GRADED ARITHMETICS

BOOK SEVEN-GRADE VIII

CHANCELLOR

ELEMENTARY SCHOOL MATHEMATICS BY GRADES

GLOBE SERIES

SEVENTH BOOK

ARITHMETIC, GEOMETRY, AND ALGEBRA

BY

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"Why do we try to make children do what we do not try to do ourselves? Instead of mastering one subject before going to another, it is almost invariably wiser to go on to a superior subject before the inferior has been mastered. . . . On the mastery theory, how much new reading or thinking should we adults do?"

"Many an educated man remembers to this day the exasperation he felt when he discovered that problems in arithmetic over which he had struggled for hours could be solved in as many minutes after he had got half way through algebra."

President Charles William Eliot, LL.D.,
Harvard University.

From "Educational Reform."

"The teachers who fear lest the child shall drift into thinking in symbols . . . are really fearing that the child shall drift into mathematics."

Professor David Eugene Smith, Ph.D.,
Mathematics, Teachers' College, Columbia University.

From "The Teaching of Elementary Mathematics."

"That algebra, arithmetic, and geometry should be taught side by side is not merely useful; it is indispensable for maintaining that unity and coördination in mathematics, without which the science loses all interest and value. A boy who has taken his arithmetic first and then his algebra and then his geometry, has his mental powers less developed than they would have been with three or four years of parallel teaching intelligently pursued."

From Laisant, "La Mathematique."
Quotation translated by Dr. D. E. Smith.

Globe School Book Company.

PREFACE

Boys and girls in the upper grammar grades of the free common schools of American towns and cities are pursuing courses of study by no means uniform. There is perhaps no general agreement in educational practice beyond the fact that it is considered desirable that boys and girls should advance from elementary to secondary schools at about fourteen years of age. There are, however, marked tendencies to agree upon several other points.

There is increasing agreement to the effect that pupils in the upper grammar grades should study what will most benefit them in discipline and in knowledge. We have come to see in the light of the new knowledge of the facts and principles of mental and moral growth, presented by genetic psychology, that what a child enjoys learning he profits by, and that what he profits by develops in him the normal life of the child, which is the guarantee of an efficient life as an adult. There is a growing tendency to decrease the range of arithmetical instruction and to introduce much more geometrical instruction. There is also a distinct tendency to rely more and more upon the various forms of "manual training" in the education of boys and girls; and this development along the lines of the industrial arts, which afford the materials of manual-mental discipline, lends itself notably to the encouragement of the study of geometry early in life.

In the primary and first grammar grades children may easily learn how to add, subtract, multiply, and divide accurately and rapidly, and common and decimal fractions, factoring, cancelling, finding least common multiples and greatest common divisors, and all the facts and operations of denominate numbers. They ought to learn to image correctly the facts involved

in ratios, percentages, and measurements, and to understand the elements of simple proportion.

It appears from the investigations of child-students and of psychologists that unless a boy learns before the age of ten or twelve how to perform the fundamental operations both correctly and quickly he seldom becomes proficient later. Early proficiency, however, can be maintained only by constant exercise. For boys and girls who are prepared in the essential elements of arithmetic this later book proposes extended instruction in percentage and commercial transactions, and in the elements of geometry. It is one of a series of handbooks for pupils, and consists of progressive lessons, arranged mainly in topics, either reviewing the treatment in earlier books, or completing the subject, or preparing for the next book.

In mathematics we have our traditions as to what ought and what ought not to be taught in the different grades. These traditions had their origins long before either courses of study were scientifically ordered or men questioned themselves as to the stages in the growth of the mind. In consequence there are many easy processes in mathematics which are postponed until after much more difficult processes have been mastered, at needlessly great costs in time and energy. It too often happens that the attack upon these more difficult processes results in such discouragement that the student never completes even the elementary school courses. It is not the purpose of this Series to overturn the accepted order of mathematical topics; but certain changes have been made in the direction suggested. The utilitarian value of the simplest geometrical exercises is not less than that of many arithmetical exercises; and their cultural value is greater because they fit more closely the powers and needs of the minds of boys and girls. It is unquestionably good pedagogy and sound common sense to develop for boys and girls fundamental geometric principles, of angles and areas, of forms and of volumes, even at the expense of an encyclopedic knowledge of the rules and methods of interest,

progressions, alligations, and scales. We cannot use tools or examine the constructions of things made with tools, whether boxes, machines, or buildings, unless we know something of geometry.

The introduction of algebra in the last grade of elementary schools is the logical outcome of the principles of the new education, and both warrants and necessitates giving considerable space to the subject in this, the last book of the series. The beauty and simplicity of algebraic methods and processes, and their superiority in many applications to topics often supposed to be within the field of arithmetic only, require that in American education we should afford to elementary school children the opportunity of studying the elements of algebra.

A great amount of material has been presented in this book so as to give the teacher an unusually large and free range of selection. No class in one year is expected even to try to solve every problem.

Author and publishers desire to acknowledge the helpful and valuable suggestions of Mr. George I. Aldrich, Superintendent of Schools, Brookline, Massachusetts, in revising the proofs of this book. We are indebted also for criticisms of methods and problems to several business men and teachers, among whom Miss Lizzie Otis, instructor in mathematics, Center Grammar School, Bloomfield, has given important assistance. No effort has been spared to make the text at once modern and practical.

W. E. C.

Bloomfield, N. J.,
June 25, 1902.

SUGGESTIONS TO TEACHERS

1. The preface explains the general purpose of this book.

2. Read also the prefaces and suggestions to teachers in each of the earlier books of this Series. It may be desirable to review some of their exercises before taking up this book systematically. The value of these exercises in awakening the pupils' interest and activity is speedily evident upon trial.

3. Read this book itself. The purposes of certain features appear only when considered in relation to other features.

4. This book deals with many practical matters. When discussing any special topic and at any time after having discussed it, welcome suggestions and information from the pupils regarding the way business men, artisans, mechanics deal with the same subject. Encourage the boys and girls to get into touch with the world of affairs. If the time of the recitation is being unduly encroached upon, postpone lengthy discussions to private talks, or, if the matter is important to all, to some suitable time "between periods," or at the beginning or end of the session. It is worth very much to boys and girls, especially to those who will not continue in school long, to be encouraged to observe and to think for themselves.

5. Remember that in our American schools, during or just after each of the fourth, fifth, sixth, seventh, eighth, and ninth years in school, from ten to twenty-five per cent of a class drop out of school. In an eighth or ninth year class of forty-five boys and girls using this book, a half dozen, more or less, will remember throughout life this instruction as the highest stage of their formal education. Some of these may be among the most promising students, sifted out from their

class by economic or social forces. For these the cultural quality of their instruction and association in school is even more important than the utilitarian. Even more than the other students those who drop out early need to know not only the processes and the methods of arithmetic, but the reasons involved. We are too apt to judge the ability of students in comparison with our own experienced skill or in comparison with the rapid work of the most forward students, who are by no means always the most thorough, the most retentive, and the most accurate. There is danger in teaching too rapidly just as there is in overdeveloping a lesson.

6. The nature of the human mind is such that when in a student's effort upon a problem he shows that he is radically deficient in the fundamental operations, it becomes the teacher's duty to give to him individual exercises. The dropped stitch in knitting is a trifle compared with an omitted process in an art. And further, if anything has been thoroughly demonstrated by the study of the psychology of children and youth, it is this, that we become proficient in addition, subtraction, multiplication, and division most easily when from eight to twelve years of age. To postpone to later years the boy's acquirement of rapidity and of accuracy is to make that acquirement yet harder for him. When we find our students compelled to add columns over and over again, because of getting different results, we know that the time when they could learn addition most quickly and surely has already passed. With every later added year the difficulty becomes greater. This book, however, does not devote very much space to the fundamental operations. Individuals who need special drill in the elements may be trained in the earlier books of this Series.

7. As all measurements involve ratio, and as measurement is an important topic of this book, it is desirable to cultivate the habit of observation in the children. Various passages in the text suggest the sort of questions we may ask from time to time in order to lead the student to make comparisons. The

habit of noticing sizes and weights is as valuable as that of noticing forms, colors, and textures, which is developed by drawing and manual training.

8. All arithmetic must be mental; but oral recitations, passing immediately from one kind of problem to another and using small numbers, insure the student's reasoning upon the problems. Reasoning is the soul of arithmetic.

9. Neatness tends to accuracy in all the written work. But it is easy to cause much unprofitable time to be spent in copying correct solutions, carelessly written. As far as possible our pupils should be induced to write neatly the first time. Even permission to copy encourages in some natures the habits of carelessness and of slovenliness. It becomes extremely important for this reason, as well as for others even greater, to study and to know the characters, needs, and powers of each individual in a class.

10. From a half to a whole page will be found usually a sufficient lesson. One hundred pages of problems make a reasonable year's work. Topics and problems are offered here in sufficient variety to permit a considerable range of choice in planning a grade's assignment. Whether problems should be given out for home-study is a question not entirely settled; but in this, the last year before the High School or Academy, some home-study seems necessary. No problems should be assigned involving processes not thoroughly explained and understood; and not too many even then.

11. Never refuse to accept a correct solution of a problem that can be explained by the student even if the solution is extremely indirect and inconvenient. But if there is a better method, make its excellence plain.

12. Many problems at first to be solved only in writing may later be solved orally. In reviews of problems on earlier pages oral explanations should be encouraged.

13. Use concrete materials and illustrative drawings as much as time permits. Arithmetic cannot be made too clear.

14. One good mode of solution well understood is worth any number of solutions but partly comprehended. On the other hand, one method of solution often throws much light upon another method.

15. Pupils who have thoroughly mastered the fundamental operations need not perform the work of all problems; let them rather indicate what must be done, giving reasons. It is well to remember that the great error of arithmetical and algebraical teaching has been too much drill on problems to the neglect of reasoning and of logic, and that the great error of geometrical teaching has been too much drill in reasoning and in logic to the neglect of problems.

16. The five principles of the recitation carried out systematically insure success in the arithmetic lesson. Let the *preparation* of the class for recitation be oral, with easy review exercises and questions. Make the *presentation* definite, with the written test-exercise after it for the *generalization*. Question here closely. Secure brief oral explanations of problems for *recapitulation*. And by further questioning bring the *application* home to the children's lives.

17. The fact that a problem is hard is not necessarily a reason for not requiring its solution. Effort is the mountain air of the soul. Difficulty lends interest. Mathematics is the main reliance of modern education to develop carefulness of mind, power of attack, and persistence. Only those boys and girls know the meaning and get the benefit of mathematics who study and master its difficulties.

18. The order of lessons in arithmetic, algebra, and geometry, as usually arranged, involves two or three lessons weekly in arithmetic and one or two in geometry and algebra each. But any one of several other plans may be followed with equal or possibly greater advantage.

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INTRODUCTORY ORAL REVIEW

1. When 3 oranges cost $10\,¢$, how much do 4 dozen cost at the same rate?

2. When milk is $8\,¢$ a quart, how many gallons can be bought for $64\,¢$?

3. John's bicycle cost $\$16\frac{1}{2}$, and his father's cost three times as much, less $50\,¢$. Find the cost of his father's.

4. James gathered 3 pecks of chestnuts one day. On the following day James and Henry together gathered $1\frac{1}{2}$ bu. They put them all into one box. How many quarts of nuts did they gather?

5. Two persons share $\$250$ in the ratio of $\frac{1}{4}$ and $\frac{1}{6}$. What is the share of each?

6. A square garden contains 900 sq. ft. How long is the fence which incloses it?

7. A man who owned $\frac{2}{3}$ of a vessel worth $\$12{,}000$ sold $\frac{3}{4}$ of his share for $\$7000$. How much was his gain?

8. $\frac{3}{5}$ of 15 is $\frac{9}{11}$ of what number?

9. Robert lost $75\,¢$, which was $\frac{3}{7}$ of all he had in his pocket. How much had he at first?

10. 810 is $14\frac{2}{7}\%$ of what number?

11. A wheat field in the form of a rectangle is 80 rods long and $\frac{1}{4}$ as wide. *a*. What is its area? *b*. If it had been in the form of a square, how much less fence would it have required to inclose it?

12. There are 3936 people watching a ball game. $16\frac{2}{3}\%$ of the number are boys. How many boys are there?

13. In an orchard $\frac{1}{5}$ of the trees bear peaches, $\frac{1}{4}$ bear pears, and the remainder, 363, bear apples. How many trees are there in the orchard?

14. A lady has pieces of red, white, and blue ribbon, containing respectively 60, 45, and 65 yards. What is the length of the longest equal strips into which she can cut them?

15. What is the simple interest on \$9500 for 4 months at 6% yearly?

16. A and B engaged in a business in which A invested \$12,000. B received \$5000 of the \$7000 which they gained. How much did B invest?

17. When a dealer sells $\frac{1}{2}$ of a ton of coal for what $\frac{2}{3}$ of a ton cost, what is the gain per cent?

18. When a sight draft on St. Louis for \$6000 sells for \$6150, what is the rate of exchange?

19. Had I paid 7% less for my house, it would have cost me \$350 less than it did. How much did it cost me?

20. The distance between opposite corners of a room is 50 ft. The room is 40 ft. long. How wide is it?

21. A man owning a house worth \$8000 had it insured for $\frac{3}{4}$ of its value at $1\frac{1}{4}\%$. How much was the premium?

22. An agent was paid \$564 as his commission for selling some merchandise. If his rate of commission was 3%, what was the value of the merchandise?

23. What is the bank discount on a 90 days' note of \$1000 when discounted at 6% on the day of date? Find the proceeds of the note.

24. $\frac{5}{6}$ of the number of marbles John has, less 13, equals $\frac{7}{8}$ of the number that Tom has. John has 66. How many has Tom?

25. A pile extends 4 ft. above the water, which is 20 ft. deep. The part above water is $\frac{1}{7}$ of that down in the bed of the river. How long is the pile?

INTRODUCTORY WRITTEN REVIEW

1. What is the cost of a house lot 50 ft. wide by 175 ft. deep at 20¢ per sq. ft.? Compare the cost at the rate of $20 per foot front. Draw the lot to scale, 1 ft. to $\frac{1}{32}$ in.

2. Without allowing for mortar, find how many bricks $8'' \times 4'' \times 2''$ there are in a wall $30' \times 6' \times 1'$.

3. A cubic foot of water weighs $62\frac{1}{2}$ lb. What is the weight of water that fell upon an acre of land to the depth of 2 inches?

4. An agent sells 25 bbl. of molasses, at $13 each, at $2\frac{1}{2}\%$ commission. What will the agent and the owner each receive?

5. A factor sells 250 bbl. of pork, at $15 each; 175 bbl. of beef, at $7 each; and 1456 lb. of butter, at 25¢ a pound; his commission is 3%. What sum should he pay the owner?

6. A man invested $8460 in the Western Manufacturing Co., and afterward sold out at $4\frac{1}{2}\%$ advance. For how much did he sell his stock?

7. Frozen water occupies $7\frac{1}{2}\%$ greater space than liquid water. After it is frozen solid, what is the volume of water that was $8' \times 7' \times 6'$ when above freezing?

8. A housekeeper bought a piece of ice $1\frac{1}{2}$ ft. $\times$ $1\frac{1}{2}$ ft. $\times$ $1\frac{1}{3}$ ft. What was the volume of water after it had melted?

9. Divide:

a. 7149.67 by 16.
b. 22222.222 by 108.
c. 1009.01 by 27.
d. 617149.234 by 121.
e. 91867.13 by 81.
f. 81.82838485 by 103.
g. 16151.712 by 144.
h. 667788.991 by 64.
i. 78998778.998 by 54.
j. 35459.263 by 98.

10. Perform the operation indicated:

a. 10.01 − 6.85.
b. 56.001 − 9.875.
c. 400 − .68954.
d. 307.025 + 96.8465.
e. 1000.21 − 609.7.
f. 3010.5 + 609.785.
g. 4.1025 − .96875.
h. 312 − .607125.
i. 419.600 − .7859.
j. 36.001 + 29.0025.
k. 418.5 + 69.765.
l. 312 − .312.
m. .0785 − .00609.
n. 10 − .680975.
o. 100 − .999.
p. 482.71 − 278.

11. X bought a suit of clothes in London for £1 5*s*. What was its cost value in our money at even rate of exchange?

12. The price of a racing automobile was 48,000 francs. What was this in our money at even rate of exchange?

13. What is the interest of $894.80 for 7 years 3 months 20 days, at 6% per annum?

14. B has 47 bushels of barley worth 73 cents per bushel, and barters it with C for 69 bushels of oats. How much does he allow per bushel for the oats?

15. What is the brokerage on:

a. \$7000 at $\frac{1}{16}\%$?
b. \$8000 at $\frac{1}{8}\%$?
c. \$8700 at $\frac{1}{4}\%$?
d. \$78500 at $\frac{1}{8}\%$?
e. \$19000 at $\frac{1}{4}\%$?
f. \$15000 at $\frac{1}{8}\%$?

16. Z bought oats at \$0.47 per bushel, and sold them at \$0.56. What was the gain per cent?

17. What is the G. C. D. of 1825 and 2555?

18. Multiply:

a. 987.761 by 567.89.
b. 7093.77 by 897.95.
c. 3141.59 by 31.416.
d. 394.959 by 4767.87.
e. 3652.42 by 4596.7.
f. 8237 by 732.8.

19. When X buys a house for \$2700 and sells it for \$3050, what is his gain per cent?

20. Divide:

a. $16 \times 5 \times 10 \times 18$ by $8 \times 6 \times 2 \times 12$.
b. $25 \times 7 \times 14 \times 36$ by $4 \times 10 \times 21 \times 54$.
c. $26 \times 72 \times 81 \times 12$ by $36 \times 13 \times 24 \times 54$.
d. $8 \times 5 \times 3 \times 16 \times 28$ by $10 \times 4 \times 12 \times 4 \times 7$.
e. $8 \times 4 \times 9 \times 12 \times 16 \times 5$ by $4 \times 6 \times 6 \times 8 \times 20$.
f. $6 \times 15 \times 16 \times 24 \times 12 \times 21 \times 27$ by $2 \times 10 \times 9 \times 8 \times 36 \times 7 \times 81$.

21. One train runs 65 mi. in 1 hr. Another runs 30 mi. in 40 min. Find the ratio of these speeds.

22. A house is 30 ft. × 32 ft. × 22 ft., without gables. At 21¢ per square yard, what is the cost of painting it?

23. It is 10 miles around a pond that is nearly a circle. What is the diameter of the pond?

24. A peddler sold at a gain of 25% goods that cost \$874. What was his gain in money?

25. Goods that cost $382.50 were sold at a loss of 4%. What was the selling price?

26. When X buys 28 yd. of cloth at $1\frac{1}{4}$ a yard, and sells it at a loss of 15%, what is the selling price a yard?

27. Z bought 3275 bu. of wheat for $3517.10, and sold the wheat at a loss of 10%. What was the selling price per bushel?

28. Find the prime factors of 4095; 39765; 832; 4793.

29. A bought 15 shares of Gold Valley stock, par value $50, at 2% advance, and sold them at 10% discount. For how much did he sell them; and how much did he lose?

30. X bought 71 shares, par value $100, at $5\frac{1}{2}$% premium. How much did they cost?

31. Perform the operation indicated:

a. 3012600 − 47609.	*g*. 2310020 ÷ 798607.
b. 4001002 × 79086.	*h*. 4136121 − 96008.
c. 706105 + 97608.	*i*. 3100210 − 796084.
d. 4001231 − 67879.	*j*. 5010213 × 608409.
e. 3184012 ÷ 74609.	*k*. 2176003 − 798074.
f. 7120031 − 670984.	*l*. 2160031 + 809765.

32. The tax assessed by a certain town is $140,000. Its property, both personal and real, is valued at $4,756,900, and it contains 2200 polls, which are assessed 50¢ apiece. What per cent is the tax rate? How much is a man's tax who pays for 3 polls, and whose property is valued at $1500?

33. Find the G. C. D. of 3252 and 4248.

34. A farmer sells 125 bu. of corn for 75¢ a bushel. The purchaser sells the corn at an advance of 20%. How much does he get for the corn?

35. A merchant buys 158 yd. of batiste, for which he pays 20¢ per yard. One half gets damaged, and he sells it at a loss of 6%; the remainder he sells at an advance of 19%. How much does he gain?

36. C bought a horse for $93, and sold it for $127. What per cent of profit did he make?

37. A man bought a farm for $6742.50, and sold it for $6000. What was his loss per cent?

38. When B purchases a house for $5700, and a horse for $275, and pays $1987.32 for household furniture and a carriage, and then sells all for $8750, what is his gain or loss per cent?

39. Perform the operations indicated:

a. 51006.08 − 3709.09.	*g.* 123456.7 − 79.8649.
b. 4102.631 ÷ 908.604.	*h.* 3020.101 − 46.089.
c. 321.0012 − 60.8974.	*i.* 21760.03 × 7.06875.
d. 23014.12 × 60.8749.	*j.* 406.0123 − 60.8597.
e. 31002.01 ÷ 3.60875.	*k.* 51080.32 ÷ 5098.76.
f. 4608.032 − 760.879.	*l.* 40010.02 + 567.89.

40. Z paid $7650 for stock at 10% below par, and sold the stock at 10% above par. How much did he gain on it?

41. M bought stock at 10% above par, but was obliged to sell it at 10% below par. He lost $12,500. What did he pay for it?

42. Find the quotients:

a. 4103817128 ÷ 54; 56; 63. *c.* 7843155120 ÷ 24; 72; 168.
b. 9324063792 ÷ 72; 84; 96. *d.* 3246789409 ÷ 44; 176; 264.

43. At 7% what is the amount of $210.25 for 2 yr. 7 mo. 20 da.?

44. Multiply:

a. 4936.02 by 809.0102.	*d*. 97.654 by 780.65.
b. 7.09020 by 300700400.	*e*. 409.7684 by .049536.
c. 4709.123 by 70770.	*f*. 837.621 by 897.95.

45. K sold a horse for $145 at a gain of 9% on the sale. What did the horse cost him?

46. What did M pay for tiles which he sold for $12 per 1000, gaining 31%?

47. Resolve 8862777 into its prime factors.

48. When 1 lb. of butter costs $22\frac{3}{4}$¢, what do 57 lb. cost?

49. $36x - 320 = 10(8 - 4)$. Find the value of x.

50. Draw a line perpendicular to another line, from a point outside of it.

51. Find the square of: 12; 16; 40; 55; 81; 90.

52. Trisect an angle of 90°.

53. How many marble slabs, 15 in. square, does it take to pave a floor 32 ft. long, and 25 ft. wide? What is the cost at $3 a square yard for the marble, and 40 cents a square yard for the labor?

54. What is the time, when the time past noon is four fifths of the time to midnight?

55. When an iron bar 5 ft. long, $2\frac{1}{2}$ in. broad, and $1\frac{3}{4}$ in. thick, weighs 45 lb., how much does a bar of the same metal weigh, that is 7 ft. long, 3 in. broad, and $2\frac{1}{4}$ in. thick?

56. A grocer bought a certain number of eggs at the rate of 4 for 3 cents, and sold them at the rate of 5 for 4 cents, by which he made 4 cents. What did he pay apiece for the eggs? What did he make on each egg sold? How many did he sell to gain 4 cents?

57. How many planks 15 ft. long and 15 in. wide, will floor a barn $60\frac{1}{2}$ ft. long and $33\frac{1}{2}$ ft. wide?

58. A gentleman gave $160 for a wagon, which was $\frac{2}{5}$ times $2\frac{2}{3}$ as much as he paid for a horse. How much did he pay for the horse?

59. A boy bought a certain number of oranges, at the rate of 3 for 2 cents; and paid for them with apples, at the rate of 5 for 2 cents. How many oranges did he buy, providing it took 120 apples to pay for them?

60. A gas company charging $1.10 per M for gas allows 10¢ per M for cash payment. What was B's bill, who used 15,000 ft. in 1 month? What was the total discount for cash payment?

61. A physician made 40 house calls and had 28 office patients in one day. His fees were $2 for outside and $1 for office patients. He discounted from $\frac{1}{2}$ of the total 20% for "family practice," and failed to collect 10% of all the day's business. What were his net receipts for that day?

62. When one pays $4.95 for $8\frac{3}{4}$ bu. of potatoes, how much does one pay per bushel?

63. From $\frac{1}{2}$ of $\frac{3}{4}$ of a day take $\frac{1}{3}$ of $1\frac{1}{2}$ hr.

64. Five persons divide $100 as follows: the first takes $\frac{1}{7}$ of $\frac{3}{4}$ of the $100; the second takes $\frac{1}{6}$ of $\frac{3}{4}$ of the remainder; the third takes $\frac{1}{5}$ of $\frac{3}{4}$ of the remainder; the fourth takes $\frac{1}{4}$ of $\frac{3}{4}$ of the remainder; and the fifth has what is left. How much does each person get?

65. What is $\frac{1}{8}$ of $\frac{3}{4}$ of $\frac{4}{7}$ of 13 wk. + $\frac{1}{2}$ of $\frac{1}{5}$ of 30 da.?

66. When D buys 113 lb. 13 oz. of butter, at $10\frac{1}{2}d.$ per pound, and uses 30 lb., for how much per pound must he sell the remainder in order to get back the cost of the whole quantity of butter?

67. At the rate of 45 mi. an hour, how long would it take a railroad train to go from Alaska to Argentina, a distance of 10,000 mi.?

68. When a certain quantity of provisions lasts 25 men 2 mo. 3 wk. 6 da., how long will it last 10 men?

69. When 112 lb. of cheese cost £2 16*s*., at the same rate how much should be paid for 1 lb.?

70. What is the cost of 1426 lb. of salt, at $9.75 a ton?

71. What are the freight charges for 3840 lb. from X to Y, at $4.50 a ton?

72. G bought 124 bbl. of turnips, each barrel containing $2\frac{1}{4}$ bu., at $33\frac{1}{3}$¢ a bushel. What was the whole cost?

73. At 1.33\frac{1}{3}$ a rod, what does it cost to build a fence 96 rd. long?

74. S wishes to put an equal part of 1066 bu. 2 pk. of potatoes into each of 474 bbl. What quantity must he put into each barrel?

75. How many packages, each holding 3 lb. 10 oz., can be made up from a chest of tea containing 58 lb.?

76. When the forward wheels of an automobile are 12 ft. in circumference, and the rear wheels are $16\frac{1}{2}$ ft., how many more times do the forward wheels turn round than the rear wheels, in running 264 mi.?

77. When a section of land is 9 mi. long, $4\frac{1}{3}$ mi. wide, how many farms of 192 acres each can be made out of it?

78. How many yards of carpeting $\frac{3}{4}$ yd. wide are required to cover a floor 27 ft. long and $17\frac{1}{2}$ ft. wide?

79. When $750 is the value of $\frac{5}{12}$ of a city lot, at the same rate what part of the lot is worth $75?

80. After having spent $\frac{1}{5}$ of his money in one store and $\frac{1}{4}$ of it in another store, Z had left 77¢. How much money had he at first?

81. Reduce $2\frac{1}{2} + 3\frac{1}{3} - 5\frac{1}{6} + 4\frac{3}{5} - 2\frac{7}{10}$ to a single fraction.

82. Simplify $2\frac{1}{2} \times \frac{3}{4} + 5\frac{7}{12} - \frac{3}{7} \times 5\frac{3}{5} - 2\frac{2}{2}\frac{3}{4}$.

83. A man paid $\$543\frac{3}{4}$ for a city lot, at $\$21\frac{3}{4}$ a square foot. How many square feet were there in the lot?

84. At $\$1\frac{2}{5}$ each, how many books can one buy for $\$37\frac{1}{10}$?

85. When $\$2\frac{3}{5}$ is paid for $5\frac{1}{5}$ bushels of corn, what is the cost of a bushel at the same rate?

86. When $2\frac{1}{2}$ apples are worth $3\frac{1}{3}$ cents, how much is 1 apple worth?

87. When $2\frac{1}{2}$ apples are worth $3\frac{1}{3}$ cents, what part of an apple can one get for 1 cent?

88. When 3 horses eat $3\frac{3}{5}$ bushels of oats in a day, how many horses would $8\frac{2}{5}$ bushels supply for the same time?

89. When 1 pt. of molasses cost $9\frac{2}{3}$¢, what do 23 pt. cost?

90. Multiply 9.235 by .48.

91. X owned thirty-four hundredths of a vessel, and sold thirty-four thousandths of his share. How much of the vessel did he then own?

92. Seventy-five times a number is three millions. What is the number?

93. Divide one fifth of a million by $5 \times 6 \times 7$.

94. How often can eighty-five be subtracted from a hundred thousand, and what is the final remainder?

95. Find the quotient and the remainder when one million five hundred thousand is divided by nine hundred thirty-three.

96. What is the three hundred twenty-third part of three hundred nineteen thousand eighty-four?

97. What is the dividend if the quotient is 563 and the divisor 192?

98. 9 yd. is what per cent of 870 yd.?

99. Square 1234, and prove the result by finding the square root of the square.

100. If 12 men can dig a trench in 16 days, how many men will be required to dig a trench 9 times as long in half the number of days?

101. In your own name, draw on the Winston National Bank a check for $45.60, payable to the order of Joseph H. Chase. Write place and date.

102. Draw your note for 60 days, at 6%, payable to the same person at the same bank.

103. Write in words: 21 439 876 298 765.

104. One angle of a triangle is 66°; another is 76°. What is the measure of the third angle?

105. Reduce to meters 81.765 kilometers.

106. On a note for $1370 at 6%, payable semi-annually, drawn Aug. 1, 1899, were endorsed three payments: Feb. 1, 1900, $300; Aug. 1, 1901, $650; and April 15, 1902, $500. What was due May 1, 1902? What was the total amount of interest earned in the whole period?

107. Write a draft on London for £1000, and draw a check to pay for it, exchange at $4.87.

108. Add:

428	
1352	
7960	
18	
705	
288	
3030	
1716	

109.

8531
949
154
25
1314
6070
2663
13

110.

909
9800
4333
4633
22806
5055
623
7

INTEREST REVIEWS

Finding the Interest

six per cent method

1. What is the interest on $67 for 2 yr. 3 mo. 7 da., at 6%?

Interest on $1 for 2 yr. $= \$.06 \times 2 = \$.12$
Interest on $1 for 3 mo. $= \$.005 \times 3 = \$.075$
Interest on $1 for 7 da. $= \$.000\frac{1}{6} \times 7 = \$.0011$
$\$.1361$

$\$.1361$
67
9527
8166
$9.1187 interest on $67 for 2 yr. 3 mo. 7 da.

Since the interest on $1 is $.1361, the interest on $67 is $.1361 multiplied by 67.

What is the interest of

2. $576.60, at 6%, for 10 mo. 18 da.?
3. $854.42, at 6%, for 3 mo. 9 da.?
4. $1153.20, at 6%, for 11 mo.?
5. $2306.54, at 5%, for 1 yr. 7 mo. 28 da.?
6. $4272.10, at 5%, for 1 yr. 10 mo. 28 da.?
7. $3689.45, at 7%, for $10\frac{1}{2}$ yr.?
8. $2245.96, at 6%, for 7 yr.?
9. $675.89, at 8%, for 3 yr. 6 mo. 6 da.?
10. $12,324, at 8%, for 3 yr. 4 mo.?
11. $15,328.20, at 9%, for 4 yr. 7 mo.?
12. $69,450 for 1 yr. 2 mo. 12 da. at 9%?

13. On a note for $1500, what simple interest was due at the following rates for the following times, viz.:

a. 6% 9 mo. 15 da.?
b. 5% 7 mo. 8 da.?
c. $4\frac{1}{2}$% 8 mo. 12 da.?
d. $5\frac{1}{2}$% 1 yr. 4 mo.?
e. $3\frac{1}{2}$% 100 da.?
f. 12% 93 da.?

Finding the Amount

1. Find the amount of $524.36 for 2 yr. 5 mo. 18 da. at 7%.

What is the amount of

2. $106.06 for 4 yr. 8 mo. 19 da. at 5%?
3. $482 for 2 yr. 3 mo. 11 da. at 6%?
4. $68.59 for 11 yr. 5 mo. 4 da. at 6%?
5. $137.50 for 3 yr. 7 mo. 8 da. at 7%?
6. $180.15 for 2 yr. 9 mo. 11 da. at 5%?
7. $132.05 for 4 yr. 9 mo. 20 da. at 9%?

Find the amount at simple interest of:

8–9. $100 for 1 yr. 7 mo. 12 da. at $5\frac{1}{2}$%; at 7%.
10–11. $125 for 3 yr. 6 mo. 25 da. at $4\frac{1}{2}$%; at 6%.
12. $1165.50 for 5 yr. 3 mo. 9 da. at 7%.
13. $714.71 for 3 yr. 7 mo. 11 da. at 6%.
14. $13.20 for 4 yr. 9 mo. 12 da. at 6%.
15–16. $20.20 for 8 yr. 8 mo. 8 da. at $5\frac{1}{2}$%; at 6%.
17–18. $25 for 3 yr. 6 mo. 20 da. at 6%; at 8%.
19. $190.50 for 3 yr. 4 mo. 10 da. at 5%.
20. $680 for 4 yr. 4 mo. at 4%.
21. $300 for 3 yr. 8 mo. at 6%.
22. $250 for 1 yr. 7 mo. at 6%.
23. $205.25 for 2 yr. 8 mo. 15 da. at 6%.
24. $150.62 for 3 yr. 5 mo. 12 da. at $4\frac{1}{2}$%.
25. $210.25 for 2 yr. 7 mo. 20 da. at 7%.
26. $57.85 for 2 yr. 3 mo. 23 da. at 5%.
27. What is the amount of $1960 for 4 years at 7%?
28. What is the amount of $3675 for 3 years at 7%?
29. What is the amount of $459 for 5 years at 8%?
30. What is the amount of $375 for 2 years at 7%?
31. What is the amount of $4049.87 for 2 years at 5%?

