of 5 gal. is 1 qt. ?

12. Add :

$a.$ $\frac{3}{2} + \frac{3}{4} + \frac{4}{5}$. $b.$ $\frac{4}{25} + \frac{8}{20} + \frac{3}{75}$. $c.$ $\frac{3}{8} + \frac{1}{5} + \frac{3}{5}$.

13. In a garden there were 185 trees. Of these 35 were apple trees and 40 pear trees. There were as many plum trees as there were apple and pear trees together. The rest were cherry trees. How many cherry trees were there ?

14. Reduce :

$a.$ 12 yd. to inches. $b.$ 2 mi. to feet.

$c.$ 3 sq. mi. to acres. $d.$ 2 A. to square feet.

e. 4 cu. yd. to cubic feet. *f.* 5 cd. to cubic feet.

g. 4 bbl. to gallons. *h.* 3 gal. to cubic inches.

i. 2 bu. to cubic inches. *j.* 4 pk. to pints.

k. 1 T. to ounces. *l.* 1 quadrant to minutes.

m. £1 to pence. *n.* 1 mo. to hours.

15. How many hundreds are there in a million?

16. In percentage, how do we find the *percentage?* the *base?* the *rate?* the *amount?* the *difference?*

17. *a.* A hen sat on 15 eggs and hatched 12 of them. What per cent of the eggs were hatched?

b. A boy who ran 2000 ft. ran 20% farther than another boy. How far did the second boy run?

c. A carpenter made a chest of drawers at a cost to him in materials and time of $12, and sold it for 25% advance. What was the selling price?

18. How many cubic inches are there in a brick 8 in. × 4 in. × 2 in.? in 10 such bricks? in 20 bricks? in 25 bricks? in 27 bricks?

19. How many cubic feet are there in a rectangular box 4 ft × 3 ft. × 2 ft.?

20. *a.* When 32 is the consequent and $\frac{1}{2}$ the ratio, what is the antecedent? *b.* When 13.5 is the antecedent and 4 the ratio, what is the consequent?

21. $1000. Boston, June 21, 1900.

For value received, we jointly and severally promise to pay to the order of Chas. T. Sherman one thousand dollars, sixty days after date, with interest.

John W. Bowen,

Thomas S. Nickerson.

What interest was due at the time of payment?

22. What is the area of a triangle 15 yd. wide at the base by 16 yd. high? Draw on the blackboard several

triangles of these dimensions to various scales. Does the shape of the triangle have any relation to its area?

23. What is the area of a parallelogram 12 yd. by 14 yd.? Draw on the blackboard several parallelograms of these dimensions to various scales. Does the shape of a parallelogram have any relation to its area?

24. What are we to find in these problems :

a. A boy sold a dog for $5, and received 40¢ as commission?

b. A man invested $175 in land and gained $58.33?

c. A householder paid a tax of $31.50 on property worth $1575?

d. A horse was sold for $133\frac{1}{3}\%$ of his cost, at a profit of $58.33?

e. What is the interest on $1575 for 4 mo. at 6% a year?

25. At the rate of two apples for 3 cents, how many apples can be bought for 36 cents?

26. A boy earned 2 dimes, a nickel, a quarter, and 8 cents. What part of a dollar did he earn?

27. A man sold 18 quarts of vinegar and $5\frac{1}{2}$ times as much molasses. How much molasses did he sell?

28. How many handkerchiefs, each costing $ .25, can be bought for $5?

29. A farmer sold $\frac{5}{8}$ of a field containing 25 acres. How many acres had he then?

30. .025 is equal to what common fraction?

31. Mr. Twinch, having $5\frac{1}{2}$ acres of land, bought $1\frac{3}{8}$ acres and then sold $2\frac{1}{4}$ acres. How many had he left?

32. Z bought butter at 20¢ a pound and sold it at a gain of 25%. What was the selling price?

33. A farmer had 240 sheep. He sold $12\frac{1}{2}\%$ to A and $\frac{1}{7}$ of the remainder to B. How many sheep did he sell?

34. What is the approximate circumference of a circle whose diameter is 21 feet?

35. The divisor is .5, the dividend .255. What is the quotient?

36. 15 is $16\frac{2}{3}\%$ of what number?

37. At $12\frac{1}{2} ¢$ a pound, how many pounds of crackers can be bought for $3?

38. How many hours from 6.15 P.M. on Thursday until noon on the following Sunday?

39. What is the difference between twice five plus fifty, and twice fifty-five?

40. Divide 75 apples between two boys so that for every apple one boy receives the other may get four.

41. What will $5\frac{7}{8}$ yards of silk cost at $4 per yard?

42. I bought 4 bolts of cloth. The first bolt contained 50 yards, the second 65 yards, the third 42 yards, the fourth 89 yards. How many yards were there in the four bolts?

GENERAL WRITTEN REVIEW

1. A boy spent 75 cents and then had left 10 cents less than he spent. How much had he at first?

2. $\frac{5}{7}$ of the distance from A to B is $15\frac{1}{2}$ miles. What is the distance between two places that are twice as far apart?

3. When $\frac{3}{5}$ of a yard of cloth cost $2\frac{1}{7}$, how many yards can be bought for $18\frac{3}{4}$?

4. Express as common fractions: .001325, .012$\frac{1}{2}$, .1274, .00275, .08.

5. Reduce to decimals: $\frac{8}{25}$, $\frac{4}{9}$, $\frac{14}{16}$, $\frac{125}{600}$, $\frac{11}{32}$.

6. A man has a piece of land 210 ft. long and 150 ft. wide. What will it cost to paint a board fence to inclose the lot, provided the fence is 6 ft. high and the cost of the painting is 10 ¢ a square yard?

7. If 5 carriages cost £26 6s. 3d., what will one cost in United States money?

8. What is the interest on $685 for 6 yr. 4 mo. at 7%?

9. How many acres in a triangular field 480 rd. base and 880 yd. in altitude?

10. A man owning $\frac{3}{7}$ of a vessel sold $\frac{1}{6}$ of his share for $1400. What was the value of the ship?

11. A man dying left $25,000 to his wife and two sons. The wife received 40%, the elder son $\frac{2}{3}$ of the remainder, and the balance was left to the younger son. What was the share of each?

12. The rate of taxation is 8 mills on the dollar. What is the tax of a man whose property is assessed at $42,750?

13. $(8.25 \times .003 \div .05) + (725.4 \div .002) = ?$

14. At $2\frac{1}{2}\%$, what is the cost of insuring property worth $7750?

15. The distance between two places is $74\frac{3}{4}$ miles. At what rate per hour does a train travel if it covers the distance in 2 hours 10 minutes?

16. What is the value of 14,700 ft. of boards at $24.50 per M.?

17. Find the sum of $\frac{4}{7}$ of a mile, $\frac{4}{7}$ of a rod, and $\frac{4}{7}$ of a yard.

18. Reduce $\frac{1}{7}$ of a rod to the decimal of a mile.

19. Multiply 16 gal. 3 qt. 1 pt. 3 gi. by 5 and to the product add 2 gal. 1 qt. 1 pt. 2 gi.

20. Reduce 14,751 oz. to higher denominations.

21. Find, in years, months, and days, the length of the life of each of these men, viz.:

	BORN	DIED
Benjamin Franklin,	Jan. 17, 1706.	Apr. 17, 1790.
George Washington,	Feb. 22, 1732.	Dec. 14, 1799.
Thomas Jefferson,	Apr. 13, 1743.	July 4, 1826.
John Marshall,	Sept. 24, 1755.	July 6, 1835.
Andrew Jackson,	Mar. 15, 1767.	June 8, 1845.
John Quincy Adams,	July 11, 1767.	Feb. 23, 1848.
Henry Clay,	Apr. 12, 1777.	June 29, 1852.
Daniel Webster,	Jan. 18, 1782.	Oct. 24, 1852.
Washington Irving,	Apr. 3, 1783.	Nov. 28, 1859.
Samuel F. B. Morse,	Apr. 27, 1791.	Apr. 2, 1872.
William Cullen Bryant,	Nov. 3, 1794.	June 12, 1878.
Robert E. Lee,	Jan. 19, 1807.	Oct. 12, 1870.
Abraham Lincoln,	Feb. 12, 1809.	Apr. 15, 1865.
Henry Ward Beecher,	June 24, 1813.	Mar. 8, 1887.

22. The Emperor Napoleon won his greatest battle at Austerlitz, Dec. 2, 1805, and the Duke of Wellington his greatest battle at Waterloo, June 18, 1815. What length of time intervened?

23. Charlemagne, Emperor of the French, was crowned at Rome on Christmas Day 800, and Napoleon, Emperor of the French, was crowned King of Italy at Milan, May 13, 1806. What length of time had passed?

24. What was your birthday? By averaging the difference in years between the birthdays of your classmates and those of many grown people, you can estimate the number of generations of men in a century. With your average as the standard, estimate the number of generations since: (*a*) the Declaration of Independence,

1776; (b) the English Revolution, 1688; (c) the landing of the Pilgrims, 1620; (d) the discovery of America, 1493; (e) the fall of Constantinople, 1453; (f) the fall of Rome, 476; (g) the assassination of Julius Cæsar, 44 B.C.; (h) the victory at Marathon, 490 B.C.; (i) the establishment of the kingdom of David, 1055 B.C.; and (j) the reign of Rameses II. as Pharaoh in Egypt, 1200 B.C.

25. At the battle of Waterloo the British forces under Wellington numbered 105,950 men, and the German forces, their allies, under Blücher, 116,000. French directly under Napoleon numbered 122,401; those under Grouchy, 33,000. Napoleon thought that the Germans were 18 miles from the British; they were really only 8 miles away. He sent Grouchy against the Germans by the wrong road. a. How many more men had Napoleon alone than Wellington? b. How many more were all the French than the British? c. But how many more had Wellington and Blücher together than Napoleon alone?

26. At the three days' battle of Gettysburg, which began July 1, 1863, the Union forces numbered about 93,500 men, besides 15,000 reserves, and the Confederate forces about 70,000. The losses were:

a. What was the total loss on each side?

	Union	Confederate
Killed,	3,072	2,592
Wounded,	14,497	12,709
Missing,	5,434	5,150

b. How many able-bodied Confederate soldiers marched back to Virginia?

c. What percentage of men were killed, wounded, missing on each side? d. What percentage formed the total loss on each side?

27. Find the difference in time :

a. Between each pair of the following dates in American history :

1492 **America was discovered by Columbus.**
1520 Magellan circumnavigated the earth.
1565 *St. Augustine was settled.*
1607 Jamestown was settled.
1620 **The Pilgrims landed at Plymouth.**
1636 Roger Williams went to Rhode Island.
1664 *The English took New Amsterdam.*
1732 **The last colony, Georgia, was established.**
1765 The Stamp Act was passed.
1773 Boston held a Tea Party.
1775 *The battle of Lexington was fought.*
1776 **The Declaration of Independence was published.**
1783 *Independence was recognized by England.*
1803 **Louisiana was purchased from Napoleon.**
1815 *The Second War with England was ended.*
1819 We bought Florida from Spain.
1823 **The Monroe Doctrine was published.**
1845 *We annexed Texas.*
1848 **We took California, New Mexico, etc., from Mexico.**
1849 Gold was discovered in California.
1863 *The Emancipation Proclamation was published.*
1865 The Four Years' Civil War ended.
1899 **We acquired Porto Rico and the Philippines.**

b. Between each pair of dates in bold face type.

c. Between each pair of dates in italics.

d. Between each pair of dates in roman type.

28. Eli Whitney invented the cotton gin in 1793. This fastened slavery on the South by making it profitable. In 1619 traders brought the first negro slaves to America. In 1863 their emancipation was proclaimed. How many years passed between each of these dates ?

29. The Romans were a great nation for a thousand years, while we Americans count our national life as begun in 1776. What is the ratio in length of years between Rome and the United States?

30. What lengths of time elapsed between the battles of Saratoga, Oct. 17, 1777, City of Mexico, Sept. 14, 1847, Gettysburg, July 3, 1863, and Santiago, July 3, 1898?

History and geography, like astronomy, afford very many facts for problems. The science of war is based on numbers. Maps are drawn by ratio. Do you not always notice carefully the *scale* on a map? Facts of time and distance are very important in history. The problems above illustrate connections between history, geography, and arithmetic.

PERCENTAGE

INSURANCE

Property insurance is a contract for payment of money for loss of property or damage to it. The obligation to pay the money is assumed by corporations called **fire insurance companies,** or **underwriters.** The consideration given is a **premium.**

Property insurance includes *fire insurance* and *marine insurance.*

A **policy** is a written contract between an insurance company and a person insured.

Personal insurance includes life insurance, insurance for a term of years, accident insurance, health insurance, and investment, or endowment insurance.

Find out all you can about these different kinds of insurance.

The sum paid for insurance is called the **premium.**

If property is insured, the premium is a certain per cent of the sum for which it is insured, and is usually paid at the time of effecting the insurance.

When life is insured, the premium is usually a sum paid annually, or at shorter intervals, during the time for which the insurance continues; and its amount is calculated from the tables of mortality used by insurance companies.

All the problems in insurance are solved in ways like those in ordinary percentage.

Face of policy = base.   Premium = percentage.

ORAL PROBLEMS

1. What will it cost to insure a ship and its cargo, valued at $150,000, at $2\frac{1}{2}\%$?

2. What is the rate of insurance when $4 is paid for a premium on property worth $1000 ?

3. A factory worth $45,000 is insured at 2%. What is the amount of the premium ?

4. The cost of insuring a house at $\frac{5}{8}\%$ is $25. For what amount is the house insured ?

5. Some furniture worth $2000 was insured at the rate of $\frac{2}{5}\%$ a year for 3 yr. What was the premium ?

6. A man had his life insured for $20,000 at $15 a thousand. How much did he pay ?

7. $400 premium is paid on a vessel insured at 2%. For how much is it insured ?

8. What is the insurance on a vessel and cargo valued at $130,000 at $\frac{1}{2}\%$?

9. If $1612 is a sum sufficient to cover the amount for which a man was insured and the premium at 2%, what was the amount insured ?

10. A dwelling house was insured for $6500 at $2\frac{1}{5}\%$. What was the premium ?

11. A man paid $60 insurance on property valued at $3000. What was the rate of insurance?

12. A $10 premium is paid for property worth $8000. What is the rate of insurance?

Written Problems

1. 4500 bbl. of flour, worth $4.75 a bbl., were insured at the rate of $4\frac{1}{2}\%$. What was the premium?

2. A grocer paid $102 a year for insuring his store at 3%, and $252.95 for insurance on the stock at 5%. What was the value of the property insured?

3. A ship and its cargo were insured for $875,000 at $3\frac{1}{2}\%$. The ship was entirely destroyed by fire. What was the actual loss to the owners?

4. A property owner paid $240 for insurance on some houses valued at $15,000. What was the rate of insurance?

5. Mr. Wall's house was destroyed by fire. He received from the insurance company $19,796 after deducting the premium. The rate of insurance was 2%. For how much was he insured?

6. Mr. C's house is worth $7880. For what amount should he insure it to cover its full value and the premium?

7. A man paid insurance on furniture worth $2160 for 3 yr. at $\frac{5}{8}\%$? At the end of that time his furniture was destroyed. What was the loss to the insurance company?

8. A merchant imported 240 pieces of cloth, each piece containing 35 yd., at $2.50 a yard. He got it insured at $2\frac{1}{2}\%$. What was his premium?

9. On a $30,000 schoolhouse a Board of Education secured $30,000 of insurance at .017 per $100 for 3 yr.

a. What was the rate per year? *b.* What was the premium? *c.* A fire destroyed three-quarters of the building. What amount did the insurance company pay?

Commission and Brokerage

An **agent**, or **commission merchant**, is a person who buys or sells goods for another person.

Commission is the percentage allowed an agent as compensation.

Brokerage is the percentage paid to a broker or dealer in stocks or bonds for buying or selling such property.

A **consignor**, or **shipper**, is a person who sends merchandise to be sold.

A **consignment** is a quantity of goods shipped.

A **consignee**, or **correspondent**, is a person to whom merchandise is sent to be sold on commission.

Value = base.   Commission = percentage.

1. At $\frac{1}{4}\%$ what is the brokerage on $10,800 par value of stock?

2. At $\frac{1}{8}\%$ what is the brokerage on $8000 par value of bonds?

3. At $\frac{1}{16}\%$ what is the brokerage on $6000 par value of shares of stock?

4. At $3\frac{1}{2}\%$ what is the commission on goods sold for $6719.50?

5. At $5\frac{1}{4}\%$ what is the commission on $47.80 worth of fruit?

6. To what does the brokerage on $7500 par value of stock amount at $\frac{1}{8}\%$?

7. To what does the commission on $234.56 worth of merchandise amount at $2\frac{1}{3}\%$?

8. At $4\frac{1}{2}\%$ what is the commission on a consignment of hay worth $555.55?

9. An agent sells 617 bu. of wheat at $1.70 per bushel. What is his commission at $2\frac{1}{2}$ %?

10. A commission merchant sells goods to the amount of $1122.30. What is his commission at $3\frac{1}{2}$ %?

11. A broker purchases stock to the amount of $8800 par value. What is his brokerage at $\frac{1}{8}$ %?

12. An agent purchases silks to the amount of $7800. What is his commission at $3\frac{3}{4}$ %?

13. An agent collects debts to the amount of $907.80. What is his commission at 5 %?

14. A commission merchant sells 7400 bbl. of flour at 7.87\frac{1}{2}$ per barrel. To what does his commission amount at $3\frac{1}{2}$ %?

15. An agent sells a farm for $7450. What is his commission at $2\frac{1}{2}$ %?

16. A broker negotiates a mortgage for the sum of $1140. What is his brokerage at $\frac{1}{2}$ %?

17. What is the commission on $1000 at $4\frac{1}{2}$ %?

18. What is the commission on $1678.30 at $2\frac{1}{4}$ %?

19. What is the commission on $7531.19 at $3\frac{3}{4}$ %?

20. Find the commission on $508.60 at $1\frac{1}{4}$ %.

21. Find the commission on $7862.50 at $1\frac{3}{4}$ %.

22. A lawyer collects debts to the amount of $878.30. What is his commission at $2\frac{1}{2}$ %?

23. A correspondent purchases teas for me to the amount of $7193.16. What have I to pay him for commission at $3\frac{1}{8}$ %?

24. A commission merchant sells goods to the amount of $6734.10. What is his commission at $2\frac{2}{3}$ %?

25. An agent sells 718 bbl. of flour at $7.13 a barrel. What is his commission at $4\frac{1}{4}$ %?

26. An agent was paid $416.25 for selling goods. What was the value of the goods sold, the rate being $2\frac{1}{2}\%$?

27. I send my agent $1963.50, which includes his 2% commission and the amount he is to invest for me in grain at $70\cancel{c}$ a bu. How many bushels will he buy?

28. A lawyer was told to collect as much as possible of a debt of $4680. He succeeded in getting 90% of the debt and charged $5\frac{1}{2}\%$ commission on the amount collected. How much did the creditor receive?

29. A boy selling eggs for a farmer received as commission $1.80 for selling 18 doz. eggs at $.30 a dozen. What was the rate of commission?

30. An agent received $67.50 for selling a consignment of goods for $5400. At what rate was he paid?

31. Mr. Quinby sent his agent $2436, which he was to invest in $800 lots after deducting his commission of $1\frac{1}{2}\%$. How many lots did he buy?

32. I wish to buy 400 bbl. of flour at $4.75 a bbl. How much money must I send my agent in order that he may purchase this quantity and still have enough money to pay his commission of $2\frac{3}{4}\%$?

33. A commission merchant received $4000 for goods which he sold. After deducting his commission of 3%, he invested the balance in wheat, this time charging 2% commission. What amount did he invest in the wheat?

34. Mr. Pyatt's agent sold for him goods amounting to $8000. He charged $2\frac{1}{2}\%$ commission and had also to pay $22.50 for freight and storage. How much did he send to his employer?

KEEPING THE CASH ACCOUNT

Bookkeeping records systematically the transactions of business.

A **cash account** is a record of transactions in which money has been received or paid for things or services.

Debtor, Dr., means that money has been received, and **Creditor, Cr.**, means that money has been paid.

A. THOMPSON, GROCER. CASH ACCOUNT.

Dr.

DATES	ITEMS	AMTS.	TOTALS
1902 Jan. 2	Bal. on hand	43 59	
"	Sold mdse.	34 21	
"	Recd on acct. of Thos. Bain	20	
			97 80
Jan. 3	Bal. on hand	21 23	

Cr.

DATES	ITEMS	AMTS.	TOTALS
1902 Jan. 2	Pd. S. W. X. Co.,	25	
"	J. W., wages	9	
"	Dec. gas.	4 15	
"	Self	5	
"	A. B. & Co.	29 50	
"	Expressage	3 92	76 57
	Bal. on hand		21 23
			97 80

There are several kinds of books that may be kept to meet the needs of a business. Among these are: *cash-book, daybook, journal, invoice book, ledger, bill book, bank book, sales book.*

There are two general methods of bookkeeping: *single entry* and *double entry.*

Find out what you can about these books and methods, and discuss them in class.

Every person who has anything to do with money finds it helpful to keep at least a cash account.

The *debit* side of a cash account records money taken in, the *credit* side money paid out.

The debits must always be more than the credits in a cash account.

CHECKS

Date, Aug. 17, 1901

To Wm. J. Mitchell

For Coal

Am't of Check, $16.50

No. 439.

No. 439

Bloomfield, N. J., Aug. 17, 1901

The Bloomfield National Bank

Pay to the order of ～～～～～～～～～～

William J. Mitchell

Sixteen ～～～～～～ 50/100 Dollars

$16.50   Estabrook Hopkins

STUB   CHECK

Mr. Hopkins has an account in his bank of deposit. This is the 439th check that he has drawn on the account. If he has $16.50 to his credit at the bank, the teller will

pay to Mr. Mitchell that amount, if Mr. Mitchell indorses
the check, that is, writes his name on the back of the check.
Or the payee, Mr. Mitchell, may indorse the check to the
order of some one else, then the bank officers will pay
this other person.   Mr. Hopkins keeps the stub of the
check, so that he may have a record of the payment.
When Mr. Mitchell has indorsed the check, and the bank
has paid it, the canceled check is given back to the de-
positor, and is legally a receipt for the payment of the
$16.50 due from Mr. Hopkins to Mr. Mitchell.

