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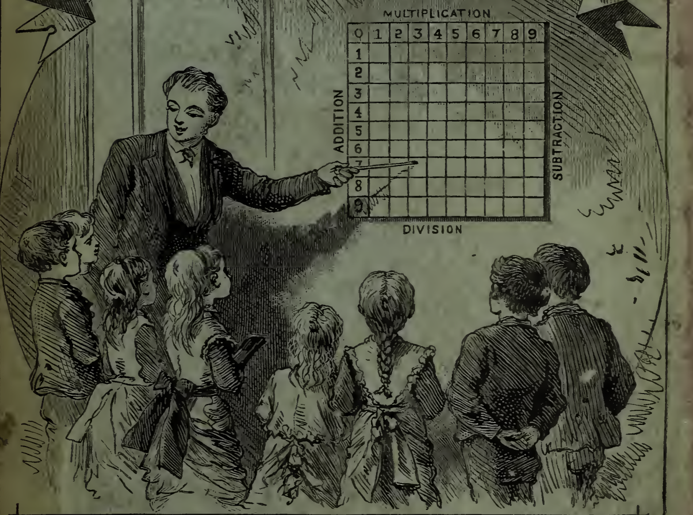
ELEMENTARY ARITHMETIC

MULTUM IN PARVO



P. O'SHEA NEW YORK

W.H.CLARK. Sc.

ELEMENTARY ARITHMETIC;

OR,

SECOND BOOK

OF A

SERIES OF MATHEMATICS.

BY

ANDREW H. BAKER, A.M., PH.D.,

NEW YORK:
P. O'SHEA, PUBLISHER,
37 BARCLAY STREET.
1878.

SMITH & McDOUGAL, ELECTROTYPERS,
82 Beekman St., N. Y.

PREFACE.

THIS book contains the Elements of Arithmetic, or perhaps more properly the Elements of Mathematical Science.

In its preparation I have endeavored to give it not only a mathematical basis, but also a scientific structure. This I have done, keeping steadily in mind that it is a book for the young, for whom the pathway of science should be made as easy and inviting as possible.

We cannot insist too strongly on the advantages of the blackboard exercises in developing the principles of science, and rendering them more easily and more thoroughly understood. In a well-drilled class, whilst one student is at the board demonstrating a theorem or solving a problem, and the other members of the class are looking on with attention, the latter learn as much as the former.

The teacher should be the guide of the class. He should not be satisfied with exemplification only, he should also endeavor to encourage and interest his pupils, carefully observing the progress made each day, each week, each month, and at the end of the year when all shall be summed up, such will be the delight of both teacher and class at the progress made, that they will begin to believe there is a royal road to learning.

CONTENTS.

The Author cannot too highly recommend to the Teacher the use of the Blackboard described on the following page. Great facility in comprehending the combinations and divisions of numbers will be acquired by this method.

BLACKBOARD EXERCISE.

1 This page represents a blackboard with the num- 37
2 bers as high as 72 painted on its margins. 38
3 There is also a box containing slips which will cover 39
4 two, three, four, etc., as high as 12, and numbered 40
5 accordingly ; one of these the student will take in his 41
6 hand and apply it to the painted numbers to perform 42
7 addition or subtraction ; thus, begin at 1 and take a 43
8 slip marked 2, then 1 and 2 are 3, 3 and 2 are 5, 5 and 44
9 2 are 7, 7 and 2 are 9, etc., counting at least the left- 45
10 hand column ; then, to perform subtraction, begin at 46
11 the bottom of the 1st column; thus, 36 minus 2 equals 47
12 34, 34−2=32, 32−2=30, 30−2=28, etc., until the top 48
13 is reached ; then taking a slip marked 3, begin with 49
14 1 or 2, or first with 1 and then with 2, and return to 50
15 the top of the column as before, by subtraction ; let 51
16 this exercise be performed with all the slips, and as 52
17 the larger numbers are taken, continue the additions 53
18 to the bottom of the 2d column, and return as before. 54
19 For multiplication and division first make a chalk 55
20 mark after every two figures up to 24, and mul- 56
21 tiply ; thus, once 2 are 2, twice 2 are 4, 3 times 2 are 57
22 6, 4 times 2 are 8, etc. ; then the number of divisions 58
23 is 12 and each division has 2 numbers ; ∴ 12 is con- 59
24 tained twice in 24, or 2 is contained 12 times, 2 is 60
25 contained once in 2, in 4 twice, in 6 three times, in 8 61
26 four times, in 10 five times, in 12 six times, etc. When 62
27 the student is familiar with multiplication and division 63
28 by 2, let the numbers be separated into 3's, then 4's, 64
29 etc., and let each be continued for 12 divisions ; when 65
30 all the divisions have been performed according to the 66
31 steps, beginning with 2 and ending with 12, a multi- 67
32 plication and division table will be made. 68
33 Rem.—In multiplication the product of any two fac- 69
34 tors is the same by making either the multiplicand and 70
35 the other the multiplier ; so also in division, the divisor 71
36 and the quotient may be substituted, as the dividend 72
is the product of the divisor and quotient.

Rem.—The numbers, continued up to 144, should be painted on the sides of the board.

ELEMENTARY ARITHMETIC.

Definitions.

1. ***Arithmetic*** is the science of numbers.

2. A ***Unit*** is a single thing; as, a book, one dollar, or simply one.

3. A ***Number*** is a unit or a collection of units; as, one, ten, five books, twenty-five dollars.

4. The numbers used in Arithmetic are all formed by combinations of the ten Arabic characters, called ***Figures;*** viz., 0, called zero or naught; 1, called one; 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine.

5. Expressing a number either in writing or figures is called ***Notation,*** and reading the expression is called ***Numeration.***

6. When numbers are used without reference to any object, they are called ***Abstract Numbers;*** as, five, twenty, etc.; but when they are applied to things, they are called ***Concrete;*** as, one book, ten men, four dollars, etc.

7. When concrete numbers express values of money, weights, measures, time, etc., they are called ***Denomi-***

nate Numbers; as, dollars, pounds, shillings, pounds of weight, ounces, hours, minutes, etc.

8. When different denominations of either kind form but one number, it is called a ***Compound Number;*** as, £4 3s. 6d., 2 lb. 1 oz. 3 pwt. and 2 gr.

9. Numbers of the same order and the same denomination are termed ***Like Numbers;*** other numbers are termed ***Unlike Numbers.***

REM.—Numbers expressing different species of the same genus are unlike, as horses and cows; while the same numbers expressed in the term of the genus are alike, as animals.

MATHEMATICAL TERMS USED IN ARITHMETIC.

1. An affirmative sentence, or anything proposed for consideration, is a ***Proposition.***

2. A self-evident proposition is called an ***Axiom.***

3. A proposition made evident by a demonstration is called a ***Theorem.***

4. When a proposition is used for developing a principle of Arithmetic, it is called a ***Problem.***

5. Propositions given merely for solution, in order to impress the principles on the mind, are called ***Examples.***

6. An obvious consequence of one or more propositions is called a ***Corollary.***

7. An established custom, or an assumption without proof, is called a ***Postulate.***

REM. 1 and 1 are 2, 2 and 1 are 3, 3 and 1 are 4, 5 and 2 are 7, 6 and 3 are 9, etc., is the postulate which forms the basis of Arithmetic.

AXIOMS.

1. If equal numbers are added to equal numbers, the sums will be equal.

2. If equal numbers are subtracted from equal numbers, the remainders will be equal.

3. If equals be multiplied by equals, the products will be equal.

4. If equals be divided by equals, the quotients will be equal.

5. If two numbers are each equal to the same number, they are equal to each other.

6. If the same number be added to and subtracted from another number, the latter number will not be changed.

7. If a number be both multiplied and divided by the same number, the former number will not be changed.

8. If two numbers be equally increased or diminished, the difference of the resulting numbers will be the same as the difference of the originals.

9. If two numbers are like parts of equal numbers, they are equal to each other.

10. The whole is greater than any of its parts.

11. The whole is equal to the sum of all its parts.

SIGNS.

1. The sign $+$, called ***plus***, is the sign of addition, and indicates that the number on the right hand is to be added to the one on the left.

2. The sign –, called ***minus,*** is the sign of subtraction, and indicates that the number on the right is to be subtracted from that on the left.

3. The sign ×, called ***into,*** is the sign of multiplication, and indicates that the numbers between which it is placed are factors of the same product.

4. The sign ÷, ***divided by,*** the left-hand number to be divided by the right hand.

5. The sign =, ***equal to,*** indicates that the numbers between which it is placed are equal.

6. 5^2, 5^3, the 2 and 3 placed to the right, a little above a number, indicates the ***power*** to which it is to be raised.

7. $\sqrt{\ }$, indicates the extraction of the ***square root;*** and $\sqrt[3]{\ }$, indicates the extraction of the ***cube root.***

NOTATION AND NUMERATION.

1st. A figure standing alone, as 1, 2, 3, holds the units place, or is of the 1st order, and is read, *one, two, three.*

2d. A number having two figures, as 14, 26, the right-hand figure holds the units place, and the left-hand figure that of tens, and they are read, *fourteen, twenty-six.*

Cor.—The right-hand figure of a number is called *units,* or the 1st order; the next figure to the left is called *tens,* or the 2d order; the third figure, *hundreds,* or the 3d order; the fourth figure, *thousands,* or the 4th order; and if a number be expressed with the nine figures in order, making 1 the right-hand figure, the figures will express their respective orders; thus,

millions,				thousands,				units.		
hundreds of	tens of	units of		hundreds of	tens of	units of		hundreds of	tens of	units of
9	8	7	,	6	5	4	,	3	2	1

If pointed in periods of three figures each, they may be read as follows: *Nine hundred and eighty-seven millions six hundred and fifty-four thousand three hundred and twenty-one.*

Rem.—The figures designate the orders.

Read the following numbers:

1	one.
21	twenty-one.
321	three hundred and twenty-one.
4,321	four thousand three hundred and twenty-one.
54,321	fifty-four thousand three hundred and twenty-one.
654,321	six hundred and fifty-four thousand three hundred and twenty-one.
7,654,321	seven millions six hundred and fifty-four thousand three hundred and twenty-one.
87,654,321	eighty-seven millions six hundred and fifty-four thousand three hundred and twenty-one.
987,654,321	nine hundred and eighty-seven millions six hundred and fifty-four thousand three hundred and twenty-one.

Rem.—The column of 1's is of the 1st order, the column of 2's is of the 2d order, the 3's the 3d order, the 4's the 4th order, etc.

Cor.—The relation of any two consecutive orders is the same, for when in addition the sum of any column reaches 10, the left-hand figure belongs to the next column or order; hence, a table may be formed, thus,

10 units	= 1 ten.
10 tens	= 1 hundred.
10 hundred	= 1 thousand.
10 thousand	= 1 ten-thousand.
10 ten-thousand	= 1 hundred-thousand.
10 hundred-thousand	= 1 million.
etc.	etc.

Addition and Subtraction.

Addition and Subtraction Table.

0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18
10	11	12	13	14	15	16	17	18	19
11	12	13	14	15	16	17	18	19	20
12	13	14	15	16	17	18	19	20	21
13	14	15	16	17	18	19	20	21	22
14	15	16	17	18	19	20	21	22	23
15	16	17	18	19	20	21	22	23	24
16	17	18	19	20	21	22	23	24	25
17	18	19	20	21	22	23	24	25	26
18	19	20	21	22	23	24	25	26	27
19	20	21	22	23	24	25	26	27	28
20	21	22	23	24	25	26	27	28	29
21	22	23	24	25	26	27	28	29	30

The square made by the ten Arabic characters forms an Addition and Subtraction Table.

Beginning with the first line, thus, Zero and zero are zero; zero and 1 are 1; zero and 2 are 2; zero and 3 are 3; zero and 4 are 4; zero and 5 are 5; zero and 6 are 6, etc.

The second line, one and zero are one; 1 and 1 are 2: 1 and 2 are 3; 1 and 3 are 4; 1 and 4 are 5, etc.

2 and zero are 2; 2 and 1 are 3; 2 and 2 are 4; 2 and 3 are 5; 3 and 4 are 7, etc.

3 and 0 are 3; 3 and 1 are 4; 3 and 2 are 5; 3 and 3 are 6; 3 and 4 are 7, etc.

Continue this, taking the first figure of the 1st column and adding it to each successive figure in the first line; the adding of zero is only nominal, as it makes no increase.

It also becomes a subtraction table, the figures of the first column being the subtrahend, and those of the first line the remainders.

Take zero from 1 and 1 remains; 0 from 2, 2 remain, etc. It may be thus expressed: $1 - 0 = 1$; $2 - 0 = 2$; $3 - 0 = 3$; $4 - 0 = 4$; $5 - 0 = 5$; which is read, 1 minus zero equals 1, etc.

Second line: take 1 from 2, 1 remains; 1 from 3, 2 remain; or, $2 - 1 = 1$; $3 - 1 = 2$; $4 - 1 = 3$; $5 - 1 = 4$, etc.

Third line; $3 - 2 = 1$; $4 - 2 = 2$; $5 - 2 = 3$; $6 - 2 = 4$; $7 - 2 = 5$, etc.

In addition, we add two numbers at a time, never more, and in the first square we have the addition of every two units that can come together; so also in subtraction.

In the second square, the units correspond with the first square, and have an additional ten.

In the third square, the units again are repeated, and another additional ten.

As a column of tens, hundreds, and every higher or lower order is added and subtracted in the same way, the above table develops every principle of addition and subtraction.

Add the column of units.

1	3	One and 2 are 3; 3 and 3 are 6; 6 and 4 are 10; 10 and 5 are 15; 15 and 6 are 21; 21 and 7 are 28; or as is customary to begin at the bottom of the column, 7 and 6 are 13; 13 and 5 are 18; 18 and 4 are 22; 22 and 3 are 25; 25 and 2 are 27; 27 and 1 are 28.
2	5	
3	6	
4	4	
5	2	
6	7	
7	9	
28	36	

$9 + 7 = 16$; $16 + 2 = 18$; $18 + 4 = 22$; $22 + 6 = 28$; $28 + 5 = 33$; $33 + 3 = 36$.

REM.—Although many numbers may be added together, in performing the operation only two at a time are added.

Add the following numbers jointly and separately; thus,

35	=	30	and	5		
24	=	20	"	4		
43	=	40	"	3		
52	=	50	"	2		21
67	=	60	"	7		200
221		200	"	21	=	221

The sum of the column of units is 21; that is, 1 unit and 2 tens; the sum of the column of tens is 20; that is, 20 tens or 2 hundred; and the two sums united make

221; precisely the same as if the column of units is first added, and the units of the sum placed under the column of units, and the tens added with the column of tens; and then the tens of the sum of the tens column placed under the column of tens, and the hundreds in place of hundreds.

COR.—As the relation of each successive order is the same, hence for every ten of any order, the 1, or left-hand figure, belongs to the next order; and the process is the same in the addition of every column; that is, one is carried to the next column for every ten in the addition of each column.

ADDITION.

3241	4365	643215
4356	5321	532684
6745	7546	478921
5364	8432	586432
19706		

754321	864321	345678
678643	678963	987654
594721	987654	321987
367543	456789	654321

SUBTRACTION.

876432	987654
543210	321234

876543	789654
654321	321043

869754
654321

Add

7654	3465321
3897	6354789
11551	9820110

Subtract

Minuend,	11551	*Minuend,*	11551
Subtrahend,	7654	*Subtrahend,*	3897
	3897		7654

Minuend,	9820110	*Minuend,*	9820110
Subtrahend,	3465321	*Subtrahend,*	6354789
	6354789		3465321

Cor. 1.—The minuend is always equal to the sum of the subtrahend and remainder, and is therefore greater than either.

Cor. 2.—Arithmetic is based upon the postulate contained in 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, *which is addition,* etc.; and the application of Axiom 6 (page 9) to this postulate proves the principle of subtraction; thus, $19 + 6 = 25$, then $25 - 6$ must be equal to 19.