32. What is the amount of $1119.48 after 2 years and 6 months at 7%?

33. What is the amount of $98.75 from Aug. 3, 1899, to Jan. 1, 1902, at 5%?

34. On May 3, 1898, Mr. A borrowed of Mr. B $950, at 6% simple interest. Since the interest remained unpaid until Oct. 15, 1901, how much money was needed to cancel the obligation at that date?

35. If a man should lend $897 at 5% interest on Sept. 17, 1902, and receive no interest until March 3, 1907, how much would then be due him?

36. What is the difference between the interest on $2750 at 7% for 2 yr. 7 mo. 18 da. and the amount of $978 at 6% for 7 yr. 9 mo. 15 da.?

37. Mr. W loaned $3760 to Mr. S on July 1, 1902, at 5%. If no interest is paid in the meantime, what sum will be necessary to settle the account Nov. 5, 1905?

Partial Payments

United States Rule

1. Jan. 1, 1899, X, Y, Z & Co., on a note on demand, borrowed of J, silent partner in the firm, $10,000 at 6% interest, payable bi-monthly. They made payments as follows, viz.:

March 1, 1899, $100. May 1, 1899, $600. Oct. 1, 1899, $200. Jan. 1, 1900, $4000. Oct. 15, 1900, $200. Jan. 1, 1901, $1000. What amount was due May 1, 1902?

2. Mr. S borrowed $3000 of Mr. X, on a note at 5% interest, payable semi-annually. He paid $100 in 6 mo., $500 in 1 yr., $1000 in 1½ yr., $300 in 2 yr., and $800 in 3 yr. How much was due in 3½ yr.?

3. Write and solve similar problems.

COMPOUND INTEREST REVIEW

SAVINGS BANK RATES

3 per ct.	3½ per ct.	4 per ct.	Yrs.
1.0300 0000	1.0350 0000	1.0400 0000	1
1.0609 0000	1.0712 2500	1.0816 0000	2
1.0927 2700	1.1087 1787	1.1248 6400	3
1.1255 0881	1.1475 2300	1.1698 5856	4
1.1592 7407	1.1876 8631	1.2166 5290	5
1.1940 5230	1.2292 5533	1.2653 1902	6
1.2298 7387	1.2722 7926	1.3159 3178	7
1.2667 7008	1.3168 0904	1.3685 6905	8
1.3047 7318	1.3628 9735	1.4233 1181	9
1.3439 1638	1.4105 9876	1.4802 4428	10
1.3842 3387	1.4599 6972	1.5394 5406	11
1.4257 6089	1.5110 6866	1.6010 3222	12
1.4685 3371	1.5629 5606	1.6650 7351	13
1.5125 8972	1.6186 9452	1.7316 7645	14
1.5579 6742	1.6753 4883	1.8009 4351	15
1.6047 0644	1.7339 8601	1.8729 8125	16
1.6528 4763	1.7946 7555	1.9479 0050	17
1.7024 3306	1.8574 8920	2.0258 1652	18
1.7535 0605	1.9225 0132	2.1068 4918	19
1.8061 1123	1.9897 8886	2.1911 2314	20

Savings banks allow their depositors interest on interest due but not paid to the depositors. This table shows the amounts of $1 on which interest at ordinary savings bank rates has accumulated for periods up to 20 years.

1. Find the amount of $395 for 16 yr. at 3½%.

2. A sailor left $1365 in a savings bank that paid 3 per cent interest. He did not return till 19 yr. had passed. What amount was then to his credit?

3. A widow deposited $1000 in bank at 4% interest, and drew out $100 a year. How long did her account last?

4. A boy saved every week $1, and put it in bank at 4%. If the interest was credited semi-annually, how much money did he have in 3 yr.?

5. A man of wealth deposited $2000 in each of 15 banks, 5 paying 3%, 5 paying 3½%, 5 paying 4%. Ten years after he called for his money at each bank. What total amount did he receive?

Many savings banks do not accept deposits of over $3000.

6. A deposited $700 in a savings bank, at $3\frac{1}{2}\%$ compound interest; B deposited $600, at 4% compound interest; and C loaned $500, at 6% simple interest. What was due each at the end of five years?

7. Compare the interest of $500 at 3%, $3\frac{1}{2}\%$, and 4% compound interest for the terms of 6, 10, 15, and 20 years.

8. The bank book of a depositor in a savings bank paying 4% interest, compounded quarterly in January, April, July, and October, showed these deposits and drafts, viz.:

Deposits	Dr.	Drafts	Cr.
Jan. 10, 1888	$5	Oct. 1, 1888	$8
Mar. 1, 1888	15	Nov. 10, 1889	25
July 3, 1888	20	July 3, 1893	5
Dec. 5, 1888	5	Aug. 3, 1893	10
Feb. 10, 1889	15	Sept. 3, 1893	5
Mar. 15, 1889	5	Oct. 3, 1893	10
Dec. 10, 1889	40	Feb. 15, 1894	50
Jan. 5, 1890	25	May 15, 1894	10
July 10, 1890	30	Sept. 1, 1901	75
Sept. 20, 1890	10		
Dec. 30, 1890	15		
Jan. 1, 1900	200		
April 4, 1902	100		
June 1, 1902	40		

Find the balances to the credit of the depositor July 1, 1889; July 1, 1895; and July 1, 1902.

Money deposited in a savings bank draws no interest till the beginning of the next interest-term after date, and money drawn out draws no interest after the end of the preceding interest-term.

9. Get savings bank books and make up the balances.

10. The Bank of Amsterdam paid 12% compound annual interest in 1626 when Governor Peter Minuit bought Manhattan Island for $24. From that time to now the rate has averaged 8%. If the money had been placed at interest, to how much would it have amounted now, *a.* at 12%; *b.* at 8%? Manhattan Island is worth nowadays $5,000,000,000. Discuss these sums.

Division by Factors

1. Divide 71469 by 35.

The factors of the dividend are 5 and 7. Dividing by 5, we get a remainder, 4, and dividing the first quotient by 7, we get a remainder, 6. To get the true remainder, we multiply 6, the last remainder, by 5, the first divisor, and add 4, the first remainder, to the product.

$35 = 5 \times 7.$

5)71469

7)14293 . . 4 = 1st rem.

2041 . . 6 = 2d rem.

$6 \times 5 = 30 + 4 = 34.$

$\frac{34}{35}$.

2. $807906 \div 84 = ?$
3. $357892 \div 96 = ?$
4. $92873 \div 144 = ?$
5. $28764 \div 225 = ?$

Multiplication by Factors

I. To multiply by 25, multiply decimally by 100 and divide by 4; or, multiply by 5×5.

6. $83765 \times 25 = ?$
7. $\$3872 \times 25 = ?$
8. $189376 \times 25 = ?$
9. $364832 \times 25 = ?$

II. To multiply by 125, multiply by $5 \times 5 \times 5$.

10. $39765 \times 125 = ?$
11. $\$149 \times 125 = ?$
12. $38765 \times 125 = ?$
13. $976425 \times 125 = ?$

III. To multiply by 625, multiply by $5 \times 5 \times 5 \times 5$.

14. $2875 \times 625 = ?$
15. $2932 \times 625 = ?$
16. $876 \times 625 = ?$
17. $1796 \times 625 = ?$

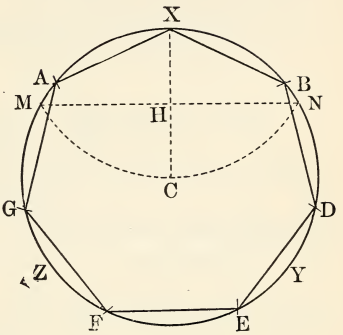
IV. To multiply by $33\frac{1}{3}$, multiply decimally by 100 and divide by 3.

18. Multiply by $33\frac{1}{3}$: 8962; 237698; \$18347261.

There are many similar "short processes" involving factors or decimals and factors combined.

To inscribe a regular heptagon in a circle

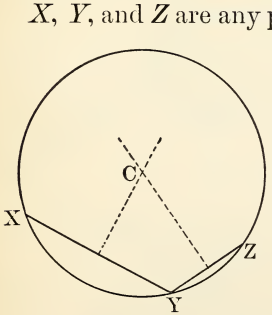
Draw any circle XYZ with any radius CX. From X as a center with the radius CX draw the arc ACB. Draw the chord MN. H is the point of intersection of XC and MN. MH is the length of the side of the inscribed heptagon. Mark on the circumference XYZ the lengths XB, BD, etc., equal to MH.



The figure $AXBDEF$ is a heptagon inscribed in the circle XYZ.

To draw a circumference through any three points not in a straight line

X, Y, and Z are any points, not in a straight line, through which a circumference is to be drawn.



Draw XY, a straight line, through X and Y; and YZ through Y and Z.

Draw perpendiculars from the middle points of XY and YZ. They intersect at C. From C as a center with the radius CY draw a circumference. It passes through X, Y, and Z.

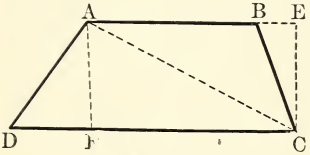
By a similar process we may *circumscribe a circumference about a triangle:* and *find the center of a given circumference.*

The area of a *square* = *square of one side*

The area of a *rectangle* = *length* × *breadth*

The area of a *parallelogram* = *base* × *altitude*

The area of a *triangle* = $\frac{base \times altitude}{2}$



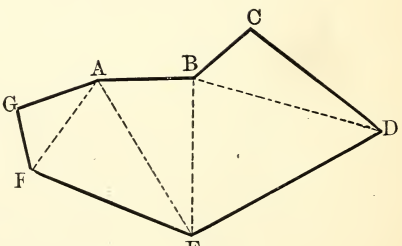
TRAPEZOID.

We find the areas of many-sided figures by dividing them into triangles. Divide $ABCD$ into two triangles, ABC and ACD. Then the area of $ABC = \frac{AB \times EC}{2}$, and the area of $ACD = \frac{DC \times EC}{2}$. But the area of $ABCD$ = the area of ABC + the area of $ACD = \frac{AB \times EC + DC \times EC}{2} = \frac{AB + DC}{2} \times EC$. The area of a *trapezoid* = $\frac{sum\ of\ bases}{2}$ × *altitude*.

To divide polygons into triangles

Polygons are figures of many sides.

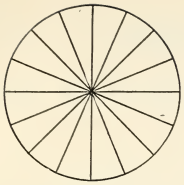
The area of a polygon may be found by finding the areas of all the triangles into which it may be divided. The area of the polygon $ABCDEFG$ equals the sum of the areas of the triangles AGF, AFE, AEB, BDE, and BCD.

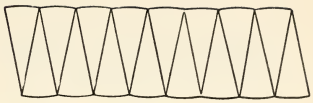


IRREGULAR POLYGON.

Dividing polygons or great surfaces into triangles is called *triangulation*. The land of all the United States has been measured by triangulation.

Cut these figures out of paper or cardboard.





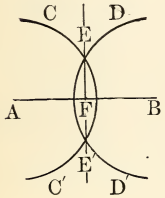
A Circle Triangulated.

To bisect a line

In geometry by *line* we mean straight line.

To bisect a line is the same as to find the point equally distant between two other points.

Open the compasses until the spread is a little greater than half the distance between the ends of the line AB; that is, the points A and B. From A as the center draw an arc CC', cutting the line AB. From B as a center draw an arc DD'. These two arcs will cut each other at E and E', since their radius is more than half of AB. Between the points E and E' draw a straight line connecting them, and cutting the line AB at F. E and E' at every point are equally distant from A and B. F is one point in that line. F is equally distant from A and from B, and divides AB into two equal parts.



Original Exercises

1. Circumscribe a regular heptagon about a circle.

2. Draw circumferences through the three angles of a triangle, (*a*) with three acute angles, (*b*) with one right angle, (*c*) with one obtuse angle.

3. By triangulation find the area of a heptagon drawn to scale $\frac{1}{4}$ in. to 1 ft.

4. Bisect various lines drawn on the blackboard.

ALGEBRA

Algebra employs letters, or figures and letters combined, to represent numbers.

An important use of *algebra* is in solving problems by simpler methods than in *arithmetic*. To accomplish this simplification *algebra* employs *factoring* and the *equation* very extensively.

I. When $a = 2$, $b = 3$, $c = 4$, $d = 5$, $e = 6$, find the values of :

1. $a + b$.
2. $a + c$.
3. $b + e + d$.
4. $b + c$.
5. $a + d$.
6. $a + c + d$.
7. $c + d$.
8. $a + e$.
9. $b + c + d$.

II. When $a = 20$, $b = 11$, $c = 9$, $d = 7$, $e = 5$, find the values of :

1. $a - b$.
2. $b - c$.
3. $a - c$.
4. $b - d$.
5. $a - d - c$.
6. $a - b - d$.

$a \times b \times c$, $a \cdot b \cdot c$, and abc all mean that a is to be multiplied by b, and that the result is to be multiplied by c, so that when $a = 2$, $b = 3$, and $c = 4$, $abc = 24$.

abc may be written acb, bca, bac, cba, or cab, since it does not matter in what order the numbers are multiplied together : $2 \times 3 \times 4 = 24$; $2 \times 4 \times 3 = 24$; $3 \times 4 \times 2 = 24$; $3 \times 2 \times 4 = 24$; $4 \times 3 \times 2 = 24$; $4 \times 2 \times 3 = 24$.

The number placed in front of a quantity shows how many times that quantity is to be added to itself. This number is called a **coefficient**. In $4a$, 4 is the *coefficient* of a, and $4a$ means $a + a + a + a$.

When no number is prefixed, 1 is always understood.

ab means the same as $1ab$.

III. When $a = 8$, $b = 4$, $c = 3$, $d = 2$, find the numerical values of the following expressions:

1. $7a - 5b$.
2. $3a + 2b + 4c$.
3. $a - b + c$.
4. $8a - 3b + 2c$.
5. $5a + 7b - 10d$.
6. $ab - 3cd$.
7. $ac - 3bd$.
8. $abcd - 6bcd$.

IV. When $a = 8$, $b = 4$, $c = 3$, $d = 2$, find the numerical values of the following expressions:

1. $2a + 3b$.
2. $7a + 2b$.
3. $4a + 3b$.
4. $9a - 6b$.
5. $7b - 5c$.
6. $8c - 6d$.
7. $5a + 4b + 2c$.
8. $8a + 5b + 3c$.
9. $a + 2b + 3c$.
10. $8b - 2c - 4d$.
11. $3a - 2b - 3c$.
12. $5b - 3c - d$.
13. $5a - 7b + 5c$.
14. $6a + 9b - 4c$.
15. $3b - 4c + 2d$.
16. $10a - 2b - 2c$.
17. $12b - 2c - 3d$.
18. $15a - 2b - 5d$.
19. $a - b + c + d$.
20. $a + b - c + d$.
21. $a - b - c + d$.
22. $3a - 2b - 3c + 6d$.
23. $7a - 3b + 4c - 2d$.
24. $10a + 10b - 5c - 5d$.

V. When $a = 10$, $b = 8$, $c = 5$, $d = 4$, find the numerical values of the following expressions:

1. $\frac{4ad}{bc}$. $\qquad \frac{4 \times 10 \times 4}{8 \times 5} = \frac{160}{40} = 4.$
2. $\frac{6a + 3d}{b - c}$. Why is this the same as $\frac{3(2a + d)}{b - c}$?
3. $\frac{a}{c} - \frac{b}{d}$.
4. $\frac{a}{b - d} - \frac{a}{c}$.
5. $\frac{3b}{6d} + \frac{bc}{ad}$.

VI. When $a = 24$, $b = 8$, $c = 4$, $d = 6$, $e = 2$, find the numerical values of the following expressions:

1. $\frac{3b}{2c}$.
2. $\frac{4a}{2b}$.
3. $\frac{ab}{cd}$.
4. $\frac{ac}{bd}$.
5. $\frac{3bc}{2de}$.
6. $\frac{9be}{3cd}$.
7. $\frac{a+b}{c}$.
8. $\frac{b+e}{e}$.
9. $\frac{3a+2b}{5c-3d}$.
10. $\frac{10b-2e}{2d+3e}$.
11. $\frac{a}{b}+\frac{b}{e}$.
12. $\frac{3a}{d}+\frac{6b}{a}$.

I. Find the value of x in these expressions:

1. $\frac{3x}{4} = 9.$

   $3x = 36,$

   $x = 12.$
2. $\frac{5x}{8} = 10.$
3. $\frac{7x}{11} = 14.$
4. $\frac{3x}{2} = 9.$
5. $\frac{4x}{3} = 12.$
6. $\frac{5x}{3} = 20.$
7. $\frac{7x}{5} = 14.$
8. $\frac{9x}{7} = 18.$
9. $\frac{10x}{4} = 15.$

II. Find the value of x in the following expressions:

1. $\frac{7x}{3} = 2x + 2.$

   Multiply each member of the equation by 3.

   $7x = 6x + 6,$

   Subtract $6x$ from each member.

   $x = 6.$
2. $\frac{4x}{x+1} = 3.$

   Multiply each member of the equation by $x + 1$.

   $4x = 3x + 3.$
3. $\frac{5x}{x-1} = 10.$
4. $\frac{3x+2}{x+4} = 2.$

5. $\frac{x^2}{2} = 8.$

6. $\frac{x^2}{3} = 12.$

7. $\frac{x+5}{x-5} = \frac{7}{2}.$

8. $\frac{x+36}{x} = 4.$

9. $6x - 17 = x + 13.$

10. $6x + 17 = 20.$

11. $5x - 10 = 14 - x.$

12. $5x - 15 = 2x + 6.$

13. $20 - 7x = 3x - 40.$

14. $16 + 8x = x + 17.$

15. A merchant had on deposit in a bank a sum of money equal to 3 times as much less $8692.84. How much had he in the bank?

Let $x =$ the sum of money on deposit,

then $x = 3x - \$8692.84.$

16. A farmer raised 7 times as many chickens less 15 in the second year he kept them than in the first, when he raised 26. How many did he raise the second year?

17. Find a number equal to 5 times itself less 64.

18. 12 plus a certain number equals $\frac{1}{2}$ the number multiplied by 4.

19. A life insurance company wrote in its tenth year 20 times as much insurance as in its first year and $1,200,000 more. In its tenth year it wrote $25,700,000. What amount of insurance did it write in its first year?

20. There were in a certain army 5 times as many infantry as cavalry plus 500, and twice as many cavalry as artillerymen less 200. Of the cavalrymen there were 2500. How numerous were the others?

21. A gardener wished to mix grass seed as follows, viz.: twice as much timothy as clover, and twice as much Kentucky blue grass as timothy, so as to have in all 50 lb. How many pounds of each did he need?

LIFE INSURANCE

In **life insurance**, corporations or associations of men agree to pay stated sums of money to the heirs or assigns of those that die. In consideration of this agreement the insured pay stated **premiums** at a rate per cent determined by reference to the **expectation of life** at the age of taking the insurance.

So many millions of individuals have lived and died that we now can say of a hundred men of a certain age how old on the average they will probably be at death. The number of years of life a man may thus anticipate is called the *expectation of life.*

The premiums pay three kinds of items: death claims, the expenses of running the corporation or association, and reserve or surplus amounts to promote the permanence of the enterprise by increasing its wealth.

Often companies pay back to their policy holders a portion of the reserve element in their premiums: a few companies have lower expenses than they have charged in the premiums, and make a profit there so as to distribute this also to their policy holders.

The contract on the part of the association to pay the claim in case of death and on the part of the insured to pay the premiums until death is the **policy**.

There are various modifications of this contract. The discussion here is of the standard "legal reserve life" policy for $1000 of mutual insurance companies.

Age	Mortality	Reserve	Expense	Premium
25	$7.41	$7.31	$5.78	$20.50
30	8.02	8.95	6.33	23.30
35	8.83	11.04	7.23	27.10
40	9.82	13.86	8.52	32.20
50	15.	20.78	12.72	48.50
60	28.29	29.27	22.34	79.90

A favorite form of policy is the twenty year endowment contract. The insured pays the premium for 20 years. If he dies before then, his beneficiary or heir gets the insurance. If he lives, he gets it.

Age	Mortality	Reserve	Expense	Premium
25	7.25	30.14	11.31	48.70
30	7.84	30.11	11.65	49.60
35	8.65	30.15	12.10	50.90
40	9.64	30.57	12.79	53.
50	14.81	31.84	15.35	62.
60	28.12	34.32	23.06	85.50

The tables given are those used by the largest American companies, averaged. Each company has slight variations from all other companies.

1. A man, 30 yr. of age, insured his life for $5000 for the benefit of his wife and children. He paid the standard life premium for 28 yr. and then died. How much money had he paid in premiums?

2. If this same man had each year put his premium money in a savings bank at 4% interest, what amount would he have left in the bank for his family?

3. A young man, 25 yr. of age, insured his life for his mother's benefit by a policy for $2000. He paid it for 3 yr. and died. How much more did the mother receive in insurance than her son paid in premiums?

4. A man, 35 yr. old, took out a 20 yr. endowment policy for $10,000 and paid its premiums for the full period. How much in all did he pay? How much did he receive?

In some States, if a policy holder of a company, after paying premiums for three years, becomes unable to pay any longer, he gets by law a policy good for a certain proportion of the face of the "paid-up" policy.

5. Two men, each 35 yr. old, took insurance of $10,000. One took life insurance. What was his annual premium? The other took endowment insurance. What was his premium? What advantage do you see in each form of insurance?

6. Mr. Stone, 30 yr. old, took out a life insurance policy for $8000. Mr. Williams, the agent, received 30% of the first year's premium as his commission for getting the policy. *a*. What did Mr. Stone pay? *b*. What did Mr. Williams receive? *c*. What was the company's net amount?

7. At the age of 39 Mr. Stone died. *a*. What was the total amount he had paid in premiums? *b*. If he had invested the money each year at 5% simple interest, to how much would the total have amounted?

8. Mr. Baldwin from the time he was 35 yr. old gave his wife every year for deposit in a savings bank at 4% compound interest $100. Also every year he paid the premiums on a life insurance policy for $4000. He died at 60 years of age. *a*. How much in all had he paid for the money his widow received from the life insurance company? *b*. What amount was to her credit in the savings bank?

9. A at the age of 25 took a twenty year endowment policy for $3000; B at the same time, at the age of 40, took a similar policy. Each policy came due twenty years later. *a*. How much had A's policy cost him? *b*. How much had B's cost?

10. C took a life insurance policy of $5000 at the age of 30 and paid its premium annually for 42 years. *a*. How much had he paid in all? *b*. In how many years did the total of his premiums equal the face of the policy?

THE STOCK MARKET

A **market** is an assembling of persons to buy and sell articles of value.

A **stock market** engages in buying and selling shares of stock and bonds of corporations.

A **corporation** is a body of persons empowered by law to engage in business. It is a legal "person," and has all the rights of a real person, with additional advantages: — of having a continual succession of stockholders so that it does not die unless it becomes bankrupt and is not reorganized; of being able to combine the wealth of many persons; and of having its losses, if any, limited in the case of each stockholder to the wealth he has invested.

Corporations are discussed at length, but more simply, in Book Six of this Series. In view of the very large number of persons now employed by business corporations and of persons who have invested in them, it is important to understand dealings in stocks and bonds. It is well to discuss in class such corporations as may be known locally or personally, even when this section of the book is not worked out arithmetically by the pupils because of its difficulty. The public welfare demands that elementary school graduates know something of these greatest business enterprises in the history of the world.

The **par value** of a share of stock in a corporation is its face value; that is, the amount that the share represents. The par value of many shares is $100. The par value of a share of stock in a mining company is usually $1. The par value of a bond, which is a mortgage on a corporation's property, and which is a security prior to a share, is generally $100 or $1000.

The income from a share of stock is its **dividend.** When a corporation has paid for all the materials that it has used, for all the labor and services that it has employed, the interest on all the money that it has borrowed, its taxes,

and all other expenses, then what remains as the profit of the business is *divided*, usually every three or six months, among the shares of the stock.

Often there are two kinds of stock, **preferred** and **common**. The dividends up to a certain per cent are paid first upon the *preferred* stock. The *common* stock takes what is left. When profits are large, dividends on common stock may be larger than on preferred.

The **market value** of stock is the price at which it is bought or sold. The **price asked** is usually a little higher than the **price bid**. When the price is higher than par, the stock is **above par** or at a **premium**; when lower, the stock is **below par** or at a **discount**.

1. Jan. 10, 1902, there appeared in the daily newspapers the following statement. Discuss each of its items.

BRIEF BALANCE SHEET, X-Y-Z-COMPANY

EXPENDITURES

Operating expenses		$6,279,247.04
Wages	$2,496,892.32	
Materials, including improvements	3,142,926.43	
Incidentals, including depreciation of plant and equipment	639,428.29	
Interest on 5% bonds ($1,200,000 outstanding)		$60,000.00
Dividends on 7% preferred stock ($2,000,000 outstanding)		140,000.00
Dividends on common stock ($1,500,000) at 6%		90,000.00
Cash on hand		178,237.51
		$6,747,484.55

RECEIPTS

Balance	$100,000.00
For merchandise	5,938,629.87
Bills receivable	663,492.13
Interest on bonds L.M.N. Co.	40,000.00
Other income	5,362.55
	$6,747,484.55

a. That day on the stock market the bonds of the Company were quoted at 110. At that price what was the entire issue worth?

b. The preferred stock was quoted at 108. What was the market value of 45 shares?

c. The common stock was quoted at 90. What was the market value of 380 shares?

Capitalists with $100 or $100,000,000 to invest look for two things: the **security** of their investment, which means their ability to get their money back when they want it, and the **per cent of income** it receives. Upon this principle explain the differences in the above market values.

1. *a.* The market value of a share of the common stock of the Atchison, Topeka & Santa Fe R.R. varied within the week ended Sept. 7, 1901, $7\frac{1}{2}\%$. The amount of stock was $102,000,000. What was the apparent difference in its market value within that week?

b. At the close of the week its market value was $72\frac{7}{8}$. What was the apparent value of all the stock?

c. During the week 141,575 shares were bought and sold. At the average price of $76\frac{1}{2}$, what was the value of these shares?

d. Of the preferred stock, amounting to $114,199,500, the closing market value was $94\frac{3}{8}$. What was the whole issue worth?

e. On this preferred stock a semi-annual dividend of

$2\frac{1}{2}\%$ was paid Aug. 1. What amount of money was required to pay the dividend on all the stock?

f. At a cost of $95 per share and a dividend of $5 a share annually, what rate per cent would an investment in preferred stock of Atchison earn?

2. *a.* The common stock of the General Electric Company was worth in the same week $260 per share. There was outstanding $24,678,800 of the stock. What was its total market value?

b. On July 15, 1901, the Company paid a quarterly dividend of 2%. What total amount was required to pay this dividend?

c. At $260 a share and an annual dividend of $8, what rate per cent did an investment in G. E. earn?

3. Stock paying 5% dividends was purchased at $125. What was the rate of income on the investment?

$\$5 \div \$125 = \frac{5}{125}$

$\frac{5}{125} = \frac{1}{25} = .04 = 4\%$

Since the stock pays 5% dividends every share yields $5, but $125 was paid for each share, therefore the income is $\frac{5}{125}$, or $\frac{1}{25}$ of the investment.

4. Canal stock was purchased at a discount of 6%. When it was paying 6% dividends, what was the rate of interest on the investment?

5. Which pays the higher rate of interest, the stock of a company paying 6% dividend, market value $103, or that paying 4%, market value $86\frac{1}{2}$? By how much?

6. When U.S. 4s (bonds paying $4 annual interest on $100) are sold at $137, what interest do they pay a purchaser?

Any corporation redeeming a bond at the time when it is due, pays its exact face value, so that it is an advantage to buy at a discount and a disadvantage to buy at a premium Discuss this.

7. A speculator bought 100 R.R. bonds at 31% and sold them in 4 days at 35. *a.* What amount of money did he make? *b.* What was his rate of profit?

8. The executors of the estate of a millionaire bought 4% $1000 bonds, Chicago, Burlington & Quincy R.R., at the following prices: 100 at 95, 200 at $95\frac{3}{8}$, 200 at $96\frac{1}{8}$, 100 at $96\frac{7}{8}$, and 100 at $97\frac{5}{8}$. They refused to buy any more in the face of the advancing market.

a. How many shares in all did they buy?

b. How much money in all did they invest?

c. What rate per cent of income would each amount invested earn, if the security was absolutely good?

9. *a.* When a bond costing $950 is redeemed at $1000, there is an extra income of $50. What rate per cent is this on $950? *b.* When a man purchases a $3\frac{1}{2}$% bond at $850, and holds it for 30 yr. and then gets, besides all the interest, $1000 for it, what extra amount does he receive? *c.* What annual rate per cent on his investment is this? *d.* What is his average annual rate per cent income from his bond?

10. A dividend of 2%, payable Oct. 1, 1901, was declared on Union Pacific R.R. common stock of $104,042,400 and on preferred stock of $99,514,700. What total amount was required to pay the dividend?

11. The N. P. & R. Company had stocks outstanding to the amount of $10,000,000, and 5% bonds to the amount of $6,000,000. Its net earnings from July 1 to Dec. 31, 1901, were $400,000. After paying semiannual interest upon the bonds, what amount was available for dividends upon the stock? The Company declared a

dividend of 2%. What surplus remained from the net earnings above interest and dividends?

12. The par value of shares of the stock of the N. P. & R. Co. was \$100, and of the bonds was \$1000. In June, 1901, X bought 75 shares at 12% below par, and 15 bonds at 4% above par. *a.* How much was his total investment? *b.* How much was his income from shares and bonds together? What annual per cent upon his investment did his income promise: *c.* from the stock? *d.* from the bonds?

13. A speculator bought in June, 1901, 100 shares of the stock of the N. P. & R. Co., and sold them in January, 1902, at \$92 each, paying $\frac{1}{16}$% brokerage at par. *a.* What was his net gain? *b.* What per cent was this upon his investment?

14. Z bought \$9000 par value of bonds at 114%, paying 6% interest. *a.* What interest was due annually? *b.* What rate per cent was this upon the investment?

15. X owned 20 shares of manufacturing stock, par value \$50, 1000 shares of mining stock, par value \$1.00, and 10 U.S. Government bonds, par value \$1000. He sold the first investment at 35% discount, the second at 1250% premium, and the third at $17\frac{1}{2}$% premium. What were his total receipts?

16. M bought 35 shares of N. Y. Central R. R. stock, par value \$100, at 160%, March 19, 1902, and sold them at 161% on April 25. On April 21 a quarterly dividend of 1% was paid. What were his total gains?

17. On April 19, 1902, the stock of the Western Union Telegraph Co. was quoted at $93\frac{1}{2}$%. D bought 180 shares of C, who paid $\frac{1}{8}$% brokerage. *a.* How much did D invest? *b.* How much did C receive?

PRACTICAL AFFAIRS

The problems in this section illustrate the use in the classroom of current newspaper items and of ordinary commercial and industrial facts. Arithmetic ought to be useful, not only in training the mind, but also in interpreting the facts and relations of everyday life.

1. A market report said, in August, 1901:

Gross earnings of all railroads in the United States reporting for July are $50,732,912, a gain of 13.2% over last year and 25.2% over 1890.

What was the amount of gross earnings: *a*. In July, 1900? *b*. In July, 1899?

What was the difference between the gross earnings: *c*. July, 1899, and July, 1900? *d*. July, 1899, and July, 1901? *e*. July, 1900, and July, 1901?

2. A daily newspaper published this paragraph:

Ottawa, Ont., Aug. 16, 1901. The first official bulletin of the census was issued to-day. The population of the Dominion is given at 5,338,883, an increase of 505,644 over the census reported in 1891. The population of the provinces is as follows:

British Columbia, 190,000; Manitoba, 246,464; New Brunswick, 331,091; Nova Scotia, 459,116; Ontario, 2,167,978; Prince Edward Island, 103,258; Quebec, 1,620,974; territories, 145,000; unorganized territories, 75,000.

a. What was the per cent of increase from 1890 to 1900?

b. What percentage of the entire population of Canada was in each of the provinces?

3. A steamer took from New York to Liverpool, in August, 1901, 305,000 oz. of silver, valued at $26\frac{3}{4}d.$ per ounce. Exchange was $\$4.88\frac{1}{2}$ per pound on demand. In our money, what was the total value of the shipment?

4. A tax of $44,800 is to be raised in a certain town. The taxable property is valued at $1,228,000, and there are 475 polls to be taxed $1.40 each. *a.* How much is the tax on $1?

b. What is the tax of each of the following persons?

J, who pays a tax on $1728, and 1 poll.

S, who pays a tax on $4335, and 2 polls.

L, who pays a tax on $7945, and no poll.

F, who pays a tax on $2794, and 1 poll.

5. In the fiscal year ending June 30, 1901, there were 32,000 persons in the Erie R. R. Company's employ, and a little more than $56\frac{1}{2}$ per cent ($16,054,456) of the total operating expenses was paid last year directly for labor.

a. What were the total operating expenses?

b. What was the average amount paid to each employee?

The net income for the year was $10,695,328 from operations, and the total net income from all sources was $12,191,405.

c. What was the total amount of operating expenses less labor costs?

d. Since $1,153,540 of this last amount was spent for permanent improvements, what were the net operating expenses less labor costs?

e. If the total net income is considered as a 4 per cent return upon the true value of the railroad, what sum represented its value in 1901?

6. 76,000 copies of a popular novel were sold in the first six months. The list price was $1.50. The discounts allowed amounted to an average of 40%. The author received 10¢ royalty on 71,575 copies. The books cost on an average to manufacture, advertise, and deliver 49¢ each. What was the publisher's net profit?