1. A large business house had on deposit in bank one
morning $182,000.   Checks were drawn that day as fol-
lows : $59.81 ; $832.90 ; $1000 ; $848.60 ; $12,500 ;
$5.85; $800; $30,000; $292.16; $348.29; $100; $86.94;
$33.58; $400; $39.96; $10; $26,792; $84.75; $105.50;
$81,297.52; $3.91; $4.63; $50.29; $100; $250; $750;
$1200; $36.37; $5000; $986; $16.28; $33.75; $500;
$29.60; $10.80; $50; $1.29; $3.71; $28; $6.

a. Copy each amount, beginning to number the checks
at 3649. *b.* Add them. *c.* What was the balance in bank
that night? *d.* If the house manufactures boots and shoes,
suggest items for which each check might be drawn.

Banks often test applicants for positions in a way indicated by *a, b, c.*

2. Against a balance of $350 write 5 checks for house-
hold expenses for a month.

3. Write 10 checks to begin a grocery business with a
deposit of $1800.

4. Balance, Mar. 1, $175.06.   Checks drawn, Mar. 1–10,
$35.40, $18.50, $19.69, $46.12.   Deposits, Mar. 11 and 12,
$463, $81.26.   Checks, Mar. 13 and 15, $379.18, $101.99.
What was the balance before the second deposit? after
the last check?

PERCENTAGE

Taxes

Taxes are amounts levied upon persons or property by government for some public purpose.

A tax on persons is called a **poll tax**, and a tax on property is called a **property tax**.

The poll tax is assessed only on voters between certain ages, as between twenty-one and seventy, and in some States is not assessed at all.

It is important to understand taxation thoroughly. Without taxes government and the culture and civilization dependent upon government would cease to exist. In history, questions of taxation have been of the greatest importance. They have led to such great changes in the government as are embodied in the Great Charter of England and in the Constitution of the United States.

In assessing taxes, where poll taxes are levied, it is necessary :

First, to estimate the value of all the property to be taxed, and to make a complete inventory of it.

Second, to find the number of polls, that is, the number of persons liable to pay a poll tax.

Third, to determine what portion of the tax is to be raised upon the polls, and to divide it equally among them. In some States the exact amount of the poll tax is fixed by law.

Fourth, to find how much must be paid on each dollar of the taxable property to raise the remainder of the tax.

The fourth operation involves dividing the amount to be raised by the estimated value of the property on which it is to be raised. This gives the rate of taxation. It is then possible to compute the tax on any single property.

Where there are no poll taxes, only the first and fourth measures are necessary.

1. A tax of $4800 is to be raised by a town. The taxable property is valued at $960,000, and there are 320 polls, each taxed $1.50. What will be the tax on each dollar, and what will be the amount of taxes for each of the following persons to pay?

A, who is assessed for $5700 and 2 polls: B, $728 and 1 poll: C, $8976 and 3 polls: D, $1147 and 1 poll.

The tax on 320 polls at $1.50 each is $480, which, subtracted from $4800, leaves $4320, to be levied on the property.

Since $960,000 is to be taxed $4320, one dollar will be taxed $\frac{1}{960000}$ of $4320, which is $4\frac{1}{2}$ mills.

A's tax on 2 polls would be twice $1.50, or $3, and on $5700 property would be 5700 times $4\frac{1}{2}$ mills, which is $25.65. This added to $3 gives $28.65 as the amount of A's tax.

The tax of the others may be found in the same way.

2. A village is taxed $537.50; the whole property of the village is valued at $35,000, and there are 50 polls assessed 25¢ apiece.

a. What per cent is the tax? *b.* How much is A's tax, who pays for 2 polls, and whose property is valued at $4240? *c.* What is B's tax, who pays for 3 polls, and whose property is valued at $3560? *d.* What is C's tax, who is assessed for 1 poll and $5350?

3. The city of Cleveland levied a tax of $1,945,600; its taxable property was rated at $243,200,000.

a. What per cent was the tax? *b.* What was A's tax, whose property was valued at $10,000? *c.* What was B's tax, who was assessed for $15,240? *d.* What was C's tax, who was assessed for $35,460? *e.* What was D's tax, who was assessed for $289,740?

4. What sum must be assessed to raise $15,757.50 net, allowing $4\frac{1}{2}$ % for arrears and commission for collection?

5. What sum must be assessed to raise $16400 net, allowing 5% for collection?

6. Allowing $3\frac{1}{2}$% for collection, what sum must be assessed to raise $33,775 net?

7. In making out a tax bill for a town or city, after finding the tax on $1, we make a table that shows the amount of tax on any number of dollars from $1 to $1000.

A town levies a tax of $5700. The town contains 30 polls, which are assessed 50¢ each, and its taxable property is inventoried at $199,500. The sum to be raised is $6000, and the tax is 3%.

a. What amount of tax must be raised to pay the tax, and 5% commission for collection? *b.* What is the tax on a dollar?

Prop.	Tax	Prop.	Tax	Prop.	Tax
$1	$.03	$10	$.30	$100	$3.00
2	.06	20	.60	200	6.00
3	.09	30	.90	300	9.00
4	.12	40	1.20	400	12.00
5	.15	50	1.50	500	15.00
6	.18	60	1.80	600	18.00
7	.21	70	2.10	700	21.00
8	.24	80	2.40	800	24.00
9	.27	90	2.70	900	27.00
10	.30	100	3.00	1000	30.00

c. In the assessment, what is A's tax, who is rated at $2256, and pays for 3 polls?

8. The same town levied a tax of $5400 the year before, with the same number of polls. Its taxable property inventoried $192,000. *a.* Allowing 5% for arrears and 2% for collection, find the tax per $1000. *b.* What was A's tax, at same assessment and number of polls?

9. In a certain town a tax of $24,400 was raised. The value of the property taxed was $3,200,000, and there were in the place 1000 polls taxed at $2 each. What was the rate of taxation? What is the amount of A's tax, who pays for one poll, and has real estate valued at $10,000, and personal property amounting to $3000?

10. The school tax in a small town is $3500 on property valued at $875,000. A resident of the town has property valued at $20,000. What tax does he pay?

11. In the town of F a tax of $32,000 was raised for the maintenance of schools. The rate of taxation was 8 mills on the dollar, and a collector was paid a fee of 1%. What was the assessed valuation of the town?

12. In the town of B the assessed valuation of the property is $1,975,360. The amount to be raised is $53,644. What is the rate of taxation, when there are 780 polls at $2 each?

13. Mr. Bright pays a tax of $1351.50, including a poll tax of $1.50. The rate of taxation is $4\frac{1}{2}$ mills on the dollar. What is the value of his taxable property?

14. A in a certain town pays a tax on property valued at $4285, at the rate of $7\frac{1}{2}$ mills on the dollar. B's property in another town is assessed at $3295, at the rate of 8 mills on the dollar. Which pays the greater tax?

15. When the rate of taxation is $6\frac{1}{2}$ mills on a dollar, what is the assessed valuation of a farm on which a tax of $45.50 is paid?

16. In the year 1900, Mr. Adams paid a property tax of $67.50, at the rate of $7\frac{1}{2}$ mills on a dollar. The next year his tax was $71.25, the rate being the same. How much greater was his assessment in 1901 than in 1900? In 1902 the rate was 9 mills. How much was his tax?

PERCENTAGE

Interest

Money paid for the use of money loaned is called **interest**. The money loaned is called the **principal**.

The *principal* and the *interest* added together form the **amount**.

In most countries laws have been passed fixing the rate of interest. Many states allow the charging of interest at a higher rate than the legal one, if a written agreement is made to pay the higher rate.

It is customary in interest calculations to consider a year as 12 mo. and a month as 30 da.

Simple interest is *interest* on the *principal* only. **Compound interest** is interest on both the principal and on any interest past due, but unpaid. Both simple and compound interest are usually computed at regular intervals.

Since savings bank depositors seldom call semi-annually or quarterly for their interest, most of these banks allow compound interest. In ordinary business there are not many instances of charging or paying compound interest, since interest is usually paid promptly when due.

It is easy to understand why we pay for goods : we keep them or use them up. But it is not quite so easy to understand why we pay interest for money which we return. There are several good reasons. Those who have capital will not let others have it unless they pay for the use of it, partly because if the owners should keep it they could "make money" with it just as the borrowers hope to do, and partly because they are not sure they will get it back when they want it. Some borrowers lose what they borrow. If the lenders did not offset these losses by the gains of profitable loans, their capital would gradually disappear. Interest is the profit on selling money to get it back after a time, just as an ordinary sale of merchandise gives an immediate profit on the wealth of the merchant.

Interest is a form of percentage.

Principal = base. Interest = percentage.

REVIEW OF THE SIX PER CENT METHOD

In interest at 6 % the rate

for 1 year is .06, for 2 mo. is .01, for 1 mo. is .005,

for 6 days is .001, and for 1 day is .0001667 or .0001$\frac{2}{3}$.

The interest upon $100 at 6 %

for 1 year is $6, for 2 mo. is $1, for 1 mo. is $.50,

for 6 days is $.10, and for 1 day is $.01$\frac{2}{3}$.

At 5 %, interest is $\frac{5}{6}$ of what it is at 6 % ; at 4 %, it is $\frac{2}{3}$; at 3 %, it is $\frac{1}{2}$; at 7 %, it is $\frac{7}{6}$; and at 8 %, it is $\frac{4}{3}$.

To find the interest upon any sum of money at any rate, find the interest at 6 % and multiply it by the ratio of the given rate to 6 %.

1. Find the interest at 5 % upon $850 for 1 yr. 7 mo. 12 da.

At 6 % the interest for 1 yr. = $850 × .06 = $51.00

for 6 mo. = $850 × .03 = $25.50

for 1 mo. = $850 × .005 = $ 4.25

for 12 da. = $850 × .002 = $ 1.70

The interest for the entire period = $850 × .097 = $82.45

$5 \% = \frac{5}{6}$ of 6 % $82.45 × $\frac{5}{6}$ = $68.71

Find the interest :

2. At 4 % upon $1290 for 2 yr. 3 mo. 8 da.

3. At 3 % upon $1,000,000 for 10 yr. 7 mo. 9 da.

4. At 8 % upon $10,500 for 6 yr. 1 mo. 26 da.

5. At 4$\frac{1}{2}$ % upon $9750 for 3 yr. 19 da.

6. At 3$\frac{3}{4}$ % upon $305,000 for 7 yr. 6 mo. 3 da.

7. At 2$\frac{1}{2}$ % upon $12,000,000 for 9 mo. 10 da.

8. At 5$\frac{1}{2}$ % upon $8630 for 4 yr. 3 mo. 28 da.

9. At 3$\frac{1}{2}$ % upon $36,600 for 1 yr. 8 da.

10. At 6$\frac{1}{2}$ % upon $800 for 11 mo. 9 da.

Annual Interest

1. What is the interest of $1250.50 for 1 yr. at 7%?

$1250.50, principal.
     .07, rate.
$87.5350, interest.

To find the annual interest on money for a year, we multiply the principal by the rate.

When the mills amount to five or more, they are counted as one cent.

Find the interest of:

2. $423 for 1 yr. at 7%.
3. $240.31 for 3 yr. at $4\frac{1}{2}$%.
4. $403.67 for 2 yr. at 5%.
5. $1640 for 1 yr. at 8%.
6. $430.45 for 2 yr. at 7%.
7. $185.06 for 4 yr. at 6%.
8. $9864.80 for 5 yr. at $4\frac{1}{2}$%.
9. $763 for 4 mo. at 7%.
10. $940.20 for 6 yr. at 4%.
11. $243.10 for 5 yr. at 8%.
12. $150,000 for 7 yr. at $3\frac{1}{2}$%.
13. $425.35 for 9 yr. at $5\frac{1}{2}$%.

Interest for a Term of Years

1. Find the interest of $619.75 for 3 yr. 6 mo. at 8%.
Int. $1 for $3\frac{1}{2}$ yr. $= .08 \times 3\frac{1}{2} = .28$.

$619.75
    .28
 495800
 123950
$173.5300

To find the interest on money for any period, we may multiply the rate for one year by the number of years, and the principal by the rate for the full term.

2. $495.95 for 5 yr. 5 mo. 5 da. at $6\frac{3}{4}$%.
3. $436.75 for 7 yr. 9 mo. 18 da. at $3\frac{4}{5}$%.

4. $2560.75 for 4 yr. 3 mo. 25 da. at $6\frac{1}{2}\%$.
5. $139.80 for $3\frac{1}{2}$ yr. at $7\frac{1}{2}\%$.
6. $1895 for $4\frac{3}{4}$ yr. at $6\frac{7}{8}\%$.
7. $375 for 3 yr. 8 mo. at 7%.
8. $446.50 for 3 yr. 3 mo. at 8%.
9. $220 for 7 mo. at $7\frac{1}{2}\%$.
10. $243.80 for 2 yr. 5 mo. at 8%.

INTEREST FOR YEARS, MONTHS, AND DAYS

1. What is the interest of $6677 for 8 yr. 8 mo. 20 da. at $5\frac{1}{2}\%$?

		To find the interest on money for any
	$6677	period, we may find the interest for
	.055	years, months, and days separately, and
	33 385	add the various items.
	333 85	
a.	367.235	interest for 1 yr.
	8	

b.	$2937.880	"	"	8 yr.	$a \times 8 = b.$
c.	183.6175	"	"	6 mo. $= \frac{1}{2}$ yr.	$a \times \frac{1}{2} = c.$
d.	61.20583	"	"	2 mo. $= \frac{1}{6}$ yr.	$a \times \frac{1}{6} = d.$
e.	15.30145	"	"	15 da. $= \frac{1}{2}$ mo.	$d \times \frac{1}{4} = e.$
f.	5.10048	"	"	5 da. $= \frac{1}{6}$ mo.	$e \times \frac{1}{3} = f.$
	$3203.10526	"	"	8 yr. 8 mo. 20 da.	

Find the interest of :

2. $974 for 1 yr. at 11%.
3. $1678.90 for 7 yr. at 9%.
4. $142.70 for 16 yr. at 8%.
5. $80.80 for 22 yr. at 7%.
6. $67.49 for 6 yr. at $2\frac{1}{2}\%$.
7. $208.60 for 11 yr. at $3\frac{3}{8}\%$.
8. $800 for 6 yr. 5 mo. 18 da. at 8%.

9. $7400 for 9 yr. 11 mo. 24 da. at $6\frac{1}{4}\%$.

10. $9680.80 for 14 yr. 4 mo. at 3%.

11. $476.76 for 10 yr. 8 mo. at $5\frac{3}{4}\%$.

12. $8900 for 6 yr. 7 mo. 28 da. at $11\frac{1}{4}\%$.

13. $8160 for 9 yr. 15 da. at $7\frac{1}{2}\%$.

14. $412.90 for 6 yr. at $4\frac{7}{8}\%$.

15. $127.40 for 3 yr. 3 mo. 3 da. at $12\frac{1}{2}\%$.

16. $80.63 for 4.78 yr. at 2.97%.

17. $106.70 for 11.113 yr. at 13.47%.

18. At 7% what is the interest of $19.41 for 1 yr. 7 mo. and 13 da. ?

19. At 5% what is the interest of $530 for 3 yr. and 6 mo. ?

20. At 8% what is the interest of $5.37 for 4 yr. and 12 da. ?

21. At 9% what is the interest of $4070 for 3 mo. ?

22. At 10% what is the interest of $3671 for 6 mo. ?

23. At 4% what is the interest of $4920.05 for 3 mo. ?

24. At 3% what is the interest of $40.17 for 3 mo. and 18 da. ?

25. At $4\frac{1}{2}\%$ what is the interest of $37.13 for 5 mo. and 12 da. ?

26. At $5\frac{1}{2}\%$ what is the interest of $489 for 3 yr. and 4 mo. ?

27. At 7% what is the interest of $700 for 1 yr. and 9 mo. ?

Find the interest

28. Of $2520 from March 5, 1898, to Dec. 25, 1902, at 6%.

29. Of $875 from Sept. 3, 1887, to Nov. 23, 1901, at 7%.

30. Of $1125 from May 2, 1892, to Feb. 17, 1899, at 6%.

31. Of $2525 from July 12, 1895, to Mar. 2, 1901, at 6%.

32. Of $978 from May 25, 1895, to Apr. 15, 1902, at 6%.

33. A man borrowed $850 on the first of June, 1897, and paid it with 6% interest on Oct. 21, 1901. What amount did he pay?

34. Mr. Holmes sailed for Europe June 15, 1897, taking with him $1500, which he had that day taken from the bank. After an absence of 2 yr. 4 mo. 15 da. he returned and again deposited $1500 in the bank. How much did he lose by withdrawing the money, the rate of interest being 4%, not compounded?

35. A note for $901.20, dated March 4, 1895, was paid May 24, 1901, with interest at 6%. What was the amount?

36. Mr. Brown left to his wife a sum of money, the yearly interest on which at 4% amounted to $582. What amount did he leave?

37. A grocer who bought goods to the amount of $1782.50, paid his bill at the end of 90 days at 6% interest. How much did he pay?

38. Mr. Wolfe, who had $4500 in the bank, drew it out and invested it in the grocery business. At the end of 5 yr. 4 mo. 12 da. he sold the business for an amount equal to the sum invested with 5% simple interest added. Find the amount that he received.

39. When X borrows $550 from a friend, and pays it back with interest in eight months, how much does he pay?

40. A man who bought a farm for $12,500 paid half the amount in cash, $\frac{1}{5}$ of the remainder without interest in 30 days, and the balance in $1\frac{1}{2}$ year with interest at 6%. What was the amount of the last payment?

41. On January 15, 1898, a man borrowed from his employer $350. At the end of 6 months he borrowed $500 more. November 15, 1898, he paid back what he had borrowed with interest at 7%. What did he pay?

42. A business man invests $3425 in such a way as to secure him an annual interest of 8%. What would be his interest for 2 yr. 5 mo. 15 da.?

Amount for Years, Months, and Days

First Method

1. What is the amount of $1752.96 for 2 yr. 5 mo. 21 da. at $5\frac{1}{2}$%?

$$\$1752.96$$
$$.055$$

12	$96.41280. Int. for 1 yr.
30	8.0344. Int. for 1 mo.
	.26788. Int. for 1 da.

$96.41 × 2 = $192.82. Int. for 2 yr.
$8.03 × 5 = 40.15. Int. for 5 mo.
$.2679 × 21 = 5.62. Int. for 21 da.

$238 59. Total interest.
$1752 96, Principal.
$1991.55, Amount.

To find the amount for a term of years, months, and days, we find the interest for the term and add it to the principal.

Find the amount of :

2. $199.48 for 16 yr. at 7%.

3. $897.50 for 3 yr. at 8%.

4. $982.35 for 4 yr. at $6\frac{3}{4}$%.

5. $1500 for 5 yr. at $5\frac{1}{4}$%.

6. $628.50 for 5 yr. at $12\frac{1}{3}$%.

7. $75.50 for 10 yr. at 6%.

8. $5040 for 2 yr. at $7\frac{1}{2}$%.

9. $1575.20 for 3 yr. 8 mo. at 7%.

10. $5000 for 2 yr. 3 mo. at $5\frac{1}{2}$%.

11. $125 for 6 mo. at $4\frac{3}{4}$%.

12. $1000 for 2 yr. at 7%.

13. $2833.25 for $4\frac{1}{2}$ yr. at 6%.

14. $5465 for 4 yr. 4 mo. at 5%.

15. $4780 for 6 yr. 6 mo. at 7%.

16. $.05 for 20 yr. 10 mo. 15 da. at 8%.

SECOND METHOD

1. What is the amount of $182.50 for $4\frac{1}{2}$ yr. at 5% ?

Rate for term $= .05 \times 4.5 = .225$

Principal $= 100\% = 1$.

Amount $=$ Principal $+$ Interest, $1 + .225 = 1.225$.

$182.50 \times 1.225 = 223.56.

To find the amount, multiply the principal by 1 plus the rate for the term.

At 6%, what is the amount of :

2. $98.50 for 5 yr. 8 mo. ?

3. $168.13 for 8 yr. 5 mo. 3 da. ?

4. $75.75 for 4 yr. 2 mo. 27 da. ?

5. $675.50 for 30 yr. 3 mo. 23 da. ?

6. What is the amount of $378.42 for 1 yr. 5 mo. 3 da. at 7%?

7. Find the amount of $45 for 12 yr. 27 da. at $6\frac{3}{4}$%.

8. What will $100 amount to in 15 yr. 6 mo. if put at interest at 4%?

9. Find the amount of $76.50 for 1000 da. at 6%.

10. Find the amount of $785.25 from April 8, 1901, to June 28, 1902, at 6% interest.

INVENTIONAL GEOMETRY

To construct a square prism
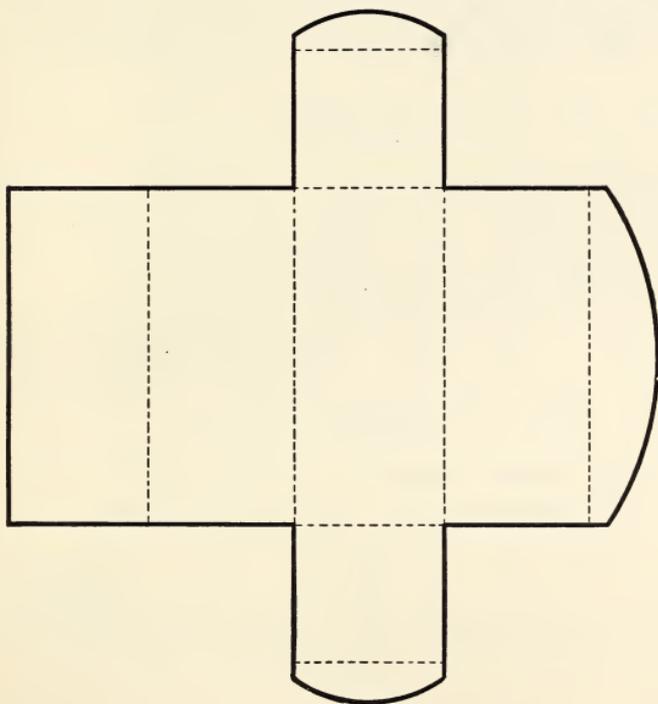
Fold at the dotted lines. Cut out of paper or cardboard.