Rem. 1.—When the figure of the subtrahend is larger than the one above it of the same order of the minuend, 1 of the next order of the minuend must be united to the figure of the minuend and then the subtraction be performed; then in order to make up for this addition to the minuend, 1 must be added to the next order of the subtrahend, and then perform the subtraction; this process is called carrying and requires all the attention of the student.

Rem. 2.—I prefer few examples, but these may be often repeated, and if thought necessary the teacher can give others in which the columns are longer.

Rem. 3.—Each order may be regarded as units, and the sum may reach one, two, or more hundred of its order.

PRACTICAL EXAMPLES

1. A farmer has 17 sheep in one pasture, 41 in another, and 57 in a third; how many sheep has he?

2. A fowler has 18 turkeys, 21 geese, 42 ducks, and 64 chickens; how many fowls has he?

3. A man has 48 cattle in one field and 36 in another;

if he take 15 from the second field and put them in the first, how many will then be in each field?

4. A granger owns 120 cattle, which are pastured in three fields; in one field there are 45 cattle, in another 30 cattle; how many cattle in the third field?

5. Willie has 41 cents in one pocket and 37 in another; if he buy a knife for 50 cents, how many cents will he have left?

6. Willie had given him two books to read, one of 153 pages and the other of 226 pages; he has read 240 pages, how many more has he to read?

7. In one basket there are 51 eggs, in another 62, and in a third 42 eggs; how many in the three baskets?

8. One hen has 15 chickens, another 17, and a third has 14; how many chickens in all?

9. Andrew learned 75 verses of poetry in one week, 94 in another, and 87 in a third; how many verses did he learn in three weeks?

10. John gave a beggar 53 cents, Willie gave him 72 cents, and Andrew gave him 65 cents; how many cents did he receive?

11. A man bought a horse for 95 dollars, a cow for 40 dollars, and a wagon for 65 dollars; how much did he invest?

12. A merchant commenced business with $9,875; the first year his net profits were $2,134, the second year $1,654, the third year $2,547, and the fourth year $2,620; what was then the amount of his funds?

13. A merchant commenced with $12,650; the first year he gained $2,163, the second $1,875, the third $1,260, and the 4th year he lost $4,105; what funds had he left?

14. A man commenced with $4500; the first year he doubled his money, but at the beginning of the next year he lost $2500; he then doubled what remained, and lost $6000; then he doubled what remained and closed business with what amount of money?

15. A man bought 3000 acres of land; he then sold to one man 324 acres, to another 236 acres, to a third 148 acres, to a fourth 465 acres, and to a fifth 634 acres; how many acres were left?

16. A merchant took with him $5000 to buy goods; he purchased dry goods for $1864, groceries for $1256; hardware for $630, hats and boots for $362; how much had he left?

17. A man owns three farms; the first contains 275 acres, the second 483 acres, and the third 1230 acres; he sells the first farm, 236 acres of the second, and 584 acres of the third; how many acres remain unsold? how many of the second farm? and how many of the third?

18. A man bought a horse for $150, and a buggy and harness for $275; he sold them, gaining $42; what did he sell them for?

19. John Jones bought a farm for $6875; he paid for repairs $2172, and then sold it for $10165; how much did he gain or lose by the transaction?

20. James Johnson bought one house for $6,540, another for $7,965, and a third for $12,384; he paid for repairs $3,165, and then sold the three houses for $31,236; how much did he gain or lose by the transaction?

21. Invested 3245 dollars in property, and sold it so as to gain 534 dollars. What was it sold for?

MULTIPLICATION.

MULTIPLICATION AND DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

As a Multiplication Table, begin with the first line; thus,

Once 1 is 1; once 2 are 2; once 3 are 3, etc. Second line, Once 2 are 2; twice 2 are 4; 3 times 2 are 6; 4 times 2 are 8, etc. Third line, Once 3 are 3; twice 3 are 6; 3 times 3 are 9; 4 times 3 are 12, etc. Recite each line similarly.

REM. 4 times 3 are 12, and 3 times 4 are 12; hence, alternating the factors does not change the product.

As a Division Table, begin with the first line; thus, 1 is contained in 1, once; in 2, twice; in 3, 3 times; in 4, 4 times, etc. Second line, 2 into $2 = 1$; 2 into $4 = 2$; 2 into $6 = 3$; 2 into $8 = 4$, etc. Third line, 3 into $3 = 1$; 3 into $6 = 2$, etc.

Rem.—As a Multiplication Table, it may also be read by the column, by which the factors are alternate, without changing the product. Any number is multiplied by 10 by adding a zero to it. As a Division Table, the first column has all the divisors, the first line all the quotients, and every number in each line is a dividend, which is always in the same line and the same column with the quotient and divisor. Any number having a zero in the units place is divided by 10 by removing the zero.

THEOREM I.

Any number is multiplied by 10 by annexing a zero to it.

Since the product of any number multiplied by 1 is equal to the number itself, the product of any number multiplied by 2 is double the number, etc.

For, as

$$10 \times 1 = 10, \text{ and } 10 \times 2 = 20, \text{ and } 10 \times 24 = 240,$$

and as alternating the factors does not change the product, hence,

$$1 \times 10 = 10, \text{ and } 2 \times 10 = 20, \text{ and } 24 \times 10 = 240.$$

∴ Any number is multiplied by 10 by annexing a zero to it.

Cor.—Any number is multiplied by 100 by annexing two zeros to it, and annexing three zeros multiplies it by 1000, etc.

THEOREM II.

The product of any two factors will have as many figures, or one less, than both factors.

1	3	3	4	9	50	500	500
1	3	4	4	9	5	5	50
1	9	12	16	81	250	2500	25000

The products of the smaller figures of units will be but one figure until above 3, when there will be two figures, but never more, as $9 \times 9 = 81$, and every additional figure annexed to each or either factor, whether small or large, will make an increase of one figure and no more; therefore the product of any two factors will have as many figures, or one less than both factors.

Cor. 1.—The product of any two figures cannot be less than one figure, nor more than two.

Cor. 2.—The product of units by units must be units, and when there are two figures, the left-hand figure will be tens. The product of tens by units must be tens, and when there are two figures, the left-hand figure will be hundreds; and if any order be multiplied by units, the right-hand figure of the product will be the same order as the multiplicand, and if there be two figures in the product, the left-hand figure will belong to the next higher order.

Cor. 3.—When the multiplier is tens, the product will be ten times as great as if the multiplier were units; that is, each product will have one zero to the right of it, holding the units place, or the first figure of the product must be placed in the column of tens; when the multiplier is hundreds, the right-hand figure must be placed in the column of hundreds; and, in general, whatever the order of the multiplier is, the right-hand figure must be in the column of that order.

Cor. 4.—If there be one or more zeros in the multiplier, the product of the next figure will be put back one figure for every zero.

Rem.—In the multiplication, each figure may be regarded as the unit of its order.

PROBLEMS.

1. $10 \times 10 = 100.$

2. $11 \times 11 = 121 = 11 \times (10 + 1) = \begin{array}{r} 11 \times 1 = 11 \\ 11 \times 10 = \underline{110} \\ 121 \end{array}$

3. $12 \times 12 = 144 = 12 \times (10 + 2) = \begin{array}{r} 12 \times 2 = 24 \\ 12 \times 10 = \underline{120} \\ 144 \end{array}$

4. Multiply 432 by $4 = (400 + 30 + 2) \times 4$.

$$\therefore \begin{array}{r} 2 \times 4 = 8 \\ 30 \times 4 = 120 \\ 400 \times 4 = \underline{1600} \\ 1728 \end{array} \quad \text{and} \quad \begin{array}{r} 432 \\ \underline{4} \\ 1728 \end{array}$$

5. Multiply 432 by $14 = 432 \times (10 + 4)$.

$$\therefore \begin{array}{r} 432 \times 4 = 1728 \\ 432 \times 10 = \underline{4320} \\ 6048 \end{array} \quad \text{or} \quad \begin{array}{r} 432 \\ \underline{14} \\ 1728 \\ \underline{432} \\ 6048 \end{array}$$

Rem.—The problems should be carefully impressed on the mind before proceeding.

6.

```
    432        432 ×   4 =  1728
    124        432 ×  20 =  8640
   ----        432 × 100 = 43200
   1728                    -----
   864                     53568
  432
  -----
  53568
```

Cor. 1.—When the multiplicand has several figures and the multiplier one that is only units, the first product of units by units will be units, or units and tens; the units must be placed in the right-hand or units place; if there be tens, it must be reserved and placed in or added to the column of tens; in the next product of tens by units, the right-hand figure will be tens, and must be united with the tens reserved, and placed in the column of tens; the left-hand figure, if there be one, must be treated as the previous one, reserved until the next product is obtained, and united with the right-hand figure; the process is the same in every successive order.

Cor. 2.—When the multiplier also has several figures, the process of each successive multiplier is the same, except that the right-hand figure of each product must be placed in the order of its multiplier. (Cor. 3, Prob. 2, page 22.)

Rem.—A multiplicand may be either an abstract or a concrete number, but a multiplier cannot be concrete, as it cannot refer to things, but merely indicates how many times the multiplicand is to be taken; but the product will be of the same name as the multiplicand; for twice $5 are $10; 3 times 20 yards of cloth are 60 yards of cloth; twice 4 are 8; 3 times 4 are 12, etc.

In computation, it is best to regard all numbers as abstract.

(7.)

```
    36425
      324
  -------
   145700
   72850
  109275
 --------
 11801700
```

(8.)

```
   26432
     104
  ------
  105728
 26432
 -------
 2748928
```

(9.)

```
    26432
     3004
   ------
   105728
 79296
 --------
 79401728
```

(10.)

```
      26432
      50004
     ------
     105728
 132160
 ----------
 1321705728
```

(11.)

```
   234
   123
   ---
   702
  468
 234
 -----
 28782
```

(12.)

```
   123
   234
   ---
   492
  369
 246
 -----
 28782
```

REM.—The product is not changed by alternating the multiplicand and multiplier.

EXAMPLES.

1. Multiply 54326 by 346.
2. Multiply 23748 by 543.
3. Multiply 46874 by 697.
4. Multiply 36975 by 476.
5. Multiply 236874 by 2134.
6. Multiply 9876325 by 356.
7. Multiply 879654 by 2175.
8. Multiply 986432 by 8704.
9. Multiply 326875 by 3005.
10. Multiply 468753 by 2100.

Examples may be added, or the same repeated, as the student will more readily comprehend by repetition than by different examples.

REM. 1.—In multiplication, two factors are given to find their product.

REM. 2.—In division, two numbers also are given to find the third; the one called the dividend corresponds to the product in multiplication, the other given number is called the divisor, and the required number is called the quotient; the two latter correspond to the factors in multiplication.

Division.

PROBLEMS.

When the product of two numbers is 4, and one of the numbers is 2, the other number is also 2; for $2 \times 2 = 4$, and 4 divided by 2, or 4 divided into 2 equal parts, each part is 2, that is, the quotient is 2.

1. $9 \div 3 = 3$.
2. $12 \div 2 = 6$.
3. $12 \div 3 = 4$.
4. $16 \div 4 = 4$.
5. $15 \div 3 = 5$.
6. $15 \div 5 = 3$.

Cor. 1.—The product of the divisor and quotient equals the dividend.

Cor. 2.—The divisor and quotient may be alternated.

$$\begin{array}{r} 24 \\ \underline{6} \\ 18 \\ \underline{6} \\ 12 \\ \underline{6} \\ 6 \\ \underline{6} \\ 0 \end{array}$$

Cor. 3.—Division is the reverse of multiplication and addition, and is similar to subtraction; for, it is separating a number into equal parts, which is the same as subtracting the same number from a larger one; that is, subtracting the divisor from the dividend and then from the remainder, repeating this process until there is no remainder, or until the remainder is less than the divisor. 6 is subtracted 4 times, hence it is contained four times. $24 \div 6 = 4$.

(1.)
```
10 ) 100 ( 10
     10
     ---
      0
```

(2.)
```
11 ) 121 ( 11
     11
     ---
      11
      11
      --
```

(3.)
```
12 ) 144 ( 12
     12
     ---
      24
      24
      --
```

(4.)
```
11 ) 121 ( 10 + 1
     110
     ---
      11
      11
      --
```

(5.)
```
12 ) 144 ( 10 + 2
     120
     ---
      24
      24
      --
```

6. $48 \div 12 = 4$.
7. $64 \div 8 = 8$.
8. $96 \div 12 = 8$.
9. $12 \times 4 = 48$.
10. $8 \times 8 = 64$.
11. $12 \times 8 = 96$.
12. $120 \div 10 = 12$.
13. $130 \div 10 = 13$.
14. $140 \div 10 = 14$.
15. $10 \times 12 = 120$.
16. $10 \times 13 = 130$.
17. $10 \times 14 = 140$.

Cor. 1.—Adding a zero to the right of a number multiplies the number by 10; taking a zero away from the right of a number divides the number by 10.

Divide 60536 by 4; thus,

```
4 ) 60536 ( 10000
    40000
    -----
    20536 (  5000
    20000
    -----
      536 (   100
      400
      ---
      136 (    30
      120
      ---
       16 (     4
       16   -----
       --   15134
```

or

```
4 ) 60536
    -----
    15134
```

The divisor 4 is contained once in the unit of the highest order of the dividend, which is one ten-thousand; into the remainder 5000 times, then 100, 30 and lastly 4.

Rem. 1.—The same result is obtained by short division, by putting the first figure of the quotient under the left-hand figure of the dividend (when it is contained in it), as it is of the same order.

Rem. 2.—If the unit of the divisor is not contained in the first unit of the dividend, then the first figure of the quotient will be of the same order as the second figure of the dividend and should be placed under it.

Divide 60536 by 14; thus,

```
14 ) 60536 ( 4324     and     214 ) 925336 ( 4324
     56                             856
     --                             ---
      45                             693
      42                             642
      --                             ---
       33                             513
       28                             428
       --                             ---
        56                             856
        56                             856
        --                             ---
4324 × 14 = 60536.          4324 × 214 = 925336.
```

Cor. 1.—Since the product of any two factors will have as many figures or one less than both factors, so in division the number of figures of the divisor and quotient will either be equal to or one greater than that of the dividend.

Cor. 2.—When the divisor is contained in the same number of figures of the dividend as is in the divisor, then the number of figures of the divisor and quotient will be one more than that of the dividend; but when it requires an additional figure of the dividend to contain the divisor, then the number of figures of the divisor and quotient will be equal to that of the dividend.

PROBLEMS.

1. Divide 9253360 by 2140; thus,   (2.)

```
214|0 ) 925336|0 ( 4324                       26432
        856                                     104
        ---                                  ------
         693          4324                   105728
         642          2140                  26432
         ---          ----                  -------
          513        17296      26432 ) 2748928 ( 104
          428        4324               26432
          ---        8648               -----
           856      -------              105728
           856      9253360              105728
           ---                           ------
```

3. Divide 987654321 by 12300.

```
123|00 ) 9876543|21 ( 80297                80297
         984                               12300
         ---                             -------
          365                             240891
          246                            160594
          ---                            80297
          1194                           ---------
          1107                           987653100
          ----                                1221
            873                          ---------
            861                          987654321
            ---
            1221, remainder.
```

REM. 1.—When the dividend is not the exact product of two integral numbers, there will be a remainder, and the dividend is equal to the product of the quotient and divisor plus the remainder.

REM. 2.—When there are the same number of zeros in dividend and divisor, beginning with the order of units they may be canceled; and when there are zeros in the divisor only, they may be omitted, and also the same number of figures in the dividend, which after the division is performed must be brought down as a part or the whole of the remainder.

EXAMPLES.