7. A city real estate owner carried, on a tenement house, a first mortgage of $23,000 at 5%, and a second mortgage of $3900 at 6%. The assessed valuation was $31,000, and the tax rate was 1.7%. What was the total of the annual fixed charges upon the property?

8. Boston sold $3,250,000 par of $3\frac{1}{2}$% bonds at 108. Find the total premium realized.

9. The following table shows the imports into the United States from Spain, and exports from the United States to Spain in each year since 1890:

Fiscal Year	Imports from Spain	Exports to Spain
1890	$5,288,537	$12,758,463
1891	6,033,481	14,619,335
1892	5,207,861	11,528,424
1893	5,694,553	13,460,083
1894	4,255,875	13,122,906
1895	3,574,126	10,927,069
1896	4,131,184	11,492,428
1897	3,631,973	10,812,745
1898	3,575,565	10,228,545
1899	3,982,363	9,077,807
1900	5,950,047	13,399,680
1901 (7 months) . . .	3,110,718	8,988,970

a. What were the total imports and exports for the ten years 1890–1900?

b. At the rate in 1901 for 7 mo., what were the total imports and exports for the year?

c. What was the difference each year between imports and exports? This difference is called **the balance of trade.**

10. The total receipts at the custom house in Cuba during the calendar year 1900 were $16,099,923, against $14,854,261 for 1899.

a. What was the amount of increase?

b. What was the per cent of increase?

The receipts from duties on imports were $14,273,141, against $13,400,649, and from duties on exports, $1,066,006, against $764,106.

c. What was the amount, and

d. What the per cent of increase in import duties?

e. What was the amount, and

f. What the per cent of increase in export duties?

The receipts at the principal custom houses were: Havana, $12,042,031; Matanzas, $454,773; Santiago de Cuba, $953,078; Cardenas, $301,930; Cienfuegos, $1,160,303.

g. What was the total at these ports?

h. What was the total at all other ports?

11. During the calendar year 1900 the aggregate value of iron, steel, and their manufactures imported into Cuba from all countries was $4,751,395, as against $2,930,845 during the calendar year 1899, an increase of $1,820,550.

a. What was the per cent of increase?

The imports of these commodities from the United States in 1900 amounted in value to $3,685,829, as against $2,395,209 in 1899.

b. What was the per cent of increase?

12. The government transport service from the United States to the Philippine Islands cost $20,000,000 for the fiscal year ended June 30, 1901. Soldiers cost on the average $136, and cabin passengers $256.

a. What per cent of the latter cost is the former?

b. If the cabin passengers numbered 2500, how many soldiers could be transported for the amount remaining?

One transport, making the trip in 25 days, burned 110 T. of coal per day, at a total cost of $15,125.

c. What did the coal cost per ton?

13. The Boston & Albany R. R. issued this statement June 30, 1901:

	1899	1900	1901
Gross earnings .	$9,946,325	$9,956,137	$9,325,035
Expenses . . .	6,425,571	6,560,230	6,777,661
Interest, taxes, etc.	1,314,452	1,363,221	615,980
Net earnings . .	3,520,754	3,386,908	2,647,374
Balance	2,206,301	2,023,685	2,031,394
Dividends . . .	2,000,000	2,000,000	2,000,000
Surplus	26,301	23,685	31,394

a, b, c. Discuss the items for each year.

d. Since the Massachusetts charter of the B. & A. R. R. forbids paying over 8% dividends, a per cent reached each of these years, what was then the total par value of the company's stock?

e. On an investment to earn $3\frac{3}{4}\%$, what was the market value of the stock?

14. The import duty on bicycles at German ports is $3.57 per kilogram. The average bicycle weighs 22 lb. What is the amount of the duty?

15. Germany imported in 1897–98 $114,000,000 worth of ironware, machinery, railroad equipment, and ships, and in 1900–01 $172,000,000. What was the per cent of increase?

16. Discuss this newspaper report, 1901, and make problems, using its figures:

Following are the imports of dry goods and general merchandise during the week, as reported to-day at the custom house, as compared with the imports of last week and those of the corresponding week a year ago:

	This week	Last week	Last year
Dry goods	$2,260,546	$1,972,857	$2,070,234
Gen. m'd'se.	6,593,341	7,771,296	8,400,081
Total	$8,853,887	$9,744,153	$10,470,315

Total imports of dry goods and merchandise in preceding weeks have been as follows:

Week ending:		Week ending:	
July 27	$9,834,164	May 11	$12,590,832
July 20	11,370,877	May 4	10,487,539
July 13	10,126,304	Apr. 27	10,968,662
July 6	9,631,531	Apr. 20	12,534,444
June 29	11,623,700	Apr. 13	10,327,550
June 22	8,885,521	Apr. 6	9,826,289
June 15	8,279,183	Mar. 30	13,874,414
June 8	12,907,190	Mar. 23	13,953,210
June 1	10,078,634	Mar. 16	10,317,929
May 25	10,561,463	Mar. 9	12,844,425
May 18	11,741,669	Mar. 2	10,954,867

For weeks corresponding to the past week in years prior to 1900, the imports were as follows:

1899	$9,586,652	1895	$10,313,458
1898	7,597,848	1894	8,117,880
1897	6,623,750	1893	8,277,565
1896	6,410,498		

a. What are the purposes for which the U. S. government collects customs duties?

b. Find the average value of imports per week for the 24 weeks ended Aug. 10.

c. What is meant by "general merchandise"?

17. Discuss these items:

Year	Total Exports	Total Imports
1894	$892,100,000	$654,900,000
1895	807,500,000	731,900,000
1896	882,600,000	779,800,000
1897	1,050,900,000	764,700,000
1898	1,231,500,000	616,100,000
1899	1,227,000,000	697,200,000
1900	1,394,400,000	849,900,000

These exports do not include gold and silver.

Find the average for the seven years:

a. Of exports. *c*. Of excess of exports over imports.

b. Of imports. *d*. Of the total of imports and exports.

e. What was the per cent of increase in 1900 of the average sum of exports and imports for the seven years?

f. Make a table of the excess of exports over imports for each of these years.

g. Make a table of the sum of exports and imports for each year.

18. The total annual domestic business of the United States is $100,000,000,000 a year in prosperous times (1900 estimate). What per cent of the domestic business was the average of the international business for the last three years of this period?

19. The total value of Chicago corporations, including their franchises, in 1900 was $268,108,312. The value of their real and personal property was $32,278,745.

a. What was the value of their franchises?

b. If their property had been assessed at $\frac{1}{5}$ of their values, since the tax rate was 7%, what would have been Chicago's revenue from the corporations?

20. A locomotive manufacturing company, employing 10,000 persons, can turn out four locomotives a day. At an average of nine hours per working day, how many hours of work are required to make a locomotive out of such materials as the company uses? What is the total labor cost of a locomotive at an average of 18.5¢ per hour per workman?

21. For the first half year of 1901 the fire insurance losses in Chicago were $2,428,573. This was 80% of the total fire loss. There were 3812 fires.

a. How many fires were there per day?

b. What was the average fire loss per day?

c. What was the average fire insurance loss per day?

d. For a population of 1,400,000 people what was the monthly fire loss per capita?

22. The City and Suburban Homes Company of New York made plans for five six-story tenements, each 52.6 feet front, at an estimated cost of $250,000. What was the estimated cost per front foot per story of these tenements?

23. The total mileage of railways in the United States at the close of the fiscal year ended June 30, 1901, was 193,345, an increase of 4051 in the twelve months immediately following July 1, 1899. In 1890 the railway mileage of the country was 163,597.

a. What was the railway mileage June 30, 1900?

b. What was the per cent of gain from June 30, 1900, to June 30, 1901?

c. What was the increase in mileage from June 30, 1891, to June 30, 1901?

d. What was the average annual per cent of gain for the period June 30, 1891, to June 30, 1901?

24. The contract price of a 6200 T. steel steamer for the Great Lakes was $275,000.

a. What was the cost per T.?

b. To earn a net profit of 8%, after allowing 5% for annual depreciation, what must the annual net earnings be?

25. The population of New Jersey in 1900 was 1,883,009 and the death rate was 17.4 per thousand.

a. How many deaths were there?

b. In 1890 the death rate was 19.7 per thousand. If 1900 had shown the same death rate, how many deaths would have occurred?

26. The rent of a certain city apartment of 8 rooms was $900, while that of another apartment of 3 rooms in a more desirable location was $1700. What was the per cent of rent per room of the lower priced apartment compared with the other?

27. The **momentum** of a body weighing 1 lb. moving 1 ft. in 1 sec. is 1 foot-pound per second.

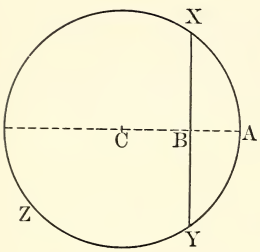
a. What is the momentum of a safe weighing 1 T. and falling 32 ft. per sec.? Answer in foot-pounds per second.

b. What is the momentum of a locomotive that weighs 180,000 lb. and is moving at the rate of 3000 ft. per min.?

c. What is the momentum of a 13 in. shell weighing 1100 lb. and moving at the rate of 2200 ft. per second?

d. What is the momentum of a steamship weighing 12,000 T. and moving at the rate of 20 miles per hour?

To bisect a chord and its arc



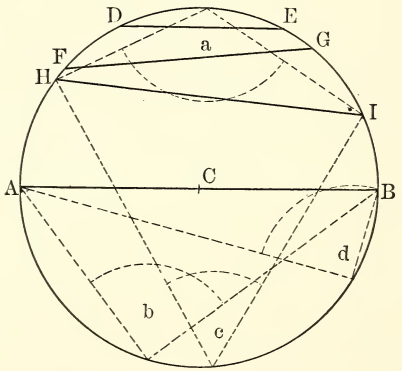
In any circle, XYZ, draw to the chord XY the radius CA perpendicular to the chord.

CA bisects the chord XY at B and the arc XY at A.

1. Are chords longer or shorter where near the diameter?

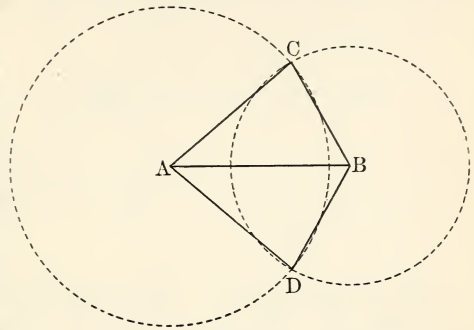
2. Compare the length of the diameter with that of any other chord.

3. The angles b and d are inscribed in the semicircle. Are they acute, obtuse, or right angles?



4. The angle a is subtended by the chord HI in the smaller segment of the circle. The angle c is subtended by the same chord, HI, but in the larger segment of the circle. Which angle is larger, a or c?

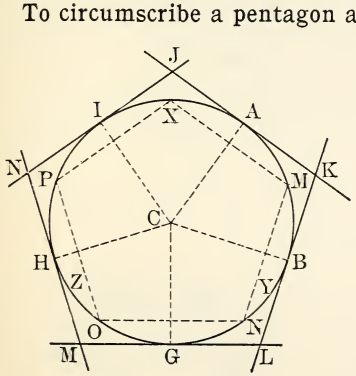
To draw a triangle with three sides given



This is possible only when neither PQ, WX, nor YZ, any side proposed, is longer than the sum of the other two sides.

Draw AB equal to WX, the longest side. From A as a center, with a radius equal to YZ, draw a circumference. Draw from B, with a radius equal to PQ, another circumference. Draw AC, CB, BD, and AD. The triangles ACB and ADB have sides equal to PQ, WX, and YZ.

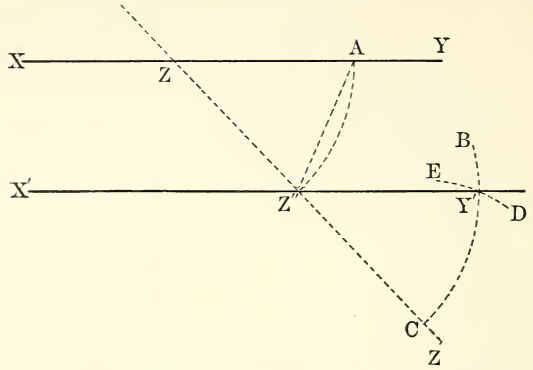
To circumscribe a pentagon about a circumference



Inscribe the pentagon $XMNOP$ in the circle XYZ. Bisect each chord and arc, XM, MN, etc., with a radius CA, CB, etc., perpendicular to the chord. At A, B, G, H, I, the middle of the arcs DM, etc., draw tangents to the circumference. The tangents are parallel to the chords XM, etc., and perpendicular to the radii CA, CB, etc. They intersect at J, K, L, M, N.

$JKLMN$ is a pentagon circumscribed about the circumference XYZ.

To draw any line parallel to any other line

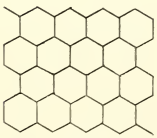


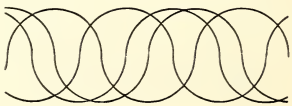
Draw XY, any line. Draw any line ZZ' through it, at any point, Z. Take any point Z'' in the line ZZ'. With ZZ'' as radius, draw the arc AZ''. From Z'' measure the chord AZ''. From Z'' describe with the radius ZZ'' the arc BC, cutting ZZ' at C. From C with the radius $Z''A$, draw the arc DE intersecting BC at Y'. Through Y' and Z'' draw $X'Y'$, which is parallel to XY.

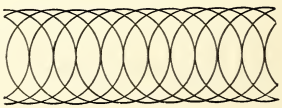
Two other methods of drawing parallels are given in Book VI.

Draw parallels by the various methods, to see by which method the most accurate result is secured.

HEXAGONS, SEMI-CIRCUMFERENCES, AND CIRCUMFERENCES





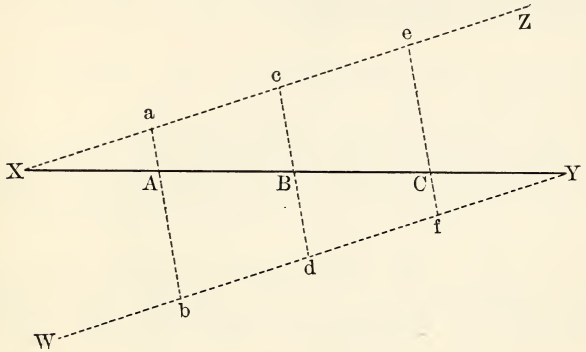


The bees make the cells of the honeycomb hexagonal.

Find how each of these figures is drawn.

To divide a line into any number of equal parts

Draw XZ, any line at any angle, ZXY, to the line XY. To divide XY into (say) four equal parts. Beginning at X, mark upon XZ three equal lengths. Draw YW parallel

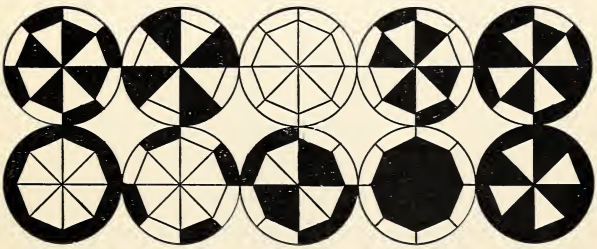


with XZ. Upon YW, beginning at Y, mark three lengths equal to the divisions of XZ. Draw ab, cd, ef. These lines divide XY into four equal parts at A, B, C.

In dividing a line into any number of equal parts mark upon the construction lines one less length than the number of parts desired.

A construction line is any line drawn in the process of developing the figure or proof but not apparent in the final result desired.

CIRCLES AND OCTAGONS



Addition

The **algebraic sum** of any number of quantities may be expressed by writing them in a row with their proper signs.

The sum of the quantities $7\ ax$, $5\ by$, $-3\ am$, $7\ bc$, and $-3\ abc$ is $7\ ax + 5\ by - 3\ am + 7\ bc - 3\ abc$.

In algebra a quantity may be either positive or negative, + or −. A positive quantity is one that is united with other positive quantities by addition; a negative quantity is united with positive quantities by subtraction, but with other negative quantities by addition.

When some of the terms in the expressions to be added are *like* quantities, these may be combined by adding the coefficients and attaching to their sum the part that expresses the *like* quantities.

$$3\ a - 4\ a + 5\ a + 7\ a - 6\ a.$$

The sum of the coefficients of positive terms is $(3 + 5 + 7) = 15$. The sum of the coefficients of the negative terms is

$$(4 + 6) = 10. \qquad (15 - 10)\ a = 5\ a.$$

For convenience we may set the quantities down in rows with the like terms under one another.

1. Add $3\ a^2x - 3\ by^2 + 2\ x$, $-3\ x + 4\ by^2 + 4\ a^2x$, $-6\ by^2 + 5\ x - 2\ a^2x$, and $5\ a^2x + by^2 + x$.

$$\begin{array}{r} 3\ a^2x - 3\ by^2 + 2\ x \\ 4\ a^2x + 4\ by^2 - 3\ x \\ -2\ a^2x - 6\ by^2 + 5\ x \\ 5\ a^2x + \quad by^2 + \quad x \\ \hline \text{Sum} = 10\ a^2x - 4\ by^2 + 5\ x \end{array}$$

When the sum of the + coefficients is the same as the sum of the − coefficients, the result is 0.

Add: **2.**
$$\begin{array}{r} 3a \\ 8a \\ -5a \\ -a \\ \hline 6a \end{array}$$

3.
$$\begin{array}{r} 5a^2b^2 \\ -2a^2b^2 \\ 4a^2b^2 \\ -4a^2b^2 \\ \hline -6a^2b^2 \end{array}$$

4.
$$\begin{array}{r} -2xy \\ 10xy \\ xy \\ -4xy \\ \hline -5xy \end{array}$$

Positive quantities may be taken to mean the money a person has owing to him, his credits; and negative quantities the money he owes, his debts. Example 3 may be taken to mean that a person owes the amount represented by the result, $-3a^2b^2$.

Or positive quantities may be compared with the degrees of heat above zero, and negative with degrees below zero.

Ideas of above and below, and of more and less than, any given standard afford means of comparison with + and − terms in algebra.

Add: **5.**
$$\begin{array}{r} -3x+3y+3 \\ x+4y-2 \\ 4x+y+4 \\ \hline -2x+5y-1 \end{array}$$

6.
$$\begin{array}{r} x^2+x+2 \\ 12x^2-4x-10 \\ -7x+5 \\ \hline -9x^2+6x+7 \end{array}$$

7. $3a+b,\ 2a+2b,\ 4a+3b,\ 5a+2b,\ a+b,\ 3a+6b.$

8. $-4x+y,\quad -2x+3y,\quad -5x+2y,\quad -6x+3y.$

9. $6p-2q,\ 7p+3q,\ 6p-4q,\ 2p+3q,\ 3p-5q.$

10. $-7y-3,\ 6y-4,\ 7y+5,\ 6y-7,\ -3y+8.$

11. $8x+6y+7z,\quad 9x+7y+8z,\quad 7x+10y+9z,$ $3x+10y+5z,\quad 8x+3y+9z,\quad 5x+7y+8z.$

12. $6p-9q+2w,\quad 3p-10q+3w,\quad 4p-9q+3w,$ $3p-10q-4w,\ 4p-6q+3w,\ 3p-5q+2w,\ 2p-7q+3w.$

13. $8ab+3cd-5c^2,\ 7ab-2cd+6c^2,\ 9ab-4cd-10c^2,$ $7ab-2cd+6c^2,\ 6ab+7cd-7c^2,\ 7ab-3cd-4c^2.$

Subtraction

The **algebraic difference** of two quantities is found by changing the signs of all the terms of the subtrahend, or by thinking of them as changed, and then proceeding as in *algebraic addition*.

From $5x^2 - 4xy + 5y^2$ take $2x^2 - 6xy + 7y^2$.

$5x^2 - 4xy + 5y^2$, minuend.

$-2x^2 + 6xy - 7y^2$, subtrahend, with signs changed.

$3x^2 + 2xy - 2y^2$, difference found, as in addition.

From $ax^2 - by^2 - 2cz^2$

take $px^2 - qy^2 + rz^2$

$ax^2 - px^2 + qy^2 - by^2 - rz^2 - 2cz^2$.

1. Subtract 3 from 12; -3 from 12; 3 from -12.

2. Subtract $3x$ from $12x$; $-3x$ from $12x$; $-3x$ from $-12x$.

3. Subtract $-3x^2y$ from $7x^2y$; from $-17x^2y$; from xy.

4. Take $4a$ from $9a$.

5. Take $-3ab$ from $10ab$.

6. Subtract $2q$ from q.

7. Subtract $3a^2b$ from $-2a^2b^2$.

8. Take $4x$ from $2a$.

9. Take $-2a$ from $-3b$.

10. Subtract 1 from x.

11. Subtract -3 from $-a$.

12. Take abc from $2bcd$.

13. Take $-z$ from xy.

14. From a take $b + c$.

15. From $x + y$ take z.

16. Take $2x - 2y - 3z$ from $6x - 2y - z$.

17. Subtract $2p - 7q - x$ from $3p - 2q + 3x$.

18. From $7a^2 - 8a - 1$ take $5a^2 - 6a + 3$.

19. Take $y^2 - xy + z^2$ from $2y^2 - 2xy + 3z^2$.

Multiplication

I. Multiply a^3 by a^2.

$a^3 = a \times a \times a.$

$a^2 = a \times a.$

$a^3 \times a^2 = a \times a \times a \times a \times a = a^5.$

II. Multiply $2\,a^2b$ by $3\,ab^3$.

$2 \times a \times a \times b \times 3 \times a \times b \times b \times b = 2 \times 3 \times a^3 \times b^4 = 6\,a^3b^4.$

To multiply like terms, add their exponents, but multiply their coefficients.

$(+a) \times (+b) = +ab.$ $\quad$ $(-a) \times (+b) = -ab.$

$(+a) \times (-b) = -ab.$ $\quad$ $(-a) \times (-b) = +ab.$

Like signs multiplied together produce plus, and unlike signs produce minus.

III. Multiply:

1. $2\,a$ by 3.
2. $8\,ac$ by bd.
3. x^2 by x^3.
4. $6\,ab^2y^3$ by $2\,b^3y^3$.
5. $3\,a^2$ by -2.
6. $-4\,ab$ by $3\,xy$.
7. $-3\,by$ by cy^2.
8. $9\,a^3b^4$ by $2\,b^2c^2$.
9. $2\,a^2b^3$ by $-3\,abc$.
10. $-2\,ab$ by $3\,a^2b^4c$.
11. $15\,xyz^2$ by $-x^2y^2z$.
12. $-5\,ax$ by $-10\,xy^2$.
13. abx^2 by $10\,a^3b$.
14. $-2\,x(1+y)$ by $-4\,y$.
15. mn^2x by $-nxz$.

IV. Find the squares of:

1. a.
2. $2\,a$.
3. $3\,ab$.
4. $6\,abc$.
5. $4\,x^2$.
6. $8\,x^3y^3$.
7. $-x$.
8. $-3\,x$.
9. $-4\,xy$.
10. $-xyz$.
11. $-9\,p^2$.
12. $12\,x^4y^4$.

V. Find the cubes of:

1. x.
2. $3\,x$.
3. $4\,xy$.
4. $2\,xyz$.
5. $5\,y^2$.
6. $6\,x^2y^2$.
7. $-b$.
8. $-4\,c$.
9. $-7\,pq$.
10. $-3\,abc$.
11. $-9\,z^4$.
12. $-10\,x^3y^3$.

Multiplying an Expression

I.

$$\begin{array}{l} x^2y^2 + a^3b^4 - xyz \\ 2\,ab \\ \hline 2\,abx^2y^2 + 2\,a^4b^5 - 2\,abxyz \end{array}$$

II.

$$\begin{array}{l} -3\,a^2 - 7\,x^3y^2 + 4\,p^4q^5 \\ -2\,a^3x^2 \\ \hline 6\,a^5x^2 + 14\,a^3x^5y^2 - 8\,a^3p^4q^5x^2 \end{array}$$

III. Multiply:

1. $7\,ab + 3\,bc$ by x.
2. $3\,pq - 2\,qr$ by a.
3. $p^2q^2 - q^3r^3$ by ab.
4. $a^3b^2 - b^2c^3$ by ab.
5. $-4\,x^2y - 2\,y^2z$ by $-3\,xyz$.
6. $-3\,a^2b^3 - 4\,c^2d^2$ by $-2\,a^2d^2$.

IV. Multiply $2\,b^2 - 3\,bc - 4\,c^2$ by $6\,bc^2 - 8\,c^3$.

$$\begin{array}{l} 2\,b^2 - 3\,bc - 4\,c^2 \\ 6\,bc^2 - 8\,c^3 \\ \hline 12\,b^3c^2 - 18\,b^2c^3 - 24\,bc^4 \\ \qquad - 16\,b^2c^3 + 24\,bc^4 + 32\,c^5 \\ \hline 12\,b^3c^2 - 34\,b^2c^3 \qquad\qquad + 32\,c^5 \end{array}$$

V. Multiply:

1. $3\,ab + 4\,b^2$ by $2\,ab - 3\,b^2$.
2. $a^4 + a^2x^2 + x^4$ by $a^2 - x^2$.
3. $x^2 - xy + y^2$ by $x + y$.
4. $a^4 + 2\,a^2b^2 + 4\,b^4$ by $a^2 - 2\,b^2$.
5. $2\,b^2 - 3\,bc - 4\,c^2$ by $6\,bc^2 - 8\,c^3$.
6. $ax + a^2x^2 + a^3x^3$ by $1 - ax$.
7. $4\,a^2b - 5\,ab^2$ by $5\,ac - 2\,b$.

Division

Like signs in division produce plus, and unlike signs produce minus. Exponents of like terms are subtracted, but coefficients are divided.

$$a \times b = ab. \quad ab \div a = b. \tag{1}$$

$$-a \times b = -ab. \quad -ab \div -a = b. \tag{2}$$

$$a \times x - b = -ab. \quad -ab \div a = -b. \tag{3}$$

$$-a \times -b = ab. \quad ab \div -a = -b. \tag{4}$$

When dividend and divisor have the same sign the quotient is positive; but when dividend and divisor have opposite signs, the quotient is negative.

$$\frac{abc}{ac} = b. \qquad \frac{12\,xyz}{-4\,y} = -3\,xz. \qquad \frac{-40\,a^2b^3c}{-5\,abc} = 8\,ab^2.$$

Divide:

1. ab by b.
2. $abcd$ by ac.
3. $6\,x^2y$ by $2\,x$.
4. $6\,pqr$ by $2\,pr$.
5. $-8\,c^2d^3e$ by $-4\,c$.
6. $-33\,m^2n^2p^2$ by $-11\,mp^2$.
7. $-p^4qxy$ by p^2x.
8. $-am^2nx^4$ by $-mnx^3$.
9. a^2b^3 by $-ab^2$.
10. $-3\,x^3$ by $-x^2$.
11. $m^2n^3xz^4$ by $mnxz^2$.
12. $15\,a^2xz$ by $30\,a^2x$.

Arranging the Terms of an Expression

In writing an algebraic expression of several terms it is customary to arrange them in the ascending or descending power of a letter or powers of letters.

Arrange:

1. $a^2 + b^2 + 2\,ab$.
2. $x^5 + x^2 + x^4 + x^3 + 1 + x$.
3. $a^4 + b^4 - a^2b^2 + 2\,ab^3$.
4. $37\,xy + x^2 + 36\,y^2$.
5. $-35\,a - 30\,a^2 + 1$.
6. $9 + y^6 - 10\,y^3$.
7. $-b - 30 - b^2$.
8. $x^4 + z^4 + 2\,x^2z^2$.

Dividing an Expression

$$-3a^2b^2c)\underline{6a^4b^3c^3 - 9a^3b^4c^2}$$
$$-2a^2bc^2 + 3ab^2c$$

I. Divide:

1. $ab - bc$ by b.
2. $-bc - cd$ by c.
3. $12a + 16b$ by 4.
4. $x^5 - x^4 + x^3$ by $-x^2$.

II. Divide $y^8 + x^8 + x^4y^4$ by $x^2y^2 + y^4 + x^4$.

We arrange dividend and divisor according to the descending powers of x.

$$\begin{array}{l} x^4 + x^2y^2 + y^4)x^8 \quad\quad + x^4y^4 + y^8(x^4 - x^2y^2 + y^4 \\ \quad\quad\quad\quad x^8 + x^6y^2 + x^4y^4 \\ \quad\quad\quad\quad \underline{\quad\quad\quad\quad\quad\quad\quad\quad} \\ \quad\quad\quad\quad - x^6y^2 \quad\quad\quad\quad y^8 \\ \quad\quad\quad\quad - x^6y^2 - x^4y^4 - x^2y^6 \\ \quad\quad\quad\quad \underline{\quad\quad\quad\quad\quad\quad\quad\quad} \\ \quad\quad\quad\quad\quad\quad\quad\quad x^4y^4 + x^2y^6 + y^8 \\ \quad\quad\quad\quad\quad\quad\quad\quad \underline{x^4y^4 + x^2y^6 + y^8} \end{array}$$

III. Divide:

1. $12ac^2 - 16a^2c + 70a^3c^3 - 8ac^2$ by $4ac$.
2. $2x^4z^4 - x^3yz^5 + 3x^4z^5 - 4x^3z^4$ by x^2z^4.
3. $x^2 + x - 30$ by $x + 6$.
4. $b^2 - 4b - 21$ by $3 + b$.
5. $x^2 - x - 56$ by $x - 8$.
6. $b^2 - 17 - 18$ by $b + 1$.
7. $a^3 - az^2 - a^2z + z^3$ by $a - z$.
8. $5x + x^2 - 84$ by $x - 7$.
9. $32a^2 - 6a + 1$ by $8a^2 - 2a - 1$.
10. $x^4 - 19x^2 + 48$ by $x^2 - 3$.

Miscellaneous Review Problems

When $a=5$, $b=3$, $c=-4$, and $d=10$, find the value of:

1. $a^2+2ab+b^2$.
2. $b^2-2bc+c^2$.
3. a^2+d^2.
4. $5b-10bc-5cd$.
5. $a^2b+b^2c+d^2$.
6. $cd-d(a+b)$.

Find the value of x in these expressions:

7. $\frac{4x}{5}=4$.
8. $\frac{x^2}{3}=27$.
9. $\frac{10x}{15}=20$.
10. $\frac{x^3}{x}=4$.
11. $\frac{5x}{2x+2}=2$.
12. $\frac{x+1}{2}=40$.
13. $\frac{x^2-1}{5}=\frac{16}{10}$.
14. $\frac{x^4}{x^2}=9$.

15. Add $4a^2x+5a^3x^2+4a^4x^3$, $2a^3x^2-3a^4x^3+6ax^4$, $10a^3x^2-7a^4x^3+9ax^4$, and $a^2x-8a^3x^2-10a^4x^3$.

16. Subtract $5a^4b^4c-7a^5b^3c^2+3a^3b^5c^3$ from $6a^4b^4c-10a^5b^3c^2-2a^3b^5c^3$.

17. From $+x^4+x^2+x^3+1+x$ take $x^3-x^4-x+1-x^2$.

Multiply:

18. a^2b^2 by $2ab$.
19. $x+1$ by $-2x^2$.
20. $-3xz^2$ by $-2x^2z$.
21. $a^2+2ab+b^2$ by $a^2-2ab+b^2$.
22. $x^2-2xy+x^2$ by $2(x-y)$.

Divide:

23. a^2-b^2 by $a-b$.
24. $m^2-6mn-9n^2$ by $m-3n$.

25. A manufacturer employed in 1902 3 times as many men less 15 as in 1897 when he employed 200 in all. How many did he employ in 1902?

26. A grocer had 1 T. of sugar put up in 20-lb. and 10-lb. packages. There were 220 more of the smaller than of the larger packages. Find the number of each.

27. Mr. Brown's annual income was $\frac{2}{3}$ that of Mr. Thompson, and Mr. Thompson's was $\frac{3}{4}$ that of Mr. Williams. Their incomes added together made $22,500. What was the income of each?

NATIONAL BANKS

National Banks are corporations which issue money, accept deposits, and make loans in accordance with the laws and under the supervision of the Government of the United States. The nature of their business is shown by the two reports of National Banks printed upon the opposite page.

In order to secure the right to issue money a National Bank is required to invest one quarter of its capital in the bonds of the United States. It must also keep as cash on hand at least a certain fixed percentage of cash upon the deposits — in large cities 25% and in small cities 15% — so as always to have funds ready to meet checks and drafts upon the deposits, and must keep in the U.S. Treasury 5% of the amount of all bank notes issued.

Questions

1. What was the total of capital stock, surplus, and undivided profits of each Bank?

2. Which Bank evidently did not own the building in which it was located?

3. Since $7500 was 5% of the circulation of bank notes of the first Bank, what was its total of bank notes issued?

4. What was the total of bank notes issued by the second Bank?

5. What is the meaning of the terms: loans, discounts, overdrafts, bonds, circulation, premiums, stocks, real estate, internal revenue, checks, exchanges, currency, dividends, certified checks?

"Reserve agent" is a bank authorized to hold elsewhere a portion of another bank's assets, such as money, notes, checks, accounts due.