Make one prism: scale, $\frac{1}{4}$ in. = 1 in.,

and another prism: scale, $\frac{1}{8}$ in. = 1 in.

To find the ratio of one line to another
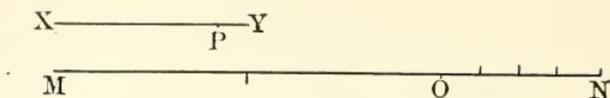
XY may be measured by the compasses upon MN twice, with the remainder ON.

ON may be measured upon XP once, with the remainder PY.

PY may be measured upon ON four times.

$ON = 4\,PY.$ $XY = ON + PY = 4\,PY + PY = 5\,PY.$

$MN = 2\,XY + ON = 2(5\,PY) + 4\,PY = 14\,PY.$

$XY : MN = 5\,PY : 14\,PY = 5 : 14.$ $MN : XY = 14 : 5.$

LEVERS

A **lever** is a rigid bar free to move upon a fixed point called the fulcrum. For convenience, we divide the lever into the **weight** arm and the **power** arm.

In any lever, the power arm is the part of the lever from the fulcrum to the point where the power is applied. The weight arm is the part of the lever from the fulcrum to the point where the weight is applied.

There are three classes of levers, called respectively levers of the first, second, and third classes.
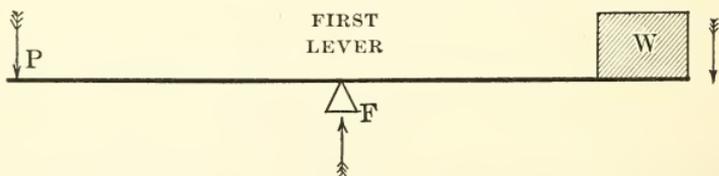
In a lever of the first class, the fulcrum is between the weight and the power.
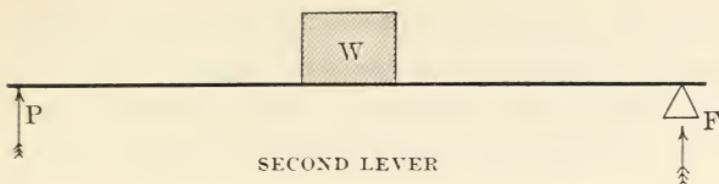
SECOND LEVER

In a lever of the second class, the weight is between the power and the fulcrum.

THIRD LEVER
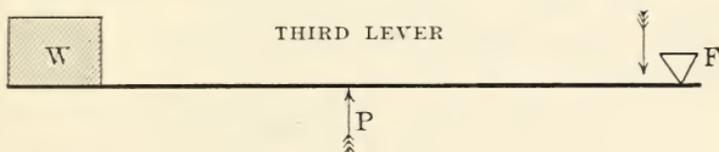
In a lever of the third class, the power is between the fulcrum and the weight.

The arrows indicate the pull or push of the weight and power, and the direction of the resistance of the fulcrum.

When one pries up a heavy stone with a crowbar, the bar is a **lever**, which is used on the principle of ratio.

Balance scales are all made on the principle of ratio as developed in the lever. Of which class of lever is the crowbar ? the balance scale ?

Principles

When the **fulcrum**, F, of the **first lever**, is halfway between the **power**, P, and the **weight**, W, then the *power* must be a little more than equal to the *weight* in order to lift it. But when the *fulcrum* separates the lever into two parts, in the ratio of P to F, 3, and F to W, 1, then a power only a little more than $\frac{1}{3}$ of the weight lifts the weight.

In all levers, **the power is to the weight as the weight arm is to the power arm.**

When the fulcrum is exactly in the middle of a lever of

the first class, a power of one pound balances a weight of one pound. When we place the fulcrum $\frac{2}{3}$ of the distance from one end, a force or power of one pound at the end of the long arm balances a weight of 2 lb. at the end of the short arm.

Problems

When the power arm is six times the length of the weight arm, a power of 2 lb. balances a weight six times as great:

$$P : W :: W \text{ arm} : P \text{ arm}$$
$$2 : x :: 1 \qquad : 6 \qquad\qquad x = 12.$$

1. In a 7-foot lever, the fulcrum is 2 ft. from one end. What force will balance a weight of 100 lb. at the end of the short arm?
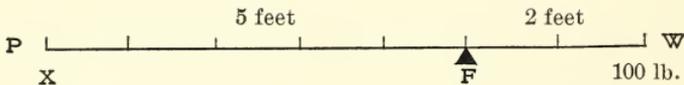
$$x : 100 \text{ lb.} :: 2 \text{ ft.} : 5 \text{ ft.}$$
$$x = 40 \text{ lb.}$$

2. A workman wished to raise a stone weighing 350 lb. His own weight used as power was 150 lb. This drawing represents the way he did it.

He needed to raise only $\frac{1}{2}$ of the weight of the stone, for one edge, A, rested on the ground, $\frac{1}{2}$ of $350 = ?$ His
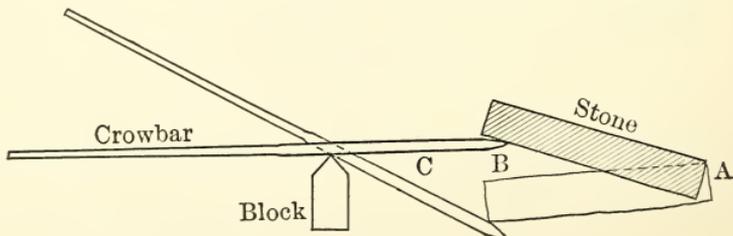
own weight thrown on the other end of the bar lifted the
stone. After getting the stone at this angle, he threw
a block under it at *B*. How heavy a weight could he
lift when he placed the block at *C*? Measure and esti-
mate.

3. A boy weighing 150 lb. sat on one end of a "see-
saw." When another boy of 90 lb. sat at the other
end, where should the fulcrum be placed?

We omit here any estimate of the weights of the two portions of the
plank or board used in "teetering."
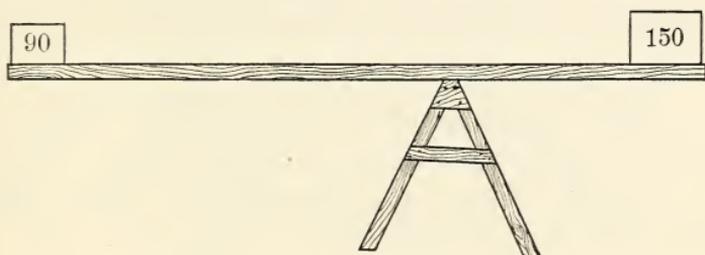
4. The fulcrum of a lever of the first class is exactly in
the middle. What power is required to balance a weight
of 20 lb.? 40 lb.? 25 lb.? Why?

5. In a lever of the second class a weight of 20 lb. is
placed at the middle point. What is the ratio of the
length of the power arm to that of the weight arm?
What force is required to balance the 20-lb. weight? A
44-lb. weight?

6. In a lever of the first class, the fulcrum is placed $\frac{5}{9}$ of
the distance from one end. What power acting at the end
of the long arm will balance a weight of 30 lb. at the end
of the short arm?

7. In a lever of the second class, what power will balance
a weight of 24 lb. placed at the middle?

8. A weight of 150 lb. is placed 3 feet from the fulcrum of a 5-foot lever of the first class. What power will balance it?

9. What weight at the end of an 8-foot lever of the third class will be sustained by a power of 24 lb. applied 3 feet from the fulcrum?

10. A lever of the first class is 7 feet in length. What weight will be balanced by a power of 16 lb. acting 3 feet from the fulcrum?

11. A lever of the third class is 9 feet long. What force applied 3 feet from the fulcrum will balance a weight of 5 lb. ?

12. A man weighing 200 lb. balanced his weight upon a teetering plank against the combined weight of 3 children by giving himself $\frac{2}{3}$ of the plank. What was the average weight of the children?

13. In a lever of the first order in which the weight arm is 3 feet, a power of 15 lb. balances a weight of 25 lb. How long is the lever?

14. In another lever of the same class a weight of 100 lb. is balanced by a 40-lb. force. The distance from the fulcrum to the weight is 5 feet. What is the length of the power arm?

15. In a 3-foot lever of the second class a weight of 30 lb. is to be balanced by a force of 20 lb. How far from the fulcrum should the weight be placed?

16. Make to scale a drawing to show a five-foot lever, balanced, of the second class, with a weight of 20 lb. one foot from the fulcrum, and find the power necessary to support it.

To construct a right cylinder
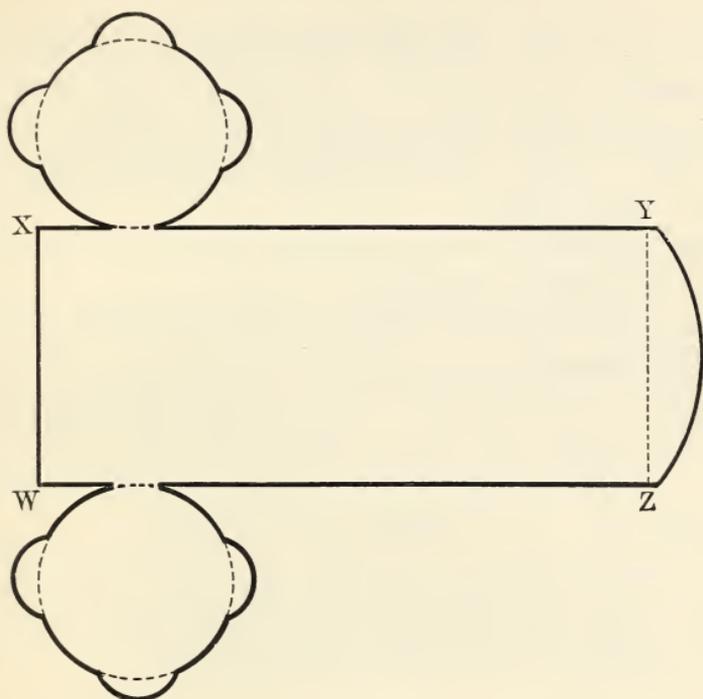
The rectangular portion should be in length $3\frac{1}{7}$ the diameter of the circles. Why?

Cut out of paper or cardboard. Fold at the dotted lines, except *YZ*. *XW* and *YZ* should meet.

Make two right cylinders, one to scale $\frac{1}{4}$ in. to 1 in., another to scale $\frac{1}{8}$ in. to 1 in. Measure the surfaces and volumes of these cylinders.
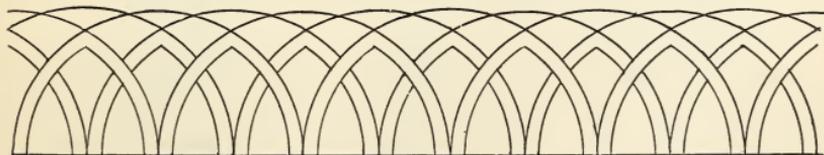
Arcs and semicircles.

PERCENTAGE

Bank Discount

A **bank** is a corporation chartered by law to receive money on deposit and to loan money. National banks also issue a paper currency, based on government bonds of the United States.

Bank discount is simple interest on the face of a *note* that is deducted in advance.

Money is loaned on **notes** which are written promises to repay loans.

$ 600.00   *Louisville, Ky., October 2, 1900*

           Three months after date I promise to pay to the order of   *John J. Audubon*

Six hundred _____ *Dollars*

at the National Broadway Bank. _____

with interest at six per cent.

Value received

No.   *Due*   *Albert Gallatin*

The **proceeds** or **cash value** of a *note* is the face of the note, less the discount.

The **term of discount** is the time the note has to run after being discounted.

When a note bearing interest is discounted, the sum from which the discount is taken is the *amount* of the note at maturity.

Days of grace, still allowed in some States, extend the time of payment. This custom is a survival of " olden days," when roads were poor and travel was so often interrupted that prompt payments on date set were uncertain.

Finding the Discount

To find the discount, *when the face, time, and rate are given:*

1. What is the bank discount on a note for $840, due 72 da. hence, discounted at 7%?

Interest of $840 at 7% for 1 yr. = $58.80.
72 da. = $\frac{1}{5}$ of a year (360 da.).
The bank discount = $\frac{1}{5}$ of $58.80 = $11.76.

2. What is the bank discount on a note of $471, due 3 mo. hence, discounted at 7%, 3 days of grace included?

Interest of $471 for 1 yr. at 7% = $32.97.
Time of discount = 3 mo. 3 da.

$32.97,   interest for 1 yr.
8.2425, interest for 3 mo.
.2747, interest for 3 da.
——————————————
$8.5172, interest for 3 mo. 3 da.

To find the bank discount, compute the interest for the term.

Find the bank discount :

	Principal	Rate of Discount	Maturity
3.	$700	7%	42 da.
4.	$840	8%	57 da.
5.	$790	5%	4 mo.
6.	$614.30	7%	2 mo.
7.	$217.20	9%	7 mo.
8.	$94.80	10%	20 da.
9.	$350	7%	3 mo.
10.	$1000	6%	60 da.
11.	$15720	6%	6 mo.

	Principal	Rate of Discount	Maturity
12.	$556.27	6%	60 da.
13.	$3456	7%	3 mo.
14.	$367.47	7%	1 yr. 1 mo. 15 da.

FINDING THE PROCEEDS

To find the proceeds of a note discounted at bank, deduct the discount from the face of the note.

What will be the proceeds of each of the following notes, if discounted at a bank?

1. $650.00.   CHICAGO, Nov. 1, 1900.

Sixty days after date I promise to pay to the order of J. R. London six hundred fifty dollars with interest at six per cent.   Value received.   F. T. BRYANT.

2. $848.25.   ST. PAUL, Aug. 3, 1900.

Ninety days after date, I promise to pay to the order of Thomas Mason, with interest at six per cent, eight hundred forty-eight and $\frac{25}{100}$ dollars.   Value received.

ALFRED DECATUR.

3. $69.28.   WHEELING, Dec. 20, 1900.

Four months from date I promise to pay to the order of William Van Buren sixty-nine and $\frac{28}{100}$ dollars with interest at seven per cent.   Value received.

JOSEPH SUMMERS.

When the rate of interest is not specified, it is understood to be the legal rate of the State in which the transaction takes place.

4. $425.25.   NEWARK. N. J., Jan. 1, 1902.

Sixty days after date, I promise to pay to the order of David Ball four hundred twenty-five and $\frac{25}{100}$ dollars. Value received.

5. $944.96.   ELIZABETH, N. J., Feb. 2, 1901.

Five months after date, I promise to pay to the order of John Cole nine hundred forty-four and $\frac{96}{100}$ dollars with interest at seven per cent.   Value received.

6. $1275.70.   BROOKLYN, N.Y., Dec. 1, 1901.

Three months after date, I promise to pay to the order of James Stoddard twelve hundred seventy-five and $\frac{70}{100}$ dollars with interest at five per cent.   Value received.

To find the face, *when the proceeds, time, and rate are given.*

1. A merchant wishes to borrow from a bank $1000, on his note, payable in 7 mo., at 6%.   For what amount must the note be drawn?

FINDING THE FACE

The interest on $1 at 6% for 7 mo. 3 da. (days of grace) is $.0355.

To get the proceeds at bank discount of $1, we subtract $.0355 from $1, and obtain $.9645.

For every $.9645 received $1 must be allowed in the note; and therefore to receive $1000, $\frac{1000}{.9645} = $1036.806 must be allowed in the face of the note, that is, the note must be drawn for $1036.806.

PROOF.
$$\begin{cases} \text{Face of note} \quad . \quad . \quad . \quad . \quad . \quad \$1036.806 \\ \text{Bank discount on } \$1036.806 \\ \quad \text{at } 6\% \text{ for } 7 \text{ mo.} \quad . \quad . \quad . \quad 36.806 \\ \text{Present value} \quad . \quad . \quad . \quad . \quad . \quad \$1000.000 \end{cases}$$

2. For what sum must a note be drawn at 8 mo. 18 da. so that discounted at 6% the proceeds shall be $670?

Interest on $1 for 8 mo. 21 da. at 6% = $.0435, and this sum taken from $1 = $.9565 = present worth of $1.

$$\frac{670}{.9565} = \$700.47 = \text{face of note.}$$

To find the face of a note, divide the whole proceeds by the proceeds of a dollar for the term.

3. For what sums must notes be drawn at each of these discounts to yield each of these proceeds?

	Time	Rate	Proceeds
a.	90 days	6%	$900
b.	100 days	5%	$10500
c.	120 days	$4\frac{1}{2}$%	$1200
d.	60 days	$5\frac{1}{2}$%	$150
e.	6 months	$6\frac{1}{2}$%	$3000
f.	1 year	4%	$7250

4. The proceeds of a note for 3 mo. 6 da., when discounted at a bank at 5%, were $3738.70. What was the face of the note?

5. The proceeds of a 60-day note, when discounted at a bank at 6%, were $1485. What was the face of the note? (No days of grace allowed.)

6. For what sum must a note be drawn at 3 mo. 18 da. so that, discounted at 6%, the proceeds shall be $1964? (No days of grace.)

7. Mr. Birch raised $1939.30 by having a note payable 1 yr. 4 mo. 24 da. discounted at a bank at 7%. What was the face of the note?

PERCENTAGE

EXCHANGE

A **draft**, or **bill of exchange**, is a written order addressed by one person to another, usually at a distance, directing him to pay a certain sum of money, at a stated time, to the order of a third person.

$695 \tfrac{75}{100}$ *Boston, Mass. Jan. 9, 1902*

At sight pay to the order of William H. Scriven

Six hundred ninety five $\tfrac{75}{100}$ *Dollars*

Value received, and charge the same to our account.

To Dodd & Mason, Bankers,
     San Francisco, Cal } *James E. Hackett & Co.*

The **drawer**, or **maker**, is the person who draws the bill, or orders the money to be paid.

The **drawee** is the person on whom the order is drawn, or who is ordered to pay the *bill of exchange*.

The **payee** is the person to whom the money is ordered to be paid.

The **buyer**, or **remitter**, is the person who purchases the bill of exchange.

An **acceptance** is the promise of the drawee to pay the draft, or bill, when due; it is made by writing the word *Accepted*, and the date, over the signature of the drawee, on the face of the draft.

An **indorsement** of a draft is a writing upon its back, by which the payee transfers his rights in the draft to another person. When the payee writes only his name on the

back of the draft, the latter becomes payable to the bearer; but if he prefixes to his signature, " Pay to the order of A—— B——," then the draft is transferred to A—— B——, who, if he transfers it, must in turn indorse it. Indorsers are separately responsible for the amount of the draft, in case the drawee fails to make payment. A draft made payable to bearer may be transferred without indorsement. Drafts are sometimes indorsed " Without recourse." This relieves the indorser from any obligation.

In many States, in order legally to bind the indorsers, it is necessary to cause an unpaid draft, as in the case of an unpaid promissory note, to " go to protest " by making, before a notary public, an affidavit as to its being due and unpaid.

A **domestic draft**, or **bill**, is one payable in the country in which it is drawn.

A **foreign bill** is one drawn in one country and payable in another.

The **rate of exchange** is the variation per dollar between the face of a sight bill and its cost, and may be at either a premium or a discount.

Exchange is said to be *at par* when the cost equals the face of the bill; *below par* when less than the face; *above par* when more.

A **sight draft** is payable when presented.

A **time draft** is payable a certain time after date, or after sight.

In some States, however, three days of grace are allowed on drafts.

A **letter of credit** is a letter from one banking house to other bankers in foreign countries, directing them to pay the person in whose favor the letter is drawn, any amount of money not exceeding the sum named in the letter, or the balance remaining from amounts already drawn.

Domestic Exchange

Sight Drafts

1. A grain dealer in Chicago wishes to send $9000 to a creditor in Boston. The rate of exchange is $\frac{1}{4}$% below par. How much must he pay for the draft on Boston?

$1 - \$.00\frac{1}{4} = \$1 - \$.0025 = \$.9975 = $ cost of $1.

$9000 × .9975 = \$8977.50 = $ cost of draft for $9000.

To find the cost of a sight draft: when exchange is below par, take from the face of the draft the percentage at the rate set; when above par, add the percentage.

2. Peter Cooper in Indianapolis wishes to pay by draft a Philadelphia creditor $3675. Exchange on Philadelphia is $1\frac{1}{4}$% premium. How much must he pay for the draft?

3. A dealer in Richmond wishes to send to Albany $8756.50. If exchange on Albany is 1% below par, how much must he pay for the draft?

4. A merchant in Denver wishes to pay in Boston $6584. Exchange is at $21\frac{1}{2}$% premium. How much must he pay for a draft for $6584?

5. A merchant in New York wishes to pay in Galveston $4653.75. Exchange between New York and Galveston is $1\frac{1}{2}$% below par. What must he pay for a draft?

6. A dealer in Newark wishes to remit $3690 to San Francisco. Exchange is $1\frac{1}{4}$% below par. What will be the face of the draft?

7. A cotton broker in Austin, Tex., wishes to send $8465 to Washington, D.C. Exchange on Washington is $1\frac{1}{2}$% premium. How much must he pay for a draft?

8. A debtor in Chicago wishes to pay $9876.40 in Providence. If exchange on Providence is 1% below par, how much must he pay for the draft?

9. What is the cost of the following draft at $1\frac{1}{2}\%$ discount?

$452.   BALTIMORE, June 21, 1900.

At sight, pay to the order of W. R. Fremont & Co., four hundred fifty-two dollars, value received, and charge the same to our account.   H. M. MASON.
To the Union National Bank, Buffalo, N.Y.