1. Divide 235643 by 123.
2. Divide 345678 by 234.
3. Divide 234567 by 891.
4. Divide 1357916 by 248.
5. Divide 369875432 by 1768.
6. Divide 487698425 by 625.
7. Divide 987654321 by 1234.
8. Divide 876543219 by 2345.
9. Divide 678956732 by 1546.
10. Divide 34567890 by 2564.
11. Divide 786954321 by 176543.
12. Divide 678900432 by 1004000.

The student must not proceed until he is familiar with division.

EXAMPLES.

1. What cost 5 lbs. of sugar at 10 cts. per lb.?

2. At 10 cts. per lb., how many lbs. can be bought for 50 cts.?

3. What cost 10 lbs. of sugar at 10 cts. per lb.?

4. At 10 cts. per lb., how many lbs. can be bought for 100 cts.?

5. What cost 15 lbs. of sugar at 10 cts. per lb.?

6. At 10 cts. per lb., how many lbs. can be bought for 150 cts.?

7. What cost 20 lbs. of sugar at 10 cts. per lb.?

8. At 10 cts. per lb., how many lbs. can be bought for 200 cts.?

9. What cost 25 lbs. of sugar at 10 cts. per lb.?

10. At 10 cts. per lb., how many lbs. can be bought for 250 cts.?

11. What cost 245 lbs. of beef at 8 cts. per lb.?

12. At 8 cts. per lb., how many lbs. of beef can be bought for 1960 cts.?

13. What cost 348 acres of land at $45 per acre?

14. At $48 per acre, how many acres can be bought for $15660?

15. What cost 3245 acres of land at $64 per acre?

16. At $64 per acre, how many acres can be bought for $207680?

17. What is the cost of 15 horses at $125 each?

18. How many horses at $125 each can be bought for $1875?

19. What is the cost of 35 oxen at $75 each?

20. How many oxen at $75 each can be bought for $2625?

21. What is the cost of 84 cows at $45 each?

22. How many cows at $45 each can be bought for $3780?

23. Multiply 54682 by 9; thus,

$$\begin{array}{rcrcr} 54682 & \times & 10 & = & 546820 \\ 54682 & \times & 1 & = & 54682 \\ \hline & & & & 492138 \end{array}$$

24. Multiply 54682 by 99.

$$\begin{array}{rcrcr} 54682 & \times & 100 & = & 5468200 \\ 54682 & \times & 1 & = & 54682 \\ \hline & & & & 5413518 \end{array}$$

25. Multiply 54682 by 999.

$$\begin{array}{rcrcr} 54682 & \times & 1000 & = & 54682000 \\ 54682 & \times & 1 & = & 54682 \\ \hline & & & & 54627318 \end{array}$$

26. Multiply 54682 by 25. Instead of 25, multiply by $\frac{100}{4}$.

$$\begin{array}{r} 4\ \underline{)\ 5468200} \\ 1367050 \end{array}$$

27. Multiply 54682 by 50. Instead of 50, multiply by $\frac{100}{2}$.

28. Multiply 54682 by 75. Multiply the result of 26th by 3.

29. Multiply 54682 by 150.

$$\begin{array}{lr} 54682 \times 100 \;= & 5468200 \\ \text{Adding } \frac{1}{2}, & \underline{2734100} \\ & 8202300 \end{array}$$

30. Divide 546825 by 25. Multiply by 4 and strike off two figures.

31. Divide 546850 by 50.

$$\begin{array}{r} 5468|50 \\ \underline{2} \\ 10937.00 \end{array}$$

Rem.—As a number is multiplied by 100 by adding two zeros, so any number is divided by 100 by cancelling the two right-hand figures, which will then form the remainder.

32. Bought 24 horses at $84 each, 54 cows at $36 each, and 364 sheep at $4 each; what did the horses cost? the cows? the sheep? what did all cost?

33. Sold all the stock of the last example at a profit of $324; what did I sell them for?

34. Bought 3064 acres of land at $25 per acre, and sold it at a loss of $1245. What did the land cost? and what was it sold for?

35. Bought 3840 acres of land at $10 per acre; divided it into twenty farms of an equal number of

acres each; twelve of the farms I sold at $15 per acre, 3 of the farms at $12 per acre, and the five remaining farms at $4 per acre; did I gain or lose, and how much?

36. A farm of 364 acres was bought for $9100 and sold at a gain of $1456; what was paid per acre? and at what rate was it sold?

37. Bought 184 acres of land for $11960, and sold it for $13064; what was the cost per acre? and at what rate was it sold?

38. The cost of 12 horses and 15 cows was $2760; the horses cost $180 each; what was the average cost of each cow?

39. In a square mile there are 640 acres; how many farms of 160 acres each in a State that has 9,000 square miles? how many in one of 46,000 sq. miles? how many in one of 257,000 sq. miles?

40. What is the value of the land in the first State at $25 per acre? in the second at $20 per acre? and in the third at $5 per acre?

41. A man having an estate worth $15,000, increases it $2500 every year for twenty-five years, when he dies, leaving it as follows: To his wife $20,000, to his eldest son $8000, to his second son $7000, to his third son $7000, and the remainder to be divided equally among his five daughters; what is the share of each daughter?

42. A merchant commences business with a capital of $25000; at the end of the first year he finds that he has increased it $5000, the second year the increase is $4500, and the same at the end of the third year, when he transfers the whole business to his three sons. What is the capital of each son? *Ans.* $13000.

Fractions.

If any thing or number is divided into equal parts, fractions are formed; and this operation is performed whenever one number is divided by another; or a mere representation of division; thus, to divide 20 by 4, place the dividend above a horizontal line, and the divisor below it; thus, $\frac{20}{4}$ represents 20 divided into 4 equal parts, and is a fraction.

So also $\frac{4}{5}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{7}{4}$, $\frac{9}{2}$, $\frac{5}{11}$, $\frac{11}{5}$, etc. The upper number is called the ***Numerator,*** and indicates the number divided; the lower number is called the ***Denominator,*** and indicates the number of parts into which the numerator is divided; the fraction itself is the quotient, and indicates how many in each part, or the size of each part; thus,

How often is 2 contained in 1? or 1 is how many times 2? or 1 is what part of 2? *Ans.* $\frac{1}{2}$.

REM.—Each of the three questions is of the same import.

3 is how many times 2? *Ans.* $\frac{3}{2}=1\frac{1}{2}$.

4 is how many times 2? *Ans.* $\frac{4}{2}=2$.

Which is also equivalent to "Divide 4 by 2." $\frac{4}{2}=2$.

PROBLEMS.

1. Divide 7 by 4 $=\frac{7}{4}=1\frac{3}{4}$.
   $\frac{7}{4}$ is one-fourth of what number? $\frac{7}{4}\times 4=\frac{28}{4}=7$.
2. Divide 12 by 3 $=\frac{12}{3}=4$.
   4 is one-third of what number? $4\times 3=12$.

3. Divide 13 by 7 $= \frac{13}{7} = 1\frac{6}{7}$.
   $\frac{13}{7}$ is one-seventh of what number? $\frac{13}{7} \times 7 = 13$.
4. Divide 15 by 4 $= \frac{15}{4} = 3\frac{3}{4}$.
   $\frac{15}{4}$ is one-fourth of what number? $\frac{15}{4} \times 4 = 15$.

When a number is partly an integer and partly a whole number, it is called a ***Mixed Number.***

By Axiom 7, when a number is multiplied and divided by the same number, its value is not changed, and this principle forms the basis of ***Cancellation.***

REM.—Fractions correspond to the division of integral numbers, with this difference:

In the latter the divisor is necessarily less than the dividend, and the dividend often contains the divisor exactly.

In Fractions, the divisor is often greater than the dividend, and is seldom contained exactly in it, although the principle is the same in every case of division.

In Division there are three terms: viz., the dividend, divisor, and quotient; so also in Fractions there are three terms, viz., the numerator, which corresponds to the dividend; the denominator, corresponding to the divisor and the fraction itself is the quotient.

FACTORING.

The product of two or more factors is found by multiplication, and the factors are restored by division.

Any number that is the product of two or more numbers is called a ***Composite Number;*** as, 12 is the product of 4 and 3; and 4 is the product of 2 and 2; the factors of 12 are therefore 2, 2, and 3, and as these factors cannot be reduced, they are therefore called ***Prime Factors,*** as any number is called prime which is not formed by other factors than itself and unity.

The prime factors are readily found; thus, take the first fifty numbers:

1,	2,	3,	~~4~~,	5,	~~6~~,	7,	~~8~~,	~~9~~,	~~10~~,
11,	~~12~~,	13,	~~14~~,	~~15~~,	~~16~~,	17,	~~18~~,	19,	~~20~~,
~~21~~,	~~22~~,	23,	~~24~~,	~~25~~,	~~26~~,	~~27~~,	~~28~~,	29,	~~30~~,
31,	~~32~~,	~~33~~,	~~34~~,	~~35~~,	~~36~~,	37,	~~38~~,	~~39~~,	~~40~~,
41,	~~42~~,	43,	~~44~~,	~~45~~,	~~46~~,	47,	~~48~~,	~~49~~,	~~50~~.

Every even number after 2 is composite, as it is divisible by 2; strike these. Every third number after 3 is divisible by 3; strike these. Every fifth number after 5; every seventh number after 7; the ninth numbers are canceled by 3; every eleventh number after 11, etc. In the above, there were none after the 7's. When more numbers are taken, the higher numbers will be required.

The prime numbers in the first fifty are,

1,	2,	3,	5,	7,	11,	13,	17,
19,	23,	29,	31,	37,	41,	43,	47.

The prime numbers are sixteen.

The following corollaries enable us readily to discover the prime factors of numbers:

Cor. 1.—Every even number is divisible by 2.

Cor. 2.—Every number whose last two figures are divisible by 4; for if these two figures are subtracted from the number, the remainder will be a certain number of hundreds, which is divisible by 4.

Cor. 3.—Every number ending in 5 is divisible by 5.

Cor. 4.—Every number ending in zero is divisible by 10, consequently by 2 and 5.

Cor. 5.—Every number is divisible by 3, when the sum of its figures taken as units is divisible by 3 ; for if from 1000 one be subtracted, the remainder is divisible by 9; if from 100 one be subtracted, the remainder is divisible by 9 ; so also of 10 ; hence if from 2000 two be subtracted, the remainder is divisible by 9 ; so also take 2 from 200, and 2 from 20, etc. ; therefore, in dividing any number of thousands, hundreds, or tens, the remainder will always be the unit of thousands, hundreds, and tens, and if the sum of these as units and also of the units of the given number equals 9, or any number of nines, the whole number is divisible by 9, and consequently by 3.

PROBLEMS.

Resolve the following numbers to their prime factors:

1. 4 = 2, 2.
2. 6 = 2, 3.
3. 8 = 2, 2, 2.
4. 9 = 3, 3.
5. 10 = 2, 5.
6. 12 = 2, 2, 3.
7. 14 = 2, 7.
8. 15 = 3, 5.
9. 16 = 2, 2, 2, 2.
10. 18 = 2, 3, 3.
11. 20 = 2, 2, 5.
12. 21 = 3, 7.
13. 22 = 2, 11.
14. 24 = 2, 2, 2, 3.
15. 26 = 2, 13.
16. 27 = 3, 3, 3.
17. 28 = 2, 2, 7.
18. 30 = 2, 3, 5.
19. 32 = 2, 2, 2, 2, 2.
20. 33 = 3, 11.
21. 34 = 2, 17.
22. 35 = 5, 7.
23. 36 = 2, 2, 3, 3.
24. 38 = 2, 19.

EXAMPLES.

Resolve the following numbers into their prime factors:

1. 60.	10. 86.	19. 106.	28. 125.
2. 64.	11. 88.	20. 108.	29. 128.
3. 65.	12. 90.	21. 110.	30. 130.
4. 70.	13. 95.	22. 112.	31. 132.
5. 72.	14. 96.	23. 114.	32. 136.
6. 75.	15. 98.	24. 116.	33. 140.
7. 78.	16. 100.	25. 118.	34. 225.
8. 80.	17. 102.	26. 120.	35. 500.
9. 84.	18. 104.	27. 124.	36. 625.

LEAST COMMON MULTIPLE.

DEF.—One number is called a ***Multiple*** of another when it is exactly divisible by that other number.

When a number is resolved into its prime factors, the original number is a multiple of all the prime factors, and of all the quotients arising from these factors divided into the original number; when the same factor occurs several times, the different products of this factor multiplied by itself must also be divided into the original number.

PROBLEMS.

Find the numbers of which the following are multiples:

1. 24. *Ans.* 2, 2, 2, 3, 6, 8, 12.
2. 25. *Ans.* 5, 5.
3. 30. *Ans.* 2, 3, 5, 6, 10, 15.
4. 32. *Ans.* 2, 2, 2, 2, 2, 4, 8, 16.
5. 36. *Ans.* 2, 2, 3, 3, 4, 6, 9, 12, 18.

6. 40. *Ans.* ~~2~~, ~~2~~, 2, 5, 4, 8, 10, 20.
7. 48. *Ans.* ~~2~~, ~~2~~, ~~2~~, 2, 3, 4, 6, 8, 12, 16, 24.
8. 54. *Ans.* 2, 3, ~~3~~, ~~3~~, 6, 9, 18, 27.
9. 56. *Ans.* ~~2~~, ~~2~~, 2, 7, 4, 8, 14, 28.

Rem.—First find the prime factors, then the quotients arising by dividing each prime factor and the different products of these factors into the original number.

PROBLEM.

Find the least common multiple of 8 and 12.

8	and	12
2, 2, 2.		2, 2, 3.

$2 \times 2 \times 2 \times 3 = 24.$

It is evident that any number which contains all the prime factors of each number is a common multiple of the given numbers; and the least common multiple must contain these factors and no others.

Cor.—Any factor must enter the L. C. M. as often as it does any given number.

PROBLEMS.

1. Find the least common multiple of 6 and 15.

6	and	15
2, 3.		3, 5.

As 3 is common to both numbers, it must be taken but once. The L. C. M. is $2 \times 3 \times 5 = 30$.

2. Find the least common multiple of 6 and 12.

6	and	12
2, 3.		2, 2, 3.

As 12 is a multiple of 6, it is evident that it contains all the prime factors of 5; hence, whenever one of the given numbers is a multiple of another given number, that other number need not be considered, and may be canceled.

EXAMPLES.

1. Find the least common multiple of 6, 15, and 42.

$\not{6}$ and	15 and	42
	3, 5.	2, 3, 7.

As 42 is a multiple of 6, the 6 may be canceled. Only 3 is common.

$$2 \times 3 \times 5 \times 7 = 210 = \text{L. C. M.}$$

REM.—Since any factor which is common to two or more given numbers must be taken only once, the method may be shortened; thus,

2. Find the least common multiple of 6, 8, 10, 12, 15.

2	$\not{6}$	8	10	12	15
2		4	5	6	15
3		2	5	3	15
5		2	5	1	5
		2	1	1	1

Since 12 is a multiple of 6, cancel 6. Since 2 is common to two or more,

$$2 \times 2 \times 3 \times 5 \times 2 = 120.$$

3. Find the least common multiple of 4, 6, 8, and 24.

Since 24 is a multiple of each of the other numbers, it is a common multiple of all the numbers.

4. Find the least common multiple of 6, 8, and 10. *Ans.* 120.

5. Find the least common multiple of 10, 12, and 14. *Ans.* 420.

6. Find the least common multiple of 9, 12, 18, 27, and 36. *Ans.* 108.

7. Find the least common multiple of 7, 14, 18, 21, and 28. *Ans.* 252.

8. Find the least common multiple of 5, 15, 20, 30, and 60. *Ans.* 60.

GREATEST COMMON DIVISOR.

The ***Greatest Common Divisor*** of two or more numbers is the largest number that will exactly divide them.

PROBLEM.

Find the greatest common divisor of 4 and 10. *Ans.* 2.

2, 2. 2, 5.