RESOURCES	
Loans and discounts	$913,952.31
Overdrafts, secured and unsecured	315.56
U. S. bonds to secure circulation	150,000.00
Premiums on U. S. bonds	1,049.54
Stocks, securities, etc.	263,444.60
Banking house, furniture and fixtures	125,000.00
Other real estate owned	5,382.75
Due from National Banks (not reserve agents)	23,066.59
Due from approved reserve agents	223,752.26
Internal revenue stamps	611.53
Checks and other cash items	12,887.84
Notes of other National Banks	2,340.00
Fractional paper currency, nickels and cents	410.50
Lawful money reserve in bank, viz.:	
Specie $59,740.48	
Legal tender notes 13,500.00	73,240.48
Redemption fund with U. S. Treasurer (5 per cent of circulation)	7,500.00
Total	$1,802,953.96

LIABILITIES	
Capital stock paid in	150,000.00
Surplus fund	125,000.00
Undivided profits, less expenses and taxes paid	25,029.67
National Bank notes outstanding	148,800.00
Due to other National Banks	36,167.49
Due to Trust Companies and Savings Banks	4,622.66
Dividends unpaid	489.00
Individual deposits subject to check	1,207,717.88
Demand certificates of deposit	157.50
Certified checks	3,619.60
Cashier's checks outstanding	1,350.16
Bills payable, including certificates of deposit for money borrowed	100,000.00
Total	$1,802,953.96

RESOURCES	
Loans and discounts	$376,461.53
Overdrafts, secured and unsecured	259.49
U. S. bonds to secure circulation	25,000.00
Premiums on U. S. bonds	700.00
Stocks, securities, etc.	64,671.25
Banking house, furniture and fixtures	2,740.00
Due from U. S. Treasurer	1,000.00
Due from National Banks (not reserve agents)	6,409.46
Due from approved reserve agents	71,630.72
Internal revenue stamps	209.38
Checks and other cash items	272.12
Exchanges for Clearing House	2,305.90
Notes of other National Banks	3,465.00
Fractional paper currency, nickels and cents	654.04
Lawful money reserve in bank, viz.:	
Specie $11,030.85	
Legal tender notes 19,340.00	30,370.85
Redemption fund with U. S. Treasurer (5 per cent of circulation)	1,250.00
Total	$587,399.74

LIABILITIES	
Capital stock paid in	100,000.00
Surplus fund	20,000.00
Undivided profits, less expenses and taxes paid	25,090.08
National Bank notes outstanding	24,500.00
Due to other National Banks	8,930.56
Dividends unpaid	444.50
Individual deposits subject to check	399,972.75
Demand certificates of deposit	55.00
Certified checks	8,406.85
Total	$587,399.74

Verify the total of each of these reports to the U. S. Treasurer.

PERCENTAGE

Insolvency and Bankruptcy

A person, firm, or corporation becomes **insolvent** when the debts exceed the assets. Courts of the United States may then declare the person, firm, or corporation **bankrupt**, and order the assets to be distributed, *pro rata*, to the owners of the debts. The settlement is made according to the principles of percentage and proportion. A **receiver** settles the affairs usually. When the business is allowed to proceed, **trustees** for the creditors may be appointed.

1. A, B & Co. were adjudged bankrupt, and the court ordered the business at an end and the assets distributed, with an allowance of $2\frac{1}{2}\%$ to the receiver. The assets were $108,695, the debts $190,000, and the expenses of settlement, besides the receiver's fee, on net assets, were $10,550. What sum did D receive to whom $15,000 was due?

$108,695, gross assets.	$98,145, net assets.
10,550, expenses.	.025, receiver's rate.
$98,145, net assets.	490.725
	1962.90
	$2453.025, receiver's fee.

$98,145.
2,453.63
$95,691.37, for distribution.

Since the debts were $190,000, and to D was due $15,000, then D's share was $\frac{15000}{190000}$, or $\frac{3}{38}$.

Since there was $96,691.37 to distribute among $\frac{38}{38}$, then D's share was $\frac{3}{38}$ of that amount.

2. The Worthington Company became bankrupt. Their assets were $18,050, and debts $47,890. Find the amounts to be apportioned to A, to whom $8700 was due, to B, a creditor for $1350, and to C, a creditor for $3875, after paying expenses amounting to $3792 and 5% to the receiver upon net assets.

3. Mr. Thompson, owner of a factory, failed, owing $32,870. His real estate was worth $12,500; his machinery $14,000; his goods on hand $1560; and his bills receivable $3690. The expenses of the receivership were $3600, and the receiver's fee on net assets was 10%. But the receiver secured only 65% of the real value of the assets. What did A finally get, to whom $1000 was due?

4. A contractor failed for $9000, with apparent assets of $7500. There were no expenses of settlement or receiver's fees, but the assets brought only $3500. What did A receive, to whom was due $785.50?

5. The Brown-Mason Company became insolvent, owing $748,000 with $595,000 assets. They compromised with their creditors, giving long term notes for 60¢ on the dollar. What was due the Richardson Company two years after, to whom a note at 6% had been given, to settle a claim of $85,000?

6. Mr. Livingston became insolvent, with debts aggregating $47,000 and assets of $3500. Find what was available to pay Mr. Osborn, to whom was due $8750, after allowing 10% of the assets for expenses of settlement.

7. A contractor failed, owing $19,000. After paying mechanics' liens and other preferred charges amounting to $4650, a balance of $5200 was left from the assets. How much was payable to X, to whom $1285 was due?

PERCENTAGE

Foreign Exchange

1. What is the cost of a bill of exchange on Liverpool for £765 11*s*. 3*d*., exchange being at $4.87 per pound sterling?

This involves what premium on the £? £1 = $4.865 in our money.

£765 11*s*. 3*d*. = £765.5625,

$4.87 × 765.5625 = $3728.29.

2. What is the cost of a bill of exchange on Marseilles for 8200 francs at $.193?

The franc is worth $.193. Is the exchange above, below, or at par?

$.193 × 8200 = $1582.60.

3. Find the cost of a 60-day bill of exchange on London for £1200, exchange being £1 for $4.86⅜.

4. How large a draft on London can be bought for £1350, the rate of exchange being £1 for $4.80?

$4.80 = £1,

$1350 = as many £'s as $4.80 is contained times in $1350, that is, 281¼,

$1350 = £281¼ = £281 5*s*.

5. Find the cost of a bill of exchange on Paris for 1500 francs at 5.16 francs for $1.

6. What was the value in London of a sight draft on New York for $1000, if $1 = 49*d*.?

7. What was the value in New York of a sight draft which cost in London £204 3*s*. 4*d*.?

8. Find the rate of exchange in Glasgow when a sight draft for $1000 on Philadelphia sells for £206 5*s*.

9. A Bradford, England, merchant exports to Boston goods which cost him £616. He pays an import duty of 12% on the goods, and to a Boston agent a commission of 7% for selling them for $7800. What should be the face of the draft by which the agent remits the proceeds of the sale, exchange being $4.86 for £1?

10. On Aug. 16, 1901, these were the rates as published in a New York paper of the next day:

Rates for actual business closed as follows: Sixty days, $4.85; demand, $4.87½; cables, $4.87¾; commercial bills, $4.84¼@$4.85¼.

Continental bills were quoted as follows: Francs, 5.18¾ and 5.16⅞ less 1/16 @ 5.17½; reichsmarks, 94 15/16 @ 95 and 95⅜; guilders, 40 @ 40 1/16 and 40¼.

a. Find out what these items mean. *b*. Make drafts of various amounts at these rates. *c*. What are the rates to-day?

11. On Aug. 17, 1900, a merchant in Sheffield, England, cabled for a certain quantity of cutlery to be imported from the United States. The sixty days' draft used in payment cost him £234 10*s*. How many dollars' worth of goods did he buy?

12. What is the rate of exchange on Paris when a sight draft of 1295 francs costs $250?

13. A sight draft which cost in Birmingham, England, £215 8*s*. 9*d*., will pay for how large a bill of goods in Lowell, Mass., exchange being $4.87 for £1?

14. Exchange being at 5.18 francs for $1, what is the cost of a bill of exchange on Paris for 13,986 francs?

EQUATION OF PAYMENTS

Equation of payments finds the time in which several debts, due at different times without interest, may be paid equitably.

The **term of credit** is the time from the incurring of a debt to the date on which it becomes due.

The **equated time** is the date on which the several debts may be paid without loss to either debtor or creditor.

The **average term of credit** is the time that must elapse before several debts, due at different times, may all be paid at once, without loss to either debtor or creditor.

To **average an account** is to find the equitable time of payment of the balance due.

A **focal date** is the date from which the computation is begun to find the equated time of the several debts and credits.

CREDITS OF ONE DATE

1. A merchant incurs a debt of $450, to be paid $100 immediately, $300 in 4 mo., and the rest in 6 mo. Find the average term of credit.

$100 × 0 mo. =
300 × 4 mo. = $1 for 1200 mo.
50 × 6 mo. = $1 for 300 mo.

1500 mo. ÷ 450 = $3\frac{1}{3}$ mo. = average term of credit.

The cash payment of $100 is first subtracted from the account. The credit on $300 for 4 mo. = credit on $1 for 1200 mo. The credit on $50 for 6 mo. = credit on $1 for 300 mo. The credit on $450 = credit on $1 for 1500 mo. The credit on $450 = $3\frac{1}{3}$ mo. since $1500 \times 1 = 450 \times 3\frac{1}{3}$.

2. X owes $600, of which $100 is to be paid in 4 mo., $200 in 10 mo., and the rest in 16 mo. If he pays the whole debt at once, what is the average term of credit?

3. B owes $600 to be paid in 12 mo., $800 to be paid in 6 mo., and $900 to be paid in 9 mo. What is the average term of credit?

4. A owes B $600: $200 to be paid in 6 mo., $150 in 8 mo., and the rest in 12 mo. What is the average term of credit?

5. D owes $300 to be paid in 60 da., $500 to be paid in 120 da., and $750 to be paid in 180 da. Find the average term of credit for the payment of the whole debt.

6. The sum of $1500 was to be paid: one sixth in 2 mo., one third in 3 mo., and the rest in 6 mo. What would be the average term of credit?

7. I owe $800: $100 is due in 2 mo., $400 in 3 mo., and the rest in 6 mo. What is the average term of credit?

8. A owes $400 payable in 3 mo., $500 payable in 6 mo., $600 payable in 8 mo. What is the average term of credit?

9. X bought goods to the amount of $400 on a credit of 3 mo., another lot of goods for $600 on a credit of 4 mo., and another lot for $800 on a credit of 8 mo. What is the average term of credit?

CREDITS OF DIFFERENT DATES

1. Find the equated time of payment of the following sales:

Jan. 6, 1900, $40 due in 3 mo.; Feb. 8, 1900, $100 due in 45 da.; Mar. 14, 1900, $75 due in 75 da.; Apr. 5, 1900, $37 due in 56 da.

We take Mar. 24, the day on which the first debt becomes payable, as the focal date.

$100 due Mar. 24,	.	$100	
40 due Apr. 5,	.	$40 \times 12 =$	480
75 due May 13,	.	$75 \times 50 =$	3750
37 due May 31,	.	$37 \times 68 =$	2516
		252	6746

$$6746 \div 252 = 26\tfrac{194}{252} \text{ da.}$$

Hence the equated time will be 27 da. from the focal date.

Mar. 24 + 27 da. = Apr. 20 = equated time.

2. Find the equated time of payment of the following bills:

Jan. 1, 1900, $175.80 due in 4 mo.; Jan. 16, 1900, $96.46 due in 3 mo.; Feb. 11, 1900, $78.39 due in 3 mo.; Feb. 23, 1900, $49.63 due in 2 mo.; Mar. 19, 1900, $114.92 due in 6 mo.

3. I owe the following bills:

Jan. 7, 1900, $375.60 due in 4 mo.; Apr. 18, 1900, $687.25 due in 4 mo.; June 7, 1900, $568.50 due in 6 mo.; Sept. 25, 1900, $300 due in 6 mo.; Nov. 5, 1900, $675.75 due in 9 mo.; Dec. 1, 1900, $100 due in 3 mo.

What is the equated time of payment?

4. A merchant bought goods as follows, on a credit of six months: Jan. 15, a bill of $3750, Feb. 10, a bill of $3000, Mar. 6, a bill of $2400, June 8, a bill of $2250.

On July 1 he gives his note for the amount. On what day should the note be made payable?

AVERAGE OF ACCOUNTS

1. The account books of A and B show that

A owes B	And that B owes A
$426.70, due Jan. 6, 1900.	$148.37, due Dec. 22, 1899.
413.65, due Feb. 2, 1900.	173.19, due Jan. 29, 1900.
169.28, due Apr. 13, 1900.	587.23, due May 7, 1900.
328.57, due Aug. 29, 1900.	658.45, due Sept. 30, 1900.

When should the balance be paid?

We take April 13, 1900, as the focal date of payment. Then A gains interest on each of his debts which become due to B before that time, and on each of B's debts which become due to him after that time; for he has the use of each for a longer time than is his right. He loses interest on each of his debts which become due to B after that time, and on each of B's debts which become due to him before that time; for he does not have the use of them for so long a time as is his right. Hence A gains the interest of:

$426.70 from Jan. 6 to Apr. 13, 97 da.	=	$6.90
413.65 from Feb. 2 to Apr. 13, 70 da.	=	4.83
169.28 from Apr. 13	=	0.00
387.23 from Apr. 13 to May 7, 24 da.	=	2.35
658.45 from Apr. 13 to Sept. 30, 170 da.	=	18.65
Sum of gains	=	$32.73

A loses the interest of:

$328.57 from Apr. 13 to Aug. 29, 138 da.	=	$7.56
$148.37 from Dec. 22, 1899, to Apr. 13, 1900, 112 da.	=	2.76
$173.19 from Jan. 29 to Apr. 13, 74 da.	=	2.14
Sum of losses	=	$12.46
Excess of A's gain over his loss, or of B's loss over his gain	=	$20.27

But the sum of A's debts is \$1338.20, and of B's is \$1567.24. \$1567.24 − \$1338.20 = \$229.04, the balance which B owes A.

The question now resolves itself into this: If by B's paying A \$229.04 April 13, 1900, A gains and B loses \$20.27 interest, when can he pay it without any gain or loss of interest? Evidently as many days after April 13, 1900, as it will take \$229.04 to gain \$20.27 interest. This is 531 da. Hence the equated time is Sept. 26, 1901.

The method of finding the equated time when there are entries on both the debit and credit sides of an account is the same in principle as that in which there are entries of one kind only.

2. Find the equated time for paying the following account:

A owes B	B owes A
\$876.37, due April 5, 1900.	\$228.13, due April 28, 1900.
\$579.48, due May 3, 1900.	\$347.16, due June 3, 1900.
\$487.83, due June 11, 1900.	\$313.27, due July 28, 1900.
\$145.38, due Aug. 8, 1900.	\$839.42, due Sept. 1, 1900.

3. Find the equated time of payment, allowing interest on each item from date.

Dr. W. T. Brooks. *Cr.*

1900			1900		
Jan. 22	To mdse.,	\$89 00	Jan. 4	By mdse.,	\$77 00
" 24	"	76 00	Apr. 16	"	40 00
Feb. 20	"	25 00	May 14	"	143 00
" 23	"	210 00			
Apr. 4	"	189 00			
May 21	"	30 00			

4. Find the equated time of payment, when six months' credit is given on the merchandise items.

HIRAM LEWIS,

Dr. in account with JAMES WARREN. *Cr.*

1900			1900		
Feb. 16	To mdse.,	$375 80	Mar. 20	By cash,	$300 00
Apr. 8	"	432 18	June 17	mdse.,	371 50
May 17	"	320 15	July 4	cash,	200 00
July 13	"	158 12	Sept. 25	mdse.,	85 20

5. I owe William Gerry, May 1, 1900, for mdse., $500;
May 15, for mdse., $400;
June 14, for mdse., $300;
July 24, for mdse., $100.

Gerry owes me March 7, 1900, for an automobile bicycle, $400;
April 2, for electrical supplies, $200;
May 6, for a dynamo, $300;
June 13, for a battery, $120.

Allowing all the items six months' credit, when will the balance of the account become due?

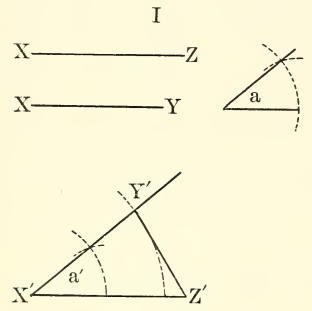
6. Find the equated time of payment for the following account, each item due a year after date, interest at 6%, viz.:

A Dr. to B	B Dr. to A
Jan. 1, 1901, $38.64.	Jan. 10, 1901, $25.
May 5, 1901, $86.90.	Aug. 18, 1901, $50.
Apr. 20, 1902, $43.75.	Sept. 15, 1901, $90.
June 1, 1902, $96.80.	July 10, 1902, $15.

INVENTIONAL GEOMETRY

I

To draw a triangle with two sides and the included angle given



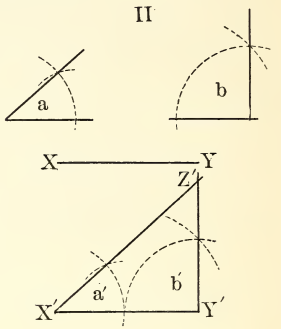
Draw $X'Z'$ equal to XZ. At X' draw the angle a' equal to a. Upon $X'Y'$ the side of the angle a, mark off the length $X'Y'$ equal to XY. Connect Y' and Z'.

The triangle $X'Y'Z'$ has two sides equal respectively to XY and XZ and the included angle a' equal to a.

This problem may be solved only when the given angle a is less than 180°.

II

To draw a triangle with two angles and the included side given



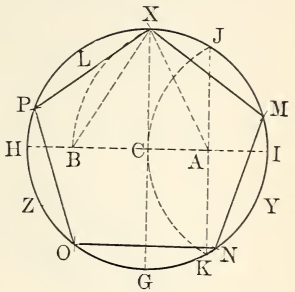
Draw $X'Y'$ equal to XY. At X' draw the angle a' equal to a; and at Y' draw the angle b' equal to b. Extend the sides of the angles, not given by $X'Z'$, until they intersect at Z'.

The triangle $X'Y'Z'$ has two angles and the included side equal to the angles and sides given.

This problem may be solved only when the sum of the two angles given is less than 180°.

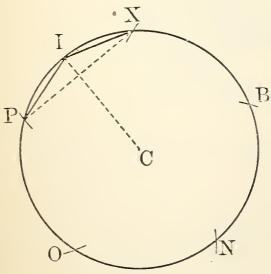
To inscribe a pentagon in a circle

From C as a center, with any radius CX, draw a circumference XYZ. Through C and X draw the diameter XG. At right angles with this diameter draw the diameter HI. From I as the center, and with the radius XC, draw the arc JCK. Draw the chord JK of the arc JCK. From A, the point of intersection of the chord JK and the diameter HI, draw the arc BLX. The chord BX of this arc BLX is equal to a side of the inscribed pentagon.



From X, with the radius BX, mark the length of an arc XM upon the circumference. With the same radius, mark also MK, KO, OP, and PX. Draw chords connecting these points.

$XMNOP$ is a pentagon inscribed in the circle XYZ.



To inscribe a dekagon in a circle

Inscribe the pentagon $XBNOP$. Draw CI, a radius perpendicular to PX. Draw the chords PI and IX. These chords are two sides of a dekagon inscribed in the circle. Complete the dekagon.

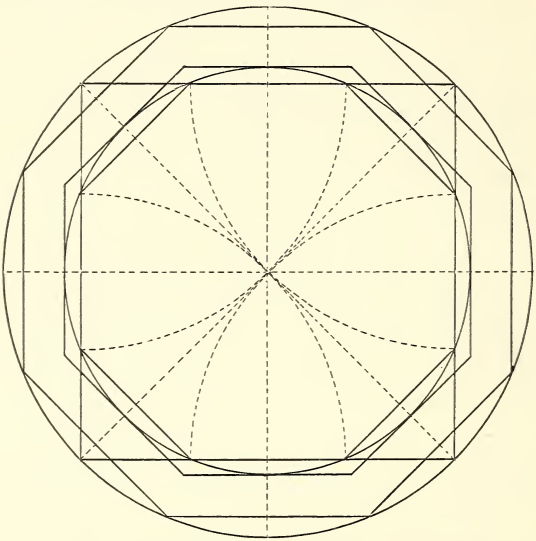
Dekagon is sometimes spelled decagon.

a. By this method, upon the blackboard, inscribe in a circle with a radius of one foot a figure with twenty sides.

b. Tangents to the circumference, parallel with the sides of the figure inscribed as in *a*, form a figure of twenty sides circumscribed about the circle. By this method, circumscribe such a figure about a circle.

To analyze a design

Every design in industrial art is based upon standard geometric forms, some of which may have been used merely as construction lines, while others may appear in the finished product.



a. Find the construction lines, noting the order in which they were drawn.

b. Find a square inscribed in the outer circle.

c. Find an octagon inscribed in the inner circle.

d. Find an octagon circumscribed about the inner circle.

e. Find an octagon inscribed in the outer circle.

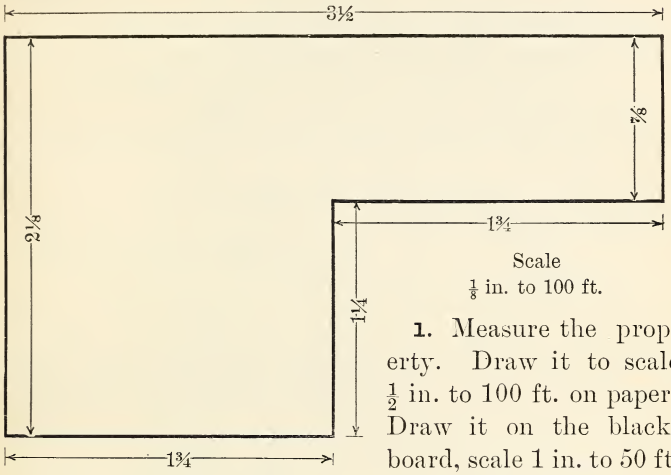
f. Find how each circle and octagon is drawn.

g. Make designs, using these figures in various combinations.

h. Make designs, using other figures in similar combinations.

To plot a piece of land in house lots

A bought a piece of land shaped like this, with the measurements indicated by the scale:



1. Measure the property. Draw it to scale $\frac{1}{2}$ in. to 100 ft. on paper. Draw it on the blackboard, scale 1 in. to 50 ft.

2. There were streets already all around the land. What other streets would you suggest? How wide should they be?

3. The owner paid $1000 an acre for the land. What was its cost to him?

4. If all the land was equally valuable for building purposes, plot it out in suitable lots and put such a price on each lot as would give the owner a gross return equal to $3000 an acre.

5. If the city required the owner to pay for streets, sewers, taxes, etc., in developing the property, in all $10,000, and he sold all the lots, about what amount of money did he make or lose?

(In developing the answers use "round numbers" only.)

Development of the Solution

In the proportion we have one great principle :
The product of the means equals the product of the extremes.
In the equation also we have one great principle :
Either member equals the other member.

We may do to one member of an equation whatever we wish when we do exactly the same thing to the other member so as to maintain the equality of the members.

I. Addition

We may add the same number to each member of an equation

$$x = 5 + 8, \quad x + 3 = 5 + 8 + 3.$$

II. Subtraction

We may take the same number from each member of an equation

$$x = 5 + 8, \quad x - 3 = 5 + 8 - 3.$$

$$\frac{15x + 18}{3} = 26.$$

$$\frac{15x + 18}{3} - \frac{13}{5} = 26 - \frac{13}{5}.$$

III. Multiplication

We may multiply each number of an equation by the same number

$$x = 5 + 8, \quad 10x = 10(5 + 8) = 50 + 80.$$

$$\frac{15x + 18}{3} = 26, \quad 15x + 18 = 78.$$

IV. Division

We may divide each member of an equation by the same number

$$x = 5 + 8, \qquad \frac{x}{3} = \frac{5 + 8}{3}.$$

$$\frac{15x + 18}{3} = 26, \quad \frac{15x + 18}{6} = 13.$$

When the member of the equation consists of several terms, the whole member must be affected exactly as the other member is affected.

$15x = 12x + 36$. Divide by 3. $5x = 4x + 12$.

If we divided by 3 only the $12x$ in the second member, we would completely change the equality of the two members of the equation.

In solving problems, we find all these processes useful.

Algebraic Equations

In **algebraic processes** we consider that the last three letters, x, y, z, of the alphabet represent unknown quantities. The purpose of many mathematical processes, whether in *arithmetic*, *geometry*, or *algebra*, is to solve problems, to find from known facts another unknown fact, called the answer.

In a simple algebraic solution we let x represent some unknown quantity, and form an equation based upon the given facts.

I. What number added to twice itself makes 72?

Let x = the number,

then $x + 2x = 72$, $3x = 72$, $x = 24$.

II. Find two numbers whose sum is 59 and whose difference is 17.

Let x = the greater number,

then $x - 17$ = the lesser number,

$x + (x - 17)$ = their sum,

$x + (x - 17) = 59$, or $2x - 17 = 59$,

$2x = 76$,

$x = 38$ = the greater number,

$59 - 38 = 21$ = the lesser number.

Proof: $38 + 21 = 59$, the sum; and $38 - 21 = 17$, the difference.

III. Two numbers differ by 8, and four times the less number exceeds twice the greater by 10. What are the numbers?

Let x = the greater number,

then $x - 8$ = the lesser number,

and $4(x - 8) - 2x = 10$,

$4x - 32 - 2x = 10$;

$\therefore\ 2x = 42$,

$x = 21$ = the greater number,

$21 - 8 = 13$ = the lesser number.

Proof: $52 - 42 = 10$.

IV. A father is twice as old as his son; twenty years ago he was three times as old. Find the present age of each.

Let x = the number of years in the son's age,

then $2x$ = the number of years in the father's age,

$x - 20$ = the number of years in the son's age, 20 yr. ago,

$2x - 20$ = the number of years in the father's age, 20 yr. ago.

Since 20 years ago the father was three times as old as his son,

$2x - 20 = 3(x - 20)$,

$2x - 20 = 3x - 60$,

$-x = -40$; or $40 = x$ = son's age,

and $2x = 80$ = the father's age.

1. Find a number such that when 12 is added to twice itself the sum is 28.

2. When we add 46 to a number, the result is three times as large as the original number. What is the original number?

3. A bought some apples, lemons, and oranges for 91¢. He paid twice as much for the lemons as for the apples, and twice as much for the oranges as for the lemons. How many cents did he expend on each kind of fruit?

4. A rectangular prism whose dimensions are in the ratio of 3, 4, and 5, contains 480 cu. in. What is each of its dimensions?

5. Divide $140 among A, B, and C, so that A may have twice as much as B, and B three times as much as C.

6. Find two numbers whose sum is 31 and whose difference is 5.

7. What two numbers are those whose difference is 14 and whose sum is 48?

8. Find two numbers whose sum is 84 and whose difference is 14.

9. $x + (x - 12) = 40$. Find x.

10. $3x - (x + 15) = 1$. Find x.

11. Two numbers differ by 3, and the sum of twice the greater added to three times the less is 41. What are the numbers?

12. The sum of two numbers is 100, and the greater exceeds three times the less by 4. Find the numbers.

13. Divide 90 into two parts, such that five times one part may be equal to four times the other part.

With these examples as models the pupils may invent others.

14. A father is three times as old as his son, and nineteen years hence he will be only twice as old. What is the present age of each?

15. In eight years a boy will be three times as old as he was eight years ago. How old is he?

16. The sum of the ages of two persons is now forty-six years, and the difference of their ages ten years since was twelve years. Find the present age of each.

17. A house and lot cost $1000, and ten times the price of the house was equal to fifty times the price of the lot. Find the price of the house and the price of the lot.

Let x = the price of the house in dollars.

18. Divide $1000 among A, B, and C, so that B shall have $100 more than A, and A four times as much as C.

19. A had $20 more than B, and after each had spent $5, A had five times as much as B. What sum had each man at first?

Comparison of Algebra and Arithmetic

The nature of the methods and processes in *arithmetic* differs from that of the methods in *algebra* as seen in the following problems:

First Comparison

1. A dealer sold a horse at a gain of $12\frac{1}{2}\%$ for $72. What was the cost of the horse to the dealer?

Arithmetical solution

If $72 = the cost and $12\frac{1}{2}\%$ more,
then $72 = $112\frac{1}{2}\%$ of the cost $= \frac{9}{8}$ of the cost.
If $72 = $\frac{9}{8}$ of the cost,
then $8 = $\frac{1}{8}$ of the cost,
and $64 = the whole cost to the dealer.

ALGEBRAIC SOLUTION

Let x = number of dollars paid to the dealer,

$x + 12\frac{1}{2}\%$ of $x = \$72$,

$x + \frac{x}{8} = \$72$, $\qquad 8x + x = \$576$,

$9x = \$576$, $\qquad x = \$64$.

SECOND COMPARISON

2. John's father is twice as old as he, and the sum of their ages is 45 years. How old is each?

ARITHMETICAL SOLUTION

If the father is twice as old as the son, then we may think of 45 yr. as divided into 3 parts, of which the father has 2 and John has 1.

$15 + 30 = 45$,

$15 \times 2 = 30$,

$\frac{1}{3}$ of $45 = 15$,

$\frac{2}{3}$ of $45 = 30$.

ALGEBRAIC SOLUTION

Let x = number of years in John's age,

then $2x$ = the father's age,

and $x + 2x = 45$, $\qquad 3x = 45$,

$x = 15$, $\qquad 2x = 30$.

Algebra is more formal in the steps of its reasoning than arithmetic. It helps most in hard problems because it is so systematic.

Make comparisons of solutions of other problems.

Factoring

1. Resolve into factors $3\,x^2 + 6\,xy + 9\,xy^2$.

$$3\,x^2 + 6\,xy + 9\,xy^2 = 3\,x(x + 2\,y + 3\,y^2).$$

2. Find the factors of $10^2\,ab^2 - 16\,a^3b^3 + 8\,ab$.

$$10\,a^2b^2 - 16\,a^3b^3 + 8\,ab = 2\,ab(5\,ab - 8\,a^2b^2 + 4).$$

Resolve into factors:

3. $a^2b + abc$.
4. $4\,xy + 8\,xz$.
5. $3\,a^2 + 9\,a^3 - 12\,a^4$.
6. $4\,a^2 - 16\,ab + 24$.
7. $3\,a^3b^4 - 21\,a^4b^3 + 27\,a^5b^2$.
8. $6\,x^2yz + 12\,xy^2z - 18\,x^3y^3z$.

   $25\,a^2b^6 + 35\,a^4b^8 - 45\,a^6b^9$.

   $42\,m^2n^2p^3 + 49\,m^3n^3p^2 - 56\,m^4n^4p$.

Monomials

In algebra an expression consisting of but a single term is called a **monomial.**

$$x;\quad 2\,x^2;\quad 4\,abx^3y^4.$$

Binomials

An expression consisting of two terms is called a **binomial.**

$$a + x;\quad x^2 + y^2;\quad ax^2 + b^2x^2y^2z^2.$$

Trinomials

An expression consisting of three terms is called a **trinomial.**

$$a^2 + 2\,ax + b^2;\quad 4\,x^2z^3 + 10\,a^5z^5 + y^3.$$

Polynomials

An expression consisting of more than two terms is called a **polynomial.**

$$a^3 + 2\,a^2b + 2\,ab^2 + b^3;\quad axz^4 + 10\,xz^5 + 10\,yz^6 + xyz^5.$$

Highest Common Factor

1. Find the H. C. F. of $8a^3$; $12ab$; $20a^2x$, and $40ad^2$.

$$8a^3 = 2^3 \times a^3,$$
$$12ab = 2^2 \times 3 \times a \times b,$$
$$20a^2x = 2^2 \times 5 \times a^2 \times x,$$
$$40ad^2 = 2^3 \times 5 \times a \times d^2.$$

Taking the product of all the common factors, we have

$$2^2 \times a = 4a = \text{the H. C. F.}$$

2. Find the H. C. F. of $8a^2b^3cd$, $16a^3b^5c^2$, and $28a^4b^4c$.

The H. C. F. of the coefficients found by the general rule is 4; it is evident that no higher power of a than a^2, of b than b^3, of c than itself, will divide the quantities; and that d will not divide them. Therefore, the divisor sought is $4a^2b^3c$.

Find the H. C. F. of:

3. $4a^2x^2$ and $10ax^3$.
4. $9abc^3$ and $12bc^4x$.
5. $4a^3b^2y^3$ and $8a^5x^2y^2$.
6. $77c^4d^6x^3$ and $44c^2d^4x^2z$.
7. $99ab^2c^4d^3x^6y^5$ and $22a^2b^4c^3d^9x^5$.
8. $17x^4y^2$, $19x^2y^3$, and $212bx^7y^4z^5$.
9. $15a^2mn$, $25am^2$, and $30amn^3$.
10. $27x^2y$, $45ax^2$, $72a^2mx^2$, and $36x^2y^2$.
11. $63a^3b^7c^4d^6$, $27a^4b^8c^5$, and $45a^2b^9c^3d$.
12. $3a^4y^3$, $6a^5x^3y^5$, and $9a^6y^4z$.
13. $8ax^2y^4z^5$, $12x^5z^6$, and $24a^3x^3z^2$.
14. $6a^2xy^2$, $12a^3y^4z^5$, $9a^5x^3y^4$, and $24a^3y^6z$.
15. $32ab^3c^6d^4z^2$, $36a^2b^4c^5d^3xz$, $148a^2b^3c^4d^3z^2$.
16. $24a^2x^4y^5z$, $36a^3x^3y^6z^4$, $54a^2x^3y^7z^2$.
17. $15ab^3cx^2$, $35a^3b^2c^2z$, $45a^2b^2cx$.

SQUARE ROOT

A **root** of any number is one of the equal factors of the number; thus, 5 is the cube root of 125, because $5 \times 5 \times 5 = 125$.

Evolution is the process of finding any required root of a number. It is the reverse of **involution.**

The **radical sign** is a character ($\surd$) that is placed before numbers to denote that a root is to be extracted.

The **square root** of a number is one of its two equal factors. Thus, 9 is the square root of 81, because 9 is one of the two equal factors of 81.

7 is the cube root of 343, because 7 is one of the three equal factors of 343.