10. A merchant in Omaha purchased a draft on New York for $1164 at 1% premium. How much did he pay?

11. What will a draft on Milwaukee cost for $4000 at $2\frac{1}{2}\%$ discount?

12. What will a draft on Topeka cost for $450 at $\frac{5}{8}\%$ discount?

TIME DRAFTS

1. What will be the cost in Philadelphia of a draft for $2000 on Topeka, payable 4 mo. 12 da. after date, exchange being at $\frac{1}{4}\%$ and the rate of interest being 6%?

The cost of $1 in Philadelphia would be $1.0025 if paid at sight, but since the banker in Philadelphia is to have the use of the money for 4 mo. 12 da., he allows the bank discount on the face of the note for that time.

$1.0025 − $.022 (bank discount) = $.9805, cost of $1 of the draft.

$.9805 × 2000 = $1961, cost of the draft.

2. What is the cost of a draft of $600 at 60 days, exchange at $\frac{1}{8}\%$ premium and interest at 6%?

Time drafts payable after sight with interest are computed upon the principles of interest on promissory notes. They are not as common now as formerly.

To find the cost of a time draft, take its proceeds at bank discount and find their cost at sight.

3. What must be paid for a draft of $875, at 90 days, exchange being at $\frac{3}{4}\%$ premium and interest at 7%?

4. What is the cost of a draft on Denver for $1500 payable in 90 days, exchange at 2% discount and interest at 8%?

5. What must be paid for a draft for $550, payable 30 days after date at $1\frac{1}{2}\%$ discount, interest 6%?

6. Find the cost of a draft on Pittsburg for $1275, payable 60 days after date, interest 6% and exchange being at $2\frac{1}{2}\%$ premium.

7. How much must be paid for a draft on Indianapolis for $725, payable 2 months after date at $\frac{5}{8}\%$ discount, interest 6%?

8. When exchange is at $\frac{1}{8}\%$ premium and interest is 7%, what is the cost of a draft for $850, payable in 6 mo. 18 da.?

9. August 16, 1901, these were the prices of domestic exchange in New York City, as published in the *New York Times* market report of August 17.

Exchange per $100 on New York at domestic centers: Chicago, 25¢ discount. Boston, $16\frac{2}{3}$ @ 10¢ discount. New Orleans commercial, $1 discount; bank, $1 premium. San Francisco, sight, 5¢; telegraphic, $7\frac{1}{2}$¢.

Make drafts for various amounts from these cities to New York.

Exchange falls below par when in the money market of the *drawer* there is a balance due to the market of the *drawee*. Exchange rises above par when in the money market of the drawer there is a balance due from the market of the drawee. Exchange is buying a credit elsewhere. When the credit is large, it is sold at a discount. When there is a debit account, credit is at a premium.

STOCKS AND BONDS

A **corporation** is a legal " person " created by one or another form of law, and authorized to act as a real person.

The **charter** of a corporation is a document reciting a grant of rights and privileges by the legislature of a state, or by its executive agents, and certified by the competent legal authority.

The **capital stock** of any incorporated company is the wealth contributed or computed as contributed by the members of the corporation to form the corporation. These contributions may be in money, lands, buildings, materials, goods, accounts, " good will," or services. The *capital stock* is divided into equal parts called *shares*.

The value of a **share** in the original contribution of capital varies. In most corporations a share is valued at $100.

The first or original value of the shares is called their *nominal value* or **par value**, and remains always the same. The *book value* of a share is the *par value* plus the pro rata share in a surplus, or less the share in a deficit in the corporation's treasury.

The price which the shares or stock of a corporation will bring if offered for sale is their **market value**, and varies from time to time.

When the *market value* equals the *par value*, the stock is at *par;* when the market value is greater, the stock is above par, and sells at a **premium** or advance ; when the market value is less, the corporation's stock is below par, and sells at a **discount**.

The profits made by a corporation are at intervals distributed among the members in proportion to the number of shares each member holds. Such profits are called *dividends*. Dividends are usually reckoned at a certain per cent of the par values of the shares.

Should there be a loss, the stockholders are sometimes required to pay a certain amount called an **assessment**.

Money may be raised by a corporation upon a loan, secured by mortgage of the property. **Bonds** are issued for the loan. They entitle their holders to a fixed rate of interest on the amount of money loaned to the corporation.

Persons who own stock in any corporation, whether by original subscription or by purchase, are called **stockholders**. They constitute the

corporation and elect **directors**, by whom a **president** and other officers are chosen.

A broker who deals in stock is called a **stock broker**. His commission for buying and selling is computed at a certain per cent (usually $\frac{1}{8}$%) on the par value of the stock.

A **coupon** is a certificate of interest, due and payable at a certain date, attached to a bond, to be cut off when due and to be presented for payment.

Oral Problems

1. When stock is selling at a premium of 17%, what is a $100 share of stock worth in money?

2. When stock is selling at a discount of 9%, what is the worth of a $100 share of stock?

3. When stock is $4\frac{1}{2}$% above par, what is the value of a $100 share of stock?

4. When stock is $9\frac{3}{4}$% below par, how much must be paid for a $100 share of stock?

5. When stock is $7\frac{3}{4}$% above par, what is a $100 share of stock worth?

6. When stock is 19% below par, what is a $100 share of stock worth?

7. When stock is $8\frac{3}{4}$% above par, what is a $100 share of stock worth?

8. When stock is at par, what is $1 stock worth?

9. When stock is at a premium of $34\frac{1}{2}$%, what is the worth of a $100 share of stock?

10. When stock is at a discount of 2%, what is the worth of a $100 share of stock?

11. The owner of 15 shares of stock, par value $100, sold them at a premium of 8%. He had paid $96 a share. What was his total gain?

Written Problems

1. What amount of stock at $7\frac{1}{2}\%$ discount can be purchased for $1200?

$$\$100 \text{ stock} = \$92.50 \text{ money,}$$
$$\$1 \text{ stock is worth } \$.925,$$
$$\$1200 \div \$.925 = \$1297.297.$$

2. How much stock can be purchased for $793 when it is selling at a premium of $17\frac{1}{4}\%$?

3. What is the value of $9000 stock at $9\frac{1}{2}\%$ discount?

4. X sold 9 shares of stock, par value $500, at a premium of $8\frac{3}{8}\%$. How much did he receive?

5. When the stock of a railroad is 12% below par, how much is $6400 stock worth?

6. When the stock of a gas works is selling at a premium of $6\frac{1}{4}\%$, C invests $2125 in it. What amount of stock does he receive?

7. When stock is 27% above par, what amount can be purchased for $8890?

8. When stock is 8% below par, what is the value of $6000 stock?

9. When a mining stock is selling at $1\frac{1}{4}\%$ premium, what must be paid for 27 shares, the par value being $25 per share?

10. When X, Y & Z Co. stock is selling at a premium of $13\frac{1}{2}\%$, what is the value of $1100 of stock?

11. A person has $9000 which he wishes to invest in A and B shares, selling at 17% discount. What amount of stock can he purchase?

12. When one invests $8500 in stock, which is selling at 11% above par, what amount of stock does one receive?

13. When A remits to his agent $17,500, with instructions to deduct his brokerage at $\frac{1}{4}$%, and invest the remainder in Great Western Railroad stock, selling at 7% premium, what amount of stock should he receive?

14. If I receive $20,000, with instructions to deduct my commission of $1\frac{3}{4}$%, and invest the balance in stock, which is then selling at 3% discount, what amount of stock should I remit to my employer?

15. A certain stock is at a premium of $5\frac{1}{2}$%. When purchased through a broker, who charges $\frac{1}{8}$% for the transaction, how much do 200 shares cost?

16. How much will 53 shares of Alpha Bank stock, par value $100, cost at an advance of 23%?

17. How much will 43 shares of Central Railroad stock cost at a discount of 78%, the par value being $50 per share?

18. How many shares of stock at an advance of 5% on the par value of $100 per share can be bought for $1995?

19. How many shares of stock, par value $100, can be bought at a discount of 5% for $1805?

20. A broker paid $1776.50 for 17 shares of stock at an advance of $4\frac{1}{2}$%. What was the par value of the stock bought?

21. A broker paid $4850 for bank stock at a discount of 3%. What was the par value of the stock bought?

INVENTIONAL GEOMETRY

To draw from any point outside a line a perpendicular to the line
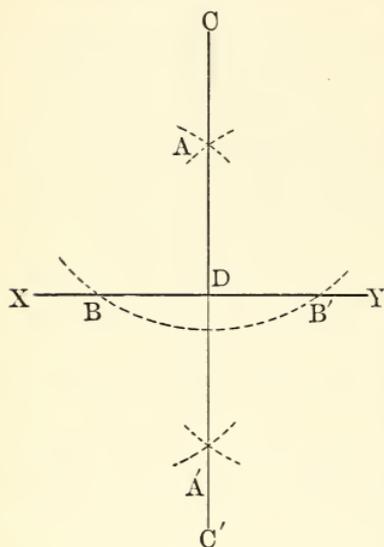
From A, the point outside the line, draw any arc BB' that has a radius long enough to cause the arc to intersect the line XY. From B and B' describe arcs with the same radius as that of the arc BB'. These arcs will intersect at A and A'. Through the points of intersection A and A' draw the line CC'.

The line CD is perpendicular to XY at D.

The line CC' bisects the chord of the arc BB'.

How many right angles are there about the point D in the lines XY and CC'?

Suggested Design

Find concentric circles and concentric arcs.

Reproduce this design and color it symmetrically and harmoniously, using water or crayon colors.
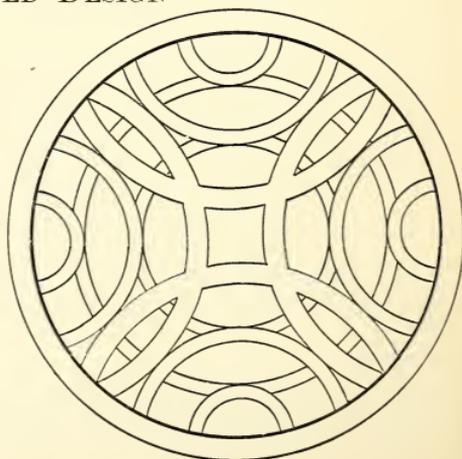
To draw a perpendicular line from any point in a line

From the point A as a center describe the arc BEB', cutting the line XY at equal distances from A at B and B'. Why are AB and AB' equal? From B and B' describe with a radius greater than AB the arcs MN and $M'N'$. The point C at which they intersect is equally distant from B and B'.
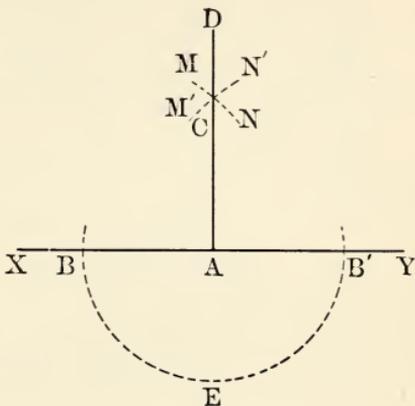
Through C and A draw the line DA.

DA is perpendicular to the line XY.

To draw a perpendicular from any point in a line

Take any point Z outside of the line XY. From Z as a center, with a radius equal to ZA, the distance from Z to A, the point at which the desired perpendicular is to begin, draw an arc $A'AC$. From A', where the arc intersects the line XY, draw a line through A' and Z to the arc $A'AC$. Through the points C at which the line $A'C$ intersects the arc $A'AC$ and A, the given point in the line XY, draw the line AD.
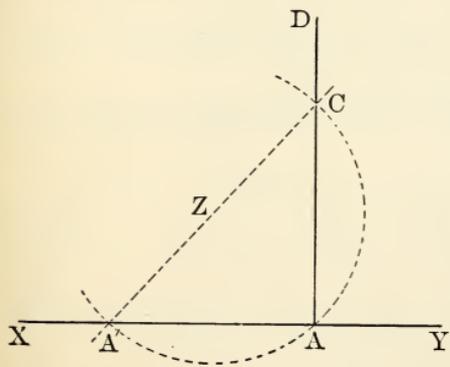
The line AD is perpendicular to XY.

Compare this method with that above.

To find the value of the angles made by the meeting of two lines
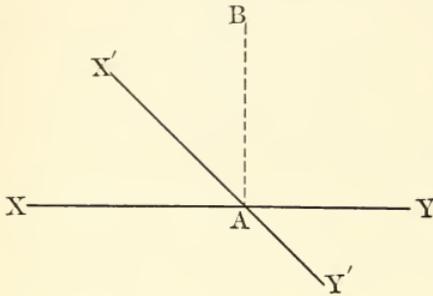
Two lines XY and $X'Y'$ meet at A. What is the value of the angles XAX' and $X'AY$ taken together?

At A, the point of intersection, erect the perpendicular AB. BAX and BAY are each right angles. $BAX + BAY = 180°$.

$$X'AY = BAX' + BAY,$$
$$XAX' = BAX - BAX'.$$

Add the two equations, member for member:

$$X'AY + XAX' = BAX' + BAY + BAX - BAX'$$
$$= BAY + BAX = 180°.$$

The value of the angles made by the meeting of two lines is upon one side of either line 180°.

The value of all the angles upon both sides of the line is $180° \times 2$, or 360°.

The arc of the diameter of a circle is 180°.

The sum of the angles about any point at the meeting of two lines is 360°.

When three or four or ten lines meet at one point, the total value of all their angles is 360°.

The angle made by a straight line is called a **straight** angle, and is 180°, or $\frac{1}{2}$ of the circumference of a circle.

When two lines meet and we know the value of one angle, how can we find the value of the other angle?

To draw a line parallel with another line

From any points, A and A', in the line XY, with a radius AB, draw arcs BB and $B'B'$.

Draw perpendiculars from A and A' and intersect the arcs at D and D'.
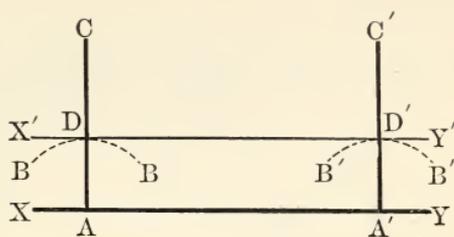
Through D and D' draw the line $X'Y'$.

The line $X'Y'$ is parallel with the line XY.

A and A' are any points in XY, and D and D' are the points in $X'Y'$ exactly perpendicular to A and A'. They are equally distant from A and A', because AD equals $A'D'$.

Parallel lines are at every point equally distant from each other by perpendicular measurement. But as A and A' are any points in XY, every point in XY must be equally distant from the point in $X'Y'$, which is perpendicularly opposite to it.

Geometric proof is based upon general constructions, true to *any*, and therefore true to *every*, case.

To draw a hexagon on a given side
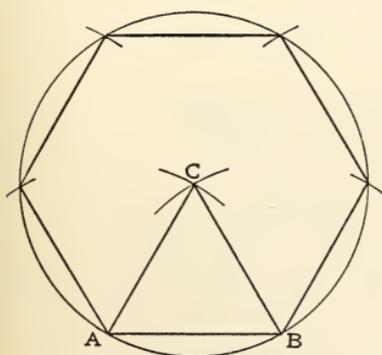
On AB, the given side, construct an equilateral triangle, ABC.

With C as center and AC as radius, describe a circle. AB is one side of a regular hexagon. Complete the hexagon.

Or, construct six equilateral triangles with C as their common apex.

To find the angular measure of the three angles of a triangle

Draw any three triangles: *ABC* with all acute angles; *DEF* with one obtuse angle; *GHI* with one right angle.
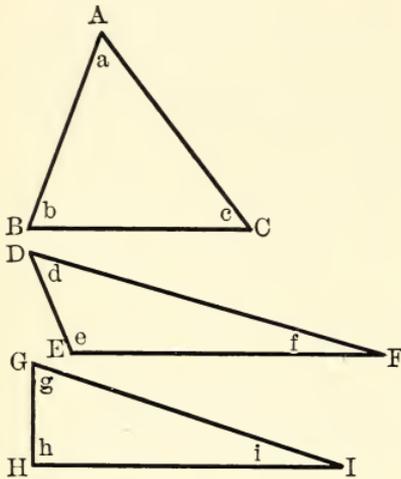
Make trials to see whether it is possible to draw any triangle with two obtuse angles; with one right angle and one obtuse angle; or with two right angles.

Two obtuse angles or one right and one obtuse angle in themselves, not counting the third angle of any triangle, are more than 180°. And two right angles equal 180°. Can the three angles of a triangle measure more than 180°?

1. Make upon paper drawings of triangles similar to those above. Cut from one of the triangles its three angles. Draw any base line upon another sheet of paper. Take in the line any point *Z*. Place the vertex of the angle *a* at *Z*: next to it the angle *c*; and last, the angle *b*. What do you notice?
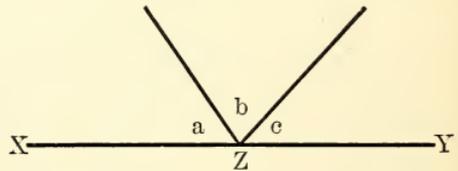
2. Make trials of the angles of the other triangles.

3. With the protractor measure the sizes of the angles of various triangles.

4. Draw various triangles with angles of given sizes.

Geometry and Industrial Art

Such geometric designs as are given here form the bases of designs for carpets, wall paper, tiling, carving, and cloth. Even floral designs in industrial art may have a geometric arrangement.
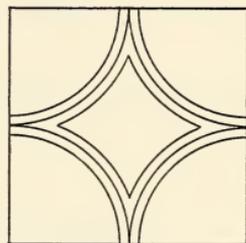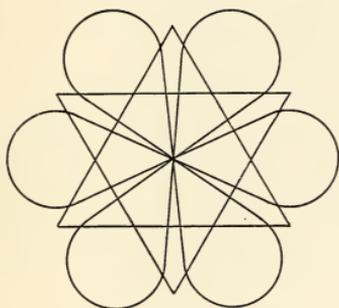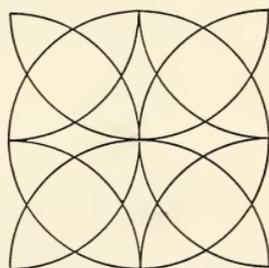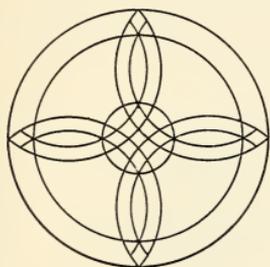
1. Find how each of these designs is formed.
2. Repeat the designs and color them.
3. For what ornamental uses would any of them be suitable?
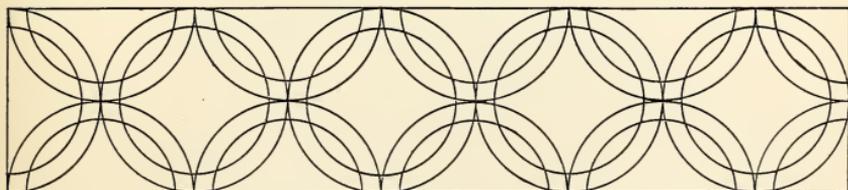
Reproduce these circles upon a larger scale.

By harmonious and symmetrical coloring, with water colors or colored crayons, these geometric designs may be made very effective.

PERCENTAGE

INTEREST

FINDING THE TIME

To find the time, *when principal, interest, and rate are given.*

1. How long must $420 be at interest at 6% to gain $32.27?

$$\$420$$

$.001 =$ Int. on $1 for 6 da. at 6%.

$6\ |\ \overline{\$.420} =$ Int. on $420 for 6 da. at 6%.

$\overline{\$.07} =$ Int. on $420 for 6 da. at 1%.

FIRST METHOD $.07 | $32.27
(BY DAYS) 30 da. | $\overline{461}$ (da.)
  12 mo. | $\overline{15}$ (mo.) $+ 11$ (da.)
  $\overline{1}$ (yr.) $+ 3$ (mo.).

The interest on $1 for 6 da. at 6% is $.001, the interest on $420 for 6 da. is $.001 × 420, or $.42, and the interest for 1 da. is $\frac{1}{6}$ of $.42, or $.07. It takes as many days to gain $32.27 as $.07 is contained times in $32.27, which is 461. 461 da. equal 1 yr. 3 mo. 11 da.

We divide the interest by the interest on the principal for one day at the rate given.

$$\$420 = \text{Prin.}$$

$$.06$$

$$\overline{\$25.20} = \text{Int. for 1 yr. at 6\%.}$$

$$\$25.20\ |\ \$32.270000$$

SECOND METHOD **1**.$2805\frac{5}{9}$ (yr.)
(BY YEARS) $\underline{12}$ mo. $= 1$ yr.
The time is **3**.$3666\frac{2}{3}$ (mo.)
1 yr. 3 mo. 11 da. $\underline{30}$ da. $= 1$ mo.
  11.000 da.

Since the principal is \$420, the interest for 1 yr. at 6%
is \$25.20. It takes as many years to gain \$32.27 as
\$25.20 is contained times in \$32.27, which is $1.280\frac{5}{9}$ yr.,
or 1 yr. 3 mo. 11 da.

**We divide the interest by the interest on the principal
for one year.**

2. How long will it take \$136.80 at 7% interest to gain
\$2.793 ?

(a) 　　　　　　\$136.80
FIRST 　　　　　　　.001 = Int. on \$1 for 6 da. at 6%.
METHOD 　　6 | \$.13680 = Int. on \$136.80 for 6 da. at 6%.
　　　　　　6 | .02280 = Int. on \$136.80 for 1 da. at 6%.
　　　　　　\$.0038　= Int. on \$136.80 for 1 da. at 1%.
　　　　　　　　7
　　　　　　\$.0266　= Int. on \$136.80 for 1 da. at 7%.

　　　　　\$.0266 | \$2.7930
　　　　　　　30 | 　105 da.
　　　　　　　　　　3 mo. 15 da.

(b) 　　　　　　　\$136.80 = Prin.
SECOND METHOD　　　　.07

　　　　　　\$9 576 = Int. for 1 yr. at 7%.