The greatest common divisor of any number of terms, must be the factor, or the product of the factors, common to the terms.

EXAMPLES.

1. Find the G. C. D. of 6 and 15. *Ans.* 3.

   2, 3. 3, 5.

2. Find the G. C. D. of 12 and 18. *Ans.* $2 \times 3 = 6$.

   2, 2, 3. 2, 3, 3.

3. Find the G. C. D. of 42 and 70.

Short method,

2	42	70
7	21	35
	3	5

Ans. $2 \times 7 = 14$.

CANCELLATION.

THEOREM.

The dividend contains all and exactly the same factors as the divisor and quotient.

Any composite number is the product of all its prime factors, and may be resolved into them. The product of any two integral numbers is a composite number and must contain all the factors of both numbers; and as a dividend is the product of its divisor and quotient, it must contain the same factors as its divisor and quotient.

Cor. 1.—The same is true if one or both divisor and quotient be fractional; for when reduced to a common denominator, their numerators may be regarded as integral.

Cor. 2.—Every factor of the divisor will cancel the same factor in the dividend.

Cor. 3.—The factors which are not canceled by those of the divisor will be the factors of the quotient.

Cor. 4.—Canceling a factor in the dividend divides the quotient by the same factor.

Cor. 5.—Canceling a factor in the divisor multiplies the quotient by the same factor.

PROBLEMS.

1. Divide 648 by 36.

$$\frac{648}{36} = \frac{\not{2}, \not{2}, 2, 3, 3, \not{3}, \not{3}.}{\not{2}, \not{2}, \not{3}, \not{3}.} = 18, \textit{ Ans.}$$

2. Divide 625 by 125.

$$\frac{625}{125} = \frac{\not{5}, \not{5}, \not{5}, 5.}{\not{5}, \not{5}, \not{5}.} = 5, \textit{ Ans.}$$

3. Divide 500 by 100.

$$\frac{50\not{0}\not{0}}{1\not{0}\not{0}} = 5, \textit{ Ans.}$$

4. A man bought 30 yards of cloth at $5 a yard; he then exchanged it for other cloth at $3 a yard. How many yards of the latter did he get?

$\frac{\overset{10}{\cancel{30}} \times 5}{\cancel{3}} = 50$ yards. The 3 of the divisor is canceled into 30 of the dividend.

5. Sold 48 cattle at \$60 each, and invested the proceeds in sheep at \$5 each; how many sheep were purchased.

$$\frac{\overset{12}{\cancel{60}} \times 48}{\cancel{5}} = 576 \text{ sheep.}$$

6. A farmer sold 150 bushels of wheat at 125 cents per bushel, and invested the proceeds in oats at 25 cents per bushel; how much oats did he get?

$$\frac{\overset{5}{\cancel{125}} \times 150}{\cancel{25}} = 750 \text{ bu. oats.}$$

7. Sold 160 acres of land at \$50 per acre, and bought another tract for the amount of sales, at \$40 per acre; how much land was bought?

$$\frac{\$50 \times \overset{40}{\cancel{160}}}{\cancel{40}} = 200 \text{ acres.}$$

8. Divide $18 \times 15 \times 16 \times 24 \times 32$ by 9, 5, 8, 12, and 16.

$$\frac{\overset{2}{\cancel{18}} \times \overset{3}{\cancel{15}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{24}} \times \overset{2}{\cancel{32}}}{\cancel{9} \times \cancel{5} \times \cancel{8} \times \cancel{12} \times \cancel{16}} = 48.$$

9. Divide $100 \times 102 \times 96 \times 45$ by $102 \times 100 \times 16 \times 9$.

$$\frac{\cancel{100} \times \cancel{102} \times \overset{6}{\cancel{96}} \times \overset{5}{\cancel{45}}}{\cancel{102} \times \cancel{100} \times \cancel{16} \times \cancel{9}} = 30.$$

ADDITION AND SUBTRACTION OF FRACTIONS.

In the addition and subtraction of abstract numbers, as we can only unite like orders; that is, units and units, tens and tens, hundreds and hundreds, etc.; and as one-half and one-half make two halves, one-fourth and two-fourths make three-fourths, $\frac{2}{5}$ and $\frac{3}{5} = \frac{5}{5}$, $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$, $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$, etc.; so fractions must have a common denominator in order to be united by addition and subtraction.

In order to reduce fractions to a common denominator, find the least common multiple of the denominators, which is a common denominator, and then multiply each numerator by the quotient arising from this common denominator divided by its own denominator; then each numerator and denominator will be multiplied by the same number.

PROBLEM.

Reduce $\frac{1}{2}$ and $\frac{1}{3}$ to the least common denominator; then add and subtract.

6 is the L. C. D.

$6 \div 2 = 3$, the multiplier of $\frac{1}{2}$. $\qquad \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$.

$6 \div 3 = 2$, the multiplier of $\frac{1}{3}$. $\qquad \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$.

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$, and $\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$.

REM.—The denominator indicates the number of parts into which a number has been divided; the numerator is the number itself.

EXAMPLES.

1. Add and subtract $\frac{3}{4}$ and $\frac{1}{2}$. 4 is L. C. D.

$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$. $\therefore$ $\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$ and $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$.

2. Add and subtract $\frac{2}{3}$ and $\frac{1}{2}$. 6 is L. C. D.

Ans. Sum, $\frac{7}{6}$; difference, $\frac{1}{6}$.

3. Add and subtract $\frac{3}{4}$ and $\frac{3}{5}$. 20 is L. C. D.

Ans. Sum, $\frac{27}{20}$; difference, $\frac{3}{20}$.

4. Add and subtract $\frac{5}{6}$ and $\frac{4}{5}$. 30 is L. C. D.

Ans. Sum, $\frac{49}{30}$; difference, $\frac{1}{30}$.

5. Add and subtract $\frac{5}{6}$ and $\frac{5}{7}$. 42 is L. C. D.

Ans. Sum, $\frac{65}{42}$; difference, $\frac{5}{42}$.

Cor.—When the denominators have no common factor, then their product is the L. C. D., and each numerator is multiplied by all the denominators except its own

6. Add $\frac{2}{3}$ and $\frac{3}{4}$. L. C. D., 12.

$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$. $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$.

$8 + 9 = 17$. Sum $= \frac{17}{12} = 1\frac{5}{12}$.

7. Add $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. Sum $= \frac{133}{60} = 2\frac{13}{60}$.

As no two numbers have a common factor, the L. C. D. is the product of all the denominators; and then as each denominator is multiplied by the other two denominators, so each numerator must be multiplied by the product of all the denominators except its own.

8. Add $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{7}{12}$.

$$\begin{array}{r|rrr} 4 & 5 & 8 & 12 \\ \hline & 5 & 2 & 3 \end{array}$$

$4 \times 5 \times 2 \times 3 = 120$.

$\frac{120}{5} = 24$. $\frac{120}{8} = 15$. $\frac{120}{12} = 10$.

$\therefore$ 5, 8, and 12 are the multipliers of the fractions.

9. Add and subtract $\frac{48}{275}$ and $\frac{64}{350}$.

$$\begin{array}{r|ll} 25 & 275 \text{ and } & 350 \\ \hline & 11 & 14 \end{array} \qquad 25 \times 11 \times 14 = 3850, \text{ L. C. D.}$$

Sum $= \frac{1376}{3850}$, and difference $= \frac{32}{3850}$.

10. Add and subtract $3\frac{2}{3}$ and $2\frac{1}{4}$.

$$3\tfrac{2}{3} = 3\tfrac{8}{12}$$
$$2\tfrac{1}{4} = 2\tfrac{3}{12}$$

Difference $= 1\frac{5}{12}$. Sum $= 5\frac{11}{12}$.

$$\text{Or,}\quad 3\tfrac{2}{3} = \frac{11 \times 4}{3 \times 4} = \frac{44}{12}$$
$$2\tfrac{1}{4} = \frac{9 \times 3}{4 \times 3} = \frac{27}{12}$$

Sum $= \frac{71}{12} = 5\frac{11}{12}$. Difference $= \frac{17}{12} = 1\frac{5}{12}$.

Rem.—Mixed numbers may be united by either of the above methods; the latter is generally preferred.

11. Cast up the following account:

				cents.
Bought	3 pairs hose,	at $37\frac{1}{2}$ cts.	=	$112\frac{1}{2}$
	5 pocket handkerchiefs,	at $62\frac{1}{2}$ cts.	=	$312\frac{1}{2}$
	3 pocket knives,	at $31\frac{1}{4}$ cts.	=	$93\frac{3}{4}$
	7 lbs. sugar,	at $12\frac{1}{2}$ cts.	=	$87\frac{1}{2}$
	9 inkstands,	at $6\frac{1}{4}$ cts.	=	$56\frac{1}{4}$
	5 qrs. paper,	at $15\frac{1}{4}$ cts.	=	$76\frac{1}{4}$
				$738\frac{3}{4}$
	Paid on account,			$549\frac{1}{4}$
	Balance due,			$189\frac{1}{2}$ cts.

The numerators of the 4ths show the value, but those of the halves must be doubled, as 4 is the common denominator.

MULTIPLICATION OF FRACTIONS.

THEOREM I.

The product of any number multiplied by a proper fraction is less than the number itself.

PROBLEMS.

1. Multiply 5 by $\frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$.

2. Multiply $\frac{3}{4}$ by $\frac{2}{3} = \frac{1}{2}$.
   2

AXIOM 7.—If any number be both multiplied and divided by the same number, the value of the original number is not changed.

If any number is multiplied by 2 and divided by 3, it is diminished.

∴ The product of any number multiplied by a proper fraction is less than the number itself.

COR. 1.—In multiplying by a fraction, the numerator is a multiplier and the denominator a divisor.

COR. 2. The factors may be alternated.

COR. 3.—In the multiplication of fractions, the product of all the numerators will be the numerator of the product; and the product of all the denominators will be the denominator of the product.

REM.—Cancellation can be applied to the multiplication of fractions, as in the division of integers.

EXAMPLES.

1. Multiply $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$, $\frac{10}{11}$, $\frac{11}{12}$.

$$\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}\times\frac{6}{7}\times\frac{7}{8}\times\frac{8}{9}\times\frac{9}{10}\times\frac{10}{11}\times\frac{11}{12}=\frac{1}{12}.$$

By analysis, $\frac{1}{2}$ of $\frac{2}{3}=\frac{1}{3}$, $\frac{1}{3}$ of $\frac{3}{4}=\frac{1}{4}$, $\frac{1}{4}$ of $\frac{4}{5}=\frac{1}{5}$, $\frac{1}{5}$ of $\frac{5}{6}=\frac{1}{6}$, $\frac{1}{6}$ of $\frac{6}{7}=\frac{1}{7}$, $\frac{1}{7}$ of $\frac{7}{8}=\frac{1}{8}$, $\frac{1}{8}$ of $\frac{8}{9}=\frac{1}{9}$, $\frac{1}{9}$ of $\frac{9}{10}=\frac{1}{10}$, $\frac{1}{10}$ of $\frac{10}{11}=\frac{1}{11}$, $\frac{1}{11}$ of $\frac{11}{12}=\frac{1}{12}$.

REM.—The expression, $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$, etc., is reduced to a simple fraction by the multiplication of their factors.

2. Multiply $\frac{7}{9}$, $\frac{10}{11}$, and $\frac{3}{5}$; thus,

$$\frac{7}{\cancel{9}_{3}}\times\frac{\cancel{10}^{2}}{11}\times\frac{\cancel{3}}{\cancel{5}}=\frac{14}{33}, \text{ product.}$$

3. Multiply $\frac{9}{16}$ and $\frac{3}{4}$.

$$\frac{9}{16}\times\frac{3}{4}=\frac{27}{64}, \text{ product.}$$

THEOREM II.

The product of two proper fractions is less than either fraction.

For if a number is multiplied by 1, the product is the same as the number. If a number is multiplied by any number greater than one, the product is greater than the number. If a number is multiplied by any number less than one, the product is less than the number.

4. Multiply $35\frac{3}{4}$ by 9.

$$\frac{3}{4}\times 9=\frac{27}{4}=\quad 6\frac{3}{4}$$
$$35\times 9\qquad\quad = 315$$
$$321\frac{3}{4} = \text{Product.}$$

5. Multiply 37 by $8\frac{1}{2}$.

$$\begin{array}{r} 37 \\ 8\frac{1}{2} \\ \hline 18\frac{1}{2} \\ 296 \\ \hline \text{Product} = 314\frac{1}{2} \end{array}$$

6. Multiply $37\frac{3}{4}$ by $11\frac{2}{3}$.

$37\frac{3}{4} \times 11\frac{2}{3} = \frac{151}{4} \times \frac{35}{3} = \frac{5285}{12} = 440\frac{5}{12} =$ Product.

Rem.—When only one of the factors is a mixed number, they may be solved as the 4th and 5th examples; but when both factors are mixed numbers, it is better to reduce them to improper fractions as in 6th example.

7. Multiply $14\frac{2}{3}$ by 4.
8. Multiply $17\frac{1}{5}$ by 6.
9. Multiply $18\frac{1}{4}$ by 9.
10. Multiply $24\frac{3}{4}$ by 8.
11. Multiply 1456 by $15\frac{1}{4}$.
12. Multiply 2375 by $27\frac{1}{3}$.
13. Multiply 3147 by $35\frac{3}{4}$.
14. Multiply $4156\frac{3}{4}$ by $2124\frac{2}{3}$.
15. Multiply $127\frac{5}{11}$ by $13\frac{3}{7}$.
16. Multiply $15\frac{4}{9}$ by $9\frac{2}{7}$.
17. Multiply $29\frac{3}{37}$ by $13\frac{2}{21}$.
18. Multiply $104\frac{2}{3}$ by $20\frac{4}{9}$.

DIVISION OF FRACTIONS.

PROBLEMS.

1. What is $\frac{1}{2}$ of $\frac{2}{3}$? *Ans.* $\frac{1}{3}$.

For, as $\frac{1}{2}$ of 2 is 1, so $\frac{1}{2}$ of $\frac{2}{3}$ is $\frac{1}{3}$.

2. What is $\frac{1}{2}$ of $\frac{1}{3}$? *Ans.* $\frac{1}{6}$.

If we were to say that $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{\frac{1}{2}}{3}$, it would not be a common fraction; but if anything is first divided into 3 equal parts, each part is $\frac{1}{3}$, and if each of these parts is divided into 2 equal parts, there will be 6 parts; each $\frac{1}{3}$ will make $\frac{2}{6}$, and $\frac{1}{2}$ of $\frac{2}{6}$ is $\frac{1}{6}$.

Cor. 1.—Dividing the numerator of a fraction divides the fraction; and multiplying the denominator also divides the fraction.

Cor. 2.—If the numerator of a fraction is divided by any number, and again if the denominator of the same fraction is multiplied by the same number, the resulting fractions will be equal.

Cor. 3.—In dividing by a fraction, the numerator becomes the divisor, and the denominator the multiplier.

GENERAL COROLLARIES.

1. The numerator of a fraction is a dividend, the denominator a divisor, and the fraction itself a quotient.
2. Any two numbers of the same denomination are divided by making the dividend the numerator and the divisor the denominator of a fraction.
3. An integral number is reduced to a fraction by multiplying it by the denominator of the fraction.
4. Two fractions are reduced to a common denominator by multiplying both terms of each fraction by the denominator of the other fraction.

THEOREM.

To divide any number by a fraction, invert the fractional divisor and make it a multiplier.

1st. When the dividend is integral.

If the dividend is multiplied by the denominator of the divisor, the product will be the numerator of the quotient, and the numerator of the divisor will be the denominator.

2d. When the dividend is also a fraction.

If the numerator of the dividend is multiplied by the denominator of the divisor, the product will be the numerator of the quotient, and if the numerator of the divisor is multiplied by the denominator of the dividend, the product will be the denominator of the quotient.

PROBLEMS.