The **index** of a root is a figure placed above the radical sign at the left, to denote what root is to be taken, that is, into how many equal factors the number is to be resolved. To express the square root, the radical sign is used without any index.

$\surd 49$, square root of $49 = 7$, since $7 \times 7 = 49$.

$\sqrt[3]{64}$, cube root of $64 = 4$, since $4 \times 4 \times 4 = 64$.

$\sqrt[4]{625}$, fourth root of $625 = 5$, since $5 \times 5 \times 5 \times 5 = 625$.

Any number whatever may be considered a power, whose root is to be extracted; but only the perfect powers can have whole numbers as roots

$$\sqrt[3]{10.648} = 2.2 \times 2.2 \times 2.2.$$

10.648 is the third power of 2.2, or $(2.2)^3$.

$256 = 16 \times 16.$ $\qquad \sqrt[4]{16} = 2.$ $\qquad \sqrt[8]{256} = 2.$

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256.$$

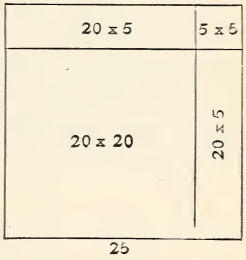
The relation of a number to its square root is shown by the analysis of algebra.

$$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$$

$$25^2 = (20+5)^2$$

$$= 20^2 + 2(20 \times 5) + 5^2 = 625$$

The square of any number of two figures equals the square of the tens, plus twice the product of the tens by the units, plus the square of the units.



We may illustrate square root geometrically. The square root of 625 is 25. Show that this illustration represents 25 squared; that is, 625 square units. Reproduce on the blackboard to larger scale, showing 25 square units in the square that is 5×5.

The square of any number contains either twice as many figures as the number itself, or twice as many, minus one.

$1^2 = 1$
$7^2 = 49$
$10^2 = 100$
$49^2 = 2401$
$125^2 = 15625$

A root consisting of one place may have one or two places in its square, and the addition of one place to the root adds two places to the square. When a number is separated into periods of two figures each, beginning at the right, there are as many periods in the number as there are figures in its square root.

The left-hand period may contain only one figure.

Find the square root of 1225.

The root consists of two figures, since there are two periods in the number. 9 is the greatest number that is the square of a whole number and is less than 12.

```
12·25(3
 9
```

We subtract 9 from 12, and annex to the 3 the second period 25, to make a dividend, and we double the first figure of the root, and write down the result as the first term of a divisor.

```
12·25(3
 9
6 | 325
```

The 6 is six tens, or 60, which is contained in 325 five times. We write five as the second figure of the root, and also annex 5 to the 6. We multiply 65 by 5, and write the product under the 325. Subtracting, we have no remainder. 35 is the square root of 1225.

```
12·25(35
 9
65 | 325
     325
```

Proof: 35

35

175

105

1225

1. What are the squares of 1? 2? 3? 4? 5? 6? 7? 8? 9?

In finding the square root of large numbers we must know the squares of the first nine numbers thoroughly.

2. What are the square roots of 16? 81? 49? 9? 25? 64? 36? 1? 4?

To find the square root of a number

Separate the given number into periods of two figures each, beginning at units' place.

Find the greatest square in the left-hand period, and write it as the first root figure. Subtract its square from the first period, and to the remainder annex the second period for a dividend.

Double the root already found, and find how many times it is contained in the dividend, omitting the right-hand figure. Annex the quotient to the root already found and to the trial divisor. Multiply the divisor thus com-

pleted by the last root figure, subtract, and bring down the next period as before.

When any trial divisor is not contained in the dividend with its last figure omitted, annex a cipher to the root already found and also to the trial divisor, bring down the next period, and find how many times the trial divisor is contained in it.

When, on multiplying a complete divisor by the last root figure, one finds that the product is greater than the dividend, the last root figure must be diminished.

If, when all the periods have been brought down, there is still a remainder, periods of decimal ciphers may be supplied, and the operation continued. The root figures corresponding to the decimal periods will be decimals.

To point off a decimal for the extraction of the square root, commence at the decimal point and go to the right, completing the last period, if necessary, by annexing a cipher.

Find the square root of 622521.

```
      62·25·21(789
        49
148   1325
      1184
1569   14121
       14121
```

49 is that greatest square of a whole number which is equal to or less than 62.

Write 7, the root of 49, in the quotient, subtract the 49 from 62, and to the remainder annex the next period, 25, for a new dividend.

Double 7, and write 14 as an incomplete divisor to the left of the dividend, using 14 to find the next quotient, or root-figure, 8; annex 8 to both the 14 and the 7.

Multiply 148 by 8, subtract the product from 1325, and to the remainder annex the third period.

Double 78, the root already found, and use the 156 as an incomplete divisor to find the next root figure 9; annex 9 to the 156 and to the 78.

Multiply the 1569 by 9, and subtract the product from 14121. Since there is no remainder, the original number is an exact square, the root of which is 789.

Find the square root of .08042896.

$$
\begin{array}{rl}
 & 0.08\cdot04\cdot28\cdot96(.2836 \\
 & \quad 4 \\
48 & \overline{\;404} \\
 & \quad 384 \\
563 & \overline{\;2028} \\
 & \quad 1689 \\
5666 & \overline{\;33996} \\
 & \quad 33996
\end{array}
$$

I. What is the square root of

1.	6561?	6.	23804641?	11.	6.5536?
2.	10201?	7.	10673289?	12.	0.00390625?
3.	3455881?	8.	20894041?	13.	17?
4.	516961?	9.	42025?	14.	37.5?
5.	182329?	10.	1014049?	15.	0.0000012321?

II. Find the square roots to three decimal places:

1.	3271.4207.	6.	1.324.	11.	.00103041.
2.	19.876.	7.	.0025.	12.	$\frac{3}{4}$.
3.	46.80231.	8.	$\frac{144}{289}$.	13.	3.
4.	4795.25731.	9.	2.	14.	4.426816.
5.	4.372594.	10.	.60032754.	15.	338.633604.

Applications of Square Root

Since the area of any rectangle is the product of the numbers representing its length and breadth, it follows that the side of a square may be found by extracting the square root of its area.

1. What is the length of one side of a square room, the area of whose floor is 1600 sq. ft.?

$$\sqrt{1600} = 40$$

To find the side of a square equal to a given area, find the square root of that area

2. A farm in the form of a square contains 160 A. What is the length of a side in rods?

3. A house lot contains 15,625 sq. ft. What is the length of a side?

4. In the right angle triangle ABC the side AC is 40 ft. long and the side CB is 30 ft. long.

Draw the triangle to scale.

$$\sqrt{40^2 + 30^2} = 50$$

The square root of the sum of the squares of these numbers is 50.

AB, the hypotenuse of the triangle, is 50 ft.

To find the side of a right angle triangle when two sides are known

1. **To find the hypotenuse**

Extract the square root of the sum of the squares of the two other sides.

2. **To find one perpendicular side**

From the square of the hypotenuse take the square of the given side, and extract the square root of the remainder.

A wire drawn taut from the top B of a steeple AB to C, 50 ft. from A, is 120 ft. long.

How high is the steeple AB?

$$120^2 = 14400 \text{ sq. ft.},$$
$$50^2 = 2500 \text{ sq. ft.},$$
$$14400 \text{ sq. ft.} - 2500 \text{ sq. ft.} = 11900 \text{ sq. ft.},$$
$$\sqrt{11900} \text{ (sq. ft.)} = 109.08 + \text{ ft.}$$

Draw illustrations to scale.

1. A rope 100 ft. long is tied on the street 60 ft. from a building. How far up the building will the rope reach?

2. Two persons start from the same place and go the one due north 50 mi., the other due west 80 mi. How far apart are they?

3. What is the distance through the opposite corners of a table top 1 yd. square?

4. What is the distance measured through the center of a cube from one corner to its opposite corner, when the cube is 3 ft. on a side?

5. A room is 16 ft. long, 12 ft. wide, and 10 ft. high. *a*. What is the diagonal distance from corner to corner measured on the floor? *b*. What is the diagonal distance measured on a wall which forms a side of the room? *c*. What is the diagonal distance measured on a wall which forms an end of the room? *d*. What is the diagonal passing through the center of the room from ceiling to floor?

6. A general has an army of 226,567 men. How many must he place rank and file to form them into a square?

7. I have 1280 A. of land in a square. How many rods square is it?

8. A piece of land is 125 rd. long and 53 rd. wide, another is $62\frac{1}{2}$ rd. long and 34 rd. wide, and a third contains 37 A. What will be the length of the side of a square field whose area will equal the three lots?

9. There are two house lots; the first is 242 ft. square, and the second is 9 times the area of the first. How many feet square is the second lot?

10. The hypotenuse of a right-angled triangle is 60 ft., and the perpendicular side is 36 ft. What is the length of the base?

11. Two steamers start from the same spot; one goes due north 360 mi., and the other due east 450 mi. What is their distance from each other then?

CUBE ROOT

The **cube root** of a number is one of its three equal factors.

Roots	Cubes	Roots	Cubes
1	1	1	1
9	729	10	1,000
99	907,299	100	1,000,000
999	987,012,999	1,000	1,000,000,000

The cube or third power of each of the numbers contains either three times as many figures as the root, or three times as many minus one or two. To determine the number of figures in the cube root of a given number:

Beginning at the right, point off the number into as many periods as possible of three figures each, and there will be as many figures in the root as there are periods.

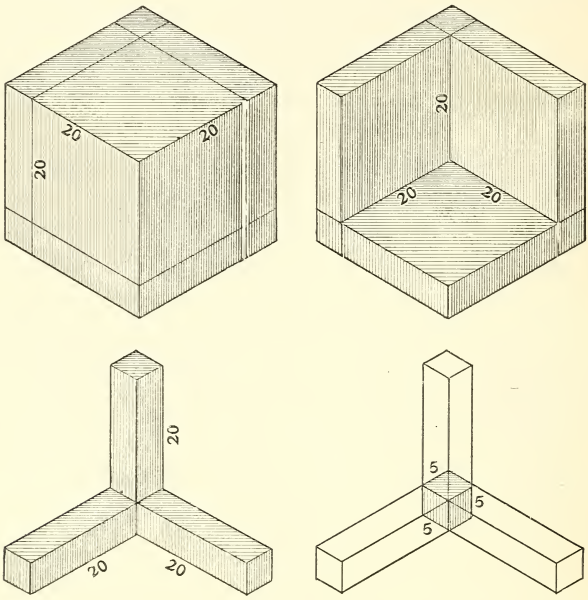
The left-hand period may contain one, two, or three figures.

The relation of a number to its cube root is shown by the analysis of algebra.

$$\begin{aligned}(a+b)^3 &= (a+b)(a+b)(a+b)\\ &= a^3 + 3\,a^2b + 3\,ab^2 + b^3\\ &= (40)^3 + 3((40)^2 \times 5) + 3(40 \times 5^2) + 5^3\\ &= 64000 + 24000 + 3000 + 125\\ &= 91125\end{aligned}$$

The cube of any number of two figures equals the cube of the tens plus three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.

We may illustrate cube root geometrically. The cube of 25 is 15625.



Take a block of soft pine wood, or a cube of clay, or a piece of raw potato, and cut it as in the illustration. The pine block or other cube, when cut, has one cube $20 \times 20 \times 20$ cubic units in volume, three rectangular prisms, each $20 \times 20 \times 5$ cubic units in volume, three rectangular prisms, each $20 \times 5 \times 5$ cubic units in volume, and one cube $5 \times 5 \times 5$ cubic units in volume.

1. Find the cube root of 41,063,625.

		41·063·625(345
		27
$3^2 \times 300 =$	2700	14063
$3 \times 4 \times 30 =$	360	
$4^2 =$	16	
	3076	12304
		1759625
$34^2 \times 300 =$	346800	
$34 \times 5 \times 30 =$	5100	
$5^2 =$	25	
	351925	17596225

Beginning at the units, we divide the number into periods of three figures. The greatest cube in 41 is 27, the cube of 3. We subtract, and bring down the next period. We square 3, multiply the square by 300, and use the result, 2700, as a trial divisor. We write 4 as the next figure of the root. We multiply the first figure, 3, by this last figure, and the product by 30, place the result under the trial divisor, and under this place the square of the last figure in the root. We now add the three results and obtain the complete divisor. Multiplying this divisor by 4, we subtract and annex to the remainder, 1759, the next period for a dividend, and proceed as before.

The cube of 52 equals

$50 \times 50 \times 50 =$	125000
$50 \times 50 \times 3 \times 2 =$	15000
$50 \times 3 \times 2 \times 2 =$	600
$2 \times 2 \times 2 =$	8
	140608

$52 \times 52 = 2704 = (52)^2$

$2704 \times 52 = 140608 = (52)^3$.

Answer and memorize:

$1^3 = ?$	$3^3 = ?$	$5^3 = ?$	$7^3 = ?$	$9^3 = ?$
$2^3 = ?$	$4^3 = ?$	$6^3 = ?$	$8^3 = ?$	$10^3 = ?$

2. What is the cube root of 429172932007?

		429·172·932·007 |7543	
		343	
Trial div. = $7^2 \times 300$ =	14700	86172	= 1st dividend.
$7 \times 5 \times 30$ =	1050		
5^2 =	25		
1st com. div. =	15775	78875	= product of com. div. by 5.
2d tr. div. = $75^2 \times 300$ =	1687500	7297932	= 2d dividend.
$75 \times 4 \times 30$ =	9000		
4^2 =	16		
2d com. div. =	1696516	6786064	= product of com. div. by 4.
3d tr. div. = $754^2 \times 300$ =	170554800	511868007	= 3d dividend.
$754 \times 3 \times 30$ =	67860		
3^2 =	9		
3d com. div. =	170622669	511868007	= product of com. div. by 3.

The greatest cube in the left-hand period is 343, of which the highest cube root is 7. We place 7 in the root and subtract 343 from the first period. This gives a remainder of 86, to which we annex the next period 172, and thus obtain 86172 for a new dividend.

We square 7 and multiply the 49 thus obtained by 300, this gives the first trial divisor 14700 which is contained in 86173 5 times.

We complete the trial divisor by adding to it $7 \times 5 \times 30 = 1050$, and $5^2 = 25$ which gives 15775 for a complete divisor. This we multiply by 5, the digit last written in the root, subtract the product and to the remainder 7297 annex the next period 932, and then proceed as before.

3. Cube, and extract the roots of the cubes:

a. 18. *b.* 23. *c.* 33. *d.* 48. *e.* 75. *f.* 89.

To find the cube root

Beginning at units, separate the number into periods of three figures each.

Find the greatest cube contained in the left-hand period and write its root for the first figure of the required root.

Subtract the cube of this root figure from the left-hand period, and to the remainder annex the next period for a new dividend.

Multiply the square of the part of the root already found by 300 *as a* trial divisor.

Find how many times the trial divisor *is contained in the dividend, and put the figure thus obtained in the root.*

To the trial divisor *add the part of the root previously found, multiplied by the last figure of the root* × 30 *and the square of the last figure of the root.*

Multiply the divisor thus completed by the last figure of the root: subtract the product from the dividend, and to the remainder bring down the next period for a new dividend. Proceed thus till the work is completed.

4. Extract the cube root of:

a. 13824.	*e.* 970299.	*i.* 245314376.
b. 32768.	*f.* 1953125.	*j.* 2803221.
c. 250047.	*g.* 15813251.	*k.* 1860867.
d. 658503.	*h.* 48228544.	*l.* 1076890625.

Applications of Cube Root

Geometry shows that **the volumes of all similar solids are to each other as the cubes of their like dimensions.**

Vol. of xyz : vol. of $abc :: x^3 : a^3$ or $y^3 : b^3$ or $z^3 : c^3$.

The volumes of cubes are to each other as the cubes of their edges.

The volumes of two spheres are to each other as the cubes of their radii, or as the cubes of their diameters, or as the cubes of their circumferences.

1. A sphere, 3 in. in diameter, weighs 8 lb. What is the weight of a sphere of the same material 4 in. in diameter?

$$3^3 : 4^3 = 8 : x.$$

$$4^3 = 64,$$

$$3^3 = 27.$$

Therefore $$x = \frac{8 \times 64}{27} = \frac{512}{27} = 18\tfrac{26}{27} \text{ lb.}$$

2. When an iron ball, 5 in. in diameter, weighs 16 lb., what is the weight of an iron ball 20 in. in diameter?

3. When a man digs a cubical pit, whose edge is 5 ft., in one day, how long does it take him to dig a similar excavation whose edge is 25 ft.?

4. How many lead balls, each $\frac{1}{4}$ of an inch in diameter, are required to make a ball 1 in. in diameter?

5. When a globe of gold, 1 in. in diameter, is worth $100, what is the diameter of a globe worth $2700?

6. 1000 bodies like the earth are required to make 1 planet the size of Saturn. Since the diameter of Saturn is 79,000 mi., what is the diameter of the earth?

7. There are three spheres, whose diameters are respectively 3, 4, and 5 in. The weight of the largest is 275 lb. Find the weight of the others.

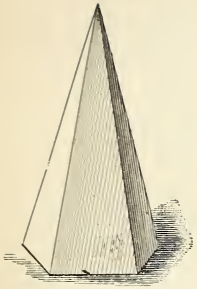
8. How many worlds like ours could be contained in the space occupied by the sun, the earth being 7912 mi. in diameter, and the sun being 886,144 mi. in diameter?

INVENTIONAL GEOMETRY

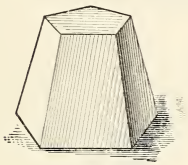
In the following last section of concrete geometry as presented in this series it is important to pursue the same objective and inventional methods as in the earlier sections. All the work should be done in class, and rather as a recreation from the other mathematics of the book than as a formal required exercise. Interest aroused in geometry leads to that familiarity with its real facts which is the best basis for later advanced work in logical demonstration. We know life itself before understanding it; and pupils ought to know these geometrical facts before inquiring too analytically and minutely into them.

REGULAR SOLIDS

Polyhedrons



RIGHT HEXAGONAL PYRAMID



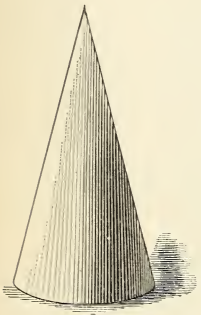
FRUSTUM OF PYRAMID

A **polyhedron** is a solid bounded by flat surfaces called **planes.**

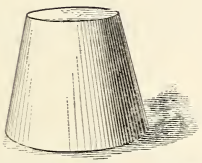
Right when used in defining polyhedrons means, *with lateral faces or sides perpendicular to the base.*

Oblique means, *not right.*

The **frustum** of a solid is a portion cut off by a plane parallel with the base, removing its apex.

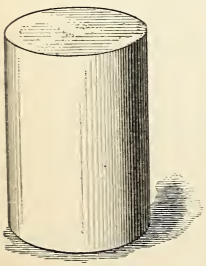


RIGHT CONE

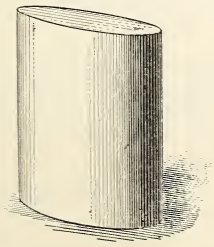


FRUSTUM OF RIGHT CONE

What are the forms of the various planes that bound these solids?

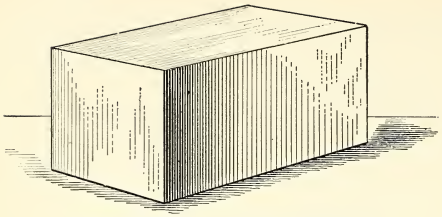


RIGHT CYLINDER

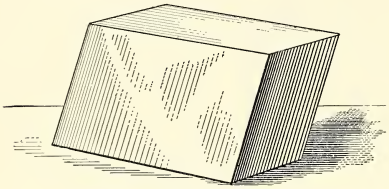


OBLIQUE CYLINDER

The illustrations on this page and on succeeding pages are brought together for the purpose of comparison and contrast.



RECTANGULAR PARALLELEPIPED



OBLIQUE PARALLELEPIPED

A **parallelepiped** is a prism, all of whose faces are parallelograms.

Cut these figures out of cardboard, with bases equal, and measure their comparative volumes by using dry sand. For methods of making these various figures, see the earlier books of this series, as well as other pages in this book.

Discuss the following facts:

The *area* of the surface of any *prism* or *pyramid* may be found by adding the sum of the areas of its various surfaces.

The *area* of the surface of a *cone* is equal to half the product of the number expressing the circumference of its base by the number expressing half its slant height plus the area of the surface of its base, which is a circle.

The *area* of the surfaces of a *cylinder* equals the product of the numbers expressing the circumference of its base and its altitude plus the areas of its bases.

The *area* of the surfaces of a *parallelepiped* equals the product of the numbers expressing the edge and the altitude of its faces plus the areas of its bases.

The *area* of the surfaces of a *regular right pyramid* equals one half the product of the numbers expressing the

perimeter of its base and its slant height plus the area of its base.

The *volume* of a *right cone* equals one third the product of the numbers expressing the circumference of its base and altitude.

The *volume* of a *regular right pyramid* equals one third the product of the numbers expressing the perimeter of its base and its altitude.

The *volume* of a *parallelepiped* equals the product of the numbers expressing its three dimensions.

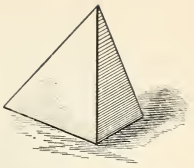
The *volume* of a *right cylinder* equals the product of the numbers expressing the area of its base and its altitude.

Make drawings to illustrate the following problems:

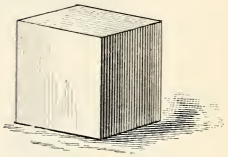
1. Find the area of the surface of a right cylinder 3 in. in diameter and 5 in. in altitude.

2. Find the volume of a regular right hexagonal pyramid, base 4 ft., altitude 3 ft.

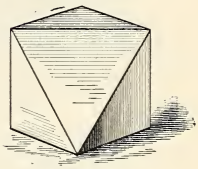
3. Find the volume of a regular right cone, base 5 ft. in diameter and height 10 ft.



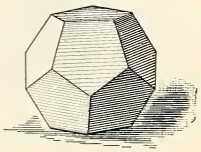
TRIANGULAR PYRAMID
TETRAHEDRON



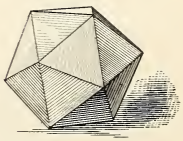
CUBE
HEXAHEDRON



OCTAHEDRON
(8 sides)



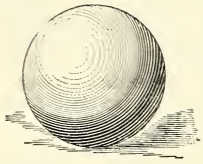
DUODEKAHEDRON
(12 sides)



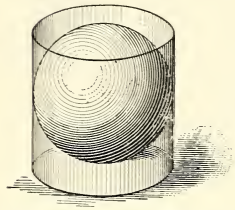
IKOSAHEDRON
(20 sides)

The Sphere

The *area* of the surface of a *sphere* equals the lateral area of the circumscribed right cylinder : equals the product of the numbers expressing its diameter and its greatest circumference.



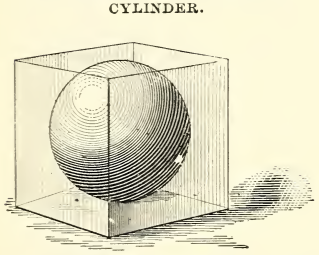
A SPHERE.



THE SPHERE INSCRIBED IN A CYLINDER.

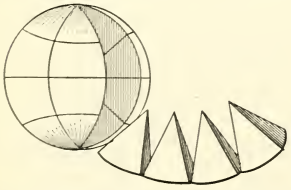
The *area* of the lateral surface of the circumscribed *cylinder* equals the number expressing its altitude (that is, the diameter of the inscribed sphere) and its base (that is, the greatest circumference of the sphere).

The *volume* of a *sphere* equals one third the product of the numbers expressing the area of its surface and its radius : equals the sum of the volumes of the pyramids into which it may be divided.



THE SPHERE INSCRIBED IN A CUBE.

The *volume* of a *pyramid* equals one third the product of the numbers expressing the perimeter of its base and its altitude. The altitude is the radius here. Take an orange, or an apple, or a sphere of clay, and divide it as shown in the illustration.



THE SPHERE CUT INTO TRIANGULAR AND QUADRANGULAR PYRAMIDS.

MENSURATION OF SURFACES

Mensuration is the measurement of magnitudes.

Measuring is applying to a magnitude a standard unit of measure, of the same kind as the magnitude, to find the number of times the unit is contained in the magnitude.

Surfaces have length and breadth.

A **square** is the space included between four equal lines, drawn perpendicular to one another.

A **point** has neither length, breadth, nor thickness, but position only.

A **line** has length, but no breadth or thickness.

A *straight line* is the shortest path between two points.

A *curved line* constantly changes its direction.

Parallel lines are everywhere equally distant from each other; they cannot meet if extended.

Two lines are *perpendicular* to each other when they meet so as to form right angles.

Two lines are *oblique* to each other when they meet so as to form acute or obtuse angles.

A **plane surface** is such a surface that, if any two points upon it are joined by a straight line, that line will lie wholly upon the surface.

A **curved surface** is one that constantly changes its direction.

A **polygon** is a plane figure bounded by straight lines.

The broken line that bounds a polygon is called the **perimeter** of the polygon.

A *polygon* of 3 sides is called a triangle; of 4 sides a quadrilateral; of 5 sides a pentagon; of 6, a hexagon, etc.

A *regular polygon* has equal sides and equal angles.

The Circle

A **circle** is a plane figure, bounded by a curve everywhere equidistant from a point within, called the **center.**

The **circumference** of the circle is the bounding curve.

An **arc** is any part of the circumference.

A **diameter** is a line passing through the center and limited by the circumference.

A **radius** is a line from the center to the circumference.

The radius of a circle is one half its diameter. The diameter is twice the radius.

To find the circumference of a circle:

Multiply the diameter by 3.1416.

Prove this ratio by numerous experiments. For example, with a steel tape, from a fixed center, draw a circumfe ence 50 ft. in radius. Measure the circumference.

1. The circumference of a circle is 15 ft. What is the diameter? Prove by experiment.

2. What is the length of the tire on a wheel whose radius is $2\frac{1}{2}$ ft.?

3. A circular garden has a diameter of 100 rd. What is the length of the fence necessary to inclose the garden?

4. An automobile wheel is 2 ft. 9 in. in diameter. Over what distance will it pass in making 8 revolutions?

5. What is the diameter of a circle whose circumference is 80 miles?

6. The earth's radius being 3962 miles, what is its circumference?

7. When the circumference of a wheel is 62.84 ft., what is the diameter?

8. When the girth of a tree is 12 ft. 5 in., what is the thickness or diameter?

To find the area of a circle:

Multiply the square of the diameter by .7854.

1. What is the area of a circle whose diameter is 9 in.?

2. What is the area:

(*a*) Of a circle whose diameter is 24 ft.?

(*b*) Of a circle whose diameter is 10 ft.?

(*c*) Of a circle whose radius is $1\frac{1}{4}$ ft.?

3. How many acres are there in a circle one mile in diameter?

4. Which is the greater area, a circle 5 ft. in diameter, or the sum of the areas of two other circles, the one being 4 ft. in diameter, and the other 3 ft.?

MENSURATION OF SOLIDS

Cylinder and Prism

A **solid** or **volume** has three dimensions, length, breadth, and thickness.

A **prism** is a solid whose ends or bases are equal and parallel polygons, and whose sides are parallelograms.

Prisms take their names from the forms of their bases, as triangular, quadrangular, pentagonal, etc.

A **parallelepiped** is a prism bounded by six parallelograms, the opposite ones being parallel and equal.

A *cube* is a parallelepiped whose faces are all equal squares.

A **cylinder** is a round volume bounded by a uniformly curved surface, the ends being equal and parallel circles.

The *altitude* of a prism, or of a cylinder, is the perpendicular distance between its bases.

The *convex surface* of a prism or of a cylinder is its entire surface less the surface of the two faces.

To find the convex surface of a prism, or of a cylinder:
Multiply the perimeter of the base by the altitude.

To find the total surface:
Add the area of the bases to the convex surface.

1. What is the total surface of a triangular prism, the width of a side being 3 ft. and the length 15 ft.?

2. What is the total surface of a square prism, each of whose sides is 9 ft. wide and 25 ft. long?

3. What is the total surface of a cylinder whose length is 18 ft. and diameter 3.5 ft.?

4. How much tin will make a cylindrical pail with a cover, the pail being 10 in. in diameter and 12 in. high?

5. What is the surface of a rectangular log 20 ft. long, whose sides are 15 in. and 18 in. broad?

To find the solid contents of a prism, or of a cylinder:
Multiply the area of the base by the altitude.

1. What is the volume of a triangular prism, whose altitude is 20 ft., and the three sides of whose ends are 5, 4, and 3 ft. respectively?

2. How many cubic feet are there in a cube whose side is 8 ft.?

3. What is the number of cubic feet in a room 30 ft. long, 20 ft. wide, and 10 ft. high?

4. How many cubic feet are there in a rectangular stick of timber 10 in. by 12 in., and 36 ft. long?

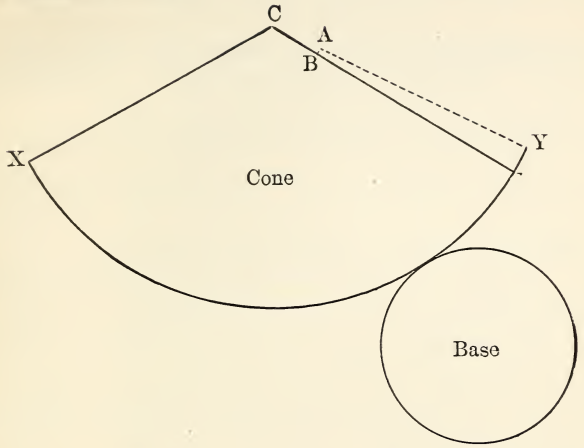
5. In a cylindrical log 14 ft. long and 14 in. in diameter, are how many cubic feet?

6. How many cubic inches are there in a round bar of iron 20 ft. long and $\frac{3}{4}$ of an inch in diameter?

7. What are the contents of a cube with side 24 in.?

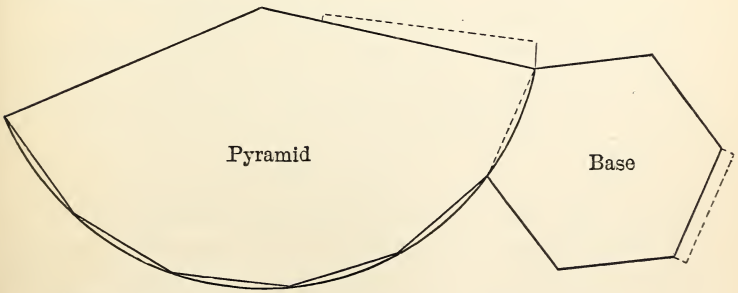
8. How many gallons of water will a cistern contain whose dimensions are 10 ft. in diameter by 18 ft. in depth?

To construct a cone



Upon any suitable heavy paper or light cardboard, with any radius CX equal to the desired slant height of the cone, draw the arc XY. Draw the radii CX and CY'. Describe the base with a radius equal to $\frac{1}{6}$ of the length of the arc XY'. $ABY'Y$ serves as means to fasten the paper with pins or paste in the form of a cone.

To construct a pyramid



Follow the method used in constructing a cone.

Pyramid and Cone

A **pyramid** is a solid or volume that has for its base a polygon, and for its other faces three or more triangles, that meet in a common point called the vertex.

Pyramids, like prisms, take their names from their bases, and are called triangular, square, quadrangular, pentagonal, etc.

A **cone** is a pyramid whose base is a circle.

The **altitude** of a pyramid or of a cone is the perpendicular distance from its vertex to its base.

The **slant height** of a pyramid is the distance from its vertex to the middle of one of the sides of the base.

The *slant height* of a cone is a straight line from the vertex to the circumference of the base.

To find the convex surface of a pyramid or of a cone:

Multiply the perimeter, or the circumference of the base, by half the slant height.

1. How many yards of cloth, that is 27 in. wide, are required to cover the sides of a pyramid whose slant height is 100 ft., and whose perimeter at the base is 54 ft.?

2. Find the convex surface of a cone, whose slant height is 50 ft., and the circumference of its base 12 ft.

If the total surface is required, add the area of the base.

The convex surface of the cone is regarded as made up of an infinite number of triangles whose vertices are at the apex of the cone, and whose bases form the circumference of the base of the cone.

3. What is the convex surface of a pyramid whose slant height is 18 in. and the perimeter of whose base is 27 in.?

4. What is the total surface of an octagonal pyramid whose slant height is 20 ft., and each side of whose base is 2 ft.?

5. What is the convex surface of a cone with slant height 30 yd., and circumference of base 24 yd.?

6. What is the total surface of a cone whose slant height is 60 in., and the radius of whose base is 50 in.?

To find the solid contents, or volume, of a pyramid or of a cone:

Multiply the area of the base by one third of the altitude.

1. What is the volume of a square pyramid each side of whose base is 30 ft., and whose altitude is 25 ft.?

2. What are the solid contents of a pyramid, the area of whose base is 207, and whose altitude is 18?

3. What are the solid contents of a pyramid, the area of whose base is 403, and whose altitude is 30?

4. What are the solid contents of a pyramid, the area of whose base is 270, and whose altitude is 16?

5. A pyramid has a rectangular base, the sides of which are 25 and 12; the altitude of the pyramid is 36. What are the solid contents of the pyramid?

6. A pyramid with a square base, each side of which is 30 ft., has an altitude of 20 ft. What are the solid contents?

7. One of the Egyptian pyramids is square at its base, and measures 693 ft. on a side. Its altitude is 500 ft. What are its solid contents, and how many miles in length of wall would it make, 4 ft. high and 2 ft. thick?

Frustums

The **frustum** of a pyramid or cone is that part of the solid which remains after cutting off the top by a plane parallel to the base.