　　　　　\$9.756 | \$2.79300
　　　　　　　　　　.29$\frac{1}{6}$ (yr.)
　　　　　　　　　　 12　mo. = 1 yr.
　　　　　　　　　　3.5　(mo.)
　　　　　　　　　　 30 da.
　　　　　　　　　　 15 da.

The time is 3 mo. 15 da.

3. In what time will $357 at 6% interest gain $29.869?

THIRD METHOD (BY PROPORTIONAL PARTS)   The interest of $136.80 for 1 yr., or 360 da., at 7% is $9.576. If it takes 360 da. to gain $9.576, it will take $\frac{2793}{9576}$ of 360 da. to gain $2.793.

$$\frac{2793}{9576} \text{ of } 360 \text{ da.} = \frac{2793 \times \overset{21}{\cancel{360}} \overset{5}{} \text{ da.}}{\underset{133}{\cancel{9576}}} = 105 \text{ da.} = 3 \text{ mo. } 15 \text{ da.}$$

When a fractional part occurs in a result, the fraction may be omitted if it be less than $\frac{1}{2}$; but if it be more than $\frac{1}{2}$, 1 is added to the number of days.

In what time will

4. $427.32 gain $19.68 at 6%?

5. $186.75 gain $12.45 at 6%?

6. $378.50 gain $4.542 at 7%?

7. $56.34 gain $18.78 at 8%?

8. $873.70 gain $17.474 at 5%?

9. $594.00 gain $60.654 at 4%?

10. $13 at 6% produce $.975?

11. $600 at 6% produce $30?

12. $45.25 at 6% produce $1.81?

13. $70.50 at 9% produce $31.725?

14. $140 at 7% produce $10.8616?

15. $700 at 7% produce $85.75?

16. $848 amount to $965 at 4%?

17. $1000 amount to $2500 at $3\frac{1}{2}$%?

18. A note at $5\frac{1}{2}$% for $636.50 amounted at the time of its settlement to $1748. How long did it run?

19. In what time will $1996 produce $370.29 when the rate of interest is 8 %?

20. For how long must $2650 be at interest at 6 % to amount to $3206.50 ?

21. Mr. Ball borrowed $984 and kept it until it had gained $288.64 at 8 %. How long did he keep the money?

22. At what time will $840, put at interest at 6 % on the first of January, 1901, amount to $915.60 ?

23. A man borrowed $480 at 3 % interest and kept it until the interest amounted to $74.28. How long did he keep the money?

24. How long will it take $475, put at 6 % interest, to gain $4.25 ?

25. A man loaned $560 and was repaid with $600.40. How long had the money been kept, the rate being 6 %?

Finding the Principal

To find the principal, *when amount, rate, and time are given.*

1. What principal at 6 % will amount to $884.125 in 1 yr. 2 mo. 10 da. ?

$\$ \ .06 \ = $ Int. on $1 for a year.

$.01 \ = $ Int. on $1 for 2 mo.

$.001\tfrac{2}{3} = $ Int. on $1 for 10 da.

$\$ \ .071\tfrac{2}{3} = $ Int. on $1 for given time at given rate.

$\underline{1.00}$

$\$1.071\tfrac{2}{3} = $ Amt. of $1 for given time at given rate.

$$\begin{array}{r|r} \$1.071\tfrac{2}{3} & \$884.125 \\ \times 3 & \times 3 \\ \hline \$3.215 & \$2652.375 \end{array}$$

$$\$825 = \text{Prin.}$$

The amount of $1 for 1 yr. 2 mo. 10 da. = 1.07\frac{1}{6}$, and if one dollar is required to gain 1.07\frac{1}{6}$, as many dollars will be required to gain $884.125 as there are times 1.07\frac{1}{6}$ in $884.125.

$$\$884.125 \div 1.07\tfrac{1}{6} = \$825.$$

We divide the principal by the amount of $1 for the time at the given rate.

2. What principal at interest at 7% will amount to $703.551 in 4 mo. 27 da. ?

$$
\begin{array}{ll}
\$.02 & = \text{Int. on } \$1 \text{ for } 4 \text{ mo. at } 6\%. \\
.0045 & = \text{Int. on } \$1 \text{ for } 27 \text{ da. at } 6\%. \\
\overline{6).0245} & = \text{Int. on } \$1 \text{ for } 4 \text{ mo. at } 6\%. \\
.0040\tfrac{5}{6} & = \text{Int. on } \$1 \text{ for } 4 \text{ mo. at } 1\%. \\
\underline{7} & \\
.0285\tfrac{5}{6} & = \text{Int. on } \$1 \text{ for } 4 \text{ mo. at } 7\%. \\
\underline{1.00} & \\
\$1.02856 & = \text{Amt. of } \$1 \text{ for given time at the} \\
& \qquad \text{given rate.}
\end{array}
$$

$$
\begin{array}{r|l}
\$1.0285\tfrac{5}{6} & \$703.5510 = \text{Amt.} \\
\times 6 & \quad\ \times 6 \\
6.1715 & 4221.3060 \\
& \$684 = \text{Prin.}
\end{array}
$$

The amount of $1 at 7% for 4 mo. 27 da. = 1.02\frac{103}{120}$, and if one dollar is required to gain 1.02\frac{103}{120}$, as many dollars would be required to gain $703.551 as there are times 1.02\frac{103}{120}$ in $703.551 = $684.

What principal will amount to

    3. $569.296 in 8 mo. 16 da. at 5% ?

    4. $573.16 in 2 yr. 9 mo. 10 da. at 6% ?

    5. $378.82 in 1 yr. 4 mo. 20 da. at 6% ?

   6. $57.72 in 8 mo. 29 da. at 10%?

   7. $899.944 in 5 mo. 14 da. at 6%?

   8. $6400 in 4 yr. 8 mo. at 6%?

   9. $4200 in 5 yr. 8$\frac{4}{7}$ mo. at 7%?

   10. $7600 in 8 yr. 8 mo. at 6%?

   11. $320 in 3 yr. 4 mo. at 6%?

   12. $1840.20 in 2 yr. 2 mo. 24 da. at 8%?

   13. A grocer bought goods and gave in payment an interest-bearing note. The note in 1 yr. 6 mo. and 27 da. amounted to $2286 at 4%. What was the face of the note?

   14. A borrowed from B a sum of money, which he kept for 2 yr. 4 mo. 15 da. At the end of that time he returned to B $342.75. The rate of interest was 6%. What was the amount B loaned A?

   15. What sum loaned at 7% interest will amount to $1996.25 in 1 yr. 6 mo. 21 da.?

FINDING THE RATE

To find the rate, *when principal, interest, and time are given.*

   1. At what rate per cent will $648 gain $81.873 in 2 yr. 3 mo. 17 da.?

$$
\begin{array}{rl}
\$648 & = \text{Prin.} \\
.01 & \\
\hline
\$6.48 & = \text{Int. for 1 yr. at 1\%.} \\
2 & \\
\hline
\$12.96 & = \text{Int. for 2 yr. at 1\%.} \\
1.62 & = \text{Int. for 3 mo. at 1\%.} \\
.306 & = \text{Int. for 17 da. at 1\%.} \\
\hline
\$14.886 & = \text{Int. for given time at 1\%.}
\end{array}
$$

Or, the interest of $648 for 2 yr. 3 mo. 17 da. at 1% is $14.886.

$$\$81.873 \div \$14.886 = 5\tfrac{1}{2}.$$

Therefore, the rate is $5\tfrac{1}{2}$ %.

We divide the interest by the interest on the principal at 1% for the time.

	Prin.	Int.	Time	Rate
2.	$325	$3.25	2 mo.	?
3.	$40	$13.36	2 yr. 9 mo. 12 da.	?
4.	$125	$32.375	3 yr. 6 mo.	?
5.	$124	$29.194	4 yr. 3 mo. 10 da.	?
6.	$36	$8.034	3 yr. 8 mo. 19 da.	?

Find the rate of interest when

7. $57.25 gains $3.038 in 1 yr. 5 mo. 18 da.

8. $855 gains $46.55 in 2 yr. 2 mo. 4 da.

9. $64.80 gains $6.246 in 11 mo. 17 da.

10. At what rate will $240 gain $29.40 in 1 yr. 9 mo.?

11. Mr. Smith borrowed $475 for 3 yr. 4 mo. At the end of that time he paid $95 interest. What was the rate of interest?

12. A man bought a bill of goods for $750. At the end of 2 yr. 10 mo. he paid the principal and $63.75 interest. What was the rate of interest?

13. What is the rate of interest, when the principal $960 will in 4 yr. 6 mo. produce $302.40 interest?

14. John borrowed from a friend $434.70. He kept it 3 yr. 4 mo. 18 da., or until the interest had amounted to $88.24. What was the rate of interest?

Finding the Principal

To find the principal, *when interest, rate, and time are given.*

1. What principal at 8% will gain $26.18 in 1 yr. 4 mo. 15 da. ?

$.08 = Int. on $1 for 1 yr. at 8%.

$.026\frac{2}{3}$ = Int. on $1 for 4 mo. at 8%.

$.003\frac{1}{3}$ = Int. on $1 for 15 da. at 8%.

$.11 Int. on $1 for given time and rate.

$.11 | $26.18 = Int. on whole amt.

$238 = Prin.

The interest of $1 for 1 yr. 4 mo. 15 da. at 8% is $.11. The principal producing $26.18 must be as many times $1 as $.11 is contained times in $26.18, or $238.

We divide the interest by the interest on $1 for the given time at the given rate.

What principal will gain

2. $8.73 in 5 mo. at 6% ?

3. $4.77 in 1 yr. 5 mo. 20 da. at 6% ?

4. $4.27 in 2 yr. 6 mo. at 5% ?

5. $116.127 in 4 yr. 4 mo. 4 da. at $7\frac{1}{2}$% ?

6. $97.72 in 6 yr. 8 mo. at 6% ?

7. $99.50 in 1 yr. 1 mo. 6 da. at 7% ?

8. $100.64 in 11 mo. 24 da. at 9% ?

9. A borrowed from B a sum of money which he kept for 5 mo. 12 da. At the end of that time the interest amounted to $91.60 at 6%. How much did A borrow ?

PROPORTION

Partnership

A **partnership** is a business association of two or more persons under an agreement to share profits and losses.

The **partners** are the individuals who associate themselves together in business.

Capital is wealth or money invested in business.

The **resources** and **assets** of a firm are the money and property it owns.

The **liabilities** of a firm are the *debts* owed by the firm.

The profit or loss made by a partnership or firm is shared among its members in proportion to the amount of capital each man has invested in the business.

In our times partnership-investments in business are not as common as corporation-investments.

Any one may buy shares in almost any corporation.

1. A, B, and C trade in partnership. A invests $250, B $750, and C $500. At the end of 6 mo. they find that they have gained $472.50. What is each man's share of the gain?

Since A's stock = $250, B's = $750, and C's = $500, the entire stock equals $250 + $750 + $500 = $1500, of which A's stock = $\frac{250}{1500} = \frac{1}{6}$; B's = $\frac{750}{1500} = \frac{1}{2}$; C's = $\frac{500}{1500} = \frac{1}{3}$. A should have $\frac{1}{6}$, B should have $\frac{1}{2}$, and C should have $\frac{1}{3}$ of the gain. $\frac{1}{6}$ of $472.50 = $78.75 = A's share; $\frac{1}{2}$ of $472.50 = $239.25 = B's share; $\frac{1}{3}$ of $472.50 = $157.50 = C's share.

2. A and B, after trading in company for 2 yr., find that their profits are $2400. If A's capital was $\frac{2}{3}$ of B's capital, how many dollars of the profit ought each man to have?

Suggestion. — Let $3x$ = B's capital.

3. A, B, and C traded in company. A put in $\frac{1}{3}$ of the stock, B put in $\frac{1}{2}$ of it, and C put in the rest. On dividing the gain, they found that C's share of it was $321. What was the gain of each of the other partners?

4. A, B, C, and D enter into business together. A puts in $2500, B $3500, C $5600, and D $6400. By the end of the first year they lost $720. What was each man's share of the loss?

5. A, B, and C were engaged together in business with a capital of $75,000, of which A furnished $35,000 and B $40,000. During the year their profits were $4500. What was the share of each?

6. Three men, X, Y, and Z, formed a partnership. X put into the business $20,000, Y put in twice as much as X, and Z 65 % of what Y put in. They gained during the year $8600. What was each partner's share?

7. Messrs. Black, White, and Brown, in business together, gain in one year $7500. Since Black put into the business $26,000, and White half as much as Black, and since the entire capital was $60,000, what was each man's share of the profit?

8. Three men hire a pasture for $360. A puts in 6 horses, B 7, and C 11. What amount should each pay?

9. A merchant, failing in business, finds that he owes A $1500, B $2000, and C $1000. He has but $900, with which to pay. How should he divide the money?

10. L, M, and N rented, for $650, a summer home, agreeing to pay rent in proportion to their families and length of stay. L's family of five staid 6 wk., M's of three staid 7 wk., and N's of seven staid 4 wk. How much was the share of each?

11. A partnership was formed with a joint capital of $50,000. The gain during a single year was $5000, of which A's share was $1500, B's $2000, and C's the remainder. What amount did each put into the business?

12. A man dying, left as he supposed $15,000 to his wife, $8000 to each of his two sons, and $4000 to his daughter. After all his debts had been paid, it was found that only $7000 remained. What amount should be given to each?

13. A, B, and C loaded a vessel with freight of which A owned 75 tons, B 45 tons, and C 53 tons. During a storm 58 tons were destroyed. How should the loss be divided?

14. Four men, working together, sold in all $56,000 worth of goods at a total gain of $8000. A contributed $18,000; B $16,000; C $9000; and D the balance. What was the share of each in the gain?

BILLS

Newark, N. J., March 11, 1902.

James O. Barnes

Bought of Swan, Brown, & Co.

Terms: 2% cash 10 days; 60 days dating; Net, 30 days.

150	yd. Percaline	@ 9	13	50		
75	" Cambric	@ 4	3	00		
65	" Silesia	@ $12\frac{1}{2}$	8	13		
					24	63

This bill can be settled as follows:

In order to take full cash discount this bill must be paid March 21, less 2% for anticipation of the dating, 60 days at 6% a year, a further discount of 1%.

If paid April 21, discounts of 2% for anticipation of 30 days, and ½ of 1% extra can be taken.

If paid May 21, a discount of 2% only can be taken, as the entire dating will have expired.

If paid June 21, the bill should be paid net or without discount.

After June 21 the bill should be paid with interest added at the rate of 6% per annum for the time passed after the maturity of the bill.

New York, March 1, 1902.

C. L. Jameson, Newark, N. J.

Bought of Smith, Lord & Co.

Terms: 2% 10 days as May 1.

6	Bbl. XXX Pastry Flour @ 4.25	25	50
3	" XXX ◈ " @ 5.00	15	00
4	" American Beauty " @ 4.50	18	00
		58	50
	Freight	1	28
		57	22
	Shipped via P.R.R. F.O.B. Newark.		

Note.— A bill dated March 1 as May 1 means that it carries a 60-day dating. The above bill could be paid May 11, less 2%.

"F.O.B. Newark" means that the charges on the invoice for transportation can be deducted from the amount of the bill, which should be taken advantage of before the discount is computed.

NEWARK, N. J., March 15, 1902.

OSBORNE, WILLIAMS MFG. CO., Buffalo, N.Y.

Bought of THE ESSEX WEAVING CO.

TERMS: 8%; 10 = 60 days dating. Via P.R.R.

6	#7 Oriental Rugs, $\frac{5}{7}$, 15.00		90	00
4	#3 " " $\frac{7}{9}$, 14.25		57	00
3	s/462 Fringe " $\frac{6}{8}$, 6.50		19	50
			166	50
	Less 10, 10, and 5, trade discount		38	38
			128	12
	F. O. B. Newark.			

NOTE. — "F.O.B." means merchandise of this bill delivered "*Free on Board*" cars at Newark, and allows of no deduction for transportation charges to the point of destination.

PHILADELPHIA, March 10, 1902.

JOHN L. RICHARDS & CO.,

East Boston, Mass.

Bought of THOMPSON & CO.

TERMS: 90 days dating; 6% for cash, 10 days.

Via P. R.R. to New York and Fall River Line.

4	#261 Metal Lamps	@ 3.00	12	00
6	#62 " "	@ 4.00	24	00
2	Doz. #145 Metal Lamps	@ 18.00	36	00
			72	00
	Less 5. 5% Trade Discount.		7	02
	F. O. B. Philadelphia.		64	98

INVENTIONAL GEOMETRY

Equilateral triangles inscribed within a circle.

A circle inscribed within the triangles.

A square inscribed within the circle.

Find a hexagon.

Find six small equilateral triangles.

Find trapezoids.
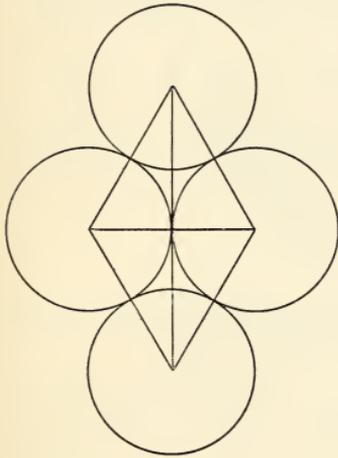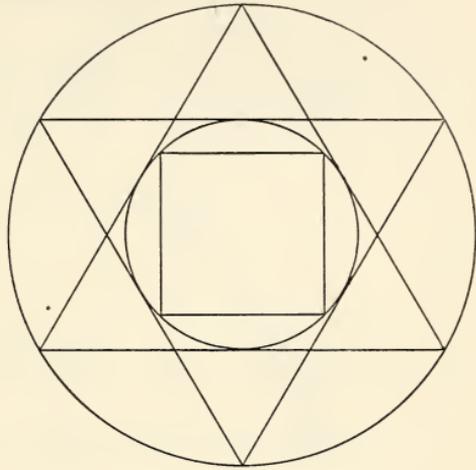
Two circles in contact.

One circle above, and another below, in contact with each of the two circles.

Their centers connected.

Find four right angle triangles.

Find four isosceles triangles.

Find a rhombus.

Find arcs, quadrants, circles, angles, perpendiculars, parallel lines, rectangles, squares, triangles, rhomboids, diagonals, diameters, tangents, sectors.
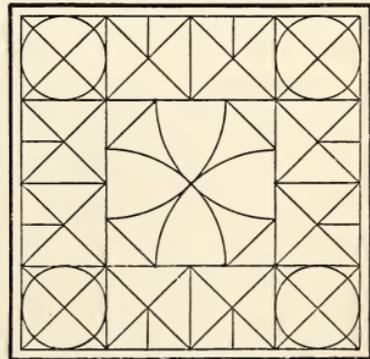
To inscribe a hexagon within a circle
To draw six equilateral triangles about a point

Draw any diameter XY.

From X as a center, with a radius equal to the radius of the circle, describe the arcs X' and X''. From Y
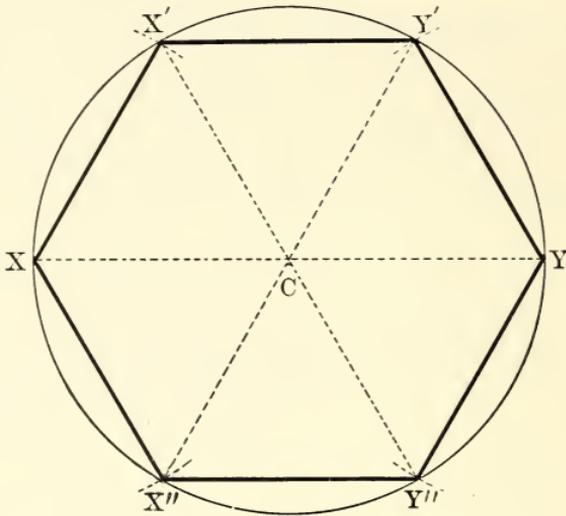
describe the arcs Y' and Y''. Draw the chords of the arcs XX', $X'Y'$, $Y'Y$, YY'', $Y''X''$, $X''X$.

The figure $XX'Y'YY''X''X$ is an inscribed hexagon.

Draw the lines CX', CY', CY'', CX''. With the diameter and the chords of the polygon these lines form six equilateral triangles.

By bisecting each chord, XX', $X'Y'$, etc., we may inscribe a twelve-sided polygon.

By drawing chords XY', $Y'Y''$, XY'', we may inscribe an equilateral triangle.

The straight sides of the hexagon make the circumference of the circle look irregular. This is an optical illusion similar to that on the opposite page.

To inscribe a square within a circle

To circumscribe a square about a circle

Draw diameters at right angles to each other. Draw chords connecting the ends of the diameters. *ABCD* is a square inscribed within the circle. Through *A* draw *EF* at an angle of 40° with *AB*. In the same angle with the corresponding sides of the inscribed square draw *FG*, *GH*, *HE*.
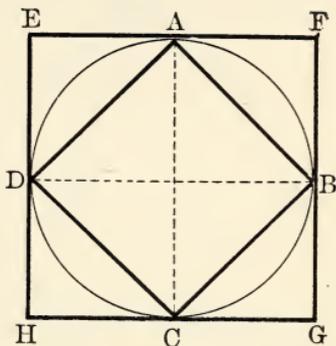
EFGH is a square circumscribed about the circle.

Do you notice that the circle does not look round? This is due to the eye, which is deceived by the nearness of the straight to the curved lines, and is called an "optical illusion." The circumscribed circle may be drawn with sides parallel with the sides of the inscribed circle.