1. Divide 15 by $\frac{3}{4}$.

$$15 \times 4 = \frac{60}{4}, \quad \text{and} \quad \frac{60}{4} \div \frac{3}{4} = \frac{60}{3} = 20.$$
$$15 \times \frac{4}{3} = 20.$$

2. Divide $\frac{15}{3}$ by $\frac{3}{4}$.

$$\frac{15}{3} \times \frac{4}{4} = \frac{60}{12}, \quad \text{and} \quad \frac{3}{4} \times \frac{3}{3} = \frac{9}{12}, \quad \text{and} \quad \frac{60}{9} = \frac{20}{3} = 6\frac{2}{3}.$$
$$\frac{15}{3} \times \frac{4}{3} = \frac{60}{9} = \frac{20}{3} = 6\frac{2}{3}.$$

Any number is divided by a fraction by inverting the fractional divisor and making it a multiplier.

EXAMPLES

1. Divide 36 by $\frac{4}{5}$.
2. Divide 12 by $\frac{2}{3}$.
3. Divide 16 by $\frac{4}{7}$.
4. Divide $\frac{3}{4}$ by $\frac{5}{6}$.
5. Divide $\frac{1}{2}$ by $\frac{2}{3}$.
6. Divide $\frac{2}{3}$ by $\frac{1}{2}$.
7. Divide $112\frac{1}{2}$ by $21\frac{2}{3}$.
8. Divide $14\frac{2}{3}$ by $9\frac{1}{6}$.
9. Divide $33\frac{1}{3}$ by $6\frac{1}{3}$.
10. Divide 15 by $\frac{7}{15}$.
11. Divide $\frac{15}{4}$ by $\frac{7}{15}$.
12. Divide $\frac{15}{19}$ by $\frac{2}{7}$.
13. Divide $\frac{13}{15}$ by $\frac{9}{11}$.
14. Divide $208\frac{3}{4}$ by $27\frac{5}{9}$.
15. Divide $109\frac{1}{8}$ by $29\frac{2}{7}$.

REM.—Mixed numbers should be reduced to improper fractions.

16. Divide $5436\frac{3}{7}$ by 3. *Ans.* $1812\frac{1}{7}$.

17. Divide $6478\frac{3}{5}$ by 9. *Ans.* $719\frac{38}{45}$.

Thus, $9\ )\ \underline{6478\frac{3}{5}}$

719, remainder $7\frac{3}{5} = \frac{38}{5} \times \frac{1}{9} = \frac{38}{45}$.

PRACTICAL EXAMPLES.

1. A man owns $\frac{3}{16}$ of a gold mine, and sells $\frac{2}{3}$ of his share for \$50,000. What is the whole mine worth at that rate? *Ans.* \$400,000.

2. The distance from Baltimore to Philadelphia is 97 miles; A starts from Baltimore to Philadelphia at the same time that B starts from Philadelphia to Baltimore; A travels $6\frac{1}{2}$ miles per hour, and B $7\frac{1}{4}$ miles. In what time will they meet, and how far will each have traveled?

3. What number multiplied by $\frac{7}{9}$ will give a product of $\frac{9}{11}$? *Ans.* $\frac{81}{77}$.

4. What number divided by $\frac{6}{7}$ will give a quotient of $\frac{7}{9}$? *Ans.* $\frac{2}{3}$.

5. If a man travel $237\frac{4}{11}$ miles in $21\frac{3}{7}$ hours; how far does he travel each hour? *Ans.* $11\frac{127}{1650}$.

6. If a traveler perform a journey in $17\frac{2}{5}$ hours, by traveling $6\frac{3}{7}$ miles in an hour, what is the length of the journey? *Ans.* $101\frac{3}{7}$ miles.

7. What number multiplied by $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{15}{2}$ will give a product of $3\frac{1}{8}$? *Ans.* $1\frac{2}{3}$.

COMPLEX FRACTIONS.

When one or both terms of a fraction are either fractions or mixed numbers, it is called a ***Complex Fraction;*** thus,

$\frac{3\frac{1}{2}}{5}$, $\frac{\frac{1}{2}}{\frac{2}{3}}$, $\frac{3}{\frac{2}{5}}$, $\frac{37\frac{1}{2}}{62\frac{1}{2}}$, $\frac{100\frac{1}{2}}{100}$, etc., are complex fractions.

REM.—When we consider that the numerator of a fraction represents a dividend, and the denominator a divisor, a complex fraction is readily reduced to a simple fraction.

1. Reduce $\frac{3\frac{1}{2}}{5}$ to a simple fraction.

$$\frac{3\frac{1}{2}}{5} = \frac{\frac{7}{2}}{5} = \frac{7}{2} \times \frac{1}{5} = \frac{7}{10}.$$

REM.—As the denominator is a divisor, it must be inverted.

2. Reduce $\frac{\frac{1}{2}}{\frac{2}{3}}$ to a simple fraction.

$$\frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}.$$

3. Reduce $\frac{3}{\frac{2}{5}}$ to a common fraction.

$$\frac{3}{\frac{2}{5}} = 3 \times \frac{5}{2} = \frac{15}{2} = 7\tfrac{1}{2}.$$

3. Reduce $\frac{37\frac{1}{2}}{62\frac{1}{2}}$ to a common fraction.

$$\frac{37\frac{1}{2}}{62\frac{1}{2}} = \frac{\frac{75}{2}}{\frac{125}{2}} = \frac{\overset{3}{\cancel{75}}}{\cancel{2}} \times \frac{\cancel{2}}{\underset{5}{\cancel{125}}} = \frac{3}{5}.$$

5. Multiply $\frac{100\frac{1}{2}}{100\frac{1}{4}} \times \frac{40\frac{1}{3}}{60\frac{1}{2}} = \frac{\frac{201}{2}}{\frac{401}{4}} \times \frac{\frac{121}{3}}{\frac{121}{2}} = \frac{\overset{67}{\cancel{201}}}{\cancel{2}} \times \frac{\cancel{121}}{\cancel{3}} \times$ $\frac{4}{401} \times \frac{\cancel{2}}{\cancel{121}} = \frac{268}{401}.$

6. Divide $\frac{\frac{1}{2}}{\frac{2}{3}}$ by $\frac{\frac{3}{4}}{\frac{4}{5}} = \frac{\frac{1}{2}}{\frac{2}{3}} \times \frac{\frac{4}{5}}{\frac{3}{4}} = \frac{1}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{5} \times \frac{\cancel{3}}{\cancel{2}} \times \frac{4}{\cancel{3}} = \frac{4}{5}.$

$\frac{1}{2} \times \frac{4}{5} \times \frac{3}{2} \times \frac{4}{3} = \frac{4}{5}.$

In changing the division to multiplication, the whole divisor must be inverted; that is, $\frac{4}{5}$ becomes the numerator and $\frac{3}{4}$ becomes the denominator, with the sign of multiplication; then again the two denominators must be inverted; then they are all in the form of multiplication.

Decimal Fractions.

Fractions whose denominators are 10, 100, 1000, etc., are rendered decimals of the same name by a little change in form; thus, a decimal point is placed on the left of the decimals, or on the right of the units, and the same relation exists between the successive orders, as in abstract numbers, but the orders themselves are reversed.

$\frac{1}{10} = .1, \qquad \frac{1}{1000} = .001,$

$\frac{1}{100} = .01, \qquad \frac{1}{10000} = .0001,$

and are read alike; thus,

one tenth, one thousandth,

one hundredth, one ten-thousandth.

Also, $\frac{3}{10} = .3$, read three tenths;

$\frac{7}{100} = .07$, seven hundredths;

$\frac{36}{100} = .36$, thirty-six hundredths;

$\frac{456}{1000} = .456$, four hundred and fifty-six thousandths.

Hence, to enumerate a decimal fraction, read it as you would an integral number, adding to this the name of the denominator, when a common fraction, which will be expressed by 1 with as many zeros attached to it as there are numbers of decimal figures.

ADDITION AND SUBTRACTION.

EXAMPLES.

1. Add .1, .01, .001, .0001, and .0001; thus:

(1.)	(2.)	(3.)
.1	Add .0234	Add 5.634
.01	.213	21.321
.001	.3146	.654
.0001	.32	.012
.00001	.6	5.364
.11111	1.4710	32.985

	(4.)		(5.)
From	4.36215	*From*	.326159
Take	1.83754	*Take*	.234573
Rem.	2.52461	*Rem.*	.091586

Cor.—As the relation of the orders are the same, and the decimals rise in value in the same direction, whilst in name they take the opposite direction; hence, addition and subtraction of decimals are performed as in Integral Numbers.

MULTIPLICATION.

THEOREM.

In the multiplication of decimals, the product will have as many places of decimals as both factors.

$$\tfrac{1}{10} \times \tfrac{1}{10} = \tfrac{1}{100} \quad \therefore \quad .1 \times .1 = .01,$$

and

$$\tfrac{1}{100} \times \tfrac{1}{10} = \tfrac{1}{1000} \quad \therefore \quad .01 \times .1 = .001.$$

	1ST COL.		2D COL.
or,	$1 \times 1 = 1$	and	$1 \times .1 = .1$
	$.1 \times 1 = .1$	and	$.1 \times .1 = .01$
	$.01 \times 1 = .01$	and	$.01 \times .1 = .001$

The first column of products is the same as the first column of multiplicands, as 1 is the multiplier. The multiplier in the second case is one-tenth, consequently the products of the second column must be one-tenth of the first.

Therefore the product of two decimal factors will have as many decimal places as both factors.

$$1 \times 1 = 1, \text{ units.}$$
$$.1 \times .1 = .01, \text{ hundredths.}$$
$$.01 \times .01 = .0001, \text{ ten thousandths.}$$

$$1 \times 1 = 1, \text{ units.}$$
$$10 \times 10 = 100, \text{ hundreds.}$$
$$100 \times 100 = 10000, \text{ ten thousands.}$$

REM.—Observe the correspondence in name, when the contrary orders are multiplied.

PROBLEMS.

1. Multiply

```
  3.156
   .215
 ------
  15780
  3156
 6312
 ------
.678540
```

2. Multiply

```
   .534
   .136
  -----
   3204
  1602
  534
 ------
.072624
```

REM.—Each product must have six decimals, hence in the second example a zero must be prefixed.

(3.)	(4.)
.01	.00001
.01	.00001
.0001	.000000001

DIVISION.

Corollaries to Theorem, Page 54.

Cor. 1.—As the product of the divisor and quotient is equal to the dividend, therefore the dividend has as many decimal figures as both divisor and quotient.

Cor. 2.—If the divisor has decimal figures and the dividend has none, or less than the divisor, as many must be added to the dividend as to make the number equal to that of the divisor, and then the quotient will be integral. If more decimals are added to the dividend, the quotient will contain as many.

PROBLEMS.

1. Divide 21.4263 by 3.12.

```
3.12 ) 21.42|63 ( 6.86+
       18 72
        2 706
        2 496
          2103
          1872
           231, remainder.
```

As the divisor has two places of decimals, the quotient will be integral for two places of decimals in the dividend; after that the quotient will be decimal.

2. Reduce the fraction $\frac{1}{4}$ to a decimal.

```
4 ) 1.00          3
     .25          5 ) 3.0
                       .6
```

Cor.—Any common fraction may be reduced to a decimal by performing the division indicated by the terms.

Denominate Numbers.

All arithmetical numbers may be considered Denominate, even abstract numbers, as every figure in each successive order, beginning at the right and going to the left, is ten times the value of the same figure in the previous order, and may be arranged in a table; thus,

10 units	= 1 ten.
10 tens	= 1 hundred.
10 hundred	= 1 thousand.
10 thousand	= 1 ten-thousand.

In the United States currency, the orders have the same relation; thus,

10 mills (*m.*)	= 1 cent (*ct.*).
10 cents	= 1 dime.
10 dimes	= 1 dollar ($).
10 dollars	= 1 eagle.

Dimes and eagles are coins, but are not regarded in computation; but only dollars ($), cents, and mills, the cents holding two places.

There is generally a decimal point placed between dollars and cents; thus, $456.295, which is numerated "four hundred and fifty-six dollars, twenty-nine cents and five mills. It may also be numerated without any change in its value, "four hundred and fifty-six thousand, two hundred and ninety-five mills.

ADDITION.

As the relations of the orders in United States money is the same as in abstract numbers, hence their application is the same; and in addition and subtraction like orders must be placed under each other, and in every other way the same methods are followed.

PROBLEMS.

$25.365	1. What is the sum of twenty-five dollars, thirty-six cents and five mills; twelve dollars, eighteen cents and four mills; nine dollars and ten cents; thirty dollars and five mills; fifteen dollars and three cents.
12.184	
9.100	
30.005	
15.030	
$91.684	

Ans., Ninety-one dollars, sixty-eight cents and four mills.

2. Add the following sums of money:

Five dollars, thirty cents and four mills.	$5.304
Three dollars and two mills	3.002
Two dollars and three cents	2.030
Seven dollars and three mills	7.003
Twelve dollars and one cent	12.010
Nine dollars	9.000
	$38.349

(3.)		(4.)		(5.)	
Add	$97.548	Add	$386.946	Add	387,642 mills.
	68.754		5372.875		548,753
	97.632		64759.654		659,864
	198.564		876943.687		3,217,634
	$462.498		$947463.162		4,713,893 mills.

REM. 1.—The sum of the last example may be numerated thus: Four millions seven hundred and thirteen thousand, eight hundred and ninety-three mills; or, thus: Four thousand seven hundred and thirteen dollars, eighty-nine cents and three mills.

REM. 2.—Mills are numerated the same as abstract numbers.

SUBTRACTION.

$287.304
194.293
$93.011

1. From two hundred and eighty-seven dollars, thirty cents and four mills, take one hundred and ninety-four dollars, twenty-nine cents and three mills. Remainder, Ninety-three dollars, one cent and one mill.

(2.)	(3.)	(4.)
$475648.364	$9,486,397.213	$21795.375
387654.875	6,397,423.875	10963.625
$87993.489	$3,088,973.338	$10831.750

(5.)	(6.)	(7.)	(8.)	(9.)
100000	100	100	100.00	100.00
99999	99	1	1.50	2.50
1	1	99	98.50	97.50

REM.—As in addition and subtraction, so also in multiplication, the process is the same as that of abstract integers and decimals; hence there is no need of further exemplification.

English money is reckoned in pounds, shillings, pence, and farthings; sometimes also in guineas; thus,

TABLE.

4 farthings (*far.*)	=	1 penny (*d.*).
12 pence	=	1 shilling (*s.*).
20 shillings	=	1 pound (£).
21 shillings	=	1 guinea.

PROBLEMS.

Reduce £1 to shillings, pence, and farthings.

£1
20
20 = shillings.
12
240 = pence.
4
960 = farthings.

£1 = 20 shillings.
£1 = 240 pence.
£1 = 960 farthings.

As there are twenty shillings in one pound, there will always be twenty times as many shillings as pounds; and as there are twelve pence in every shilling, there will be twelve times as many pence as shillings; and four times as many farthings as pence.

Cor.—A higher denomination is reduced to a lower one by multiplication.

Reduce 960 farthings to pence, shillings and pounds; thus,

4) 960 farthings.
12) 240 pence.
20) 20 shillings.
1 pound.

As four farthings make one penny, there will be one-fourth as many pence as farthings, one-twelfth as many shillings as pence, and one-twentieth as many pounds as shillings.

Cor.—A lower denomination is reduced to a higher one by division.

Reduce 1095 farthings to pence, shillings, and pounds.

```
 4 ) 1095 farthings.
12 )  273 . . . 3 far.
20 )   22 . . . 9d.
       £1 2s. 9d. 3 far.
```

The first remainder is farthings, the second pence, and the third shillings.

```
Reduce    £1  2s.  9d.  3 far.   to farthings.
          20
          --
          22 shillings.
          12
          --
         273 pence.
           4
        ----
        1095 farthings.
```

In reducing a higher denomination to a lower one, begin by multiplying by the number of the next lower denomination that makes one of the higher, and if it be a compound number, add to the product the number of the lower denomination, and continue this process until you reach the lowest denomination.