A *frustum* is often called a *truncated* cone or pyramid.

The convex surface of the frustum of a cone is regarded as composed of an infinite number of trapezoids whose longer bases make up the circumference of the lower base of the frustum, and whose shorter bases make up the circumference of the upper base.

To find the convex surface of a frustum of a pyramid or of a cone:

Multiply the sum of the perimeters, or circumferences, by one half the slant height.

To find the total surface, **add to this product the areas of both ends, or bases.**

1. What is the total surface of the frustum of a square pyramid whose slant height is 10 ft., each side of whose lower base is 3 ft. 4 in., and of the upper base 2 ft. 2 in.?

2. A square pyramid has its top cut off 20 ft. slant height from the base. The length of each side at the base is 8 ft., and at the top 4 ft. Find the total surface of the pyramid.

3. The slant height of a frustum of a cone is 12 ft., the circumference of the base 18 ft., and that of the upper end 9 ft. What is the total surface?

4. What is the convex surface of the frustum of a pyramid whose slant height is 6 ft., and the perimeters of whose bases are 5 ft. and 15 ft.?

5. What is the total surface of the frustum of a pentagonal pyramid, one side of whose lower base is 4 ft., one side of the upper base 2 ft., and slant height 7 ft.?

To find the cubic contents or volume of a frustum of a pyramid or of a cone:

Find the area of each end of the frustum; multiply these two areas together, and extract the square root of the

product. To this root add the two areas, and multiply this sum by one third of the altitude.

1. What is the volume of a frustum of a square pyramid, whose height is 30 ft., and each side at the bottom being 20 ft., and at the top 10 ft.?

2. What are the cubic contents of a stick of timber 20 ft. long, the diameter at the large end being 12 in., and at the small end 6 in.?

3. A stick of timber is a frustum of a cone 42 ft. long, 3 ft. in diameter at the large end, and 6 in. at the small end. How many cubic feet does it contain?

4. What is the volume of a beam 24 ft. long, 15 in. square at one end, and 6 in. square at the other?

5. A measure in the form of a frustum of a cone has its top diameter 6 in., and a bottom diameter 9 in., and is 12 in. deep. How many cubic inches will it contain?

6. A stick of timber in the form of a frustum of a pyramid is 30 ft. long, and 30 in. square at one end, and 13 in. square at the other. How many cubic feet does it contain?

THE SPHERE

A **sphere** is a body bounded by a uniformly curved surface, all the points of which are equally distant from a point within called the center.

The **diameter** of a sphere is a straight line passing through the center of the sphere, and terminated at both ends by the surface.

The **radius** of a sphere is a straight line drawn from the center to any point in the surface.

To find the surface of a sphere:

Square the diameter and multiply it by 3.1416.

1. Find the surface of a sphere whose diameter is 9 in.

9 in. × 3.1416 = 28.2744 in., circumference.

28.2744 in. × 9 = 254.4696 sq. in., surface.

2. Find the surface of a sphere whose diameter is 12 ft.

3. Find the surface of a sphere whose diameter is 7 in.

4. Find the number of square inches in the surface of a sphere whose diameter is 2 ft.

5. Find the number of square feet in the surface of a sphere whose diameter is 14 ft.

6. Find the number of square inches in the surface of a sphere whose diameter is 3 ft.

7. What is the area of a sphere whose radius is 50 in.?

8. Since our earth is a sphere whose radius is 4000 mi., what is its surface?

To find the volume, or cubic contents, of a sphere:

Multiply the surface by $\frac{1}{6}$ of the diameter or $\frac{1}{3}$ of the radius.

Or: **Multiply the cube of the diameter by .5236.**

1. Find the volume of a sphere whose diameter is 12 ft., 4 ft.; 6 ft.; 14 in.

2. When the diameter of a globe is 5 ft., how many cubic feet does it contain?

3. How many cubic inches are there in a signal ball, the diameter of which is 12 in.?

4. What is the volume of the earth, whose diameter is 8000 mi.?

5. When the radius of a sphere is 17 ft., what are its cubic contents?

6. How many cubic inches are there in a baseball 5 inches in diameter?

7. What is the volume of a globe whose circumference is 47.124 inches?

ALGEBRA

Least Common Multiple

1. Find the L. C. M. of $6a^2$, $8a^3b$, $12a^3b^2$.

$$6a^2 = 3 \times 2 \times a^2,$$
$$8a^3 = 2 \times 2 \times 2 \times a^3,$$
$$12a^3b^2 = 2 \times 2 \times 3 \times a^3 \times b^2,$$
$$2 \times 2 \times 2 \times 3 \times a^3 \times b^2 = 24a^3b^2, \text{ the L. C. M.}$$

2. Find the L. C. M. of $9ax^2$, $a + b$, and $18a^3x$.

$$9ax^2 = 3^2 \times a \times x^2,$$
$$a + b = (a + b),$$
$$18a^3x = 3^2 \times 2 \times a^3 \times x.$$

$3^2 \times 2 \times a^3 \times x^2 \times (a+b) = 18a^3x^2(a+b) = 18a^4x^2 + 18a^3bx$, the L. C. M.

Find the L. C. M. of:

3. $12a^2x^2$, $6a^3$, and $8x^4y^2$.
4. $3a^2b$, $9abc$, and $27a^2x^2$.
5. $6c^2nz^2$, $9n^4z$, and $12c^3n^2z^3$.
6. 15, $6xz^2$, $9x^2z^4$, and $18cx^3$.
7. $3a^4b^2c^6$, $6a^7b^4cd^2$, and $10abcx^5$.
8. $16abx$, $80ab^4x^2$, and $35a^7bx^4$.
9. $4a^2x^2y^2$, $8a^3xy$, $16a^4y^3$, and $24a^5y^4x$.
10. $ax - bx$, $ay - by$, and x^2y^2.

Least Common Denominator

1. Reduce $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{m}{n^2}$ to a common denominator.

$$\frac{a}{b} = \frac{a \times d \times n^2}{b \times d \times n^2} = \frac{adn^2}{bdn^2}, \quad \frac{c}{d} = \frac{c \times b \times n^2}{bdn^2} = \frac{bcn^2}{bdn^2},$$

$$\frac{m}{n^2} = \frac{m \times b \times d}{bdn^2} = \frac{bdm}{bdn^2}.$$

Reduce to equivalent fractions with L. C. D.:

2. $\frac{a}{m}$, $\frac{b}{n}$, and $\frac{c}{o}$.

3. $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{1}{4}$.

4. $\frac{x}{y}$ and $\frac{x+a}{b}$.

5. $\frac{2}{3}$, $\frac{3a}{4}$ and $\frac{x+y}{b}$.

6. Reduce $\frac{3a^2b}{4cx^2}$, $\frac{y}{2x}$, and $\frac{5x^2}{3ac^2}$ to L. C. D.

Reduce to equivalent fractions having the L. C. D.:

7. $\frac{3x}{4}$, $\frac{4}{6}$, $\frac{12x^2}{15}$.

8. a, $\frac{3b^2}{4}$, and $\frac{5c^3}{6}$.

9. $\frac{3x}{2a}$, $\frac{2b}{3c}$, and d.

10. $\frac{2a}{3bc}$, $\frac{3x}{cd}$, and $\frac{5y}{6bd}$.

Addition of Algebraic Fractions

1. $\frac{3a}{m^2}+\frac{a-m}{m^2}=\frac{3a+a-m}{m^2}=\frac{4a-m}{m^2}$.

2. Add $\frac{5a}{3b}$ and $\frac{3c}{b^2x}$.

$\frac{5a}{3b}=\frac{5abx}{3b^2x}$, and $\frac{3c}{b^2x}=\frac{9c}{3b^2x}$. The sum is $\frac{5abx+9c}{3b^2x}$.

Add:

3. $\frac{x}{y}$, $\frac{z}{xy}$, and $\frac{y}{x}$.

4. $\frac{a}{b}$, $\frac{a+b}{c}$.

5. $\frac{x}{2}$, $\frac{x}{3}$, and $\frac{x}{4}$.

6. $\frac{x-2}{3}$ and $\frac{4x}{7}$.

7. $\frac{1}{a+b}$ and $\frac{1}{a-b}$.

8. $\frac{x}{x+y}$ and $\frac{y}{x-y}$.

9. $\frac{12b-a}{35c}$ and $\frac{3a-b}{7c}$.

10. $5x+\frac{x-2}{3}$ and $\frac{2x-3}{5x}$.

Subtraction of Algebraic Fractions

1. $\frac{a+b}{m^2x} - \frac{a-b}{m^2x} = ?$

$$\frac{a+b-a+b}{m^2x} = \frac{2b}{m^2x}.$$

2. From $\frac{3bx+x}{b}$ subtract $\frac{ex-(x-a)}{c}$.

Reducing the fractions to the L. C. D., we have

$$\frac{3bcx+cx}{bc} - \frac{bcx-(bx-ab)}{bc} = \frac{3bcx+cx}{bc} - \frac{bcx-bx+ab}{bc}$$

$$= \frac{2bx+cx+bx-ab}{bc}.$$

3. From $\frac{7a-4}{3}$ take $\frac{11a-7}{5}$.

$$\frac{7a-4}{3} - \frac{11a-7}{5} = \frac{35a-20}{15} - \frac{33a-21}{15}$$

$$= \frac{35a-20-(33a-21)}{15} = \frac{35a-20-33a+21}{15} = \frac{2a+1}{15}.$$

4. Simplify $\frac{2x}{5} + \frac{7x}{10} - \frac{2x-3}{15}$.

$$\frac{2x}{5} + \frac{7x}{10} - \frac{2x-3}{15} = \frac{24x}{60} + \frac{42x}{60} - \frac{8x-12}{60}$$

$$= \frac{24x+42x-(8x-12)}{60} = \frac{24x+42x-8x+12}{60}$$

$$= \frac{58x+12}{60} = \frac{2(29x+6)}{2(30)} = \frac{29x+6}{30}.$$

Simplify:

5. $\frac{12x}{7} - \frac{3x}{5}$.

6. $5y - \frac{3y}{8}$.

7. $\frac{3}{4x-y} - \frac{2}{c}$.

8. $8x - \frac{3x}{5}$.

9. $\frac{3x}{7} - \frac{2x}{9}$.

10. $\frac{6c}{x+y} - \frac{5}{x^2-y^2}$.

11. $\frac{4a}{b-c} - \frac{10a}{d}$.

12. $\frac{7a}{8} - \frac{3a}{10}$.

13. $\frac{3a}{b^2x^2} - \frac{7x-2}{x^2}$.

14. $\left(6x + \frac{x-3}{4}\right) + \frac{2x-1}{3x}$.

15. $\frac{4x}{a-b} + \frac{3a}{a+b}$.

16. $\frac{x-7}{3} + \frac{5x}{7} - \frac{2x}{21}$.

17. $\frac{4}{3x-2y} - \frac{3}{c+d}$.

18. $\frac{2cx}{a+b} - \frac{5cx}{a-b}$.

19. $\frac{x+a}{a-x} + \frac{a}{a^2-x^2}$.

20. $\frac{x^2}{ab+1} - \frac{2x^2}{ab-1}$.

21. $\frac{1}{a^2-b^2} - \frac{1}{a^2+b^2}$.

22. $\frac{x}{b} - \frac{y}{c} - \frac{z}{a+c}$.

Lowest Terms

1. Reduce $\frac{4abc}{6a^2bd}$ to its lowest terms.

$$\frac{4abc}{6a^2bd} = \frac{2ab \times 2c}{2ab \times 3ad} = \frac{2c}{3ad}.$$

2. Reduce $\frac{15a^2c^2}{25acd}$ to its lowest terms.

$$\frac{15a^2c^2}{25acd} = \frac{3(5aacc)}{5(5acd)}.$$

3. Canceling the common factors, 5, a, and c, we have,

$$\frac{15a^2c^2}{25acd} = \frac{3ac}{5d}.$$

Multiplication of Algebraic Fractions

1. Multiply $\frac{3\,a^2b^2}{4\,x^3y^3}$ by $\frac{8\,x^4y^3}{9\,a^3b^2}$.

$$\frac{3\,a^2b^2}{4\,x^3y^3}\times\frac{8\,x^4y^3}{9\,a^3b^2}=\frac{3\,a^2b^2\times 8\,x^4y^3}{4\,x^3y^3\times 9\,a^3b^2}=\frac{24\,a^2b^2x^4y^3}{36\,a^3b^2x^3y^3}$$

$$=\frac{12\,a^2b^2x^3y^3(2\,x)}{12\,a^2b^2x^3y^3(3\,a)}=\frac{2\,x}{3\,a}.$$

Find the product:

2. $\frac{3\,x}{4\,y}\times\frac{2\,a}{3\,b}$.

3. $\frac{x^2}{y^2}\times\frac{x}{y}$.

4. $\frac{3\,ab}{10\,b^2}\times\frac{4\,ab^2}{9\,b^2c}\times\frac{5\,ac}{8\,a^2b}$.

5. $\frac{8\,ab}{15\,b^2}\times\frac{3\,a^2}{16\,a^2b}$.

6. $\frac{14\,bc}{25\,c^2}\times\frac{5\,bc^2}{7\,c^2}$.

7. $\frac{a^2x}{b^2y}\times\frac{2\,cd^2}{3\,ab}\times\frac{4\,xy}{7\,x^2}$.

Division of Algebraic Fractions

1. $\frac{3\,a}{2\,b}\div\frac{5\,c}{4\,d}=?$ $\quad\frac{3}{2}\div\frac{5}{4}=\frac{3}{2}\times\frac{4}{5}=\frac{6}{5}$.

$\frac{a}{b}\div\frac{c}{d}=?$ $\quad\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$.

$$\frac{3\,a}{2\,b}\div\frac{5\,c}{4\,d}=\frac{3\,a}{2\,b}\times\frac{4\,d}{5\,c}=\frac{12\,ad}{10\,bc}=\frac{6\,ad}{5\,bc}.$$

2. $\frac{2\,a}{3\,x}\div\frac{b}{3\,y}=?$ $\quad\frac{2\,a}{3\,x}\div\frac{b}{3\,y}=\frac{2\,a}{3\,x}\times\frac{3\,y}{b}=\frac{2\,ay}{3\,b}$.

Find the value of:

3. $\frac{3\,ac}{4\,bd}\div\frac{6\,a}{11\,b}$.

4. $\frac{4\,a^2}{3\,mz}\div\frac{8\,ab}{m^2}$.

5. $\frac{8\,a^4b}{15\,xy^3}\div\frac{2\,a^3}{3\,xy^2}$.

6. $\frac{2\,a^5b^4}{5\,xy}\div\frac{8\,ab}{15\,xy}$.

7. $\frac{a}{1-a}\div\frac{a}{5}$.

8. $\frac{2\,x}{ab}\div\frac{3\,xy}{ab}$.

Fractional Equations

1. Solve the equations, $\frac{x-7}{2}+\frac{x}{9}=\frac{x+7}{3}$,

L. C. D. $= 18$.

Simplifying, $9(x-7)+2x=6(x+7)$

$$9x-63+2x=6x+42$$

$$9x+2x-6x=63+42.$$

Or, $$5x=105,$$

$$\therefore\ x=21.$$

2. Solve the equation, $\frac{1}{2}(27-2x)=\frac{9}{2}-\frac{1}{10}(7x-54)$.

This equation is the same as

$$\frac{27-2x}{2}=\frac{9}{2}-\frac{7x-54}{10},$$

L. C. D. $= 10$.

$$5(27-2x)=45-(7x-54).$$

Simplifying, $135-10x=45-7x+54,$

$$\therefore\ 10x+7x=45+54-135.$$

Or, $$-3x=-36.$$

Or, $$36=3x,$$

$$\therefore\ x=12.$$

Solve the following equations:

3. $\frac{x}{2}=4.$ 4. $\frac{2x}{5}=6.$ 5. $\frac{x}{2}+\frac{x}{3}=15.$

6. $\frac{x}{3}-\frac{x}{4}=24.$ 7. $\frac{x}{5}+\frac{x}{8}=17-\frac{x}{10}.$

8. $\frac{x}{2}+\frac{x}{3}=13-\frac{x}{4}$.

9. $\frac{x}{2}+\frac{x}{3}=\frac{x}{4}+7$.

10. $\frac{4x}{5}-\frac{5x}{7}=x-8$.

11. $\frac{x}{2}+\frac{x}{3}=x-7$.

12. $\frac{2x}{3}-\frac{3x}{4}=\frac{5x}{6}-4\frac{1}{2}-\frac{13x}{24}$.

13. $\frac{x-3}{4}+\frac{x-1}{3}=x-4$.

14. $\frac{x-6}{5}=\frac{x-5}{4}+\frac{1-x}{6}$.

15. $\frac{x}{2}+\frac{1+x}{3}+\frac{2+x}{4}+\frac{3+x}{5}=4$.

16. $\frac{2-x}{3}+\frac{3-x}{4}+\frac{4-x}{5}+\frac{5-x}{6}+\frac{3}{4}=0$.

Stating the Equation

1. When $\frac{3}{4}$ of a number is increased by 6, the sum is 15. Find the number.

Let x = the number,

then $\frac{3}{4}x+6=15$.

Therefore $\frac{3}{4}x=15-6=9$.

Therefore $\frac{1}{4}x=9\div 3=3$.

Therefore $x=3\times 4=12$.

2. A post sunk in a pond is $\frac{1}{4}$ of its length in water, and $\frac{1}{3}$ of its length in mud, and stands 10 ft. out of water. Find the length of the post.

Let x = length of post.

$x=\frac{1}{4}x+\frac{1}{3}x+10$.

$x=\frac{7}{12}x+10$.

Therefore $x-\frac{7}{12}x=10$.

$\frac{5}{12}x=10$.

Therefore $\frac{1}{12}x=10\div 5=2$.

$x=2\times 12=24$.

3. Divide the number 100 into two parts such that a fourth of one part shall exceed a third of the other part by 11.

Let $x =$ one part,

and $100 - x =$ the other part.

Then $\frac{x}{4} - \frac{1}{3}(100 - x) = 11.$

Then $3x - 400 + 4x = 132.$

$3x + 4x = 132 + 400.$

$7x = 532.$

Therefore $x = 76 =$ one part.

$100 - 76 = 24 =$ the other part.

4. Divide the number 25 into two parts so that a fourth of the one shall exceed one third of the other by 1.

5. The sum of two numbers is 35 and their difference exceeds one fifth of the smaller number by 2. Find the numbers.

6. In a pond one fourth of the length of a post appears above the water, and one third of its length is embedded in the mud. If the water is 7 ft. deep, what is the length of the post?

7. A and B begin business with equal sums of money. At the end of a year A has gained \$50, and B has lost a third of his capital. It is then found that A has twice as much as B. How much had each at first?

8. Three numbers amount to 60; the first is one fifth of the third and two thirds of the second. Find all the numbers.

9. A clerk spends $\frac{1}{3}$ of his salary for board, $\frac{1}{5}$ for clothes and books, and $\frac{1}{10}$ in other expenses, and has \$308 left. Find his income.

10. A and B together have $3240, but B has only $\frac{7}{8}$ as much money as A. What is the money of A? Of B?

11. My watch is worth $114 more than the chain, and $\frac{2}{3}$ the value of the watch is 7 times the value of the chain. What is the value of each article?

12. A certain man added to his estate $\frac{1}{4}$ its value, and then lost $7600. But he afterwards gained $600. His property then amounted to $2000. What was the value of his estate at first?

13. Of a force of marines, $\frac{2}{3}$ are on duty, $\frac{1}{8}$ are sick, $\frac{1}{5}$ of the remainder are absent on furlough, and the rest, which is 380, are on shore. What is the number of men in the force?

14. In a quantity of gunpowder, the nitre was 10 lb. more than $\frac{2}{3}$ of the whole quantity, the sulphur $4\frac{1}{2}$ lb. less than $\frac{1}{6}$ of the whole, and the charcoal 2 lb. less than $\frac{1}{7}$ of the nitre. What was the weight of the gunpowder?

15. In an orchard, $\frac{1}{2}$ are apple trees, $\frac{1}{4}$ peach trees, $\frac{1}{6}$ plum trees, 100 cherry trees, and 100 pear trees. How many trees in the orchard?

16. If from $\frac{1}{3}$ of my height in inches 12 inches be subtracted, $\frac{1}{5}$ of the remainder will be 2. What is my height?

17. A man left half of his property to his wife, one sixth to each of two sons, one twelfth to a brother, and the remainder, $600, to a hospital. What was the value of his property?

18. This year the rent of a house is $12\frac{1}{2}\%$ greater than it was last year. This year it is $540. What was the rent last year?

19. Two boys had together $28. One had 125% as much as the other. How much had each?

20. The cost of one bicycle was $\frac{2}{3}$ the cost of another, whose cost was $\frac{3}{4}$ that of a third. The cost of all was $135. What was the cost of each?

21. A father is 53 yr. old, and a son is 28 yr. How many years ago was the son $\frac{1}{2}$ as old as the father?

22. A balloon, empty, weighed 120 lb.; full, it carried 200 lb. What was the lifting power of the gas that it contained when full?

23. $75 left in a savings bank for a certain period gained $5.40. How much money must be deposited to gain $40.50 in an equal period?

24. A, B, and C worked for 15 days together. B received $\frac{4}{5}$ and C $\frac{2}{3}$ as much wages as A. They received in all $111. What wages did each receive per day?

Proving the Answer

To prove the answer to an algebraic problem substitute for the unknown quantity in the equation the value found. To illustrate:

In **24**, above, the equation is $\left(x+\frac{4x}{5}+\frac{2x}{3}\right)\times 15=\$111$.

x, A's wages per diem, $=\$3$. Therefore,

since $\left(\$3+\frac{\$3\times 4}{5}+\frac{\$3\times 2}{3}\right)\times 15=\$111$, $x=\$3$.

When the equation proves, the value found is correct.

25. The first boy is $\frac{2}{3}$ the age of the second, who is $\frac{3}{4}$ the age of the third, who is $\frac{4}{5}$ the age of the fourth. All their ages together are 7 times the age of the first boy. The first and third boys' ages together are 18 yr. Find the age of each boy.

Problems long in Arithmetic may be short in Algebra.

Examples in Percentage and Interest

Percentage = base × rate

Amount = base + percentage Base = percentage ÷ rate

Difference = base − percentage Rate = percentage ÷ base

1. A received $780 upon a note at 6% for 8 mo. What was its face?

Let x = its face;

then $\$780 = x + (x \times .04) = x + .04x.$

$\$780 = x(1 + .04) = x \times 1.04.$

$$x = \frac{\$780}{1.04} = \$750.$$

2. S received $1242 to pay a 7 mo. note at 6% interest. What was the face of the note?

3. With what amount did W pay in full a note for $2500 at 5% after 9 mo.?

4. With $1985.50 Z paid in full a note for $1900 that ran 1 yr. What was the rate of interest?

$\$1985.50 - \$1900 = \$85.50.$

Let x = the rate.

$\$85.50 = \$1900 \times x.$

$$x = \frac{\$85.50}{\$1900} = .045 = 4\tfrac{1}{2}\%.$$

5. With $11,481.25 M paid a note for $11,000 that ran 10 mo. What was the rate of interest?

6. With $1485.97 K paid a note for $1425 that ran 7 mo. 10 da. What was the rate of interest?

7. What amount was due on a 90 day note for $19,600 at $4\frac{1}{8}\%$?

8. A commission merchant received a check for $975.28, with which to pay a bill for flour at $4.25 per barrel, including a 2% commission on the purchase. How many barrels were purchased?

SUGGESTION. Let x = the amount of the purchase.

$$x + \frac{2\,x}{100} = \$975.28.$$

9. T sent to a customer a bill for $103.83 for buying 30 doz. watermelons, at 3% commission. What was the average price per melon?

10. A bill collector paid in $180.84 for one week's work in collecting for his employer, after deducting his commission of 15%. What was the amount he collected?

REVIEW OF PARTIAL PAYMENTS

MERCHANTS' RULE

1. On a 6% note for $800 dated Mar. 15, 1901, there were paid the following sums: July 1, $20; Sept. 20, $400; Dec. 1, $25. What was due Feb. 15, 1902?

2. X drew a note on demand to the order of Y for $15,000 at 5% interest quarterly, and paid at the end of the first quarter, $400; at the end of the second, $2000; and at the end of the third, $300. What was due at the end of the year?

3. Note for $475; on demand; interest monthly; rate 6%. Paid in 1 mo., $60; in 2 mo., $75; in 3 mo., $25; in 4 mo., $125; and in 5 mo., $100. What was due at the end of a half year?

4. Note for $1800; on demand; interest quarterly; rate 8%. Paid at end of 1st quarter, $600; of 2d quarter, $50; and at end of 3d quarter, $25. At the expiration of 10 mo. the payee demanded payment in full. What amount was due?

PERCENTAGE

Present Worth

The **present worth** of a debt, payable at a future time without interest, is such a sum as, being put at legal interest, will amount to the given debt at that time.

True discount is the difference between a debt and its present worth.

The terms, *present worth*, *discount*, and *debt*, are equivalent to *principal*, *interest*, and *amount*.

1. What is the present worth of $409, due 3 mo. hence, money being worth 7%?

7% per annum = $1\frac{3}{4}$ per cent for 3 mo.,
$.0175 = interest on $1 for 3 mo. at 7%.
Amount of $1 at given rate and for given time = $1.0175,
$409 \div 1.0175 = \$401.965$.

2. What is the true discount on a note for $794.63, due in 27 da., if money is loaned at 8%?

Amount of $1 at 8% for 27 da. = $1.005917,
$794.63 \div \$1.005917 = \$789.955$ = present worth,
$794.63 − $789.955 = $4.675 = true discount.

To find the present worth of any sum, payable at a future time without interest, we find the amount of $1 for the given time, at the given rate, and divide the given sum by this amount. The quotient is the present worth.

The present worth subtracted from the given sum gives the true discount.

What is the true discount on :

3. $740 for 3 mo., money at 7%?
4. $90 for 2 mo., money at 9%?
5. $250 for 6 mo., money at 6%?
6. $714.20 for 11 mo., money at 11%?

Find the value of :

7. $911.40 payable in 5 mo. at 11% discount.

8. $571.43 payable in 4 mo. at 7% discount.

9. $947.60 payable in 2 yr. at 4% discount.

10. $888.93 payable in 1 yr. 4 mo. at 7% discount.

11. $7146.90 payable in 47 da. at 10% discount.

12. $710 payable in 2 mo. at 7% discount.

13. $1100 payable in $1\frac{1}{2}$ mo. at 7% discount.

14. $6714.83 payable in $2\frac{1}{3}$ mo. at 6% discount.

15. What is the present worth of a note for $962, payable in one year, money being worth 4% interest?

16. What is the present worth of $2202, payable in 5 years and 9 months, money being worth 6% interest?

17. What sum will discharge a debt of $1003.50, due in 8 months, allowing 6% discount?

18. What sum in cash will now pay a debt of $716, due 7 months hence, allowing a discount of 8%?

19. What sum will now pay a debt of $1342.50, due 125 days hence, money being worth $6\frac{1}{2}$% interest?

20. If a legacy of $2400 is left to me on the 3d of May, to be paid on next Christmas Day, what should I get as present payment, allowing 5% discount?

21. Find the discount on a bill of $2202, at 5%, payable 9 months hence.

22. What is the present worth of a non-interest-bearing note for $4360, payable in 1 year 5 months, money being worth 6% interest?

23. What is the present worth of a non-interest-bearing note for $1647, due 11 months hence, at 6%?

RATIO

When a ratio is expressed in the form of a fraction, the numerator may become the antecedent and the denominator the consequent.

Ratio is either **direct** or **inverse**, **simple** or **compound**.

A *direct ratio* is one arising from the division of the antecedent by the consequent.

An *inverse* or *inverted ratio* is one arising from the division of the consequent by the antecedent.

The inverse ratio of 15 to 3 is $3 : 15$, or $\frac{3}{15} = \frac{1}{5}$.

An inverse ratio is sometimes called a *reciprocal*.

The reciprocal of a quantity is 1 divided by that quantity.

The reciprocal of 8 is $\frac{1}{8}$; of $\frac{2}{7}$, $\frac{1}{\frac{2}{7}} = \frac{7}{2}$.

A *simple ratio* is one that has but one antecedent and one consequent.

$9 : 3$; $7 : 11$; $18 : 2$ are simple ratios.

A *compound ratio* is a ratio that is produced by compounding, or multiplying together, the corresponding terms of two or more simple ratios.

The simple ratio of $9 : 3$ is 3.
The simple ratio of $24 : 2$ is 12.
The ratio compounded of these is $216 : 6 = 36$.

Ratios are compounded by multiplying together all the antecedents for a new antecedent and all the consequents for a new consequent.

The ratio compounded of $2 : 7$; $2 : 3$; $5 : 11$; $4 : 3$ is $2 \times 2 \times 5 \times 4 : 7 \times 3 \times 11 \times 3$, or $80 : 963$.

Find the inverse ratio of the following numbers:

1. 7 to 21.
2. 12 to 2.
3. 27 to 6.
4. 9 to 36.
5. 19 to 57.
6. 23 to 92.

7. 81 to 9.
8. 187 to 17.
9. 6 da. to 4 wk.
10. 11 min. to 30 sec.
11. 4 lb. to 12 oz. Av.
12. 3 qt. to 43 gal.

Find the reciprocal ratio of the following numbers:

13. 7 to 42.
14. $\frac{1}{8}$ to $\frac{1}{2}$.
15. 42 to 28.
16. 17 to 68.
17. 19 to 17.
18. $\frac{1}{24}$ to $\frac{1}{36}$.
19. 72 to 18.
20. 512 to 32.
21. $\frac{1}{4}$ to $\frac{7}{8}$.
22. $\frac{2}{3}$ to $\frac{4}{5}$.

Compound these ratios:

23. 5 : 7 and 9 : 3.
24. 8 : 9 and 15 : 7.
25. 16 : 90 and 3 : 42.
26. 9 : 80 and 10 : 95.

Find the ratios compounded of the following ratios:

27. 2 to 3, 5 to 7, and 1 to 7.
28. 8 to 6 and 17 to 3.
29. 9 to 8, 7 to 6, 4 to 3, and 2 to 1.
30. 1 to 7, 1 to 3, 3 to 1, and 5 to 1.
31. 2 to 5, 3 to 7, 4 to 5, 21 to 2, and 1 to 9.

Principles of Ratio

Since the antecedent of a couplet is a dividend, the consequent a divisor, and the ratio the quotient, it follows that:

I. **Multiplying the antecedent of a couplet or dividing the consequent by any number multiplies the ratio by that number.**

The ratio of 28 to 112 $= \frac{1}{4}$.

The ratio of 28×3 to $112 = \frac{3}{4} = \frac{1}{4} \times \frac{3}{1}$ = three times the ratio of 28 to 112.

II. **Dividing the antecedent of a couplet or multiplying the consequent by any number divides the ratio by that number.**

The ratio of 64 to 16 = 4.

The ratio of $64 \div 2$ to $16 = 32 : 16 = 2 = 4 \div 2$ = half the ratio of 64 to 16.

III. **Multiplying or dividing both antecedent and consequent of a couplet by the same number does not alter the value of the ratio.**

The ratio of 18 to 6 is 3.

The ratio of $18 \times 7 : 6 \times 7 = 126 : : 42 = 3$ = ratio of $18 \div 2 : 6 \div 2 = 9 : 3$.

When several ratios are to be compounded together, we may cancel the common factors in the terms before multiplying the corresponding terms together.

1. Compound together 4 : 17; 34 : 55; 11 : 2; 13 : 7; and 21 : 65.

17 is contained in 34 2 times, and this 2 cancels 2, the third consequent; 11 is contained in 55 5 times; 13 is contained in 65 5 times; 7 is contained in 21 3 times. The only antecedents left are 4 and 3, which, multiplied together, make 12; and the only remaining consequents are 5 and 5, which, multiplied together, make 25. The ratio of 12 to 25 is, therefore, the ratio compounded of all the given ratios.

4 : 17
5
34 : 55
2
11 : 2
13 : 7
5
21 : 65
3

$4 \times 3 : 5 \times 5$
or
12 : 25

2. Compound the following ratios: 7 : 16; 24 : 3; 9 : 49; 14 : 11; 22 : 13.

3. Find the ratio compounded of the following ratios: 1 : 7; 16 : 23; 29 : 14; 98 : 319; 253 : 64.

IV. **If the terms of two or more couplets, of the same ratio, be added together, the resulting couplet will have the same ratio.**

The ratio of $6 : 2 = 3$; the ratio of $21 : 7 = 3$; and the ratio of $33 : 11 = 3$; and the ratio $6+21+33$ to $2+7+11$, or of 60 to 20, is also 3.

If $6 : 2 = 21 : 7 = 33 : 11$, then $6 + 21 + 33 : 2 + 7 + 11 = 6 : 2$.

V. **If from the terms of any couplet the terms of another couplet, of the same ratio, be subtracted, then the resulting couplet will have the same ratio.**

The ratio of 35 to 5 is 7, and the ratio of 14 to 2 is 7. So also the ratio of $35 - 14 : 5 - 2$; that is, of $21 : 3$, is 7; or, if $35 : 5 = 14 : 2$, then $35 - 14 : 5 - 2 = 35 : 5$.

VI. **An integral ratio is diminished by adding the same number to both terms.**

The ratio of $48 : 8 = 6$, an integer. $48 + 12 : 8 + 12$, or $60 : 20 = 3$, a result which is less than the ratio $48 : 8$.

PROPORTION

A, C, a, and c stand for any four original antecedents and consequents.