THE ELLIPSE

The ellipse is a figure in which every point *X* in the circumference is distant from two points, called its foci, in such a way that the sum of *AX* and *BX* is always the same. It may be drawn by setting pins at *AB*, connecting them by a loose string, with a pencil stretching the string tight, and tracing the curve as far away from *A* and *B* as the string permits.
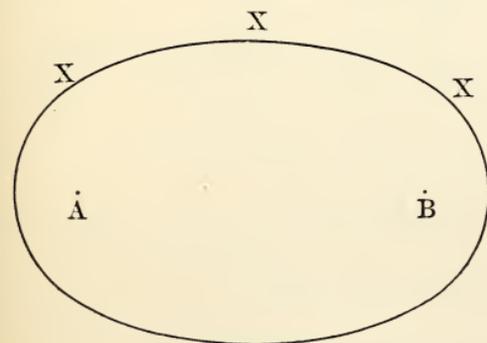
The earth and all the other planets of our solar system revolve around the central sun in ellipses. Find out some facts about the signs of the elliptical orbits of the Earth, Mars, Venus, Saturn, and Jupiter.

To construct an hexagonal pyramid
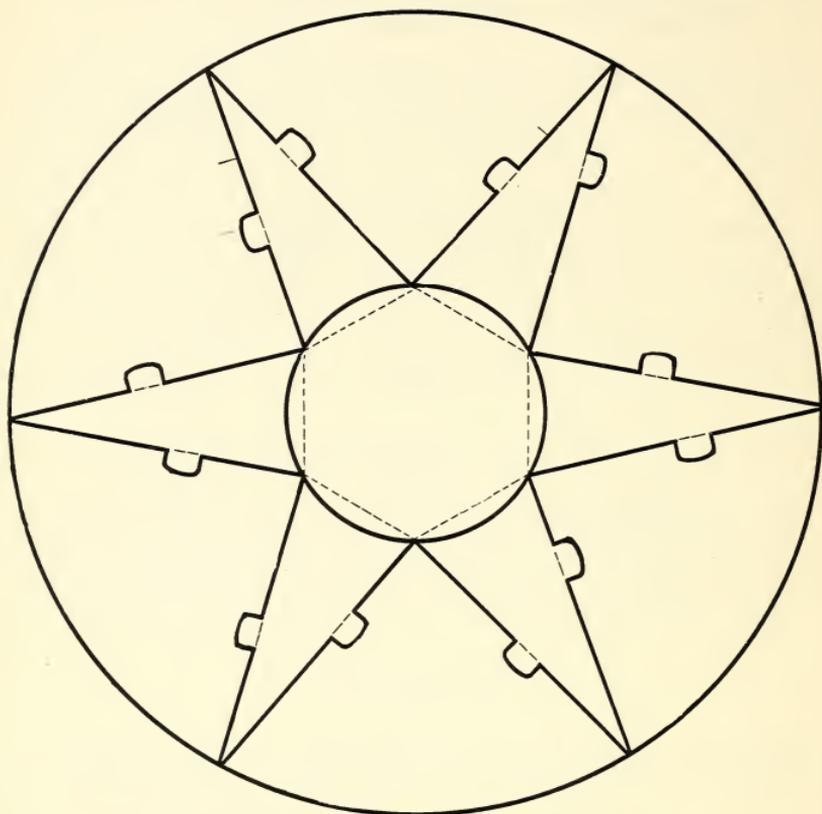
The two circumferences are merely construction lines.

The chord of an arc of 60° equals the radius of the circle.

Fold at the dotted lines.

Measure the areas of the surfaces.

Measure the volume.

How many isosceles triangles are there?

Make two pyramids,

$$\text{Scales}: \begin{cases} \frac{1}{4} \text{ in. to } 1 \text{ in.} \\ \frac{1}{8} \text{ in. to } 1 \text{ in.} \end{cases}$$

To trisect a line

From one end X of the line XY draw any line XZ that makes an acute angle with it. Upon XZ measure three equal lengths, XA, XA', XA''. From A'' draw to Y, the other end of XY, a straight line AZ. Draw from A' the line $A'B'$ parallel with $A''Y$, and from A the line AB also parallel with $A''Y$.
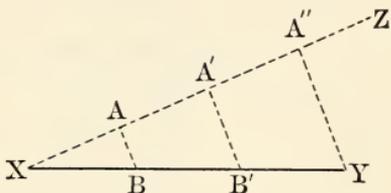
The distances in XY, XB, BB', and $B'Y$ are equal. The line XY is trisected by the parallels.

By the same method a line may be divided into four, five, or any other number of equal parts.

To trisect an angle of 90°

From A, the vertex of the right angle, draw with any radius the arc BC, cutting the sides of the angle at B and at C. From B, with the same radius, describe an arc $B'B''$, cutting the arc BC at A'. Describe from C the arc $C'C''$, cutting BC at A''. The lines AA' and AA'' trisect the arc BC and the angle BAC.

Find by the compasses whether the lines AA' and AA'' trisect also the chord BC.

A line trisecting an arc trisects also the chord and the angle.

Without reference to these pages let the members of the class develop on the blackboard by way of review all the principles of inventional geometry that they know.

National Banks

The six National Banks of New York City with the largest deposits July 15, 1901, were as follows, viz.:

Name	Deposits	Capital	Surplus	Annual Dividends
City	$127,623,300	$10,000,000	$6,170,700	6%
Commerce	73,050,800	10,000,000	6,758,700	8%
Park	66,422,900	2,000,000	3,853,600	15%
First	59,875,185	10,500,000	11,102,746	10%
Hanover	61,131,900	3,000,000	5,434,900	10%
Chase	52,332,800	1,000,000	2,513,400	12%

1. What was the total of their deposits? of their capital and surplus taken together?

Four Banks presented the following facts, viz.:

Name	Deposits	Capital	Surplus	Annual Dividends
Chemical	$26,383,800	$300,000	$6,516,200	150%
Fifth Avenue	11,107,000	100,000	1,362,300	100%
New Amsterdam	7,857,400	250,000	558,300	60%
New York County	4,496,400	200,000	597,400	50%

2. What was the ratio of the capital to the deposits of each of these ten Banks?

3. What was the ratio of the surplus to the capital of each of the ten Banks?

4. The market values of shares of the stock of the Banks, par value $100, were as follows, viz.:

Chemical	. . .	$4000	Chase	. . . .	$700
City	. . . . .	675	New York County		1500
Hanover	. . .	700	Park	. . . . .	575
Fifth Avenue	. .	3250	First	. . . . .	750
Commerce	. . .	375	New Amsterdam	.	1400

Compare these values with the dividends.

COMPOUND INTEREST

Compound interest is interest on both principal and overdue, unpaid, or accrued interest. The simple interest may be added to the principal annually, semi-annually, quarterly, monthly, or otherwise, as agreed or as established by law.

Savings Banks and Building and Loan Associations pay compound interest.

Payment of compound interest on ordinary debts in most States cannot be legally enforced.

1. What is the compound interest of $700 for 2 yr. at 4% half yearly?

Since the interest is payable half-yearly, there are four payments.

Interest of $700 at 4% = $28,

$700 + 28 = $728 = principal for second half year.

Interest of $728 at 4% = $29.12,

$728 + 29.12 = $757.12 = principal for third half year.

Interest of $757.12 at 4% = $30.285,

$757.12 + $30.285 = $787.405 = principal, fourth half year.

Interest of $787.405 at 4% = $31.496,

$787.405 + $31.496 = $818.90 = amount at end of fourth half year.

$818.90, amount.

$700.00, principal.

Difference, $118.90, the compound interest.

We find the interest upon the principal for the first period. The amount becomes the new principal upon which interest for the second period, if any, is found. From the last amount we subtract the original principal. The difference is the compound interest for the term.

What is the compound interest of :

2. $1000 for 3 yr. at 5% per annum?

3. $800 for 4 yr. at 6% per annum?
4. $900 for 5 yr. at 6% per annum?
5. $600 for 2 yr. at 3% half yearly?
6. $250 for 2 yr. at $2\frac{1}{2}$% half yearly?
7. $880 for $1\frac{1}{2}$ yr. at $1\frac{1}{4}$% quarterly?
8. $500 for 3 yr. at $5\frac{1}{2}$% per annum?
9. $400 for 2 yr. at $2\frac{1}{2}$% half yearly?
10. $714.90 for 2 yr. at $1\frac{1}{2}$% quarterly?
11. $794.60 for $1\frac{1}{2}$ yr. at $1\frac{3}{4}$% half yearly?

TABLE

Showing the amount of $1 at interest compounded annually from 1 year to 20 years.

Year	3 per cent	4 per cent	5 per cent	6 per cent
1	1.030000	1.040000	1.050000	1.060000
2	1.060900	1.081600	1.102500	1.123600
3	1.092727	1.124864	1.157625	1.191016
4	1.125508	1.169858	1.215506	1.262476
5	1.159274	1.216652	1.276281	1.338225
6	1.194052	1.265319	1.340095	1.418519
7	1.229873	1.315931	1.407100	1.503630
8	1.266770	1.368569	1.477455	1.593848
9	1.304773	1.423311	1.551328	1.689478
10	1.343916	1.480244	1.628894	1.790847
11	1.384233	1.539454	1.710339	1.898298
12	1.425760	1.601032	1.795856	2.012130
13	1.468533	1.665073	1.885649	2.132928
14	1.512589	1.731676	1.979931	2.260903
15	1.557967	1.800943	2.078928	2.396558
16	1.604706	1.872981	2.182874	2.540351
17	1.652847	1.947900	2.292018	2.692772
18	1.702433	2.025816	2.406619	2.854339
19	1.753500	2.106849	2.526950	3.025599
20	1.806111	2.191123	2.653297	3.207135

1. What are the amount and the compound interest of $3400 at 5% for 15 yr.?

By the **table** the amount of $1 at 5% for 15 yr. = $2.07893.

$$\$2.07893 \times 3400 = \$7068.362 = \text{amount.}$$
$$\underline{3400.000} = \text{principal.}$$
$$\overline{\$3668.362} = \text{interest.}$$

Find the compound interest of :

2. $875 for 11 yr. at 6%.

3. $643.98 for 13 yr. at 4% half yearly.

4. 1¢ at 6% per annum for 45 yr.

5. $78.20 for 7 yr. at 3% quarterly.

6. $777.77 for 9 yr. at 5% half yearly.

7. $75,439.75 for 4 yr. at $4\frac{1}{2}$%.

8. $540.50 at 6% for 3 yr. 6 mo. 15 da.

9. $200 for 1 yr. 7 mo. 9 da. at 5%.

10. $761.75 for 4 yr. at 6%.

11. $76.18 for 2 yr. 8 mo. 9 da. at 5%.

12. $884 for 7 yr. at 4%.

13. $721 for 9 yr. at 5%.

14. $1,000,000 for 10 yr. at 10%, payable annually.

15. $1800 for 14 yr. at 8%, payable semi-annually.

16. $9401.50 for 19 yr. 4 mo. at 9%, payable semi-annually.

17. $1176.80 for 10 yr. 10 mo. 10 da. at 10%, payable quarterly.

18. $8025 for 18 yr. 7 mo. 9 da. at 8%, payable semi-annually.

PARTIAL PAYMENTS

A **partial payment** is the payment in part of the amount due on a note or other written obligation bearing interest.

UNITED STATES RULE

Find the interest on the principal to the time of the first payment; when the payment equals or exceeds this interest, add the interest to the principal, and subtract the payment from the sum; the remainder forms a new principal.

When the payment is less than this interest, compute the interest on the principal to the time of that payment which with all preceding payments exceeds the interest due on the principal to the time of that payment; then add the interest to the principal, and subtract the sum of the payments; the remainder forms a new principal, with which proceed as before.

This method is in use chiefly for notes running for periods longer than one year.

1. $750.00.   CINCINNATI, April 7, 1896.

For value received, I promise to pay to the order of Benjamin Thompson, seven hundred and fifty dollars, on demand, with interest.

                              ROBERT T. SMITH.

On this note are the following indorsements:

    Jan. 17, 1897. Received $100.
    Mar. 13, 1898. Received $25.
    Feb. 19, 1899. Received $30.
    Aug. 3, 1899. Received $200.

What was due on the note at the time of settlement, Aug. 14, 1900?

$750.00 = 1st principal, April 7, 1896.
  35.00 = Int. 280 da. at 6 %.
$785.00 = Amt. due Jan. 17, 1897.
 100.00 = 1st payment.
$685.00 = Balance due Jan. 17, 1897.
  68.50 = Int. 20 mo. ⎫
  34.25 = Int. 10 mo. ⎪ 2 yr. 6 mo. 17 da.
   1.713 = Int. 15 da. ⎬
    .228 = Int. 2 da. ⎭
$789.69 = Amt. due Aug. 3, 1899.
 255.00 = 2d, 3d, and 4th payments.
$534.69 = Bal. due Aug. 3, 1899.
  32.08 = Int. 1 yr.
   .98 = Int. 11 da.
$567.75 = Amount due Aug. 14, 1900.

Since the first payment was greater than the interest at the time due, we find the amount of the note to that time, and deduct from it the payment. The remainder is a new principal. The second payment, $25, is evidently less than the interest then due; for the time is over one year, while the principal, between $600 and $700, gives more than $36 interest per year. The interest to the time of the third payment is greater than the second and third payments together. But the fourth payment, together with the second and third, is very evidently more than sufficient to pay the interest then due; therefore we find the amount of the new principal to the time of the fourth payment, and subtract from it the sum of the second, third, and fourth payments, thus getting our third principal. This is on interest till the time of settlement; and hence its amount is the sum due.

2. $850.00.   DES MOINES, April 7, 1896.

For value received, I promise to pay to the order of Winfield Abbott, eight hundred fifty dollars, on demand, with interest.

                                        WALTER TAYLOR.

On this note were the following indorsements:

Jan. 7, 1898.	Received $83.
May 1, 1899.	Received $20.
Aug. 28, 1899.	Received $300.

How much was due on the note Oct. 13, 1900? (6% interest.)

3. $1000.00.   BALTIMORE, Nov. 28, 1896.

I promise to pay to the order of Henry King, one thousand dollars, on demand, with interest.   Value received.

                                        CHARLES COLEMAN.

On this note were the following indorsements:

July 23, 1897.	Received $80.
June 27, 1898.	Received $20.
April 2, 1899.	Received $25.
Dec. 20, 1899.	Received $500.

How much was due Aug. 14, 1900?   (6% interest.)

4. $1275.00.   DENVER, Sept. 29, 1895.

For value received, we promise to pay Green & Co., twelve hundred and seventy-five dollars, on demand, with interest at six per cent.   HENRY CHASE & CO.

Is this note negotiable or non-negotiable?   See Fifth Book.

Indorsements : Sept. 22, 1897.   Received $400.
              May 25, 1898.   Received $200.
              Nov. 13, 1900.   Received $150.

What is the balance due March 1, 1901 ?

5. $3000.00.   DETROIT, April 3, 1897.

For value received, I promise to pay to the order of John
Perry, three thousand dollars, on demand, with interest
after four months at 6 per cent.   JAMES BEMIS.

Indorsements : Nov. 1, 1897.   Received $500.
              Mar. 25, 1899.   Received $50.
              July 18, 1899.   Received $600.
              Jan. 1, 1900.   Received $1000.

The note was settled Nov. 8, 1900.   How much was due ?

6. $3469.32.   WHEELING, Feb. 6, 1893.

For value received, I promise to pay to the order of
Henry Jones, three thousand four hundred sixty-nine and
$\frac{32}{100}$ dollars, with interest at 6 per cent.

                         ROBERT HOUGHTON.

On this note were indorsed the following payments :

        May 16, 1896.   Received $545.76.
        May 16, 1898.   Received $1276.00.
        Feb. 1, 1899.   Received $2074.72.

What remained due Aug. 11, 1900 ?

7. A 6% note of $635.84, dated Sept. 5, 1896, had
indorsed on it the following payments :

Nov. 13, 1898, $416.08 ;
May 10, 1899, $152.00.   What was due March 1, 1900 ?

8. $350. PHILADELPHIA, May 1, 1896.

For value received, I promise to pay to the order of William Payne, three hundred fifty dollars, with interest at 6 per cent. PETER M. MARTIN.

Dec. 25, 1896, there was indorsed $50; Aug. 22, 1898, $15; June 4, 1899, $100.

How much was due April 5, 1900?

9. 108\frac{43}{100}$. MILWAUKEE, Dec. 9, 1895.

For value received, I promise to pay to the order of Louis Sanders, one hundred eight dollars and forty-three cents, on demand, with interest at 7 per cent.

E. S. BROOKS.

March 3, 1896, there was indorsed $50.04; Dec. 10, 1896, $13.19; May 1, 1898, $50.11.

How much remained due Oct. 9, 1900?

THE MERCHANTS' RULE

Find the amount of the principal for the entire time.

Find the amount of each payment from the time that it was made to the time of settlement.

From the first amount subtract the sum of the amounts of the several payments.

This method is used in business transactions when partial payments are made on notes paid in full within one year of their date.

1. $500.00. PORTLAND, July 8, 1899.

Ten months after date, I promise to pay to the order of Thomas P. Noyes, five hundred dollars with interest. Value received. WILLIAM WEST.

Indorsements: Sept. 23, 1899. Received $60.
Nov. 20, 1899. Received $100.
Jan. 17, 1899. Received $50.

How much was due May 11, 1900? (6% interest.)

```
$500.00  = principal.
  25.25  = int. 10 mo. 3 da.
$525 25  = amt. of note to May 11, 1900.
  60.00  = 1st payment, Sept. 23, 1899.
   2.28  = int. 7 mo. 18 da.
 100.00  = 2d payment, Nov. 20, 1899.
   2.85  = int. 5 mo. 21 da.
 200.00  = 3d payment, Jan. 17, 1900.
   3.80  = int. 3 mo. 24 da.
  50.00  = 4th payment, Feb. 8, 1900.
    .775 = int. 3 mo. 3 da.
$419.705 = amt. of payments, May 11, 1900.
$525.25  = amt. of note May 11, 1900.
$105.55  = bal. due May 11, 1900.
```

2. $728.00.   MINNEAPOLIS, Sept. 7, 1899.

On demand, I promise to pay to the order of James Robinson, seven hundred twenty-eight dollars with interest. Value received.   ALBERT CLARK.

Indorsements: Oct. 3, 1899.   Received $80.
              Dec. 1, 1899.   Received $190.
              Feb. 4, 1900.   Received $100.

How much was due Aug. 2, 1900?

3. 1525\frac{25}{100}$.   ATLANTA, Nov. 9, 1899.

On demand, we jointly and severally promise to pay to the order of James E. Benton, one thousand five hundred twenty-five and $\frac{25}{100}$ dollars, with interest at five per cent. Value received.   CHARLES T. WYMAN.
                                   N. P. TYLER.

Indorsements: Feb. 23, 1900.   Received $375.
              April 6, 1900.   Received $250.
              June 17, 1900.   Received $425.

How much was due at settlement, Oct. 20, 1900?

Review of Stocks and Bonds

1. On Aug. 10, 1901, U.S. Government Bonds paying 4% a year, and payable in 1925, were worth about $113. At that price, upon a par value of $100, what interest would they return to an investor? How many years had the bonds to run?

2. On the same day the $100 bonds, paying 4% a year, of the Chicago, Burlington & Quincy R.R. were worth $97½. At that price what interest would they earn?

3. What was the cost of 875 $100 bonds, paying 6%, of the International Paper Company, at 111%? What interest would they return?

4. Find the cost of the following stocks at these rates:

Shares	Name	Market Rate	Shares	Name	Market Rate
88	Canadian Pacific R.R.	107¾	246	Republic Steel Co.	19¼
760	Erie R.R.	36⅛	100	Tennessee Iron Co.	59⅝
467	General Electric Co.	249¼	1000	Union Pacific R.R.	94⅜
355	Illinois Central R.R.	143	3695	U.S. Steel Corporation	41½
279	Metropolitan Street Ry.	164¼	33	W. U. Telegraph	91¾
1225	Missouri Pacific R.R.	99¼			
58	Pacific Mail Co.	38			
3000	Pennsylvania R.R.	142¾			
595	People's Gas Co.	108⅜			

These were prices on Aug. 10, 1901, on the New York City Stock Exchange. Find out what you can regarding the Exchange.

5. The gross earnings of the Louisville & Nashville R.R. for the fiscal year ending June 30, 1901, were $28,022,207. The operating expenses were $18,233,034. What were the net earnings? When bonds and stocks averaged 5% return, what was the value of the railroad?

6. The highest quotation of the Delaware, Lackawanna, & Western R.R. for the week ending Aug. 10, 1901, was $227½, the lowest, $220. What was the difference in market value of 38 shares at the two quotations?

Large Transactions

Most of the transactions of the business of the world are retail, involving small amounts. But there are many great transactions, now that the world, and especially our country, has grown so rich.

1. 274 persons bought 14,325,000 bu. of corn at an average price of 46¢ per bushel, and sold it at a net profit of $1,146,000. For what average price per bushel did they sell the corn? What was its cost to them? What was the average profit per bushel?

2. The United States Steel Corporation issued $500,-000,000 preferred stock, and $350,000,000 of bonds. When it pays 8% dividend on the preferred stock, and 5% interest on the bonds, what amount is left for dividends on the common stock out of net earnings of $83,000,000? What rate per cent would such a dividend be?

3. Our national government pays pensions amounting to $144,000,000 a year. What is the average cost per capita in a population of 76,300,000 people?

4. The Boston Electric Light Company exhibited, on June 29, 1901, assets of $5,646,377, and these liabilities: capital stock, $3,000,000; bonds, $1,520,000; general liabilities, $1,396,377. What was the apparent surplus or deficit? Subtract the sum of the bonds and the general liabilities from the assets. What was the ratio of the difference to the common stock? At this ratio what was the book value of a share of the stock, par value $100?

5. The 100,000 shares of the Calumet and Hecla Mining Company's stock fell in six months in 1901 $160 per share, to a market value of $700 a share. What was the total loss in market value? At the beginning of the decline, what was the total market value of the stock?

COMPLEX FRACTIONS

A **complex fraction** is a fraction containing a fraction or mixed number in one or both of its terms. $\dfrac{\frac{1}{2}}{\frac{1}{8}}, \dfrac{\frac{7}{2}}{19}, \dfrac{3\frac{1}{5}}{\frac{7}{13}}$.