In reducing a lower to a higher denomination, divide by the number of the lowest denomination that makes one of the next higher, and if there be a remainder, it will be of the lowest denomination, etc.

Cor.—In the computation of compound numbers, instead of carrying a unit to a higher order for every ten, as in abstract numbers, a unit is carried to a higher denomination as often as the sum reaches the number that it takes of the lower denomination to make one of the next higher denomination; thus, as 4 farthings make 1 penny, as often as the sum of the farthings reaches four, one must be carried to the pence; and as 12 pence make 1 shilling, in computing pence as many must be carried to shillings as the number of times 12 is contained in the number of pence; 1 from shillings to pounds for every 20.

In division, the order is reversed, as then we begin with the highest denomination and descend.

EXAMPLES.

	£	*s.*	*d.*	*far.*
1. Add	3	8	7	3
	5	9	6	2
	6	11	9	1
	8	15	11	3
	24	5	11	1

The sum of the first column is 9 farthings, which is 2 times 4 and 1; the 1 is farthings, and must be placed under the farthings; the 2 is carried to the next denomination and added with the pence, the sum of which is 35; that is, 2 times 12 and 11, that is, 2 shillings and 11 pence; the 2 is added with the shillings, making the sum 45, which is £2 5s.; the shillings are placed under the shillings and the 2 carried to the pounds, the sum of which is 24.

	£	s.	d.	far.
2. From	54	6	5	1
	28	7	6	3
	£25	18s.	10d.	2 far.

As you cannot subtract 3 farthings from 1 farthing, you must borrow 1 penny, which is 4 farthings; this 4 and the 1 make 5; then 3 from 5, 2 remains; the 1 penny borrowed must be carried to the 6, which makes 7, which cannot be subtracted from 5; 1 shilling, that is, 12 pence, must be borrowed and added to the 5, which makes 17; 7 from 17, 10 remains; 1 shilling to carry to 7 makes 8, which cannot be taken from 6; 1 pound, that is, 20 shillings, must be borrowed and added to the 6, making 26, from which subtract 8 and 18 remains; and £1 to carry to 28, making 29, which is subtracted from 54 and 25 remains.

REM.—When the subtrahend is less than the minuend, the difference can be taken directly.

	£	s.	d.	far.
3. Multiply	4	6	5	3
by				5
	£21	12s.	4	3

4	6	5	3
5	5	5	5
21	20) 32	25	) 15
	£1 12s.	3	3d. 3 far.
		12) 28	
		2s. 4d.	

COR.—Multiply each denominate number, and divide the product by the number of this denomination that it takes to make one of the higher, and carry the number of times it is contained to the higher denomination, and place the remainder under its kind.

4. Multiply £48 12s. 7d. 2 far. by 6.

5. Divide 4) £5 6s. 3d. 1 far. by 4.

£1 6s. 6d. $3\frac{1}{4}$ far.

4 is contained in 5, once and £1 over; this £1 is 20 shillings, which added to the 6 shillings make 26 shillings, into which 4 is contained 6 times and 2 shillings over; this 2 shillings is 24 pence, which added to the 3 pence, makes 27 pence, in which 4 is contained 6 times and 3 pence over, which is 12 farthings, and 1 more make 13, in which 4 is contained $3\frac{1}{4}$ times.

6. Divide £754 15s. 9d. 3 far. by 27.

```
27 ) £754  15s.  9d.  3 far. ( £27
      54
     ---
     214
     189
     ---
      25
       20
     ----
      515 ( 19s.  Add the 15s.
      27
      ---
      245
      243
      ---
        2
       12
       --
       33 ( 1d.  Add the 9d.
       27
       --
        6
        4
       --
       27 ( 1 far.  Add the 3 far.
       27
       --
```

Quotient = £27 19s. 1d. 1 far.

7. Multiply £5 4s. 6d. 1 far. by 35.

£5	4s.	6d.	1 far.
			35
£182	18s.	2d.	3 far.

35	35	35	4) 35
5	4	6	8d. 3 far.
175	140	210	
7	18	8	
£182	20) 158 (7	12) 218	
	140	18s. 2d.	
	18		

REM.—Observe these solutions carefully; for if they are understood, there is no further difficulty in denominate numbers; the principle is the same in all, the tables alone differ.

EXAMPLES.

1. In 2 dollars, how many cents? How many mills?

$$\$2 \times 100 = 200 \text{ cents.}$$
$$2 \times 1000 = 2000 \text{ mills.}$$

2. In 5 dollars, how many cents? How many mills?
3. In 7 dollars, how many cents? How many mills?
4. In 5 dollars 15 cents, how many cents? How many mills?

$\$5 = 500$ cents.
15
515 cents $= 5150$ mills.

5. In 6 dollars 15 cents and 3 mills, how many mills?
6. In 500 cents, how many dollars? $\frac{500}{100} = \$5$, *Ans.*
7. In 625 cents, how many dollars and cents?

Ans. $6.25.

8. In 5325 mills, how many dollars, cents, and mills?
Ans. $5.325.

9. In 63257 mills, how many dollars, cents, and mills?

10. In 75325 cents, how many dollars and cents?

11. If 1 bushel of wheat cost $1.125, what will 8 bushels cost?

12. If 1 bushel of wheat cost $1.05, what will 10 bushels cost?

13. If 1 bushel of wheat cost $1.05, what will 100 bushels cost?

14. If 8 bushels of wheat cost $9, what cost 1 bushel?

15. If 8 bushels of wheat cost $9, what cost 35 bushels?

16. If 10 bushels of wheat cost $10.50, what cost 53 bushels?

17. Bought dry goods for $243.37; groceries for $146.294; hardware for $71.96; notions for $21.512. What was the amount of the bill? Sold the same at a profit of $157.192. What did I sell the whole for?

18. If 5 lbs. sugar cost 50 cents, what will 6 lbs. cost? 7 lbs.? 8? 9? 10? 11? 12?

19. If 6 lbs. cost 72 cts., what will 7 lbs. cost? 8 lbs.? 9? 10? 11? 12?

20. In 15 farthings, how many pence?
Ans. $3\frac{3}{4}$ pen

21. In 18 farthings, how many pence? How many pence in 21 far.? 23? 25? 27? 29? 31? 33? 34? 35?

22. How many shillings in 25 pence? in 28? 35? 38? 45? 51? 56? 65?

23. How many pounds in 35 shillings? in 40? 50? 60? 65? 70? 75? 80? 85? 90? 95? 100? 105? 110? 120?

24. How many farthings in £9 13s. 9d. 3 far.?

25. How many pounds, shillings, pence, and farthings in 37864321 farthings?

26. Multiply £4 8s. 9d. 3 far. by 9.

27. Divide £25 9s. 4d. 1 far. by 13?

AVOIRDUPOIS, OR COMMERCIAL WEIGHT,

is used in commercial transactions, when goods are bought or sold in quantity, and for all metals except gold and silver.

TABLE.

16 drams (*dr.*)	=	1 ounce (*oz.*)
16 ounces	=	1 pound (*lb.*).
25 pounds	=	1 quarter (*qr.*).
4 quarters	=	1 hundredweight (*cwt.*).
20 cwt.	=	1 ton (*T.*).

EXEMPLIFICATION.

```
     1 T.
    20
    --
    20 cwt.                16 ) 512000 dr.
     4                     16 )  32000 oz.
    --                     25 )   2000 lbs.
    80 qrs.                 4 )     80 qrs.
    25                     20 )     20 cwt.
  ----                               1 T.
  2000 lbs.
    16
 -----
 32000 oz.
    16
------
512000 dr.
```

```
        T. cwt. qr. lb. oz. dr.
Reduce  3   4   2   8   6   10
       20
       --
       64
        4                      16 ) 1653354
      ---                         ---------
      258                       16 ) 103334 . . . 10 dr.
       25                          --------
     ----                          25 ) 6458 . . .  6 oz.
     6458                              -----
       16                              4 ) 258 . . .  8 lb.
   ------                                 ----
   103334                             20 ) 64 . . .  2 qr.
       16                                 ---
  -------                                3   4   2   8   6   10
  1653354 drams.                         T. cwt. qr. lb. oz. dr.
```

Reduce 1653354 drams to the original denominations.

TROY WEIGHT

is used for gold, silver, and jewels; also in philosophical experiments.

TABLE.

24 grains (*gr.*)	= 1 pennyweight (*pwt.*).
20 pennyweights	= 1 ounce.
12 ounces	= 1 pound.

```
   1 lb.
  12                       24 ) 5760 gr.
  --                            ----
  12 oz.                    20 ) 240 pwt.
  20                             ---
  ---                        12 ) 12 oz.
  240 pwt.                        --
   24                              1 lb.
 ----
 5760 qr.
```

Reduce 5 lb. 6 oz. 10 pwt. 16 gr.

```
   5 lb.   6 oz.   10 pwt.   16 gr.
  12
  66      Add the 6 oz.
   20
 1330     Add the 10 pwt.
   24
31936     Add the 16 gr.
```

Reduce 31936 grs. to the original denominations.

```
24 ) 31936 gr.
 20 ) 1330 . . . 16 gr.
    12 ) 66 . . . 10 pwt.
          5 lb.  6 oz.  10 pwt.  16 gr.
```

DIAMOND WEIGHT.

Used for diamonds and other precious stones.

TABLE.

16 parts	=	1 grain	=	.8 grain Troy.
4 grains	=	1 carat	=	3.2 grains Troy.

APOTHECARIES' WEIGHT

is used by druggists in putting up prescriptions; the pound, ounce, and grain are the same as in Troy Weight.

TABLE.

20 grains	=	1 scruple (℈).
3 scruples	=	1 dram (ʒ).
8 drams	=	1 ounce (℥).
12 ounces	=	1 pound.

1 lb.
12
12 ℥
8
96 ʒ
3
288 ℈
20
5760 gr.

20) 5760 gr.
3) 288 ℈
8) 96 ʒ
12) 12 ℥
1 lb.

APOTHECARIES FLUID WEIGHT

is used for liquids in medical prescriptions.

TABLE.

60 minims (♏)	=	1 fluid dram (f ʒ).
8 fluid drams	=	1 fluid ounce (f ℥).
16 fluid ounces	=	1 pint (O.).
8 pints	=	1 gallon (*Cong.*).

For ordinary use, 1 teacup = 2 wine glasses = 8 table-spoons = 32 tea-spoons = 4 f ℥.

COMPARISON OF WEIGHTS.

1 lb. Avoirdupois	=	7000 gr. Troy.
1 lb. Troy	=	5760 gr. Troy.

LINEAR MEASURE

is used for lengths and distances.

TABLE.

12 inches (*in.*)	=	1 foot (*ft.*).
3 feet	=	1 yard (*yd.*).
5½ yds., or 16½ ft.	=	1 rod (*rd.*).
40 rods	=	1 furlong (*fur.*).
8 furlongs	=	1 mile (*m.*).
3 miles	=	1 league (*lea.*).

MARINER'S MEASURE.

6 feet	=	1 fathom.
120 fathoms	=	1 cable length.
880 fath., or $7\frac{1}{3}$ cable lengths	=	1 mile.

REM.—1 nautical league = 3 equatorial miles = 3.45771 statute miles. 60 equatorial miles = 69.1542 statute miles = 1 equatorial degree (°). 360° = the circumference of a circle. 360 equatorial degrees = the circumference of the earth.

CLOTH MEASURE.

$2\frac{1}{4}$ inches (*in.*)	=	1 nail (*na.*).
4 nails or 9 in.	=	1 quarter.
4 quarters	=	1 yard.
3 quarters	=	1 ell Flemish.
5 quarters	=	1 ell English.
6 quarters	=	1 ell French.

SURVEYOR'S MEASURE OF LENGTH.

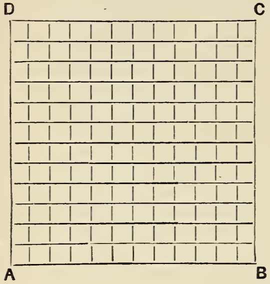
$7\frac{92}{100}$ inches	=	1 link (*l.*).
25 links	=	1 pole (*p.*).
100 links, 4 poles, 66 feet	=	1 chain (*ch.*).
10 chains	=	1 furlong.
8 furlongs, or 80 chains	=	1 mile.

LAND MEASURE.

40 perches	=	1 rod.
4 rods	=	1 acre.
640 acres	=	1 square mile, termed a Section.

SQUARE MEASURE.

A ***Square*** is a surface bounded by four equal sides, its angles are also equal; thus,



This figure, ABCD, represents a square foot, considering each of the small spaces as an inch. The sides, AB, BC, AD, and DC, each equal to 12 inches in length. The angles, A, B, C, and D, are equal; that is, if one is placed on the other, the sides respectively will coincide.

AB is 12 inches long, and every inch in width makes 12 square inches; and the 12 inches in width, which is either AD or BC, makes $12 \times 12 = 144$ sq. in.; hence the correspondence of linear and square measure; thus,

LINEAR MEASURE.

12 inches = 1 foot.
3 feet = 1 yard.
$5\frac{1}{2}$ yards = 1 rod, pole, or perch.

SQUARE MEASURE.

$12 \times 12 =$ 144 square inches = 1 square foot.
$3 \times 3 =$ 9 square feet = 1 square yard.
$5\frac{1}{2} \times 5\frac{1}{2} =$ $30\frac{1}{4}$ square yards = 1 square rod; also called perch.

CUBIC MEASURE.

$12 \times 12 \times 12 =$ 1728 cubic inches = 1 cubic foot.
$3 \times 3 \times 3 =$ 27 cubic feet = 1 cubic yard.

CUBIC MEASURE.

A ***Cube*** is a solid figure bounded by six equal squares; the square on page 72 represents the base or any other side, as the sides are all equal; the length of all the edges are equal, and the angles are all equal. If the above figure have 12 inches altitude added to it, every inch will make 144 cubic inches and 12 inches in altitude, $144 \times 12 = 1728$, which is the number of cubic inches in a cubic foot.

TABLE.

1728 cubic inches	=	1 cubic foot.
27 cubic feet	=	1 cubic yard.
16 cubic feet	=	1 cord foot.
128 cubic feet, or 8 cord feet	=	1 cd. of wood, bark, etc.

40 cubic feet of round timber, or 50 cubic feet of hewn timber = 1 Ton.

A perch of stone is $16\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet wide, and 1 ft. high = $24\frac{3}{4}$ solid feet.

LIQUID MEASURE.

This measure is used for all liquids.

TABLE.

4 gills (*gi.*)	=	1 pint (*pt.*).
2 pints	=	1 quart (*qt.*).
4 quarts	=	1 gallon (*gal.*).

In all liquids, except ale, beer, and milk, the gallon is 231 cubic inches.

In ale, beer, and milk, it is 282 cubic inches.

Rem.—In the former $31\frac{1}{2}$ gallons is called a barrel, 63 gallons a hogshead, 42 gallons a tierce, 84 gallons a puncheon, and 126 gallons a pipe, and 2 pipes a tun. In the latter, 36 gallons = a barrel, and 54 gallons = a hogshead; these, however, are not measures, but only vessels.

DRY MEASURE

is used for grain, fruits, vegetables, coal, salt, etc.

TABLE.

2 pints	=	1 quart.
8 quarts	=	1 peck (*pk.*).
4 pecks	=	1 bushel (*bu.*)
	=	2150.42 cubic inches.

The wine gallon of United States	=	231	cu. in.
The beer gallon of United States	=	282	cu. in.
The dry gallon of United States	=	268.8	cu. in.
Imperial gal. of Great Britain for dry and liquid measures	=	277.274	cu. in.
Dry bushel of United States	=	2150.42	cu. in.
Imperial bushel of Great Britain	=	2218.192	cu. in.