If	$A : C = a : c.$
By **alternation**	$A : a = C : c.$
By **inversion**	$C : A = c : a.$
By **composition**	$A + C : C = a + c : c.$
By **division**	$A - C : C = a - c : c.$
By **conversion**	$A : A + C = a : a + c,$
or	$A : A - C = a : a - c.$

Or if $15 : 6 = 10 : 4$,

By *alternation* $15 : 10 = 6 : 4$.

By *inversion* $6 : 15 = 4 : 10$.

By *composition* $15 + 6 : 6 = 10 + 4 : 4$,

or $21 : 6 = 14 : 4$.

By *division* $15 - 6 : 6 = 10 - 4 : 4$,

or $9 : 6 = 6 : 4$.

By *conversion* $15 : 15 + 6 = 10 : 10 + 4$,

or $15 : 21 = 10 : 14$.

Or $15 : 15 - 6 = 10 : 15 - 4$,

or $15 : 9 = 10 : 6$.

COMPOUND PROPORTION

A **compound proportion** is an equality between a compound ratio and a simple ratio.

$7 : 11$ compounded with $22 : 21 = 34 : 51$, is a compound ratio, or

$7 \times 22 : 11 \times 21 = 34 : 51$, or $7 \times 22 \times 51 = 11 \times 21 \times 34$.

Compound proportion is sometimes called the **double rule of three.**

In *compound proportion* there are three or more ratios, one of which is imperfect.

In every proportion the product of the means is equal to the product of the extremes.

1. If 18 men dig a trench 30 yd. long in 24 da., working 8 hr. a day, how many men will dig a trench 60 yd. long in 64 da., working 6 hr. a day?

FIRST METHOD

A. If 18 men dig 30 yd., how many will dig 60 yd. in the same time?

$$\overset{\text{men}}{18} : x = \overset{\text{days}}{30} : 60. \qquad 30\,x = 18 \times 60. \qquad x = \frac{18 \times 60}{30}.$$

B. If x men require 24 da. to dig the trench of 60 yd., how many are required to dig it in 64 da., working the same number of hours a day?

$$x : 64 = y : 24. \qquad 64\,y = 24\,x. \qquad y = \frac{24\,x}{64}.$$

C. If y men require 8 hr. a day to dig a trench of 60 yd., working 24 da., how many are required to dig it, working 6 hr. a day?

$$y : 6 = z : 8. \qquad 6\,z = 8\,y. \qquad z = \frac{8\,y}{6}.$$

A. $x = \frac{60}{30}$ of 18 men $= 36$ men.

B. $y = \frac{24}{64}$ of $\frac{60}{30}$ of 18 men $= \frac{27}{2}$ men.

C. $z = \frac{8}{6}$ of $\frac{24}{64}$ of $\frac{60}{30}$ of 18 men $= 18$ men.

$$\frac{z}{18} = \frac{8}{6} \times \frac{24}{64} \times \frac{60}{30}.$$

SECOND METHOD

1st cause is to 2d cause as 1st effect is to 2d effect.

$$\left\{\begin{matrix} 18 \text{ men} \\ 24 \text{ da.} \\ 8 \text{ hr.} \end{matrix}\right. : \left\{\begin{matrix} ? \text{ men} \\ 64 \text{ da.} \\ 6 \text{ hr.} \end{matrix}\right. :: 30 \text{ yd.} : 60 \text{ yd.}$$

Cancellation:

$$\frac{\overset{3}{\cancel{18}} \times \overset{3}{\cancel{24}} \times 8 \times \overset{6}{\cancel{60}}}{\underset{10}{\cancel{30}} \times \underset{8}{\cancel{64}} \times 6} = 18 \qquad \therefore\ 18 \text{ men.}$$

But every fraction is a ratio.

$$z : 18 = \begin{cases} \frac{8}{6} \\ \frac{24}{64} \\ \frac{60}{30} \end{cases} \quad \text{or} \quad \begin{matrix} \text{yd.} & \text{yd.} \\ 30 & : & 60 \\ \text{da.} & & \text{da.} \\ 64 & : & 24 \\ \text{hr.} & & \text{hr.} \\ 6 & : & 8 \end{matrix}$$

$$z = \frac{\overset{2}{60} \times \overset{3}{24} \times 8 \times \overset{3}{18}}{30 \times 64 \times \underset{8}{6}} = 2 \times 3 \times 3 = 18.$$

The answer is equal to the continued product of the third term and all the second terms, divided by the continued product of all the first terms.

2. If 24 men can cut 90 cd. of wood in 6 da., working 9 hr. a day, how many cords can 8 men cut in 36 da., working 12 hr. a day?

FIRST METHOD

$$\left.\begin{matrix} \text{men} & & \text{men} \\ 24 & : & 8 \\ \text{da.} & & \text{da.} \\ 6 & : & 36 \\ \text{hr.} & & \text{hr.} \\ 9 & : & 12 \end{matrix}\right\} = \begin{matrix} \text{cd.} & & \text{cd.} \\ 90 & : & x \end{matrix} \qquad \left.\begin{matrix} \overset{4}{\cancel{24}} & : & \overset{2}{\cancel{8}} \\ \cancel{6} & : & \cancel{36} \\ 9 & : & 12 \end{matrix}\right\} \begin{matrix} 10 \\ = 90 : x \\ \therefore\ x = 10 \times 2 \times 12 = 240. \end{matrix}$$

Here the imperfect ratio is $90 : x$. If 24 men cut 90 cd., will 8 men cut more or fewer cords?

First ratio is 24 : 8.

If in 6 da. 90 cd. of wood are cut, will more or fewer be cut in 36 da.? More. The second ratio is 6 : 36. Lastly, if in a day 9 hr. long 90 cd. are cut, will more or fewer cords be cut in a day 12 hr. long? More. The third ratio is 9 : 12.

SECOND METHOD

The effect of 24 men working 9 hours a day for 6 days is 90 cords of wood cut, and the effect of 8 men working 12 hours a day for 36 days, which is an unknown number of cords, bears the same relation to its cause as the first effect does to its cause; hence the statement:

1st cause is to 2d cause as 1st effect is to 2d effect.

$$\left\{\begin{matrix}24 \text{ men}\\ 9 \text{ hr.}\\ 6 \text{ da.}\end{matrix}\right. : \left\{\begin{matrix}8 \text{ men}\\ 12 \text{ hr.}\\ 36 \text{ da.}\end{matrix}\right. :: 90 \text{ cd.} : ? \text{ cd.}$$

Cancellation:

$$\frac{\overset{4}{\cancel{8}} \times \overset{10}{\cancel{12}} \times 90 \times \overset{6}{\cancel{36}}}{\underset{2}{\cancel{24}} \times \cancel{9} \times \cancel{6}} = 240 \quad \therefore \text{ cd.}$$

3. If 248 men, in $5\frac{1}{2}$ da. of 11 hr. each, dig a trench of 7 degrees of hardness, $232\frac{1}{2}$ yd. long, $3\frac{2}{3}$ yd. wide, and $2\frac{1}{3}$ yd. deep, in how many days, each 9 hr. long, will 24 men dig a trench of 4 degrees of hardness, $337\frac{1}{2}$ yd. long, $5\frac{3}{5}$ yd. wide, and $3\frac{1}{2}$ yd. deep?

FIRST METHOD

$$\left.\begin{matrix}\text{men} & \text{men}\\ 24 & : 248\\ \text{hr.} & \text{hr.}\\ 9 & : 11\\ \text{da.} & \text{deg.}\\ 7 & : 4\\ \text{yd.} & \text{yd. long}\\ 232\frac{1}{2} & : 337\frac{1}{2}\\ 3\frac{2}{3} & : 5\frac{3}{5} \text{ wide}\\ 2\frac{1}{3} & : 3\frac{1}{2} \text{ deep}\end{matrix}\right\} = 5\tfrac{1}{2} \text{ da.} : x; \quad \text{or,} \quad \left.\begin{matrix}\frac{24}{1} & : \frac{248}{1}\\ \frac{9}{1} & : \frac{11}{1}\\ \frac{7}{1} & : \frac{4}{1}\\ \frac{465}{2} & : \frac{675}{2}\\ \frac{11}{3} & : \frac{28}{5}\\ \frac{7}{3} & : \frac{7}{2}\end{matrix}\right\} = \tfrac{11}{2} : x.$$

$$x = \left(\frac{24}{1}\frac{8}{1} \times \frac{11}{1}\frac{1}{1} \times \frac{4}{1} \times \frac{67}{2}\frac{5}{2} \times \frac{28}{5}\frac{8}{5} \times \frac{7}{2} \times \frac{11}{2}\frac{1}{2}\right) \div \left(\frac{24}{1} \times \frac{9}{1} \times \frac{7}{1} \times \frac{465}{2} \times \frac{11}{3}\frac{1}{3} \times \frac{7}{3}\right)$$

$$= \frac{24}{1}\frac{8}{1} \times \frac{11}{1}\frac{1}{1} \times \frac{4}{1} \times \frac{67}{2}\frac{5}{2} \times \frac{28}{5}\frac{8}{5} \times \frac{7}{2} \times \frac{11}{2}\frac{1}{2} \times \frac{1}{2}\frac{1}{4} \times \frac{1}{9} \times \frac{1}{7} \times \frac{2}{465} \times \frac{3}{11} \times \frac{3}{7}$$

$$= 4 \times 3 \times 11 = 132 \text{ da.}$$

SECOND METHOD

1st cause is to 2d cause as 1st effect is to 2d effect.

$$\left\{\begin{array}{l}248 \text{ men} \\ 5\frac{1}{2} \text{ da.} \\ 11 \text{ hr.}\end{array}\right. : \left\{\begin{array}{l}24 \text{ men} \\ ? \text{ da.} \\ 9 \text{ hr.}\end{array}\right. :: \left\{\begin{array}{l}7 \text{ degrees} \\ 232\frac{1}{2} \text{ yd.} \\ 3\frac{2}{3} \text{ yd.} \\ 2\frac{1}{3} \text{ yd.}\end{array}\right. : \left\{\begin{array}{l}4 \text{ degrees} \\ 337\frac{1}{2} \text{ yd.} \\ 5\frac{3}{5} \text{ yd.} \\ 3\frac{1}{2} \text{ yd.}\end{array}\right.$$

Since the product of the means is equal to the product of the extremes,

$$\frac{4 \times 337\frac{1}{2} \times 5\frac{3}{5} \times 3\frac{1}{2} \times 248 \times 5\frac{1}{2} \times 11}{7 \times 232\frac{1}{2} \times 3\frac{2}{3} \times 2\frac{1}{3} \times 24 \times 9} = 132 \quad \therefore 132 \text{ da.}$$

4. If a regiment of 679 soldiers consume 702 bbl. of flour in 336 da., how many barrels will an army of 22,407 soldiers consume in 112 da.?

5. If 12 tailors in 27 da. can make 130 suits of clothes, how many tailors in 19 da. can make 4940 suits?

6. If 180 bricks, 8 in. long and 2 in. wide, are required for a walk 20 ft. long and 6 ft. wide, how many bricks will be required for a walk 100 ft. long and 4 ft. wide?

7. What will it cost to keep 2 horses for 11 mo., if it cost $84 to support 3 horses for 7 mo.?

8. How long will it take 7 men to reap 6 A., if 1 man can reap $345\frac{5}{7}$ sq. yd. in an hour?

9. If 20 men in 3 wk. earn $900, in what time will 12 men earn $1500?

10. How many hours a day would 30 men have to work in order to do a piece of work in 16 da., if 25 men can do the same piece of work in 24 da., working 8 hr. a day?

11. If 1500 copies of a book of 11 sheets require 66 reams of paper, how many reams will be required for 5000 copies of a book of 25 sheets?

12. In how many days of 8 hr. each can 7 men reap a field whose length is 1800 ft. and breadth 960 ft., if 5 men can reap a field whose length is 800 ft. and breadth 700 ft. in 14 da. of 9 hr. each?

13. If 20 masons build a wall 50 ft. long, 2 ft. thick, and 14 ft. high in 12 da. of 7 hr. each, in how many days of 10 hr. each will 60 masons build a wall 500 ft. long, 4 ft. thick, and 16 ft. high?

14. If 10 men can perform a piece of work in 24 da., how many men will perform another piece of work 7 times as great in one fifth of the time?

15. What is the weight of a block of stone 12 ft. 6 in. long, 6 ft. 6 in. broad, and 8 ft. 3 in. deep, when a block of the same stone 5 ft. long, 3 ft. 9 in. broad, and 2 ft. 6 in. deep weighs 7500 lb.?

16. How many days will 90 bu. last 6 horses, if 120 bu. of oats last 14 horses 56 da.?

17. If a wall 28 ft. high can be built in 15 da. by 63 men, how many men can build a wall 32 ft. high in 8 da.?

18. How many pounds of thread will be required to make a piece of linen 45 yd. long and 1 yd. wide, if 1 lb. of thread makes 3 yd. of linen $1\frac{1}{4}$ yd. wide?

19. If 3 lb. of worsted make 10 yd. of cloth $1\frac{1}{2}$ yd. wide, how many pounds of thread will be required to make a piece of linen 45 yd. long and 1 yd. wide?

20. How many horses can move 2640 tons of stone in 18 da., if 12 horses move in 5 da. 880 tons?

PROPORTIONAL PARTS

A quantity may be divided into two or more parts, which have to one another the same ratio that certain given numbers have respectively. The quantity is then said to be divided proportionally, and the parts are called **proportional parts.**

When $108 is divided into the parts, $24, $36, $48, it is divided into parts which have the same ratio as 2, 3, 4, respectively.

1. Divide $144 into parts proportional to 3, 4, 5.

$3 + 4 + 5 = 12$, and $\frac{3}{12}$, $\frac{4}{12}$, $\frac{5}{12}$, are proportional to 3, 4, 5, and their sum is 1. When we take $\frac{3}{12}$, $\frac{4}{12}$, $\frac{5}{12}$ of $144, these parts are together equal to the whole sum and are proportional to 3, 4, 5.

$\frac{3}{12}$ of $144 = $36; $\frac{4}{12}$ of $144 = $48; $\frac{5}{12}$ of $144 = $60; and $36, $48, $60 are together equal to $144, and are proportional to 3, 4, 5.

2. Divide $1275 among 3 persons, whose shares are to be in the proportion of 3, 5, and 7.

$\$\frac{1275}{15} = \$85$.

One of the persons receives $85 × 3, or $255;
the second receives $85 × 5, or $425;
the third receives $85 × 7, or $595.

3. Divide $837 among three partners, whose shares are to be in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$.

The shares are to be in the proportion of $\frac{15}{30}$, $\frac{10}{30}$, and $\frac{6}{30}$, that is, in the proportion of 15, 10, and 6.

Amount of one share out of 31 shares $= \$\frac{837}{31} = \$27$.

One of the partners receives 15 × $27, or $405;
the second receives 10 × $27, or $270;
the third receives 6 × $27, or $162.

4. A merchant invests $700 in trade, and at the end of 3 yr. takes a partner who invests $1900. At the end of 4 yr. more they have gained $500. How ought this profit to be divided?

The merchant employs $700 for 7 yr., which is equivalent to employing $4900 for 1 yr.

The partner employs $1900 for 4 yr., which is equivalent to employing $7600 for 1 yr.

Their profits are divided in the ratio of $4900 : $7600, that is of 49 : 76.

The merchant's share is $\frac{49}{125}$ of $500 = $196,

and the partner's share is $\frac{76}{125}$ of $500 = $304.

5. Divide $60 into two parts proportional to 11 and 9.

6. Divide $2500 into parts proportional to 2, 3, 7, 8.

7. Divide $8470 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

8. Gunpowder is made of saltpeter, sulphur, and charcoal, in parts proportional to 75, 10, and 15. How many pounds of each are contained in 12 cwt. of gunpowder?

9. The sides of a triangle are as 3, 4, 5, and the sum of the lengths of the sides is 480 yd. Find the length of each side.

10. Divide $640 between A, B, and C, so that A may have three times as much as B, and C as much as A and B together.

11. Divide 100 apples between three boys, so that the first may receive 7 as often as the second receives 8, and the third may receive 5 as often as the second receives 4.

12. A bankrupt owes £ 272 10*s.* to A, £ 354 5*s.* to B, and £ 490 10*s.* to C: his assets are £ 418 19*s.* $4\frac{1}{2}d.$ What should each of the creditors receive?

13. A certain sum is divided among A, B, and C. A gets 3 times as much as B, and B 3 times as much as C; A receives £20 more than B. What is the sum divided?

14. 1921 soldiers were distributed among 4 Philippine cities in proportion to the number of inhabitants in each, the population being 4150, 12,450, 24,900, and 29,050 respectively. How many soldiers were sent to each city?

15. Find the area of a field whose perimeter is a quarter of a mile, and whose length is eight thirds of its breadth.

16. The sides of a triangle are proportional to 3, 4, and 5; the smallest side is 2 yd. Find its perimeter.

17. The sides of a triangle are proportional to 1, $\frac{2}{3}$, and $\frac{3}{4}$, and the largest side is 1 yd. Find the two other sides.

18. A and B are partners; A furnished \$840, and B \$1080 capital. What should be their shares of \$432 profit?

19. A and B enter into partnership, A furnishes \$1500 and B \$1875 capital. What should be A's share in a loss of \$270?

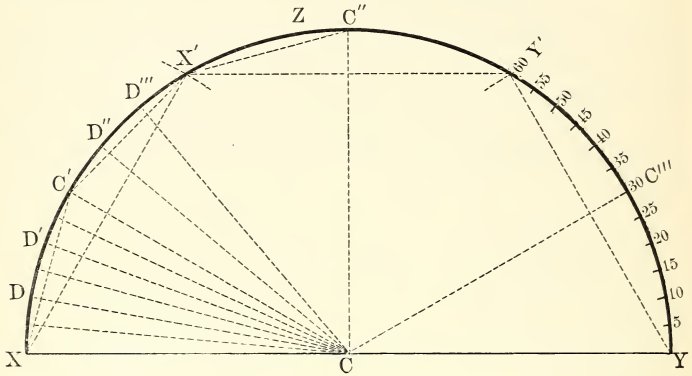
20. The gains of a partnership of A, B, and C for a year are \$4620$\frac{1}{2}$; A furnishes \$1875, B \$1500, and C \$1250 capital. What profit does A receive?

21. A and B enter into partnership; A furnishes \$400 for 8 mo., B \$600 for 4 mo. What should A's share be in \$350 profit.

22. A, B, and C hire a pasture for a year for \$16; A puts in 80, B 120, and C 200 sheep respectively. What share of the rent should each man pay?

23. A and B undertake a contract, A furnishes 5 men for 20 da., and in addition 6 men for 15 da., B furnishes 10 men for 12 da., and also 9 men for 20 da. A receives \$332.50; what should B receive?

To draw a protractor marked to 5°, or to draw half a circumference and to divide it into degrees



Take any line XY as diameter. Bisect it. C is the middle point. From C as a center describe with a radius $\frac{1}{2}$ of XY, that is, XC or YC, the circumference XZY.

From X as a center, with a radius equal to XC, describe an arc, cutting the circumference at X'.

Describe a similar arc from Y.

Draw the chords XX', $X'Y'$, and $Y'Y$.

Each arc, XX', $X'Y'$, $Y'Y$, is 60°.

Bisect each chord. Draw the radii CC'', CC''', CC''''.

Each arc XC'', $C''X'$, etc., is 30°.

Trisect each chord, of XC'', $C''X'$, etc.

Each arc, XD, DD', $D'C'$, etc., is 10°.

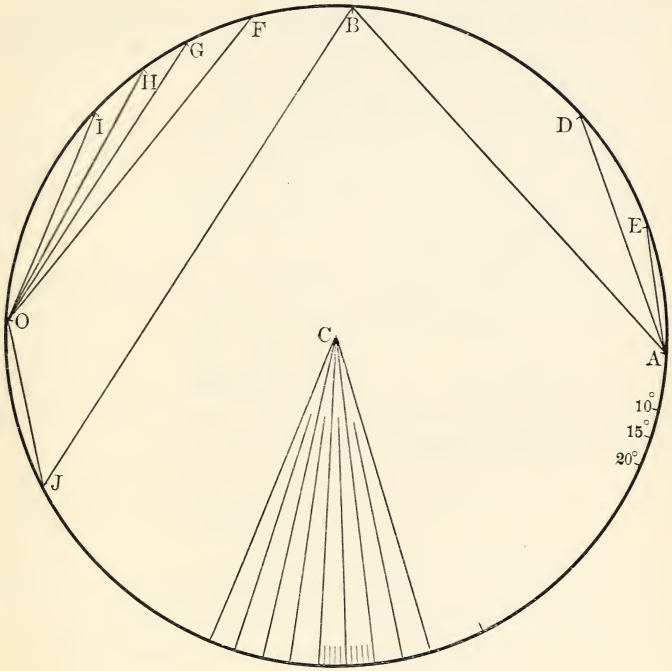
Bisect each chord. The new arcs are of 5° each.

Bisecting these new chords of 5° we have arcs of $2\frac{1}{2}$°.

Trisecting each chord of 5° we have arcs of $1\frac{2}{3}$°.

If we mean to find 1°, we may subtract by measurement the arc of $1\frac{2}{3}$° from the arc of $2\frac{1}{2}$°. The difference is $\frac{3}{4}$°. We may take $\frac{3}{4}$° four times and get 3°. This we may trisect, getting 1°.

To inscribe any regular polygon in a circle



AB is the side of the inscribed square. Its arc is 90°.

AD is the side of the inscribed octagon. Its arc is 45°.

AE is the side of the inscribed 16-sided polygon. Its arc is $22\frac{1}{2}°$.

OF is the side of the inscribed pentagon. Its arc is 72°.

OG is the side of the inscribed hexagon. Its arc is 60°.

OJ is the side of the inscribed duodekagon. Its arc is 30°.

BJ is the side of the inscribed equilateral triangle. Its arc is 120°.

OH is the side of the inscribed heptagon. Its arc is $51.4+^\circ$ or $51^\circ\ 25\frac{1}{2}'$.

OI is the side of the inscribed nonagon. Its arc is 40°.

By using the protractor we may inscribe in a circle a regular polygon of any number of sides.

The arc of an 18-sided polygon is 20°; of a 20-sided polygon is 18°; of a 24-sided polygon is 15°; of a 25-sided polygon is $14^\circ\ 24'$; of a 30-sided polygon is 12°; of a 36-sided polygon is 10°; of a 60-sided polygon is 6°; etc.

Complete these figures.

The chord whose length equals the radius is the side of a hexagon inscribed in the circle. A radius bisecting its arc marks the arc of the chord that is the side of an inscribed duodekagon. Continued bisection gives 24-sided, 48-sided, 96-sided polygons. Follow out similar relations of polygons and chords.

Review Exercises

1. Draw on the blackboard a regular polygon with 16 sides, inscribed within a circle 30 in. in diameter.

2. Find by triangulation the area of the above polygon.

3. Draw on the blackboard a protractor marked to 2°, with base line 30 in. long.

4. Draw any line and divide it into parts that are to each other as 3, 7, and 5.

5. Draw a rectangle $8'' \times 12''$, and parallel with its side, and inclosing it, draw a second rectangle with four times the area of the first.

6. Make various designs combining the circle and the pentagon.

TWO UNKNOWN QUANTITIES

Equations may be formed in which there are two or more quantities with unknown values.

To find the values of the unknown quantities, it is necessary to have as many different equations as there are unknown quantities.

Elimination so combines two equations, involving two unknown quantities, as to develop from them a single equation containing only one unknown quantity.

There are three principal methods of elimination:

By *addition* or *subtraction*.
By *substitution*.
By *comparison*.

BY ADDITION AND SUBTRACTION

1. The sum of two numbers is 8, and their difference 6. What are the numbers?

Let x = one of the numbers,
and y = the other.

Their sum $x + y = 8.$ (1)

Their difference $x - y = 6.$ (2)

Adding (2) and (1) together, $2x = 14$; $\therefore x = 7.$

Subtracting (2) from (1), $2y = 2$; $\therefore y = 1.$

7 and 1 are the two numbers.

BY SUBSTITUTION

2. A fruit grower paid 5 men and 3 boys $21 for working a day; he afterwards hired 7 men and 5 boys for $31 a day. What were the wages of each man? of each boy?

Let x = the wages of a man,
and y = the wages of a boy.

$$5x + 3y = 21, \quad (1)$$

and $$7x + 5y = 31. \quad (2)$$

We find the value of x in the first equation, and substitute that value in the second.

$x = \frac{21 - 3y}{5}$. Substituting this value in (1),

$$\frac{7(21 - 3y)}{5} + 5y = 31. \quad (3)$$

$$147 - 21y + 25y = 155. \quad (4)$$

$$-21y + 25y = 155 - 147. \quad (5)$$

$$4y = 8. \quad (6)$$

$$y = 2. \quad (7)$$

Substituting in equation (1) this value of y, we have

$$x = \frac{21 - 6}{5} = \frac{15}{5} = 3. \quad (8)$$

$2 is a boy's wages; and $3 is a man's wages.

BY COMPARISON

3. I have two small silver cups, and a silver cover which is worth $10. When the cover is put on the first cup, the value of both cup and cover is then twice the value of the second cup; but when the cover is put on the second cup, the value of both is then three times that of the first cup. What is the value in dollars of each cup?

Let x = the value of the first cup,

and y = the value of the second cup,

then $10 + x = 2y$, (1)

and $10 + y = 3x$. (2)

From (1), $x = 2y - 10$, (3)

From (2), $x = \frac{10 + y}{3}$. (4)

Equating these values of x,

$$2y - 10 = \frac{y + 10}{3}. \qquad (5)$$

$(5) \times 3$, $\quad 6y - 30 = y + 10. \qquad (6)$

Transposing, $\quad 5y - 30 = 10. \qquad (7)$

Combining, $\quad 5y = 40. \qquad (8)$

Therefore, $\quad y = 8.$

Substituting in (3) the value of y, we have,

$$x = 2 \times 8 - 10 = 16 - 10 = 6.$$

\$6 is the value of the first cup, and \$8 that of the second.

4. Solve the equations:

$$\left.\begin{aligned} \frac{x+2}{7} + \frac{y-x}{4} &= 2x - 8, \\ \frac{2y - 3x}{3} + 2y &= 3x + 4. \end{aligned}\right\}$$

Clearing of fractions,

$$4x + 8 + 7y - 7x = 56x - 224;$$

and $\quad 2y - 3x + 6y = 9x + 12,$

or $\quad 7y - 59x = -232, \qquad (1)$

and $\quad 8y - 12x = 12. \qquad (2)$

From (1) $\times$ 8, $\quad 56y - 472x = -1856. \qquad (3)$

From (2) $\times$ 7, $\quad 56y - 84x = 84. \qquad (4)$

Subtracting (3) from (4), $388x = 1940$;

Therefore, $\quad x = 5.$

Substituting in (2), $\quad 8y - 60 = 12;$

$$8y = 72; \text{ and } y = 9.$$

Solve the equations:

5. $x+y=37$, $x-y=1$

6. $2x+3y=7$, $4x-5y=3$

7. $2x+3y=10$, $8x-7y=2$

8. $4x+9y=51$, $8x-13y=9$

9. $5x+6y=76$, $4x-3y=14$

10. $3x-4y=7$, $-2x+5y=7$

11. $4x+7y=62$, $3y-2x=8$

12. $5x+6y=137$, $13x-4y=23$

13. $3x-y=3$, $9x-5y=-45$

14. $3x+2y=39$, $3y-2x=13$

15. $3x+5y=370$, $5x+3y=590$

16. $7x+2y=8$, $9y+50x=-1$

17. $\frac{2x}{3}+y=16$, $x+\frac{y}{4}=14$

18. $\frac{x}{2}+\frac{y}{3}=13$, $\frac{x}{5}+\frac{y}{8}=5$

19. $\frac{x}{2}+\frac{y}{3}=29$, $\frac{x}{3}-\frac{y}{2}=2$

20. The sum of two numbers is 100, and twice the less exceeds the greater by 5. Find the numbers.

21. Find two numbers whose sum exceeds four times their difference by 1, and such that twice the greater number exceeds three times the less by 1.

22. A cattle raiser at one time sold 9 beeves and 7 calves for \$300; and at another time, 6 beeves and 13 calves for the same price per head and received \$350. What was the price of each?

23. When the greater of two numbers is added to $\frac{1}{3}$ of the less, the sum is 37; but when the less is diminished by $\frac{1}{4}$ of the greater, the difference is 20. What are the numbers?

24. A boy has two ponies, and a harness worth \$50. When the harness is put on the first pony, his value is double that of the second; but when the harness is put on the second pony, his value is three times that of the first pony. What is the value of each pony?

25. When to the ages of A and B 18 years are added, the result is double the age of A; but when from their difference 6 years are subtracted, the result is the age of B. What are their ages?

26. A, B, and C have $2300 together. A has $\frac{1}{2}$ as much as B plus $100, and B has 3 times as much as C. How much has each?

27. One number exceeds another by 3, while its square exceeds by 99 the square of the other. Find the numbers.

28. A has 3 times as many sheep as B and 35 less than C. All have 315. How many has each?

29. There are 3 times as many pupils in one school as in another. When 80 are transferred to the other school, the first is twice as large as the second. What is the number of pupils in each?

30. The sum of the ages of mother and child is 60 yr. The difference is 3 times the child's age. Find their ages.

SUGGESTION. This problem, like many another, may be solved with either one or two unknown quantities.

Let $x =$ mother's age, or Let $x =$ the child's age,
and $y =$ child's age, then $60 - x =$ the mother's age,
then $x + y = 60$, and $60 - x - x = 3x$.
and $x - y = 3y$.

31. The width of a room is $\frac{2}{3}$ of its length. Were the width 6 feet more and the length 6 feet less, the room would be square. Find its dimensions.

32. Divide an inheritance of $1,200,000 among 10 men, 20 women, and 40 children, in such a way that each man may receive $3000 more than each child, and each woman twice as much as each child.

33. Two tally-ho coaches, planning to meet, start at the same time from Albany and Boston, 200 mi. apart. One travels $9\frac{1}{2}$, the other $9\frac{1}{4}$ mi. per hr. In how many hr., and at what point between the two places, do they meet?

34. William is 20 yr. older than John, and in two years will be twice as old as the latter. What is the age of each?

THE METRIC SYSTEM

Neither Great Britain nor the United States has a system of weights and measures based upon one standard unit. But our national government has legalized the use of the **metric system**, first developed in France, and now used upon the continent of Europe.

The French people have developed a decimal system, now generally used in Europe for all weights and measures, that is simpler than our various tables. It is better than our methods, partly because it is much easier. Our decimal money is certainly easier than English money.

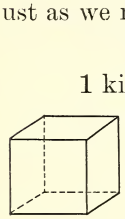
The unit of the metric system is the **meter** of length. The meter is almost exactly .000,000,01 of the distance from the equator to either pole.

A *kilometer* is 1000 meters.

Most distances in Europe are measured in kilometers, just as we measure distances in miles.

1 meter = 39.37+ in. = 1 yd. $3\frac{2}{3}$ in.

1 kilometer = 3280.9 ft. = .62+ mile.



Cubic centimeter

This figure illustrates the essential fact about the **metric system**. A centimeter is $\frac{1}{100}$ of a **meter**, just as our cent is $\frac{1}{100}$ of a dollar.

The volume of a cubic centimeter of water makes the weight known as a **gram**.

Ten centimeters make a **decimeter**, $\frac{1}{10}$ of a meter.

We know from using "decimal" so often that "deci" means tenth.

A decimeter equals 3.94 in.; that is, a trifle less than 4 inches. See the illustration on the opposite page.

The volume of a cubic decimeter of anything, liquid or dry, is called a **liter**.

Make cubes of cardboard or heavy paper, a cubic centimeter for the *gram* and a cubic decimeter for the *liter*.

In measuring wood the cubic meter is called a **ster**.

In measuring land the square dekameter (100 sq. meters) is called an **ar**.

These are the fundamental facts in the metric system. Everything else grows out of them by reason of the system itself.

The **liter** is the unit of volume.

Is a liter more or less than 64 cu. in.?

A liter is more than a liquid, but less than a dry, quart in volume.

The **gram** is the unit of weight.

A kilogram (1000 grams) is the weight of a liter of water. It weighs 2.2 lb.

1. An automobilist in Germany traveled 80 kilometers (80 Km.) in an hour. How many miles was this?

2. A professional runner in France ran .5 Km. in 90 seconds. How many feet did he run in a second? Was this faster time than a quarter mile in $47\frac{3}{5}$ seconds?

3. Which is greater, a cubic meter or a cubic yard? Give the answer in cubic inches.

4. Draw on the blackboard one meter divided into centimeters.

The use of the metric system is increasing in our own country. It is important both in domestic trade in scientific apparatus, and in our import trade for many kinds of merchandise from many foreign lands.

In work with the metric system it is not best usually to give any home lessons. In the classroom there should be much blackboard and object instruction, and nearly all the problems should be oral.

Complete mastery of the metric system is not necessary, but it is desirable to know the fundamental facts and principles.

Scientists and scholars invented the metric system, and are using it constantly. They took from the Latin language the syllables meaning tenth, hundredth, thousandth, and from the Greek language the syllables meaning ten, hundred, thousand, ten thousand, and made with the standard facts these tables: viz.

Meaning of Syllables

milli-	0.001	thousandth
centi-	0.01	hundredth
deci-	0.1	tenth
	1.	standard
deka-	10.0	ten
hekto-	100.0	hundred
kilo-	1000.0	thousand
myria-	10000.0	ten thousand

mill means	$\frac{1}{1000}$ of $1.	$0.001
nt means	$\frac{1}{100}$ of $1.	$0.01
deci- means	$\frac{1}{10}$ of $1.	$0.1

We had deka in dekagon, often spelled decagon; figure with ten sides.
Meter we have in thermometer and barometer; it means *measure*.
Myria we have in our common word *myriads*.
Only hekto- and kilo- are unfamiliar.