1. Reduce $\dfrac{\frac{1}{2}}{\frac{3}{8}}$ to a simple fraction.

$$\tfrac{1}{2} \div \tfrac{3}{8} = \tfrac{1}{2} \times \tfrac{8}{3} = 1\tfrac{1}{3}.$$

2. Reduce $\dfrac{7\frac{1}{5}}{8\frac{4}{9}}$ to a simple fraction.

3. Multiply $3\tfrac{1}{5} \times \tfrac{5}{7} \times 4\tfrac{2}{3} \times 3\tfrac{3}{8}$.

4. $\tfrac{7}{11} \times \tfrac{4}{5} \times 3\tfrac{1}{2} \times \tfrac{55}{98} = ?$

5. Divide $\tfrac{3}{4}$ of $\tfrac{7}{8}$ by $\tfrac{15}{16}$ of 7.

SUGGESTION. $\tfrac{3}{4}$ of $\tfrac{7}{8} \div \tfrac{15}{16}$ of $7 = (\tfrac{3}{4} \times \tfrac{7}{8}) \div (\tfrac{15}{16} \times \tfrac{7}{1})$.

6. Simplify $\dfrac{1\frac{3}{4}}{2\frac{1}{2}}$.

$$\frac{1\frac{3}{4}}{2\frac{1}{2}} = \frac{\frac{7}{4}}{\frac{5}{2}} = \frac{7}{4} \div \frac{5}{2} = \frac{7}{\underset{2}{4}} \times \frac{2}{5} = \frac{7}{10}.$$

7. Find the value of $\dfrac{15\frac{3}{4} - 4\frac{2}{3} \times 1\frac{5}{6}}{\frac{1}{5} \div \frac{3}{70} + 2\frac{19}{36}}$.

Numerator $= 15\tfrac{3}{4} - \left(\dfrac{14}{3} \times \dfrac{11}{\underset{3}{6}}\right) = 15\tfrac{3}{4} - \tfrac{77}{9} = 15\tfrac{3}{4} - 8\tfrac{5}{9} = 7\tfrac{7}{36}.$

Denominator $= \left(\dfrac{1}{5} \div \dfrac{3}{70}\right) + 2\tfrac{19}{36} = \left(\dfrac{1}{5} \times \dfrac{\overset{14}{70}}{3}\right) + 2\tfrac{19}{36}$

$\qquad\qquad = \tfrac{14}{3} + 2\tfrac{19}{36} = 4\tfrac{2}{3} + 2\tfrac{19}{36} = 7\tfrac{7}{36}.$

8. Find the value of $\dfrac{1}{3 + \dfrac{1}{7 + \frac{1}{16}}}$.

$$7 + \frac{1}{16} = \frac{112}{16} + \frac{1}{16} = \frac{113}{16}.$$

Therefore, $\quad \dfrac{1}{7 + \frac{1}{16}} = \dfrac{1}{\frac{113}{16}} = \dfrac{1}{1} \times \dfrac{16}{113} = \dfrac{16}{113}.$

Hence, $\quad 3 + \dfrac{1}{7 + \frac{1}{16}} = 3 + \dfrac{16}{113} = \dfrac{339 + 16}{113} = \dfrac{355}{113}.$

And, $\quad \dfrac{1}{3 + \dfrac{1}{7 + \frac{1}{16}}} = \dfrac{1}{\frac{355}{113}} = \dfrac{113}{355}.$

9. Find the value of $\dfrac{\frac{100}{3}}{66 \times \frac{2}{6}}.$

Simplify the following complex fractions :

10. $\dfrac{15}{\frac{3}{7}}.$ **11.** $\dfrac{6\frac{1}{8}}{7\frac{5}{9}}.$ **12.** $\dfrac{87\frac{1}{2}}{\frac{7}{8}}.$

13. $\dfrac{\frac{3}{7}}{15}.$ **14.** $\dfrac{\frac{5}{6}}{\frac{4}{5}}.$ **15.** $\dfrac{\frac{8}{9}}{4\frac{1}{2}}.$

16. $\dfrac{\frac{8}{9}}{\frac{15}{16}}.$ **17.** $\dfrac{1\frac{5}{9}}{\frac{9}{16}}.$ **18.** $\dfrac{8\frac{4}{7}}{12}.$

19. $\dfrac{12}{\frac{3}{7}}.$ **20.** $\dfrac{\frac{3}{7}}{14}.$ **21.** $\dfrac{4\frac{7}{8}}{9}.$

22. $\dfrac{\frac{4}{5}}{\frac{6}{7}}.$ **23.** $\dfrac{6\frac{2}{3}}{\frac{2}{3}}.$ **24.** $\dfrac{7\frac{11}{12}}{3\frac{5}{6}}.$

25. $\dfrac{\frac{3}{4} \text{ of } \frac{1}{2}}{\frac{3}{7}}.$ **26.** $\dfrac{\frac{2}{9} \text{ of } \frac{3}{5}}{\frac{1}{4} \text{ of } 2\frac{4}{9}}.$ **27.** $\dfrac{25}{8\frac{3}{7}}.$

28. $\dfrac{14\frac{9}{10}}{\frac{7}{9} \text{ of } 15}.$ **29.** $\dfrac{214\frac{3}{4}}{25\frac{11}{12}}.$ **30.** $\dfrac{\frac{21}{25}}{4\frac{9}{10}}.$

31. $\dfrac{\frac{4}{7} \text{ of } \frac{2}{3} \text{ of } 5\frac{1}{4}}{\frac{15}{16} \text{ of } 48}.$ **32.** $\dfrac{3\frac{1}{4}}{6\frac{1}{2}} \times 72\frac{1}{2} \times \frac{2}{3} \times \frac{3}{5} \times 9\frac{3}{8}.$

33. $\dfrac{8\frac{2}{7}}{4\frac{1}{7}} \div \frac{1}{2} \div \frac{1}{4} \times 8\frac{2}{3}.$ **34.** $\frac{1}{2} \times \frac{9}{11} - \dfrac{8\frac{2}{3}}{4\frac{1}{5}} \times 2\frac{1}{10}.$

Divide :

35. $6\frac{1}{2}$ by $4\frac{3}{8}$.

36. $3\frac{5}{6}$ by $\frac{2}{3}$ of $\frac{5}{8}$.

37. $3\frac{1}{2} \div 2\frac{1}{3} = ?$

38. $2\frac{4}{63} \div \frac{2}{3}$ of $1\frac{5}{9} = ?$

39. $4\frac{3}{4} \div 1\frac{2}{3} = ?$

40. $\frac{1}{2}$ of $\frac{3}{4} \div \frac{7}{8} = ?$

41. $12\frac{1}{2} \div 7\frac{1}{4} = ?$

42. $\frac{4}{5}$ of $\frac{5}{8} \div \frac{7}{20} = ?$

43. $9\frac{3}{8} \div 2\frac{1}{5} = ?$

44. $\frac{11}{15} \div \frac{1}{7}$ of $\frac{2}{3} = ?$

45. $10\frac{4}{9} \div \frac{7}{12} = ?$

46. $9\frac{1}{6} \div \frac{1}{2}$ of $5 = ?$

47. $5\frac{5}{7} \div 8\frac{1}{3} = ?$

48. $4\frac{5}{7} \div 2\frac{3}{4} = ?$

49. $\frac{7}{36} \times \frac{14}{27} = ?$

50. $12\frac{9}{10} \div \frac{1}{3}$ of $6\frac{3}{4} = ?$

51. $\frac{20}{21} \div \frac{1}{2}$ of $\frac{4}{5} = ?$

52. $\frac{3}{4}$ of $26\frac{2}{3} \div 2\frac{6}{7} = ?$

Simplify :

53. $\frac{2}{7}$ of $\frac{3}{8} \div 4\frac{3}{4}$.

54. $7\frac{3}{8} \div \frac{1}{2}$ of $\frac{3}{4}$ of $6\frac{1}{2}$.

55. $\frac{6}{7}$ of $\frac{4}{5} \div \frac{3}{5}$ of $\frac{7}{9}$.

56. $3\frac{3}{5}$ of $8\frac{1}{2} \div 6\frac{2}{5}$ of $5\frac{2}{7}$.

57. $4\frac{1}{2}$ of $3\frac{1}{3} \div 2\frac{1}{4}$ of $6\frac{1}{4}$.

58. $\frac{5}{9}$ of $7\frac{3}{11} \div \frac{4}{11}$ of $17\frac{3}{7}$.

59. $1\frac{7}{28}$ of $\frac{10}{13}$ of $\frac{3}{4}$ of $1\frac{2}{5} \div \frac{5}{6}$ of $\frac{3}{28}$ of $\frac{3}{4}$ of 5.

60. $\frac{1}{3}$ of $\frac{6}{7} \div \frac{5}{7}$ of $1\frac{2}{13}$.

61. $\frac{3}{5}$ of $2\frac{1}{2} \div \frac{4}{5}$ of 9.

DAILY AFFAIRS

1. A furnace pipe is to be 18 feet long and 9 inches in diameter. Without allowance for overlapping at the joints find how many square feet of galvanized iron are required to make it.

2. A toy maker wished to make 10,000 cubical blocks 2 inches in each dimension. How many feet of board did he require, without allowance for waste ?

3. The enrollment of a class for a month of 20 days was 39 pupils. They attended in all 715 days. What was the average per cent of attendance to enrollment ?

4. X could walk without resting 4 miles the first hour, 10% less the next hour, 10% less the third hour than the second, and 10% less each succeeding hour than the preceding. After resting, he could repeat the morning rates. One day he walked four hours, then rested for an hour, walked two hours, then rested again, and completed his journey by walking for five hours more. How far did he go?

5. Z with a bicycle could go without resting 12 miles for an hour, then 15% farther the next hour, and in succeeding hours could go $12\frac{1}{2}$% less than in each preceding hour. After resting he could repeat these rates. One day he rode two hours, rested, rode an hour, rested, rode again for three hours, then rested, and completed his journey by riding for four hours. How far did he go?

6. A contractor knew that 8 of his men could do a certain piece of work in 12 days. He had to do in 9 days another piece of work that was 75% larger. How many men did he require?

7. A piece of cloth in the making shrunk from 67 yd. to $59\frac{1}{2}$ yd. What was the per cent of shrinkage?

8. The average lung capacity is 280 cu. in. for a person 5 ft. 7 in. in height. What percentage of the normal capacity is that of one whose measurement is 215 cu. in.?

9. There were 132 bananas on a bunch that cost $1.20. $\frac{1}{2}$ were sold at the rate of 20 ¢ per dozen; $\frac{1}{3}$ at 1 ¢ each; and the rest brought an average of $\frac{1}{2}$ ¢ each, not including 8 that were spoiled. What was the gain on the bunch?

10. An orchard, carefully tilled and trimmed, yielded 215 bbl. of apples at $1.40 per barrel. Another orchard, uncared for, yielded 150 bbl. at $1.15 per barrel. The cost of the care of the first orchard was 20% of the value of the yield. Compare the net returns.

11. How many cubic feet of fresh air are required per hour by the normal person who breathes 16 times per minute, changing the air in his lungs to the amount of 20 per cent of his capacity?

12. How many bushels of grain are there in a bin 15 feet square, filled to the depth of $4\frac{1}{2}$ ft.?

13. Without allowance for waste, what, in cubic feet, was the harvest of a circular pond 1 mile in diameter, frozen 18 in. in depth?

14. How many sods $12'' \times 18''$ can be cut from a plot $40' \times 50'$? What is their total value at $25 ¢$ per sq. yd.?

15. How many square feet of flagging, 4 ft. wide, are required for a corner lot $148' \times 175'$?

16. Compare the total cost for the above sidewalk in stone at $18 ¢$ per sq. ft., and in concrete at $1 per sq. yd.

17. At $38 ¢$ per square foot find the cost to glaze all the windows of a house, requiring 248 panes of glass $12'' \times 15''$ and 196 panes $14'' \times 18''$.

18. Find the number of linear feet of pine board, $10''$ wide, required to floor a piazza $28'' \times 10''$. What is its cost @ $70 per M. board measure?

19. How many T. of copper can be obtained from 1500 T. of ore that yields 9% metal?

20. Z was offered as compensation for his services as president of a corporation either 5% of the net earnings, which were usually $80,000, or the income from $75,000 worth of stock, that paid usually 6% dividends. Which was the larger amount?

21. A certain town grew in wealth 25% in a decade, and in population $33\frac{1}{3}\%$. At the beginning of the decade its wealth was $4,000,000 and its population was 6000.

What was its average wealth per capita at the beginning of the decade ? What at the end ?

22. When the freight charges on 125 head of cattle, averaging 900 lb., are $200 for 150 miles, find the charges at the corresponding rates on 375 head, averaging 1050 lb., for 1200 miles.

23. The ratable property of a certain city is $175,000,000. Find the amount of its total school tax at $6.40 per thousand.

24. A clock loses 40 sec. per day. Find its loss per hour and per week. What is its rate per cent of error ?

25. Find the different prices to paint a fence 256 ft. long, @ 6 ¢ per linear foot for one coat, and @ 11 ¢ for two coats.

26. When a horse is fed 9 qt. of oats per day, what amount is needed as the supply for 3 mo. ?

27. X bought 10 shares of stock, par value $100, at 4% premium, that for 3 yr. paid dividends of 5%. He then sold the stock at $8\frac{1}{2}$% premium. What was the total gain on the investment, including dividends ? What rate per annum was this ?

28. Since at the sea level the atmospheric pressure is 15 lb. per sq. in., and at the height of 3 mi. is $\frac{1}{2}$ as much, what is the total difference of pressure upon the body of a man whose surface is 10 sq. ft., and who goes from the seacoast up 3 mi. to the top of Mount McKinley ?

29. Mrs. Jones sold 360 eggs at 15 ¢ per dozen, and took her pay in sugar at $5\frac{1}{4}$ ¢ per pound. To how many pounds was she entitled ?

30. When 8 oz. of iron make a horseshoe, find the number of pounds of iron required weekly by a smith who shoes daily an average of 15 horses.

31. To A was due $600, with interest, 4 mo. hence. Anticipating its payment, he borrowed at the bank on his note for $600, at 6% interest. What was the amount due to him on the debt at its maturity? What amount did he receive from the bank?

32. A ship's coal capacity was 270 T. 24 T. were used daily. What amount remained at the end of an 8-da. voyage, when the ship started with a load of 90% of its capacity of coal?

33. A city's water supply averages 150 gal. daily per house. When the town has gained 20% in its number of houses, and has lost 10% in its water supply, what is its daily average per house?

34. A tenant one year rented a farm for $175 and the taxes, $35. His crop was worth in cash $950. The next year he paid ⅓ of the crop only, which was worth $900. Compare his net returns in these two years.

35. A grocer owned a store worth $8000, and two business horses and wagons worth $400. He owned also a house worth $6000 and a driving horse and two carriages worth $650. What per cent of his money was invested in business? Since his income was 20% of his total business capital, and he saved annually 20% of it, what was the total amount of his family expenditure?

36. One man earns $3500 a year, while another has a 4½% income on $60,000. Which man has the larger amount to spend? By how much?

37. A cow is tied to a stake first with a 16-ft. rope and later with a 24-ft. rope. Compare the number of square feet to graze with the different ropes.

THE METRIC SYSTEM
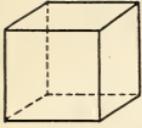
The **unit** of the *metric system* is the **meter** of length.

$\frac{1}{10}$ of a meter is a decimeter.

$\frac{1}{100}$ of a meter is a centimeter.

Cubic centimeter

The weight of a cubic centimeter of water is one **gram**. The volume of a cubic decimeter is one **liter**.

Make cubes of cardboard or of heavy paper, of the sizes of the liter and of the gram.

A meter equals 39.37+ in., or 3 ft. $3\frac{2}{3}$ in.

A decimeter is slightly less than four inches, as the illustration shows.

The metric system provides for measurements of lengths, volumes, and weights, by decimal divisions and multiplications of standard units.

centi-   means   hundredth   .01
deci-   means   tenth   .1
deka-   means   ten   10.
hekto-   means   hundred   100.
kilo-   means   thousand   1000.

1. In a kilogram are how many grams?
2. In a dekaliter are how many liters?
3. In a hektometer are how many meters?
4. What part of a liter is a deciliter?
5. What part of a gram is a centigram?
6. What part of a kiloliter is a deciliter?
7. What part of a hektogram is a decigram?
8. In a kiloliter are how many centiliters?

FOUR INCHES
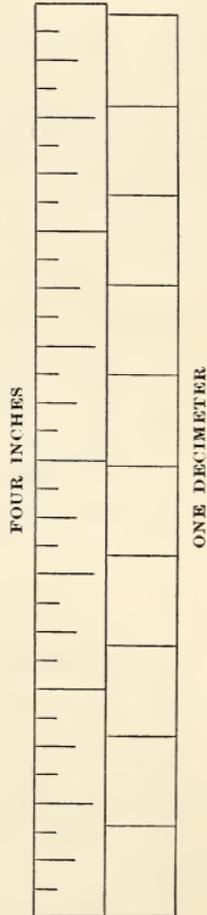
ONE DECIMETER

Long Measure

1 centimeter (cm.)	= .01 meter.
1 decimeter (dm.)	= .1 meter.
1 meter (m.)	= **standard**.
1 dekameter (Dm.)	= 10 meters.
1 hektometer (Hm.)	= 100 meters.
1 kilometer (Km.)	= 1000 meters.

Measure of Capacity

1 centiliter (cl.)	= .01 liter.
1 deciliter (dl.)	= .1 liter.
1 liter (l.)	= **standard**.
1 dekaliter (Dl.)	= 10 liters.
1 hektoliter (Hl.)	= 100 liters.
1 kiloliter (Kl.)	= 1000 liters.

Measure of Weight

1 centigram (cg.)	= .01 gram.
1 decigram (dg.)	= .1 gram.
1 gram (g.)	= **standard**.
1 dekagram (Dg.)	= 10 grams.
1 hektogram (Hg.)	= 100 grams.
1 kilogram (Kg.)	= 1000 grams.
1000 Kg.	= 1 metric ton [T.].

Other Equivalents in Customary Measures

1 kilometer = .62 + miles.	1 liter = 1.06 liquid quarts.
1 kilogram = 2.2 + pounds.	1 metric ton = 2205 pounds.

Written Exercises

1. In three meters are how many inches?

2. In a kilometer are how many feet?

3. Reduce to grams and add: 5 Kg., 2 Hg., 100 dm., and 5000 cm.

4. Reduce to liters and add: 3 Hl., 5 Dl., 15 dl., and 200 cl.

5. From 1 Km. take 2050 dm.

6. Which is the greater distance: 5 Km. or 3 mi.?

7. From ten metric tons of coal were taken ten ordinary tons. How many pounds of coal remained?

8. X bought 100 *long tons* of coal in the United States at $3.40 per *long ton*, and sold it in France at 28 francs per metric ton. When exchange was 5.4 francs for $1, what was the difference between the cost and the selling price?

9. One man ran a quarter of a mile in 49 sec. Another ran 300 meters in 35 sec. At the same rate, which could run 100 yards faster? By how many seconds?

10. An automobilist traveled 60 Km. in 1 hour. How many miles is this?

11. Z bought, at 40¢ per l., 250 l. of wood alcohol for chemical uses, and sold it at 60¢ per qt. What was his gain?

12. A wholesale merchant bought 12,000 m. of silk ribbon at 1.5 francs per meter, when exchange was 5.3 francs per dollar. What was the total cost in our money?

13. Find in Kg. the weight of a barrel of flour of 196 lb.

14. At 65¢ per l., what is the cost of 6.5 Dl. of molasses?

15. From 5 metric tons of hay a horse was fed at the rate of 75 Hg. per day, and two cows were fed at the rate of 9 Kg. per day. How many days did the hay last?

FINAL ORAL REVIEW

Make blackboard drawings to illustrate the geometrical questions.

1. What is the ratio of 12 to 144? 15 to 75? 50 to 205? 1000 to 1,000,000? 625 to 25? 200 to 4?

2. How many generations have passed since Charles the Hammer conquered the Moors at Tours in France in 732?

3. X bought, at 2% discount for cash, a bill of goods amounting to $144. What cash was due?

4. An agent collected debts to the amount of $700. What was his commission at $12\frac{1}{2}\%$?

5. A boy having $12\frac{3}{5}$ miles to go, walked $7\frac{1}{2}$ miles and rode the rest of the way. How far did he ride?

6. When it is 6 P.M. at Greenwich, what time is it at a place 90 degrees west of Greenwich? 30° east?

7. How many feet of fencing are needed to inclose a piece of land half a mile square? How many to inclose half of a square mile?

8. X bought 3 lb. 4 oz. of butter at 24¢ a pound. What did it cost him?

9. How many square yards are there in a rectangle 9 ft. long by 3 ft. wide?

10. The difference in time between two places is 3 hr. 4 min. What is their difference in longitude?

11. How many yards of carpet a yard wide are needed to cover the floor of a room 15 ft. by 12 ft. wide?

12. Two places are situated 75° 60′ 45″ apart. What is the difference in time between them?

13. The area of a parallelogram is 108 sq. ft. Its base is 12 ft. What is the altitude?

14. Find the cost of 2 bu. 1 qt. of nuts at 10¢ per qt.

15. At the rate of $7\frac{1}{3}$¢ for 5 oranges, what must I pay for 30 oranges?

16. After spending 45% of his money for books and 30% for tools, a boy had left $.75. How much had he at first?

17. 12 men in $5\frac{1}{2}$ days can do a piece of work. How long will 60 men working at the same rate require?

18. What number increased by $37\frac{1}{2}\%$ of itself equals 44?

19. When the fulcrum of a lever of the first class is $\frac{3}{4}$ the distance from one end, what power at the end of the short arm will balance a weight of 60 lbs.?

20. An agent received $250 for collecting rents. How much did he collect, his commission being 5%?

21. Harry is 16 yr. of age, $\frac{3}{4}$ of Harry's age is $\frac{1}{4}$ his father's age, and twice his father's age is 10 yr. more than his grandfather's age. How old is the grandfather?