TIME TABLE.

60 seconds (*sec.*)	=	1 minute (*m.*).
60 minutes	=	1 hour (*hr.*).
24 hours	=	1 day (*da.*).
7 days	=	1 week (*wk.*).
30 days	=	1 month (*mo.*).
365 days	=	1 common year.
366 days	=	1 leap year.

ANGULAR OR CIRCULAR MEASURE

is applied to angles and circumferences, reckoning latitudes and longitudes, etc.

TABLE.

60 seconds (″)	= 1 minute (′).
60 minutes	= 1 degree (°).
30 degrees	= 1 sign (*S.*).
12 signs or 360 degrees	= 1 circumference.

Apparently the sun makes an entire revolution of the earth in 24 hours,* and consequently travels 15° in 1 hour; therefore,

1 hour of time	= 15° longitude.
1 minute of time	= 15′ longitude.
1 second of time	= 15″ longitude.

MISCELLANEOUS TABLE.

12 units	= 1 dozen.
12 dozen	= 1 gross.
12 gross	= 1 great gross.
20 units	= 1 score.
24 sheets of paper	= 1 quire.
20 quires	= 1 ream.
196 lbs.	= 1 barrel of flour.
200 lbs.	= 1 barrel of pork.

When a sheet of paper is folded into two leaves, or 4 pages, and a book made in this way, it is called a folio.

4 leaves is called a quarto.
8 leaves is called an octavo.
12 leaves is called a duodecimo.

* *Really,* the revolution is that of the earth on its own axis.

PRACTICAL QUESTIONS.

1. Reduce £3 9s. 11d. 3 far. to farthings.
2. Reduce £12 15s. 8d. to pence.
3. Reduce £7 0s. 2d. to pence.
4. Reduce 2354 farthings to the higher denominations.
5. Reduce 543 pence to the higher denominations.
6. Reduce 731 shillings to the higher denominations.
7. Reduce 3 T. 6 cwt. 2 qr. 12 lb. 6 oz. and 9 dr. to drams.
8. Reduce 672432 drams to the higher denominations.
9. Reduce 5 lb. 8 oz. 9 pwt. 15 gr. to grains.
10. Reduce 64324 grains Troy to the higher denominations.
11. Reduce 2 lb. 6 ℥ 4 ʒ 2 ℈ 10 gr. to grains.
12. Reduce 6742 gr., Apothecaries weight, to the higher denominations.
13. Reduce 3 lea. 2 mi. 5 fur. 24 rods 2 yd. 1 ft. 6 in. to inches.
14. Reduce 802456 inches to the higher denominations.
15. Reduce 4 yards 3 qrs. 2 na. and 2 inches to inches.
16. Reduce 5 ells Flemish to inches.
17. Reduce 4 ells English to inches.
18. Reduce 3 ells French to inches.
19. Reduce 4 sq. rods 8 sq. yd. 105 sq. ft. and 112 sq. in. to square inches.
20. Reduce 3 cu. yd. 12 cu. ft. and 1236 cu. in. to cubic inches.
21. A pile of wood is 16 ft. long, 4 ft. high, and the length of the wood is 4 feet. How many cords of wood?

$$\frac{16 \times 4 \times 4}{8 \times 4 \times 4}$$

22. Reduce 25 gal. 3 qt. 1 pt. and 3 gills to gills.

23. Reduce 9 bu. 3 pk. 4 qt. 1 pt. to pints.

24. How many years, months, and days, from April 15th, 1842, to June 20th, 1850?

yr.	mo.	da.
1850	6	20
1842	4	15
8	2	5

25. How many years, months, and days, from October 25th, 1845, to August 18th, 1850?

yr.	mo.	da.
1850	8	18
1845	10	25
4	9	23

REM.—In this question, instead of using the eighth and tenth months, some authors prefer calling them 7 months and 9 months; but if there is an inaccuracy in the months, there is also in the years; hence if we read the above thus: The one thousand eight hundred and fiftieth year, the 8th month and 25th day, there is no inaccuracy.

In computations of time we always take 30 days as a month.

26. The difference in time of two places is 2 hr. 2 min. and 2 sec.; what is the difference in longitude?

hr.	min.	sec.
2	2	2
		15
30°	30′	30″

Ans. Thirty degrees, 30 minutes, and 30 seconds.

27. If the difference of longitude of two places is 15°, the difference of time will be one hour; the eastern place will have the latest time. If the difference in longitude is 16° 24′ 30″, what is the difference in time?

15) 16° 24′ 30″ (1 hr. 5 min. 38 sec.
15
1
60
84 (5
75
9
60
570 (38
45
120
120

10 cts. = $\$\frac{1}{10}$.
$12\frac{1}{2}$ cts. = $\$\frac{1}{8}$.
$16\frac{2}{3}$ cts. = $\$\frac{1}{6}$.
20 cts. = $\$\frac{1}{5}$.
25 cts. = $\$\frac{1}{4}$.
$33\frac{1}{3}$ cts. = $\$\frac{1}{3}$.
$37\frac{1}{2}$ cts. = $\$\frac{3}{8}$.
50 cts. = $\$\frac{1}{2}$.
$62\frac{1}{2}$ cts. = $\$\frac{5}{8}$.
$66\frac{2}{3}$ cts. = $\$\frac{2}{3}$.
75 cts. = $\$\frac{3}{4}$.
$87\frac{1}{2}$ cts. = $\$\frac{7}{8}$.

28. Multiply 576 by 100 = 57600.

29. Multiply 576 by 25 = $\frac{1}{4}$ = 14400. Take $\frac{1}{4}$ of the above.

30. Divide 576 by 100 = 5.76.

31. Divide 576 by 25 = 23.04. Multiply by 4.

32. Multiply 576 by 50 = $\frac{1}{2}$ (57600) = 28800.

33. Divide 576 by 50 = 5.76×2 = 11.52.

34. Multiply 576 by $.12\frac{1}{2}$ = $\frac{1}{8}$ = \$72.

35. Multiply 576 by $.16\frac{2}{3}$ = $\frac{1}{6}$ = \$96.

36. Multiply 576 by $.33\frac{1}{3}$ = $\frac{1}{3}$ = \$192.

37. Multiply 576 by $.62\frac{1}{2}$ = $576 \times 5 \div 8$ = 360.

38. Multiply 576 by $.87\frac{1}{2}$ = $576 \times 7 \div 8$ = 504.

39. What cost 342 yds. muslin at $12\frac{1}{2}$ cts. per yard?

8) 342
$\$42\frac{6}{8} = \$42\frac{3}{4}$.

40. What cost 342 yds. linen at $37\frac{1}{2}$ cts. per yard? Multiply by 3 = $\$128\frac{1}{4}$.

41. What cost 342 yds. linen at $62\frac{1}{2}$ cts. per yard? Multiply by 5 = $\$213\frac{3}{4}$.

42. What cost 342 yds. linen at $87\frac{1}{2}$ cts. per yard? Multiply by 7 = $\$299\frac{1}{4}$.

43. What cost 548 yds. muslin at $16\frac{2}{3}$ cts. per yard?

$$6\,)\,548 \quad \$91\frac{1}{3}$$

44. What cost 345 yds. muslin at 20 cts. per yard?

$$5\,)\,345 \quad \$69$$

45. What cost 496 yds. linen at $33\frac{1}{3}$ cts. per yard?

$$3\,)\,469 \quad \$156\frac{1}{3}$$

46. What cost 496 yds. linen at $66\frac{2}{3}$ cts. per yard? Multiply by 2 = $\$312\frac{2}{3}$.

47. What cost 500 yds. linen at 25 cts. per yard? $500 \div 4 = \$125$.

48. What cost 500 yds. linen at 75 cts. per yard? Multiply by 3 = $375.

49. What cost 500 yds. linen at 50 cts. per yard? $500 \div 2 = \$250$.

50. Bought 648 yards muslin at $12\frac{1}{2}$ cts. a yard, and sold it at $16\frac{2}{3}$ cts. per yard. What was the profit?

$\frac{1}{6}$ of 648 = $108
$\frac{1}{8}$ of 648 = 81
$27, Profit.

51. Bought 500 yds. cloth at 20 cts., and sold it at 25 cts.; what profit?

$\frac{1}{4}$ of 500 = $125
$\frac{1}{5}$ of 500 = 100
$25, Profit.

52. Bought 480 yds. cloth at $66\frac{2}{3}$ cts., and sold it at $87\frac{1}{2}$ cts.; what was the profit?

$$\tfrac{1}{8} \text{ of } 480 = \$60; \quad \tfrac{7}{8} = \$420$$
$$\tfrac{1}{3} \text{ of } 480 = 160; \quad \tfrac{2}{3} = \underline{320}$$
$$\$100, \text{ Profit.}$$

53. Bought 480 yds. at $37\frac{1}{2}$ cts., and sold it for 50 cts. per yard; what was the profit?

$$\tfrac{1}{2} \text{ of } 480 = \$240$$
$$\tfrac{1}{8} \text{ of } 480 = 60; \quad \tfrac{3}{8} = \underline{180}$$
$$\$60, \text{ Profit.}$$

54. Bought 600 yds. cloth at \$1 per yard, and sold it for \$1.25 per yard; what was the profit?

$$600 \times 1\tfrac{1}{4} = \$750$$
$$600 \times 1 = \underline{600}$$
$$\$150, \text{ Profit.}$$

55. How many yards of cloth, at $12\frac{1}{2}$ cts. per yard, can be bought for \$240? 8 yds. can be bought for every dollar.

$$240 \times 8 = 1920 \text{ yds.}$$

56. How many yards for $16\frac{2}{3}$ cts.? 20 cts.? 25 cts.? $37\frac{1}{2}$ cts.? 50 cts.? $62\frac{1}{2}$ cts.? 75 cts.? $87\frac{1}{2}$ cts.?

$$37\tfrac{1}{2} = \tfrac{3}{8}; \quad \overset{80}{\cancel{240}} \times \tfrac{8}{\cancel{3}} = 640 \text{ yds.};$$
$$\text{for } 62\tfrac{1}{2} \text{ cts.} = 240 \times \tfrac{8}{5}.$$

57. Sold 500 barrels flour at $\$6.62\frac{1}{2}$ per barrel, and invested the proceeds in different kinds of dry goods, averaging $87\frac{1}{2}$ cts. per yard. What were the proceeds of the flour, and how many yards of goods did I get?

RATIO.

Two fractions can be formed of any two integral numbers, the one a proper fraction, the other an improper fraction. These fractions are also called ***Ratios.***

As the product of any number multiplied by the proper fraction is less than the number itself, the fraction is called a ***Diminishing Ratio;*** and as the product of any number multiplied by the improper fraction is greater than the number itself, it is called an ***Increasing Ratio.***

PROBLEMS.

1. If 5 lbs. sugar cost 50 cents, what will 9 lbs. cost ?

It is evident that 9 lbs. will cost more than 5, and just as much more as is indicated by the increasing ratio formed by the two like terms, 5 lbs. and 9 lbs.

If 5 lbs. cost 50 cts., 9 lbs. will cost $\overset{10}{\not{5}\not{0}}$ cts. $\times \frac{9}{\not{5}} = 90$ cts.; this may be further demonstrated thus,

5 lbs. = 50 cts.
1 lb. = 10 cts.
9 lbs. = 90 cts.

In a problem of ratios, the one ratio is given, and one of the terms of the other ratio, to get the second term; thus, in the above:

Given,	5 lbs. sugar	and	50 cents.
Required,	9 lbs. sugar	and	?

The ratio of the money will be the same as of the sugar. As the required sugar is more than the given, the ratio must be increasing; that is, $\frac{9}{5}$. $\therefore$ $50 \times \frac{9}{5} = 90$, the ratio of the required money to the gain, $\frac{90}{50} = \frac{9}{5}$, the same as of the sugar.

EXAMPLES.

1. If 5 bushels of wheat cost $6.25, what will 8 bushels cost?

Given	5 bu.	and	$6.25.
Required,	8 bu.	and	

$$\$6.25 \times \frac{8}{5} = \$10.00.$$

$$\frac{8}{5} = 25)\frac{1000}{625} = 5)\frac{40}{25} = \frac{8}{5}.$$

The ratios of the wheat and of the money is the same.

REM.—Ratios can only be formed by two like terms.

2. If 5 bushels of oats cost $1.50, what will 21 bushels cost?

$$\frac{21}{5} = \frac{630}{150} = \frac{21}{5};$$ the ratio is the same.

Given	5 bu.	and	$1.50.
Required,	21 bu.	and	

$$\$1.50 \times \frac{21}{5} = \$6.30.$$

REM.—Write the given terms in a line and the like term of the required immediately under that of the given. One term of the required is wanting, and the given like term may be called the term of demand, and should be placed first and multiplied by the ratio, having for its numerator the required term of the ratio, and for its denominator the given term.

As a general thing, an increase in the required term of the ratio will take more of the unknown to accomplish it; an increased amount of goods will cost a greater sum of money; an enlarged piece of work, an additional sum of money; and the greater the work, the longer time to perform it, etc. In examples of this kind, the ratios are direct, and the required term of the ratio holds the place of the numerator and the given term that of the denominator, and the product of the ratio and the odd given term is the term required.

3. If a man travel 40 miles in 8 hours, how many miles will he travel at that rate in 18 hours?

Given	40 miles	and	8 hours.
Required,	? miles	and	18 hours.

$$\overset{5}{\not{4}\not{0}} \text{ miles} \times \frac{18}{\not{8}} = 90 \text{ miles.}$$

4. If 15 bushels of wheat yield 3 barrels of flour, how many bushels will yield 10 barrels of flour?

Given	15 bu.	and	3 barrels of flour.
Required,	?	and	10 barrels of flour.

SOLUTION.

$$\overset{5}{\not{1}\not{5}} \times \frac{10}{\not{3}} = 50 \text{ bushels.}$$

5. If a man travel 30 miles in 2 days, how long will it take him to travel 240 miles?

Given	30 miles	and	2 days.
Required,	240 miles	and	?

6. If a staff 4 feet long cast a shadow 3 feet, what is the height of a steeple which casts a shadow 90 feet?

	STAFF.	SHADOW.
Given	4 ft.	3 ft.
Required,	?	90 ft.

7. If the interest of $100 for one year is $5, what would be the interest of $500 for the same time?

	PRINCIPAL.		INTEREST.
Given	$100	and	$5.
Required,	$500	and	?

8. If $\frac{2}{3}$ of a barrel of flour cost $4, what will $4\frac{2}{3}$ barrels cost?

Given	$\frac{2}{3}$ barrel	and	$4.
Required	$4\frac{2}{3}$ barrel	and	?

$$4 \times \frac{4\frac{2}{3}}{\frac{2}{3}}. \qquad \$4 \times \frac{\overset{7}{\cancel{14}}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{2}} = \$28.$$

REM. $4\frac{2}{3}$ is a multiplier, and $\frac{2}{3}$ is a divisor; the $\frac{2}{3}$ must be inverted.

9. If $4\frac{1}{2}$ bushels of wheat cost $5.40, what will $8\frac{3}{4}$ bu. cost? $9\frac{1}{2}$ bu.? $23\frac{3}{4}$? $31\frac{2}{3}$? $47\frac{1}{2}$? $39\frac{1}{6}$? $58\frac{3}{4}$? $97\frac{1}{2}$? $106\frac{3}{4}$?

10. If 8 bushels of wheat cost $10, what will be the cost of $9\frac{1}{2}$ bu.? $10\frac{1}{4}$ bu.? $15\frac{3}{5}$ bu.? $37\frac{1}{5}$ bu.? $95\frac{3}{5}$ bu.? $125\frac{3}{5}$ bu.? $150\frac{4}{5}$ bu.? $279\frac{2}{5}$ bu.?