Long Measure

A **millimeter** (mm) = 0.001 of a meter
A **centimeter** (cm) = 0.01 " "
A decimeter (dm) = 0.1 " "
A **meter** (m) = **Standard**
A dekameter (Dm) = 10 meters
A hektometer (Hm) = 100 "
A **kilometer** (Km) = 1000 "
A myriameter (Mm) = 10000 "

Square Measure

A square millimeter (^{qmm}) = 0.000001 of a square meter
A **square centimeter** (^{qcm}) = 0.0001 " " "
A square decimeter (^{qdm}) = 0.01 " " "
A **square meter** (^{qm}) = **Standard**
A square dekameter (^{qDm}) = 100 square meters
A **square hektometer** (^{qHm}) = 10,000 " "
A **square kilometer** (^{qKm}) = 1,000,000 " "

Land Measure

A square meter (^{qm}) = a centar.
A square dekameter (^{qDm}) = an ar.
A square hektometer (^{qHm}) = a hektar.

"q" means square, from *quadré*, squared.

Cubic Measure

A cubic millimeter ($^{cu\ mm}$) = 0.000000001 of a cubic meter.
A **cubic centimeter** ($^{cu\ cm}$) = 0.000001 " " "
A cubic decimeter ($^{cu\ dm}$) = 0.001 " " "
A **cubic meter** ($^{cu\ m}$) = **Standard**

Wood Measure

cubic meter ($^{cu\ m}$) = **ster** (st).
10 decisters (^{dst}) = 1 ster (st).
10 sters = 1 dekaster (^{Dst}).

Measure of Capacity

A milliliter (^{ml}) = 0.001 of a liter.
A **centiliter** (^{cl}) = 0.01 " "
A deciliter (^{dl}) = 0.1 " "
A **liter** (^{l}) = **Standard**
A dekaliter (^{Dl}) = 10 liters.
A **hektoliter** (^{Hl}) = 100 "
A kiloliter (^{Kl}) = 1000 "

Measure of Weight

A **milligram** (mg) = 0.001 of a gram.
A centigram (cg) = 0.01 " "
A decigram (dg) = 0.1 " "
A **gram** (g) = **Standard**
A dekagram (Dkg) = 10 grams.
A hektogram (Hg) = 100 "
A **kilogram** (Kg) = 1000 "
A **metric ton** (T) = 1000 kilograms.

A **cubic centimeter** of water weighs a **gram.**
A **liter**, cubic decimeter, of water weighs a **kilogram.**
A **cubic meter** of water weighs a **ton** (1000Kg).

Our common measures have their equivalents in the metric measures. Sometimes we need to know them, for transfer and exchange are not uncommon. It is well to remember these standard equivalents: by calculation all others may be derived from them.

Long Measure

Meter = 39.37+ inches.
or 1.1– yards.

Kilometer = .62+ miles.
= 3281– feet.

Square Measure

Square meter = 1.2+ square yards.

Ar = .25– acre.

Measure of Capacity

Liter = 1.06 liquid quart.

Cubic Measure

Cubic meter = 1.33 cubic yards.

Measure of Weight

Kilogram = 2.2+ pounds.

Wood Measure

Ster = .75– cord feet.

The *kilogram* is often called a **kilo.** The **metric ton** (**tonneau**) weighs 2205 pounds, that is 35 pounds less than our *long ton* of 2240 pounds.

Our nickel (5¢) weighs 5 g. and is 2 cm. in diameter.

Oral Problems

1. What is the measure:

a. of the yard in meters? *b.* of the mile in kilometers? of the square yard in square meters? of the acre in ars? of the liquid quart in liters? of the cubic yard in cubic meters? of the cord foot in sters? of the pound in kilograms? of the tonneau in our short ton?

2. Get a tape, and mark it with ink in meters and decimeters, and measure: books, desks, floors, yards, sidewalks, doors, windows, blackboards, pencils, etc.

3. How many:

a. decimeters are in 3 hektometers? in 2 kilometers? *b.* centimeters are in 4 meters? in 4 kilometers? *c.* kilometers are in 6 myriameters? in 12 myriameters?

4. What part:

a. of a meter are 3 centimeters? 5 millimeters? *b.* of a gram are 4 milligrams? 6 decigrams? *c.* of an ar are 8 centars? *d.* of a ster are 7 decisters?

5. Compare in ratio:

a. 5 dekaliters and 5 deciliters. *b.* 2 hektometers and 2 centimeters.

6. Transfer these numbers:

a. 10 decigrams to dekagrams.

b. .2 hektoliters to centiliters.

c. 4 sters to dekasters.

d. .000005 cubic decimeters to meters.

e. .04 ars to square meters.

f. .052 kilometers to centimeters.

7. Write the numbers in **6** with their denominations abbreviated as in the Tables.

8. Discuss the Tables of Equivalents, using familiar facts.

Written Problems

1. Express 31415.92 g. in succeeding denominations from the highest to the lowest.

Since 10 units of one denomination make 1 unit of the next higher denomination, and the units' figure represents grams, the tens' figure will represent dekagrams, the hundreds' figure hektograms, . . . ; hence,

31415.92 g. = 3 Mg. 1 Kg. 4 Hg. 1 Dg. 5 g. 9 dg. 2 cg.
or = 31 Kg. 415 g. 92 cg.
or = 31 Kg. 415.92 g.

2. Express 9 hektars 25 ars 8 centars as a decimal of a square kilometer.

A hektar is a square hektometer, and 100 qHm. make 1 qKm.; hence,

9 hektars 25 ars 8 centars = 9.2508 qHm.
= .092508 qKm.

Express:

3. 1000 cm. in meters; 1000 cm. in decimeters; 1000 cm. in dekameters.

4. 1000 m. in kilometers; 1000 m. in dekameters; 1000 m. in decimeters.

5. 12 Kl. in cubic dekameters; 15 Kl. in cubic meters; 21 Kl. in cubic millimeters.

6. 123,456,789 mm. in decimeters; in meters; in kilometers.

7. 1200 ars in hektars; 1200 ars in centars; 1200 ars in dekars.

8. 12.345 milliars in ars; 5678 centars in dekars; 1 hektar in centars.

9. 1 qdm. in milliars.

10. 1 ar 5 centars in square centimeters.

11. 1 dst. in cubic decimeters.

12. 4 cu. m., 4 cu dm., 4 cu. cm. in sters.

13. Add 14. 6 m., 227 cm., 162.3 dm., 1634 km.; express the result in meters.

14. Express in centimeters the difference between 5.678 Km. and 1364.89 dm.

15. Multiply 12 Km. 5 m. 8 cm. by 96.

16. Change: *a.* 1 yd. to centimeters; *b.* 100 mi. to kilometers; *c.* 50 cm. to yards; *d.* 200 Km. to miles.

17. A train is running at the rate of 66 Km. per hour. How many meters does it go at that rate in one second?

18. A train running at the rate of 60 Km. per hour passes over 20 spaces between telegraph poles in one minute. Find the distance between two consecutive poles in meters.

19. Express in centimeters the height of a man 5 ft. $10\frac{1}{2}$ in. tall.

20. Express in millimeters the height of a barometer which stands at 29.5 in.

21. Mercury is 13.5 times as heavy as water. What is the weight in kilograms of 85 liters of mercury?

22. How long will it take a man to walk from X to Z, a distance of 65 Km., at the rate of 80 m. per minute?

23. Taking a centimeter as $\frac{2}{5}$ of an inch, find the number of millimeters in 1 yd.

24. Reduce 84,759 mm. to kilometers.

25. Reduce 42.9 Km. to dekameters.

26. Reduce 729 qKm. to square meters.

27. How many metric tons are there in 8,297,600 hg. ?

28. How many ars in 92,756 square dekameters ?

29. Which is more expensive, 2.53 yd. of velvet at $2 a yard, or the same amount at $3 a meter ?

30. What will it cost to plaster the walls and ceiling of a room 5.1 m. long by 4.6 m. wide by 3 m. high, at 30¢ a square yard ?

31. A milkman sold 44.52 qt. of milk. How many liters did he sell ?

32. What is the weight in kilograms of $2\frac{1}{2}$ bbl. of flour, each barrel weighing 196 lb. ?

33. A rectangular piece of land 72.5 m. long by 75 m. wide is worth how much at $90 an acre ?

34. How many cubic dekameters in 119,700 cu. yd. ?

35. How many liters of water in a tank 6 m. long, 54 dm. wide, and 114 cm. deep ?

36. What is the weight in grams of 8.2 cu. dm. of water ?

37. Find in cubic meters the contents of a box 2.66 yd. long, 1.33 yd. wide, and a yard high.

38. How many sections, each 1 m. 5 cm. long, in 1 Km. ? Express the remainder in millimeters.

39. Find the number of square meters of carpet required to cover a floor, the dimensions of which are 6 m. 1.75 dm. by 4 m. 12 cm.

40. The area of a rectangle is 255.5 sq. m. Its length is 2 Dm. 1 m. 4 dm. What is its width ?

41. Find the sum of 12.64 ars, .0468 hektars, one million milliars. Express the result in ars.

42. What will it cost to excavate a cellar 5.4 m. long, 5.2 m. wide, and 3.3 m. high at 12¢ a cubic meter?

43. When a wall 15 m. long, 1 m. thick, and 3.4 m. high contains 51,000 bricks, how many bricks of the same size are required for a wall of which the volume is 45 sters?

44. Given that 1 cu. cm. of water weighs 1 g., find in kilos the weight of one ster of earth which is 2.5 times as heavy as water.

45. Each linear dimension of a block of metal is increased .002 of itself by heating. Find the volume of a block which before heating measured 2 dm. × 1.8 dm. × 1.5 dm.

46. One ster of water is poured into an empty tank 2 m. long and 1.25 m. wide. Find the depth of the water in the tank.

47. An empty tank is 4 m. long and 2.5 m. wide; water is poured in at the rate of 1 ster per minute; in what time will the water be 1 dm. deep?

48. Two brothers, one living in France, the other in Canada, bought land. The first bought 800 ars at 200 francs per ar; and the second bought 160 acres at $40 per acre. With exchange at 5 fr. for $1, what was the relative cost of the two farms? What was the size of the second farm compared with the first? What was its cost per acre compared with the first?

49. A liter equals .91 of a dry quart. What amount remains from 100 liters of grain, from which 30 bu. have been taken?

50. A tank holds 1000 cm. of water. What is the weight of the tank when full? How many gal. does it contain?

NATIONAL TAXES

The chief sources of revenue for the National Government of the United States are duties laid upon manufactures, such as the **internal revenue** duties on beer, whiskey, tobacco, and patent medicines, and the **import duties** on merchandise brought here from other countries.

1. At 90¢ per gallon, what revenue was derived from the tax upon the product of a distillery, amounting to 7000 bbl. annually?

2. At $\frac{3}{8}$¢ per bottle, what revenue was derived from the tax upon the product of a patent medicine laboratory, sending out 1700 doz. bottles annually?

3. At 55% ad valorem duty, what was the customs tax on 1500 yd. of silk appraised at 75¢ per yard?

4. At 25% ad valorem and with a specific duty of $4.50 per pound added, what was the total customs cost to import 1200 lb. of cigars appraised at $5 per pound?

5. An importer bought 2000 yd. of velvet at $1.50 per yard in our money, paid an ad valorem duty of 60%, and sold the lot at $3.50 per yard. What was the gain, without allowance for other costs?

6. What is the cost to import 800 T. of tin plate at $2\frac{1}{8}$¢ per pound customs duty?

7. At 11¢ per pound duty, what is the cost to import 5000 lb. of wool?

8. An importer, paying 50% ad valorem duty, brought in $1500 worth of cotton goods, $900 worth of hosiery, and $1700 worth of gloves as invoiced. The customs officers appraised all the goods at 10% advance. What was the total cost of the goods at the port of entry, not including costs of carriage?

ARITHMETICAL PROGRESSION

An **arithmetical progression** is a series of numbers increasing or decreasing by a constant common difference: 1, 4, 7, 10, 13, etc.; 29, 25, 21, 17, etc.; 1000, 900, 800, etc.

1. The first term in an ascending series is 6, and the common difference is 4. What is the tenth term?

Since 6 is the first term, the tenth term equals $6 + 4$, the common difference, multiplied by 9; or 42.

2. A boy received on the first day 2¢, on the next 3¢ more than on the first, and so on for 12 days. What did he receive on the twelfth day?

3. Find the first term in a descending series, of which the eighth term is 74, and the constant difference is 9.

GEOMETRICAL PROGRESSION

A **geometrical progression** is a series of numbers each of which is a multiple of the preceding or succeeding term: 4, 12, 36, etc.; 7, 42, 252, etc.; 100, 10, 1.

1. The first term is 3, and the ratio is 3. Find the fifth term.

Since 3 is the first term, and 3 is the ratio, then the fifth term equals 3 multiplied by 3^4; or 243.

2. A merchant doubled his capital every 3 yr. He began with $2000. How much had he at the end of 12 yr.?

3. A man loaned $500 at interest so that his original loan was doubled every 12 yr. How much was due in 36 yr.?

4. A father so invested $100 for his daughter when she was a year old that the amount was doubled every 8 yr. How much did she receive when 25 yr. old?

ANNUITIES

An **annuity** is a fixed sum payable at equal periods of time for years, for life, or for life and afterwards to one's heirs or assigns.

British *consols* are annuities payable forever; that is, **perpetuities.** They are not *bonds* whose principal is to be redeemed at a certain time.

1. A offered to sell a perpetual annuity of $275 at a sum to yield 4%. $275 ÷ .04 = $6875.

2. At 3% find the cost of a perpetuity to yield $300.

3. At $3\frac{1}{2}$% what is the cost of a perpetuity to yield $750?

Problems in annuity are usually problems in *geometrical progression.* In actual business, annuities are calculated by tables, that give the capitalized value of the income at various rates.

FINAL REVIEW

Miscellaneous Problems

1. How many cubic feet of oxygen are there in a room $28 \times 32 \times 11$, when oxygen is 21% of all the air?

2. When each child uses 25 cu. ft. of air per breath and breathes 16 times a minute, how many cubic feet of air do 50 children breathe per minute?

3. The boiler of a steam engine is certified to withstand 225 lb. pressure per sq. in. The boiler is 20 ft. long by 5 ft. in diameter. What is the total pressure upon its surface?

4. A patented prepared paint is guaranteed to cover 30 sq. ft. of surface per gal. How many gal. are required to paint with one coat a house 35 ft. by 28 ft. by 21 ft.?

5. A can perform a piece of work in 6 da. of 10 hr. each, and B can do the same in 8 da. of 11 hr. In how many days of 11 hr. can A and B together do the work?

6. A rides 14 miles an hour on his bicycle. After he has been riding $\frac{3}{4}$ of an hour, B follows him at the rate of 20 miles an hour. How long before B will overtake A?

7. A broker bought 48 shares of Michigan Mine stock, at 14% discount, and sold them at 6% advance. How much did he make by the operation?

8. When a purchaser employs a broker to buy 55 shares of X-Z Mill stock, which is 20% below par, and pays him $\frac{1}{8}$% brokerage, how much does the stock cost in all?

9. What is the yearly income of a man who invests $8547 in U.S. 5% bonds at 97%, brokerage $\frac{1}{8}$%?

10. At the Simplon Tunnel contract price of 70,000,000 francs, what was the average price per mile of the tunnel, which is 20 kilometers long?

11. M purchases 723 yards of velvet in Paris and pays $4.25 a yard; he pays 7% for insurance, $23.70 for carriage, $2.70 for harbor dues, $3.16 for wharfage and storage, and a custom duty of 22%, and then sells the velvet for $5270. What is his gain or loss per cent?

12. Three persons, A, B, and C, engage to build a certain piece of wall for $244.16. While A can build 10 rd., B can build but 6, and C but $4\frac{1}{2}$. When the wall is half completed C ceases to labor upon it, and A and B finish the work. What part of the $244.16 ought each to receive?

13. A broker sold on consignment 376 shares bank stock, par value $100 per share, at an advance of 7%. He charged 25¢ per share for his services. How much ought he to remit to the person consigning the stock?

14. Find the area of a trapezoid whose parallel sides are 51 ft. and 37 ft. 6 in., and the perpendicular distance between them 20 ft. 10 in. Draw the figure to any suitable scale.

15. Required the area of a trapezoid whose parallel sides are 41 yd. and 24.5 yd., when the perpendicular distance between them is 21.5 yd.

16. What did I pay for $89\frac{1}{2}$ pounds of lard upon which I lost 14% by selling it at $8\frac{9}{10}$ ¢ per pound?

17. By selling cotton for $9490 I gained 43%. What did I pay for the cotton per bale of 630 lb.?

18. I sent $972.63 to a stockbroker, directing him to invest it in Broad River Railroad stock. He bought the stock at a premium of 7%, the par value being $100 per share, and charged a commission of 1% on the money invested. How many shares did he buy?

19. What is the area of a trapezoid whose bases are 9 ft. and 21 ft., and whose altitude is 16 ft.?

20. What is the area of a trapezoid whose bases are 43 rd. and 65 rd., and whose altitude is 27 rd.?

21. A train going at 45 mi. per hr. left New York at 9 A.M. A second train going at 60 mi. per hr. left New York at 11.30 A.M. At what distance from New York was the second train due to catch the first?

22. There was an increase of $37\frac{1}{2}$% in the population of a certain city within the last decade. At the end of the decade the population was 42,500. What was it 10 yr. earlier?

23. A can do a piece of work in 6 da., B can do 6 times as much in 16 da., and C 10 times as much in 24 da. In what time can they all do the first piece of work?

24. What length of rope will fasten a horse to a post in the center of a square field, a half acre in area, so that he can graze upon it without stepping across its bounds?

25. $5\frac{1}{2} \times \frac{7}{11} \times \frac{6}{7} = ?$

26. $12\frac{3}{5} \times \frac{2}{6} \times 7 = ?$

27. $\frac{6}{7} \times \frac{7}{11} \times 5\frac{1}{2} \times \frac{5}{6} = ?$

28. $\frac{3}{4} \times \frac{2}{5} \times 1\frac{2}{3} \times \frac{10}{11} = ?$

29. $2\frac{1}{7}$ of $14 \times 1\frac{1}{3} \times 2\frac{1}{4} = ?$

30. $\frac{5}{8}$ of $\frac{7}{12}$ of $\frac{9}{16} = ?$

31. A man bought a house for $1850.50, and sold it for $1517.41. What was the rate per cent of loss?

32. How many planks 15 feet long, and 15 inches wide, will floor a barn $60\frac{1}{2}$ feet long, and $33\frac{1}{2}$ feet wide?

33. When it is 10 A.M. at Stockholm, Sweden, it is 3 hr. 24 min. 58 sec. A.M. at Wheeling, W. Va. The longitude of Wheeling is 80° 42′ W. What is the longitude of Stockholm?

34. In a certain city the average rate of growth in school attendance was $8\frac{1}{2}\%$ a year. In 1902 the city required 88 teachers for 3800 children, and $61,600 for teachers' salaries. Find the number of teachers and the amount of money needed at the same rate of expenditure and growth for the year 1903.

35. A pile of wood is 45 m. long, 1.5 m. wide, and 8 m. high. Find its value at $2 per ster.

36. If 468 yards of sheeting can be made in a cotton mill in one hour, how many yards can be made in the same mill in 313 hours?

37. What is the difference between $3\frac{3}{8}+7\frac{5}{6}$ and $4+2\frac{11}{12}$?

38. $x + y = 425$. $x - y = 175$. Solve.

39. $x \times y = 5.25$. $x - y = 2.35$. Solve.

40. What is the G. C. D. of 647 and 2750?

41. At $1.90 per gross, and an allowance of 3 pencils per child per year, what is the annual cost of pencils for 7500 children?

42. X bought 729 barrels of flour for $2916. For what must he sell it per barrel in order to gain 28 per cent?

43. When $\frac{11}{13}$ of a vessel costs $6294\frac{3}{11}$, what does $\frac{7}{16}$ of the vessel cost?

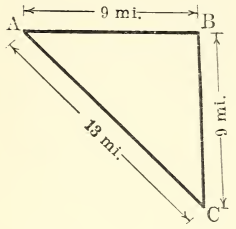
44. A can build a wall in 10 days which B can build in 7 days and C in 12 days. If all three work together, in what time can they build the wall?

45. Solve: (*a*) $7x - 3y = 3$, $5x + 7y = 25$. (*b*) $3x + 5y = 5$, $4x - 3y = 26$.

46. Z bought $\frac{3}{5}$ of a 12-acre lot and sold $\frac{1}{3}$ of his purchase. How much had he remaining?

47. When $3\frac{1}{4}$ bu. of oats are required to sow an acre, how many bushels are required to sow $7\frac{1}{5}$ A.?

48. *a*. If cavalry can advance three times as fast but only twice as far as infantry, in a country where the men can walk 16 mi. in 5 hr., how far can the cavalry advance in that time?



b. There were 2200 infantry and 450 cavalry at *A*. The general in command wished all to be ready together at *C* by noon. The infantry were to march direct, the cavalry to go by way of *B*. At what time should each division start? The infantry could march 3 mi. per hour.

49. On a map of Alaska, drawn to scale $\frac{3}{16}$ in. = 100 mi., the Pribilof Islands are $3\frac{1}{8}$ in. from Fort Wrangel in a straight line. By way of Unalaska Island they are $3\frac{5}{16}$ inches distant. What is the distance in a straight line over land and sea? by sea alone?

50. The Civil War cost our people, directly and indirectly, at least $10,000,000,000. Our population was about 35,000,000 in 1865. What was the cost per capita? If the negro slaves were worth $400 each on the average, would Seward's, and later Lincoln's, desire to buy them on appraisements and set them free have been an economical measure when there were 4,000,000 slaves?

51. At 75¢ per milliar find the cost of painting the walls and ceiling of a room 5 m. long, 4 m. wide, and 3.5 m. high.

52. A rectangular garden 15 m. long and 12 m. broad is surrounded by a path 1.5 m. wide. Find the area of the path in centars.

53. Find in sters the volume of a cube 3 m. square.

54. A certain mill property for the want of care deteriorated annually 5%. Its original value was $180,000. What was it worth after 10 years?

55. Factor $5\,m^2 + 10\,mn + 5\,n^2$.

56. X bought 50 gallons of molasses at 75¢ a gallon, 10 gallons of which leaked out. At what price per gallon must the remainder be sold to clear 10% on the cost?

57. At $22 per M., board measure, find the cost of hemlock siding for a house $34' \times 26' \times 21'$ with an ell $16' \times 14' \times 19'$. Make a 10% allowance for doors, windows, etc.

58. A man having $1500 spends 20% and then he expends $660; what per cent was the $660 of the money that remained after the first expenditure?

59. A man gave $\frac{3}{4}$ of his estate to his wife, $\frac{2}{3}$ of the remainder to a son, and what then remained to a daughter, her gift being $1000. What was the value of the estate?

60. The area of a square field is 18.49 Ha. Find the perimeter in meters.

61. How high must wood be piled on a car which is 28 ft. long and 8 ft. wide, that it may carry 56 cords?

62. When 46 cows yield on an average 2.5 gallons of milk a day, and when each gallon produces 1.1 pounds of cheese, (*a*) how many pounds can be made in 5.7 months of 30 days each, and (*b*) how much will the cheese be worth at 15¢ a pound?

63. I purchased three lots of goods as follows, on six months' credit:

June 6, $650, July 8, $890, Aug. 1, $7940.

What is the equated time of payment?

64. 32 men agree to construct 28 mi. 192 rd. of road; after completing one half of it, one fourth of the number of men left the company. What distance did each man construct before and after these men left?

65. Change 5832000 sq. in. to square rods.

66. Reduce 157 sq. rd. 26 sq. yd. to square inches.

67. A man bought 96 apples at the rate of 4 for 3¢ and exchanged them for oranges at 4¢ apiece. How many oranges did he receive?

68. A number of overcoats at $38 each and twice the same number of hats at $5 each are sold for $4800. Find the number of hats sold.

69. A rope 220 rods long is to be divided into lengths of $5\frac{1}{2}$ rods each. How many lengths will there be?

70. 5 mi. 147 rd. 13 ft. are how many feet?

71. 369874 cu. in. are how many cubic yards?

72. 4 A. 96 sq. rd. 25 sq. yd. are how many square feet?

73. A piece of marble weighing 2 T. 14 cwt. 18 lb., is worth 3¢ per lb. What is the value of the stone?

74. $110.75. Cleveland, March 14, 1900.

Three months after date, I promise to pay James Minturn, or bearer, one hundred ten and $\frac{75}{100}$ dollars, with interest. Value received.

Alexander Hammond.

This note was discounted at the National Merchants' Bank at 5%. What were the proceeds?

75. When A and B, with C working half time, can build a wall in 21 da.; B and C, with D working half time, in 24 da.; C and D, with A working half time, in 28 da.; D and A, with B working half time, in 32 da.; in what time can (*a*) all together build it? (*b*) each one alone?

76. At 6%, what is the amount of $173 for 3 yr. 1 mo. 15 da.?

77. At 7¢ a pound, what is the value of 245 beeves, averaging in weight 984 pounds each?

78. In an orchard there are 136 pear trees. If each tree yields on an average 17 bushels of pears, what would be the value of the orchard yield at $1.26 a bushel?

79. At 5% what is the amount of $182.50 from Jan. 1, 1896, to July 1, 1900?

80. A shopkeeper buys a piece of cloth containing 42 yd. for £22 10*s*., of which he sells 27 yd. for £15 15*s*. (*a*) How many yards has he left, and (*b*) what have they cost him?

81. A man bought a house for $6425. At the end of five years it had increased in value 110 per cent. What was it worth then?

82. $715.00. New York, Aug. 1, 1900.

Thirty days after date, I promise to pay to the order of Thomas M. Fillmore, at the Chemical Bank of New York, seven hundred fifteen dollars, with interest. Value received.

William S. Sawyer.

What amount was due at the maturity of this note?

83. Change $\frac{3}{512}$ and $\frac{3}{51200}$ to decimals.

84. From $\frac{3}{4}$ of $\frac{6\frac{4}{5}}{12}$ take $\frac{1}{2}$ of $\frac{1\frac{7}{8}}{5}$.

85. *a*. $\frac{13}{15} - \frac{13}{12} = ?$ *b*. $\frac{5}{12} - \frac{5}{18} = ?$ *c*. $\frac{7}{12} - \frac{7}{10} = ?$

86. The perimeter of a rectangular cornfield in Illinois is 296 rods, and its width is 40 rods. How many acres does it contain?

87. There are two pastures, one of which contains 124 acres, and the area of the other is to the former as 5 to 4. How many rods square is the latter?

88. A room is 24 ft. long, 18 ft. wide, and 12 ft. high. What is the distance from one of the lower corners to an opposite upper corner?

89. (*a*) How far does a plowman travel in plowing an acre of land, using a plow that throws a furrow 12 in. wide? (*b*) Find by ratio from the answer, his distance when using a 16 in. plow.

90. A firm failed with liabilities of $2976.51 and assets of $1862.27. There were two large creditors, A for $750 and B for $525. The receiver was allowed 5% on gross assets. What did each of these creditors receive?

91. A corporation went into bankruptcy. Its liabilities were $1,895,695, and its assets realized 82% of their estimated value, $1,290,000. The receiver was allowed $10,000 as a fee. The expenses of settlement were $82,965. (*a*) What did the X-Y-Z Co. receive, to which was due $78,956? (*b*) Find the receipts of the firm of M & Q, to whom the debt was $18,620.50.

92. A merchant purchased an equal number of yards of silk and of kersey, paying $166.75 for both fabrics. He gave $4.50 a yard for the silk, and $2.75 a yard for the kersey. How many yards of silk did he buy? How many yards of kersey?

93. A wall is to be built to the height of 27 ft., and 9 ft. in height of it are built by 12 men in 6 da. How many men must be employed to finish the remainder in 4 da.?

94. In how many days of 7 hr. each can a man travel 390 mi., if he takes 3 da. of 14 hr. each to travel 130 mi.?

95. When a clover field of 16 A. can support 6 horses for 4 mo., how many acres are there in a similar field that can support 12 horses for 9 mo.?

96. In what time will 48 men build a wall 864 ft. long, 5 ft. high, and 3 ft. wide, when 32 men can build a wall 36 ft. long, 8 ft. high, and 4 ft. wide in 45 da.?

97. What number added to $\frac{7}{8} + \frac{5}{12}$ gives $2\frac{1}{8}$?

98. Divide $\frac{8}{9}$ by $\frac{5}{6} - \frac{7}{11}$.

99. A sold 64 shares of Reno Railroad stock, at $10\frac{1}{2}\%$ premium. How much money did he receive?

100. A man purchased molasses for $1.25 a gallon, but he was obliged to sell it at 20% a gallon less than he gave for it. He sold in all $289\frac{3}{4}$ gal. How much did he lose?

101. A boy bought a bicycle for $16\frac{1}{2}$. He paid $2\frac{3}{4}$ for a new tire, $1.85 for a lantern, $1.98 for a brake. He then sold it for 3% less than it had cost him. For how much did he sell it?

Find the amount of:

102. $384.10 for 1 yr. 6 mo. 15 da. at 7%.

103. $250 for 2 yr. 4 mo. 12 da. at 6%.

104. $300 for 3 yr. 16 mo. 18 da. at 6%.

105. $2.02 for 10 yr. 10 mo. 10 da. at 6%.

106. $98 for 5 yr. 7 mo. at 9%.

107. $225 for 5 yr. 8 mo. 24 da. at 6%.

108. $144.25 for 6 yr. 5 mo. at 4%.

109. $878.75 for 2 yr. 9 mo. 6 da. at 5%.

110. $1492.50 for 1 yr. 10 mo. 12 da. at 5%.

111. $36.75 for 2 yr. 4 mo. 12 da. at 7%.

112. When 6 shoemakers in 4 wk. make 36 pairs of men's shoes and 24 pairs of women's shoes, how many pairs of each kind would 18 shoemakers make in 5 wk. ?

113. How many worlds like the moon would it take to make a sphere the size of our earth, the moon's diameter being 2083 mi., and the earth's 7917 mi. ?

114. What is the difference in cubic feet between half a cubic yard and a half-yard cube ?

115. If the gas which fills a balloon 26.7 ft. in diameter has a lifting force of 700 lb., how large must be the balloon that, when filled with the same quality of gas, will have a lifting force of a ton ?

116. A father who had a daughter 5 yr. old wished to be sure to have $400 to give her each of the years she was 18, 19, 20, and 21 yr. old, when he hoped she would be studying in a college away from home. He deposited in a savings bank that year, and every succeeding year for 10 yr., $125. If the interest was credited annually at 4%, would she be able to draw out $400 every year ?

117. When the interest on 568.25 fr. for 3 yr. is 127.86 fr., what is the rate of interest ?

118. Mr. Cox was in need of money to complete the purchase of machinery. A friend took his note for 90 days. If he had the note discounted at bank on day of date at 5% and received $2500, what was its face ?

119. How many cubic feet are there in a block of marble, of which the length is 3 ft. 2 in., breadth 2 ft. 8 in., and height or thickness 2 ft. 6 in. ?

120. Find in hektars the area of a rectangular field, in length 4 Hm. and in breadth 2 Hm. 7 Dm.

121. A and B have together $2400. A loses $\frac{1}{4}$ of his money; while B gains a sum equal to $\frac{1}{4}$ of his money, and finds that his money is then just equal to that of A. How much money had each at first?

122. Seven years ago the age of A was just three times that of B; and seven years hence, A's age will be just double that of B. What are their ages?

123. Eleven men, who were neighbors in a town, engaged in various business transactions. Which of them made the largest per cent of gain? Which suffered the greatest per cent of loss?

1. A bought a quantity of wood for $790 and sold it for $750.

2. B bought hay for $24 per ton and sold it at $26.25.

3. C bought 279 barrels of pork at $17.80 per barrel and sold them for $5570.

4. D bought 212 barrels of apples at $2.30 per barrel and sold them for $600.

5. E bought 93 barrels of vinegar at $7.40 per barrel and sold them for $651.

6. F bought 205 horses at an average cost of $93.40 each and sold them for $20,987.

7. G bought a farm for $7400 and sold it for $6250.

8. H bought tea at 60¢ a pound and sold it at $87\frac{1}{2}$¢ a pound.

9. I bought lard at 13¢ and sold it at 11¢ a pound.

10. J bought flour at $6.20 a barrel and sold it at $7.80.

11. K bought cloth at $2.75 per yard and sold it at $3.10.

124. A merchant bought two bolts of velvet for $236, the first bolt at $4 and the second at $7 per yard; but the goods were damaged, and he sold $\frac{1}{8}$ of the first bolt and $\frac{2}{5}$ of the second bolt for $160 in all, and thereby lost $8 on what he sold. What was the number of yards in each bolt?

SUGGESTION. Solve by algebra, using x and y.

125. A boy agreed to work for 60 days at 25¢ the first day and an increase of 3¢ a day. What were his wages the last day? How much did he earn all together?

126. What length of fence is required to inclose a square farm containing 85 acres?

127. Find the net amount of a bill of goods amounting to $975 on which trade discounts of 15%, 10%, and 8% are allowed.

128. Mr. Slocum borrowed $950 at 5% simple interest on June 1, 1897, and paid $95 July 1, 1899. How much was required to settle the debt on April 19, 1902?

129. Mr. Bolton sent his agent $4062.20 with instructions to invest in sugar after deducting his commission of $3\frac{1}{2}\%$. If sugar was quoted at $4\frac{1}{2}$¢ a pound, how many pounds were purchased?

130. Mr. Bostwick, who lives in New York, has purchased $2500 worth of cotton. What is the cost of the 60 days' draft on New Orleans with which he pays for it, when exchange is at $1\frac{1}{2}\%$ premium and interest is charged at 6% per annum?