22. Fred can build a chicken coop in 4 days, and Charles in 5 days. How long would it take them working together?

23. William sold $\frac{5}{7}$ of his rabbits and had 4 left. How many had Nelson, who had twice as many as William had at first?

24. An agent received $25.75 as commission for selling goods at 5%. What was the value of the goods sold?

25. Edna paid 35¢ for a game, which was $\frac{5}{12}$ the amount Oscar gave for a ball. Both the game and ball were paid for with a two-dollar bill. How much change should the children receive?

26. How many bags each containing 2 bu. 1 pk. 2 qt. will 1000 bu. of wheat fill?

27. What is the area of a triangle whose base is 60 ft. and whose altitude is 40 ft.?

28. When $\frac{6}{7}$ of an acre of land cost $1.20, what will half an acre cost?

29. A number diminished by $\frac{1}{3}$ of itself is equal to 64. What is the number?

30. A real estate agent collected rent to the amount of $4500. What was his commission at 2%?

31. When $\frac{2}{5}$ of a yard of ribbon cost 15¢, what do $5\frac{1}{5}$ yd. cost?

32. What part of a barrel of flour worth $4.80 can I buy with the money I receive from selling 12 doz. eggs at 20¢ a dozen?

33. At 4¢ a foot, what will it cost to put a moulding around a room 30 ft. long by 28 ft. wide?

FINAL WRITTEN REVIEW

1. A farmer has three olive orchards: from the one he obtains 224 bu. 2 pk. 2 qt.; from another one-half as much, increased by 76 bu. 3 pk. 1 qt.; and from the third one as much as from the other two, lacking 84 bu. 2 pk. 7 qt. How much did he obtain from the three fields?

2. I bought from one dealer 8 bu. 2 pk. 3 qt. of grass seed; from another 3 times as much, and 2 bu. 2 qt. more; and from another 3 times as much as of the second, less 1 bu. 1 pk. 6 qt. How much did I buy of each respectively, and how much of all?

3. A farmer by improved methods increased the yearly net financial return of his farm from $375 to $1200. If the income represented 10% of the farm's value, what was the difference in its values?

4. (*a*) The earth has a surface area, land and water, of 200,000,000,000 sq. mi. and a volume of 266,600,000,000,000 cu. mi. The ratio of the moon's surface to the earth's is 1 to 14, and of its volume to the earth's is 1 to 49. What is the moon's surface? volume?

4. (*b*) The surface of a sphere equals its diameter squared multiplied by 3.1416. Its volume equals its surface multiplied by one sixth its diameter. Find the earth's surface and volume.

The earth's average diameter is 7917.5 miles.

5. The diameter of the moon's orbit around the earth averages 485,000 miles. The diameter of the sun is over 865,000. If the earth's center were at the sun's center, how far inside the sun would the moon be ?

6. The diameter of the moon is 2163 miles, while that of the earth is 7926 miles at the equator. What is the ratio of the diameters ?

7. Draw circles with radii in the ratio of 4 to 7. If the smaller indicates the moon's orbit, what dimension of the sun does the larger circle indicate ?

8. Draw on the blackboard two lines to indicate the ratio of the earth's diameter to the sun, calling these 8000 and 880,000 respectively.

9. A "spot" on the sun, measured in 1894, was 80,000 miles deep. What was the ratio of this depth to the sun's radius ?

10. The heat of the earth's interior seems to increase 1° Fahrenheit with every 51 feet of descent. If the rate continues, what is the temperature of the earth's center ? How does this compare with 15,000°, which is believed to be the temperature of the sun's surface ?

11. Jupiter moves once around the sun in 4333 days, but rotates once on its own axis in 10 hours. How many times does it turn on its axis in going once around the sun ?

12. The average diameter of Jupiter is about 88,000 miles. What is its ratio to the earth's diameter ? to the sun's ?

13. The ratio of the distance of Jupiter from the sun as compared with the earth's distance is 5.20 to 1. The earth is distant nearly 93,000,000 miles. What is Jupiter's distance?

14. The ratio of the diameter of Mars to that of our moon is 2 to 1. The ratio of the diameter of Venus to that of the earth is 77 to 79. The ratio of the diameter of Saturn to that of the earth is 737 to 79. Find all these diameters, calling the moon's diameter 2150 miles and the earth's 7900.

15. The nearest star, Alpha Centauri, seen in the southern sky, is 25,000,000,000,000 miles away. Since light travels at the rate of 186,000 miles a second, how long does it take light to come to us from this star? What is the ratio of the distance of Alpha Centauri from us compared with the distance of the sun? How long does it take light to come to us from the sun?

16. At the rate of 186,000 miles a second, how far does light travel in 1 year?

17. The sun with all the planets and moons is moving through the heavens at the rate of 11 miles per second. How far is this in a year? in 100 years?

18. A star a thousand times as far away as the nearest star may have ceased to shine how many years before its light ceased to reach our earth?

19. When the expenses of a town are $850, and $100 is allowed as the amount assessed that may be in arrears, what sum must be assessed to raise the total amount needed, with 5% commission for collecting it?

20. What sum must be assessed to raise a net amount of $8500, with 1% commission for collection and $200 allowed for arrears of taxes?

21. The length of a rectangular pasture that contains 15 acres is 990 ft. How many rods is it in width?

22. Find the sum of $\frac{1}{3}$ bu. $\frac{1}{5}$ pk. $\frac{1}{4}$ qt. and 1 pt.

23. What does it cost to plaster the walls of a room 21 ft. long, 18 ft. wide, and 9 ft. high, at 36¢ a square yard?

24. How much must one pay for a pile of wood 36 ft. long by 12 ft. wide by 8 ft. high, at $3.75 a cord?

25. A man earned $2400 in a year. He spent $\frac{1}{8}$ of it for coal, .5 of the remainder for rent and other expenses, and $5\frac{1}{2}\%$ of the balance for some furniture. What was the cost of the furniture? How much did he save?

26. Two men together purchased a barrel of flour containing 196 lb. for $4.72. The first took 49 lb. of the flour, and the second 147 lb. How much should each pay?

27. A milkman sold on Monday 7 gal. 3 qt. 1 pt. 3 gi. of milk; on Tuesday 5 gal. 2 qt. 1 pt. 2 gi.; on Wednesday 4 gal. 2 qt. 2 gi.; on Thursday 6 gal. 1 pt. 2 gi.; and on Friday 7 gal. 3 qt. How much did he receive for it at 6¢ a quart?

28. If my horses eat in a single day 4 pk. 4 qt. of oats, how long will 27 bu. 2 pk. 4 qt. last them?

29. A dealer sold 15 gal. 1 qt. 1 pt. 3 gi. of vinegar from a barrel containing 20 gal. 3 qt. How much remained?

30. Reduce $\frac{32}{33}$ to a decimal.

31. A traveler bought two suits of clothes, paying for each £3 10*s*. 9*d*. How much did they cost in United States currency?

32. What decimal part of a day are 15 sec.?

33. Reduce $\frac{2}{3}$ of a mile to units of lower denominations.

34. Find the cost of a block of marble $8\frac{1}{4}$ ft. long by 4 ft. 6 in. wide by 2 ft. thick, at $15.20 per cubic yard.

35. The difference in time between two places is 5 hr. 15 min. 10 sec. How far apart are the places?

36. Reduce 2 qt. 1 pt. 1 gi. to the decimal of a gallon.

37. $\frac{1}{3}$ of a man's property is in houses, $\frac{2}{5}$ in lots, and the remainder, $8296, is in a bank. How much is the man worth?

38. A and B together own $\frac{5}{9}$ of a ship. Of this part, A owns $\frac{2}{5}$ and B the remainder, which is valued at $3600. What is the value of the whole ship?

39. Reduce 75.231 inches to units of higher denominations.

40. A man borrowed $360 for 3 yr. 6 mo. at simple interest and loaned it at compound interest to be paid semi-annually. What was his gain?

41. It takes 364 rd. of fence to inclose a circular field. What is the area of the field?

42. When it is 4 P.M. at San Francisco, 122° 26′ 15″ west longitude, what time is it at Philadelphia, 75° 10′ west?

43. $\frac{3}{8}$ of a foot is what part of a mile?

44. Mr. John W. Baldwin bought of the Doremus Stove Co. 5 lb. coffee @ 35¢, $3\frac{1}{2}$ lb. butter @ 36¢, 8 lb. lard at $6\frac{1}{2}$¢, 14 lb. ham @ $12\frac{1}{2}$¢, and 10 lb. sugar at $6\frac{1}{2}$¢. Make out a receipted bill.

45. Divide 24 millionths by 18 thousandths and multiply the quotient by 2.05.

46. A man had $790.36 changed to its equivalent in English money. How much did he receive?

47. By selling land for $375 an acre I lose 25%. For how much should I sell it in order to gain 20%?

48. A man loaned at 7% interest a sum of money which in 3 yr. 4 mo. 24 da. amounted to $643.76. What was the principal?

49. A, B, C, and D hired a pasture containing 275 acres at $1.20 an acre. A pastured 100 sheep, B 125, C 75, and D 200. How much rent should each pay?

50. Reduce .00175 to a common fraction in its lowest terms.

51. One side of a square field measures 13 rd. 5 yd. 2 ft. How many feet of fence are needed to inclose the field?

52. How many feet and inches are there in .375 of a rod?

53. Write the following numbers in Roman notation:

568   1902   1729   3900

54. Divide, using cancellation:

138 × 74 × 56 × 90 by 30 × 46 × 14 × 6.

55. Mr. Chase thought that his gas bill for January was too high. The bill was $3.80. He had burned, according to his gas meter, 2,800 cu. ft. of gas at $1.10 per M. Was he correct in thinking there was a mistake? When the bill was made right, he paid it and received a discount of 10¢ per 1000 cu. ft. for payment within 5 da. How much did he pay?

56. Reduce 130 rd. 4 yd. 2 ft. to the decimal of a mile.

57. Find the cost of a pile of wood 40 ft. long by 12 ft. wide by 4 ft. high at $4.90 a cord.

58. How many kilograms are in 32.91 metric tons?

59. A man bought a piano for $150. How much must he ask for it so that he may gain 20% and yet reduce by 10% the asking price?

60. A boy collecting stamps, by accident destroyed 40% of the number he had. He then gave his cousin 30 stamps, which was 25% of the remainder. How many had he at first?

61. Where should the fulcrum of a 7-foot lever be placed so that a power of 300 lb. may balance a weight of 125 lb.?

62. A commission merchant sold 750 bbl. of flour at $5.25 a barrel. What was his commission at $4\frac{1}{2}\%$?

63. From $1145\frac{5}{9}$ take $925\frac{7}{8}$.

64. A man who owned $\frac{5}{8}$ a vessel sold $\frac{2}{3}$ of his share for $12,765. What was the value of the part he had owned?

65. *a.* Find the G.C.D. of 364 and 392.

b. Find the L.C.D. and add: $\frac{2}{5}$, $\frac{7}{12}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{6}{25}$.

66. A boy rides 480 mi. in $5\frac{1}{3}$ da. How far, at that rate, would he ride in $7\frac{4}{5}$ da.?

67. Multiply five hundred-thousandths by one hundred twenty-five millionths, and divide the product by five ten-thousandths.

68. Two boys picked 8 bu. 2 pk. 1 qt. of nuts. They kept 12 qt. and sold the remainder at 12¢ a pint. How much did they make?

69. Mr. Sloane received $5075 with which to purchase cotton at $12\frac{1}{2}$¢ a pound after deducting his commission of $1\frac{1}{2}\%$. How many pounds did he purchase?

70. A man's insurance premium at $\frac{1}{4}\%$ was $124.80. What was the amount for which he was insured?

71. Two men, 2448 mi. apart, travel toward each other, one at the rate of 37.4 mi. a day, and the other 30.6 mi. a day. In how many days will they meet?

72. A man bought 7 horses at $80 each, and 8 at $125. He sold all at $130 each. What was his gain per cent?

73. The minuend is 6075.5, the subtrahend is 495.0275. What is the remainder?

74. On a note for $750 at 6% X paid in 30 days $100, in 60 days afterwards $75, and in $\frac{1}{2}$ yr. still later $20. What was due 30 days after the last payment?

75. Find the cost of a draft for $800 payable in 90 da., exchange being at $\frac{1}{8}$% premium and interest 6%.

76. In a 5-foot lever of the first class the fulcrum is 2 ft. from the weight. What power will balance a weight of 96 pounds?

77. An agent in New Orleans received $32,445 with which to purchase 350 bales of cotton after deducting his commission of 3%. What was the cost of a bale of the cotton?

78. X bought 125$\frac{1}{2}$ yd. of carpet at 1.62\frac{1}{2}$ a yard, receiving discounts of 10% and 2% for cash. What was his bill?

79. How long will it take $2000 to amount to $3000 at 4% compounded semi-annually?

80. Mr. Hooper lost 30% of his property through failures and 20% of the remainder by fire. He still had $8700. How much had he at first?

81. With yourself the payee, and your friend the maker, write a negotiable note for $1000 with interest at 6%.

82. Property valued at $54,400 was insured for $1224. What was the rate of insurance?

83. 120 shares of stock were bought at 90 and sold at 95$\frac{1}{4}$. What was the gain when the brokerage was $\frac{1}{8}$% for each transaction?

84. John borrowed $150 from his father on the first of January, 1899, and paid it back with interest at 6% July 21, 1901. What amount did he pay?

85. What principal gains $40.50 in 9 mo. 12 da. at 6%?

86. What must I pay for a sight draft upon Cleveland for $8500 at ¼% premium?

87. How many bags, each containing 2 bu. 1 pk. 2 qt., will be needed to hold 13 bu. 3 pk. 4 qt. of oats?

88. A piece of land 120 rd. long and 60 rd. wide cost $250 an acre. It was sold at a loss of 16%. What was the selling price?

89. A room 27 ft. long by 25 ft. wide was carpeted with carpet 1 yd. wide, the strips running lengthwise. What was the cost of the carpet at 87¢ a yd., provided there was no waste in matching?

90. On a bill of goods amounting to $800, what is the difference between 25% off and 15% and 10% off?

91. At the rate of $7\frac{1}{2}$ mills on the dollar, what is the tax on property valued at $17,284?

92. What is the amount of $1347.50 for 2 yr. 6 mo. 18 da. at 5%?

93. Write a non-negotiable note for $600, making Howard L. Briggs the payee, payable on demand without interest.

94. The altitude of a parallelogram is 14 ft., and its parallel sides are 20 and 16 ft. Draw it to scale. How many square yards are there in the area?

95. How many liters of water are there in a tank 8 m. long, 72 dm. wide, and 152 cm. deep?

96. A and B start in business with a capital of $15,000. At the end of a year they found they had gained $4280, of which $3168 belonged to A. What capital was furnished by each?

97. A merchant paid $180 for insuring some grain at $2\frac{1}{4}\%$. For how much was the grain insured?

98. Find the interest of $1141.20 for 6 yr. 8 mo. at 7%.

99. Mr. Boyd spent for his summer vacation the interest of $6000 from Feb. 2, 1900, to Apr. 4, 1902, at 6%. How much did he spend?

100. Find the G.C.D. of 84, 126, 189, 63.

101. How many square yards in a triangle whose base is 320 yd. and whose altitude is 198 feet?

102. The distance across a circular field is 37 ft. What is the distance around the field?

103. A room 30 ft. long and 27 ft. wide was carpeted with carpet 27 in. wide at $1.20 a yard. What was the cost if the strips ran lengthwise and there was a waste of an eighth of a yard on each strip?

104. Two men raised 640 bu. of wheat on a farm which they rented together for $280, A's share of the rent being $210. How many bushels of the wheat did each own?

105. The assessed valuation of the property in a certain town is $4,080,000. The amount of tax to be raised is $26,520. What is the rate of taxation?

106. At 5% compound interest, what is the amount of $1280 for 3 yr. 5 mo. 18 da.?

107. Mr. Camp deposited in a savings bank paying 4% compound interest, $100 annually for 9 years. How much were his deposits at the end of 10 years?

TABLES OF WEIGHTS AND MEASURES

LINEAR MEASURE

12 inches (in.)	= 1 foot (ft.).	$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod (rd.).
3 feet	= 1 yard (yd.).	320 rods, or 5280 feet	= 1 mile (mi.).

SQUARE MEASURE

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet	= 1 square rod (sq. rd.).
160 square rods (43,560 sq. ft.)	= 1 acre (A.).
640 acres (1 section)	= 1 square mile (sq. mi.).
36 sections (sq. mi.)	= 1 township (U.S. Survey).

CUBIC MEASURE

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.).
27 cubic feet	= 1 cubic yard (cu. yd.).
16 cubic feet	= 1 cord foot (cd. ft.).
8 cord feet, or 128 cubic feet	= 1 cord (cd.).

LIQUID MEASURE

4 gills (gi.)	= 1 pint (pt.).
2 pints	= 1 quart (qt.).
4 quarts	= 1 gallon (gal.).
$31\frac{1}{2}$ gal.	= 1 barrel (bbl.).
2 bbls.	= 1 hogshead (hhd.).
1 gal.	= 231 cubic inches.

DRY MEASURE

2 pints (pt.)	= 1 quart (qt.).
8 quarts	= 1 peck (pk.).
4 pecks	= 1 bushel (bu.).
1 bushel	= 2150.42 cubic inches.

AVOIRDUPOIS WEIGHT

16 ounces (oz.)	= 1 pound (lb.).
100 pounds	= 1 hundredweight (cwt.).
2000 pounds	= 1 ton (T.).
2240 pounds	= 1 long ton.

CIRCULAR MEASURE

60 seconds ($''$)	= 1 minute ($'$).
60 minutes	= 1 degree ($°$).
90 degrees	= 1 quadrant.
360 degrees	= 1 circumference (circ.).

Time Measure

60 seconds (sec.)	= 1 minute	(m.).
60 minutes	= 1 hour	(h.).
24 hours	= 1 day	(d.).
7 days	= 1 week	(wk.).
30 days	= 1 month	(mo.).
365 days	= 1 common year	(yr.).
366 days	= 1 leap year	(l. yr.).
100 years	= 1 century	(C.).

Miscellaneous Table

12 units	= 1 dozen	(doz.).
12 dozen	= 1 gross	(gr.).
12 gross	= 1 great gross	(g. gr.).
20 units	= 1 score	(sc.).
24 sheets	= 1 quire	(qr.).
20 quires	= 1 ream	(rm.).
2 reams	= 1 bundle	(bun.).
5 bundles	= 1 bale	(B.).

Equivalents

1 fathom	= 6 ft.
1 perch	= 24¾ cu. ft.
7.5 gal.	= 1 cu. ft.
196 lb.	= 1 bbl. flour.
200 lb.	= 1 bbl. beef.
56 lb. corn	= 1 bu.
60 lb. wheat	= 1 bu.
60 lb. potatoes	= 1 bu.
32 lb. oats	= 1 bu.
1 link	= 7.92 in.
8 furlongs	= 1 mi.

1 carat = 3.168 gr. Troy.

1 carat means $\frac{1}{24}$, as used in alloying gold.

20 carat = $\frac{20}{24}$ gold.

3 sizes in shoes = 1 in.

1 hand	= 4 in.
3 knots	= 1 league.
1 bu.	= 1.25 cu. ft.
1 stone	= 14 lb.
1 qt. liq.	= 57.75 cu. in.
1 qt. dry	= 67.2 cu. in.

1 cu. ft. pure water weighs 1000 oz. avoirdupois.

$1 silver weighs 412½ gr.

$10 gold weighs 258 gr.

U. S. gold and silver coins are 9 parts pure metal, 1 part alloy.

1 nautical mile = 1.15 ordinary miles = 1 knot = 6086 ft.

60 nautical miles = 1 degree = 69.16 + mi. at the equator.

Troy Weight

24 grs. (gr.)	= 1 pennyweight	(pwt.).
20 pennyweights	= 1 ounce	(oz.).
12 ounces	= 1 pound	(lb.).

Apothecaries' Weight

20 grains (gr.) = 1 scruple (℈).	8 drams = 1 ounce (℥).
3 scruples = 1 dram (ℨ).	12 ounces = 1 pound (℔).

The pound Troy and the Apothecaries' pound each weigh 5760 grains. The pound Avoirdupois weighs 7000 Troy or Apothecary grains.

The ounce Troy and the Apothecaries' ounce are each 480 grains. The ounce Avoirdupois is $437\frac{1}{2}$ grains.

Surveyor's Measures

Long Measure

7.92 in.	= 1 link	(l.).
100 links	= 1 chain	(ch.).
80 chains	= 1 mile	(mi.).

Square Measure

16 sq. rd.	= 1 sq. ch.
10 sq. ch.	= 1 A.
640 A.	= 1 sq. mi.

Aliquot Parts of 100

$$\frac{100}{3} = 33\frac{1}{3}$$
$$\frac{100}{6} = 16\frac{2}{3}$$
$$\frac{100}{8} = 12\frac{1}{2}$$
$$\frac{100}{12} = 8\frac{1}{3}$$
$$\frac{100}{16} = 6\frac{1}{4}$$

Book Measure

folio (fol)	= 2 leaves to sheet.
quarto (4to)	= 4 leaves to sheet.
octavo (8vo)	= 8 leaves to sheet.
duodecimo (12mo)	= 12 leaves to sheet.

In practice printers' sheets vary in size.

United States Money

10 mills (m.)	= 1 cent	(¢).
10 cents	= 1 dime	(d.).
10 dimes	= 1 dollar	($).
10 dollars	= 1 eagle	(E.).

English Money

4 farthings (far.)	= 1 penny	(d.).
12 pence	= 1 shilling	(sh.)
20 shillings	= 1 pound or sovereign	(£).

Other Foreign Money Facts

French franc	= (nearly) 20¢.
Spanish peseta	= (nearly) 20¢.
Italian lire	= (nearly) 20¢.
100 centimes French	= 1 franc.

German mark = (nearly) 24¢.
German mark = (nearly) 1 shilling English.
100 pfennig German = 1 mark.
Russian rouble = 60¢.

For Metric Measures see p. 160.

It is interesting and profitable in many ways for pupils to invent problems, using these facts.

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