11. If $5\frac{3}{4}$ acres of land cost $230, what is the cost of $6\frac{1}{2}$ acres? $7\frac{3}{4}$ acres? $12\frac{1}{2}$ acres? $13\frac{1}{8}$ acres? $17\frac{2}{8}$? $18\frac{3}{8}$? $19\frac{4}{8}$? $20\frac{5}{8}$? $37\frac{6}{8}$? $49\frac{7}{8}$?

12. If $2\frac{1}{5}$ acres of land cost $110, what will $\frac{3}{4}$ of an acre cost? $\frac{1}{2}$ acre? $\frac{1}{4}$ acre? $\frac{1}{5}$ acre? $\frac{2}{5}$ acre? $\frac{3}{5}$ acre? $\frac{4}{5}$ acre? $1\frac{1}{5}$ acres? $1\frac{2}{5}$ acres?

13. If $\frac{5}{12}$ of a yard of cloth cost $\frac{9}{10}$ of a dollar, what will $\frac{2}{7}$ of a yard cost?

Given	$\frac{5}{12}$ yd.	and	$\$\frac{9}{10}$.
Required,	$\frac{2}{7}$ yd.	and	?

$$\frac{9}{10} \times \frac{2}{7} \times \frac{12}{5}.$$

14. If $\frac{1}{2}$ yard of cloth cost \$2, what will 3 ells F. cost?

$$2 \times \tfrac{9}{2} =$$

15. If $\frac{3}{4}$ yard of cloth cost \$2.25, what will 5 ells English cost?

$$2.25 \times \tfrac{25}{3}.$$

What will 5 ells French cost?

In the preceding examples, the ratios were all direct; as in those cases any increase in the required term of the ratio demanded a similar increase of the unknown; but there are cases which require the ratio to be inverted, such as, the more men employed, the less time will be required to perform a piece of work; the more hours employed in the day, the less days; the wider the material, the less yards it will take to make a garment, etc.

These cases of inverse ratio are readily detected by asking this question: "Will an increase of the required term of the ratio demand an increase in the unknown term?" If it does, the ratio is direct; but if an increase in the required term of the ratio demand a diminution of the unknown term, the ratio must be inverted; thus,

PROBLEM.

If 4 men can do a piece of work in 10 days, how long will it take 8 men to do the same work?

Given	4 men in 10 days.
Required,	8 men in ?

$$\overset{5}{\cancel{10}}\text{ days} \times \frac{\cancel{4}}{\underset{2}{\cancel{8}}} = 5\text{ days.}$$

COR.—It is evident that 8 men will do it in less time; that is, in one-half the time that it will take 4, which ratio is expressed by the diminishing ratio of 4 and 8, that is, $\frac{4}{8} = \frac{1}{2}$, in which the given term is the numerator of the ratio and the required term the denominator.

EXAMPLES.

1. If 5 men can dig a ditch in 20 days, how many men will dig it in 25 days?

Given	5 men	and	20 days.
Required,	?	and	25 days.

$$\not{5} \text{ men} \times \frac{\overset{4}{\not{20}}}{\underset{\not{5}}{\not{25}}} = 4 \text{ men.}$$

REM.—An increase in the 25 will require less men.

2. If 6 horses eat a certain quantity of hay in 30 weeks, how many horses will consume the same quantity of hay in 9 weeks?

Given	6 horses	and	30 weeks.
Required,	?	and	9 weeks.

$$\overset{2}{\not{6}} \text{ horses} \times \frac{\overset{10}{\not{30}}}{\underset{\not{3}}{\not{9}}} = 20 \text{ horses.}$$

3. If a man perform a journey in 12 days, when the days are 9 hours long, how many days of 12 hours will it take him?

Given	12 days	and	9 hours.
Required,	?	and	12 hours.

4. How many yards of lining $\frac{3}{4}$ yd. wide will it take to line 3 yards of cloth $\frac{6}{4}$ yd. wide?

Given	$\frac{6}{4}$ yard wide	and	3 yards long.
Required,	$\frac{3}{4}$ yard wide	and	?

$$3 \times \frac{6}{3} = 6 \text{ yds.}$$

COMPOUND RATIO.

When there are two or more ratios, it is termed ***Compound Ratio;*** thus,

If 3 men in 12 days build 40 rods of wall, how many rods will 9 men build in 24 days?

Given	3 men,	12 days,	40 rods.
	9 men,	24 days,	?

$$40 \times \frac{\overset{3}{9}}{3} \times \frac{\overset{6}{24}}{\underset{4}{12}} = 240 \text{ rods.}$$

REM.—Each ratio is direct.

If 12 men dig a ditch 20 rods long in 18 days by working 8 hours a day, how many men will dig a ditch 40 rods long in 24 days, working 6 hours a day?

Given	12 men,	20 rods,	18 days,	8 hours.
	?	40 rods,	24 days,	6 hours.

$$12 \text{ men} \times \frac{\overset{2}{40}}{20} \times \frac{\overset{6}{18}}{\underset{3}{24}} \times \frac{8}{6} = 24 \text{ men.}$$

EXEMPLIFICATION.—The longer the trench, the more men it will take, and the ratio is direct; but the greater the number of days and the more hours of each day, the less men would be required; hence these two ratios are inverse.

COR.—Each ratio must be dealt with as in the preceding article.

EXAMPLES.

1. If 3 men in 8 days of 9 hours each build a wall 20 ft. long, 2 ft. thick, and 4 ft. high, in how many days of 8 hours each will 12 men build a wall 100 ft. long, 3 ft. wide, and 6 ft. high?

Given	3 men,	8 days,	9 hrs.,	20 ft. l.,	2 ft. w.,	4 ft. h.
	12 men,	?	8 hrs.,	100 ft. l.,	3 ft. w.,	6 ft. h.

$$\cancel{8} \text{ days} \times \frac{3}{\cancel{12}} \times \frac{9}{\cancel{8}} \times \frac{\overset{5}{\cancel{100}}}{\cancel{20}} \times \frac{3}{2} \times \frac{\overset{3}{\cancel{6}}}{4} = \frac{9 \times 5 \times 3 \times 3}{4 \times 4}$$

$$= \frac{405}{16} = 25\tfrac{5}{16} \text{ days.}$$

REM.—The men and hours are inverse, the other ratios direct.

2. If 6 men mow 12 acres of grass in 2 days of 10 hrs. each, how many hours a day must 8 men work to mow 40 acres in 4 days? *Ans.* $12\frac{1}{2}$ hours a day.

3. If 6 horses eat 36 bushels of oats in 18 days, how many bushels will be sufficient for 12 horses 24 days? *Ans.* 96 bushels.

4. If \$100 in 12 months gain \$6, how long will it take \$500 to gain \$15?

Given	\$100 prin.,	12 mo.,	\$6 int.
Required,	\$500 prin.,	?	\$15 int.

$$\overset{2}{\cancel{12}} \text{ mo.} \times \frac{1}{\cancel{5}} \times \frac{\overset{3}{\cancel{15}}}{\cancel{6}} = 6 \text{ months.}$$

5. If 6 men manufacture 300 pairs of shoes in 30 days, how many men will make 900 pairs of shoes in 60 days? *Ans.* 9 men.

PERCENTAGE.

Per Cent. means per hundred, and is generally expressed fractionally; thus, 5 per cent., 6 per cent., marked 5% and 6%, is expressed $\frac{5}{100}$, $\frac{6}{100}$, etc., or .05, .06; thus, $\frac{5}{100}$ of 100 $= \cancel{100} \times \frac{5}{\cancel{100}} = 5$, and $\frac{6}{100}$ of 100 is 6.

EXAMPLES.

1. What is 5% of 200? $\overset{2}{\cancel{200}} \times \frac{5}{\cancel{100}} = 10$, *Ans.*
   What is 5% of 300? *Ans.* 15.
   What is 5% of 400? *Ans.* 20.
2. What is 5% of 245? $2.45 \times \frac{5}{\cancel{100}} = 12.25$, *Ans.*

The 100 is canceled in the 245 by pointing off two places of decimals.

3. What is 6% of 300? $\overset{3}{\cancel{300}} \times \frac{6}{\cancel{100}} = 18$, *Ans.*
   What is 6% of 400? *Ans.* 24.
   What is 6% of 500? *Ans.* 30.
4. What is 6% of 368? $3.68 \times \frac{6}{\cancel{100}} = 22.08$.

COMMISSION, OR BROKERAGE.

The business of a commission merchant or broker is to make purchases and sales, on which he receives a percentage.

PROBLEM I.

A purchase of $100 worth of goods, at 1% commission, will cost $101; that is, $\frac{101}{100}$ of the amount of the purchase.

PROBLEM II.

In a sale of goods for \$100, at 1% commission, the owner will realize \$99; that is, $\frac{99}{100}$ of the amount of sale.

PROBLEM III.

When stocks, bonds, drafts, or currency, are purchased at a discount of 2%, the cost of \$100 worth will be \$98; that is, $\frac{98}{100}$ of the face of the bond, etc.; but when they are purchased at a premium of 2%, the cost of \$100 worth is \$102; that is, $\frac{102}{100}$ of the face.

PROBLEM IV.

In the exchange of currency, when there is a premium on the funds on hand, as that of English money to be exchanged into United States, the premium in favor of England is about 9%; it is computed as follows:

$$\text{Eng. £} \times \frac{40}{9} \times \frac{109}{100} = \text{\$ U. S.},$$

and

$$\text{\$ U. S.} \times \frac{9}{40} \times \frac{100}{109} = \text{£ Eng.}$$

that is, England gets \$109 for every \$100 of her money, and the United States must pay \$109 of her money for \$100 English money.

REM.—Ordinarily, an English £ equals $\frac{40}{9}$ U. S. dollars.

EXAMPLES.

1. A broker sold goods to the amount of \$6000, at 2% commission, and invested the balance of the proceeds, after deducting 2% on the amount of purchase; what was the owner's portion of the sale, and what amount of goods were purchased?

$\$6000 \times \frac{98}{100} = \$5880 =$ owner's portion of the sale.

$\$5880 \times \frac{100}{102} = \$5764.70\frac{10}{17} =$ amt. of goods purchased.

$$\therefore\ 6000 \times \frac{\overset{49}{\cancel{98}}}{\cancel{100}} \times \frac{\cancel{100}}{\underset{51}{\cancel{102}}} = 6000 \times \frac{49}{51}$$

$$= \$5764\tfrac{12}{17}, \text{ amt. of goods purchased.}$$

Rem.—Observe the difference in the ratios of the sale and purchase.

2. What is the cost of a bond for \$5000 at 5% discount, stocks whose face indicate \$2000 at 4% premium, \$1000 currency at 2% discount, and \$3000 gold at 8% premium?

$$\begin{array}{rcr} \$50\cancel{0}\cancel{0} \times \frac{95}{\cancel{100}} & = & \$4750 \\ 20\cancel{0}\cancel{0} \times \frac{104}{\cancel{100}} & = & 2080 \\ 10\cancel{0}\cancel{0} \times \frac{98}{\cancel{100}} & = & 980 \\ 30\cancel{0}\cancel{0} \times \frac{108}{\cancel{100}} & = & 3240 \\ \hline & & \$11050 \end{array}$$

Cor.—When brokerage is paid in the exchange of money, the percentage is on the amt. purchased, which, if the rate is 2%, is $\frac{100}{102}$ of the funds on hand.

3. If a broker makes sales to the amount of \$500, on which he receives 3%, what is his commission?

$$\$\overset{5}{\cancel{5}}\cancel{0}\cancel{0} \times \frac{3}{\cancel{100}} = \$15.$$

2. What is the cost of a draft for \$1000 at a premium of $\frac{1}{2}$%?

$$\$\overset{5}{\cancel{1}}\cancel{0}\cancel{0}\cancel{0} \times \frac{100\frac{1}{2}}{200} = \frac{201}{\cancel{2}\cancel{0}\cancel{0}} = \$1005.$$

3. What is the face of a draft at $\frac{1}{2}$% premium, costing \$1005?

$$\$\overset{5}{\cancel{1}}\cancel{0}\cancel{0}\cancel{5} \times \frac{100}{100\frac{1}{2}} = \frac{200}{\cancel{2}\cancel{0}\cancel{1}} = \$1000.$$

4. A broker makes sales for $4325, at 2%; what is the brokerage, and what does the owner realize?

$$43.25 \times \tfrac{2}{100} = \$86.50, \text{ commission.}$$

$$43.25 \times \tfrac{98}{100} = \frac{\$4238.50}{\$4325.00}, \text{ owner realized.}$$

5. A merchant sells to a broker $3275 uncurrent funds at 5% discount; what does he realize?

$$\begin{array}{c} 163.75 \times 19 \\ \$\cancel{3275} \times \tfrac{\cancel{95}}{\cancel{100}} = \$3136.25. \\ 20 \end{array}$$

6. An architect charges $1\frac{1}{2}\%$ for plans and specifications, and $2\frac{1}{2}\%$ for superintending a building, the cost of which is $10000; what is the architect's fees? *Ans.* $800.

7. A broker has 2% commission, and 3% for guaranteeing payment; what does he receive on sales amounting to $42325? *Ans.* $2116.25.

8. I sent my broker $4000 to purchase goods; what amount of goods did he purchase after deducting commissions at 2% on the amount of goods? $\$4000 \times \frac{100}{102}$.

For every $102 he gets $100 worth of goods.

9. Bought a draft on New York, the face of it $500 premium, $\frac{1}{4}\%$; what is the whole cost and the premium?

$$\$500 \times \frac{100\frac{1}{4}}{100} = \frac{401}{400} = \qquad \$501.25$$

$$\begin{array}{rr} & 500.00 \\ \text{Premium,} & \$1.25 \end{array}$$

10. Sold goods to the amount of $4444, and invested the proceeds, after retaining my commissions, which were 2% on the sales, and 1% on the investment; what was the amt. of investment?

$$\begin{array}{c} 44 \\ \$\cancel{4444} \times \tfrac{98}{\cancel{100}} \times \tfrac{\cancel{100}}{\cancel{101}} = \$4312. \end{array}$$

INTEREST.

Interest is an allowance for the use of money. It is reckoned by percentage; thus, 5%, 6%, etc., meaning for a year, when not otherwise expressed; for any other time it is as the ratio of the time; thus, the interest of \$100 at 6% is \$6 for a year, for two years \$12, and for six months \$3.

PROBLEMS.

1. Find the interest of \$150, at 6%, for 1 year.

$$\$150 \times \frac{6}{100} = \$9.$$

For 8 months.

$$\$1.50 \times \frac{\cancel{6}}{1\cancel{00}} \times \frac{\overset{4}{\cancel{8}}}{\underset{2}{\cancel{12}}} = 1.50 \times \frac{4}{1\cancel{00}} = \$6.00.$$

For 6 months.

$$\$1.50 \times \frac{\cancel{6}}{1\cancel{00}} \times \frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{12}}} = \$4.50.$$

For 14 months.

$$\$1.50 \times \frac{\cancel{6}}{1\cancel{00}} \times \frac{\overset{7}{\cancel{14}}}{\underset{2}{\cancel{12}}} = 1.50 \times \frac{7}{1\cancel{00}} = \$10.50.$$

Cor.—At 6%, the rate per cent. for any number of months is $\frac{1}{2}$ the number of months; thus, for 8 months it is 4%, for 6 months it is 3%, and for 14 months 7%.

2. Find the interest of \$150, at 6%, for 129 days?

$$\$150 \times \frac{\cancel{6}}{100} \times \frac{129}{\underset{60}{\cancel{360}}} = \$150 \times \frac{129}{6000} = \$3.225.$$

Cor.—The interest of a sum of money for any number of days is equal to the product of the sum of money and the number of days divided by 6000; or, if the number of dollars be multiplied by the number of days and this product divided by 6, the quotient is the interest in mills; point off three decimals and it is reduced to dollars, cents, and mills.

If the rate of interest is 7%, add $\frac{1}{6}$; if 8%, add $\frac{1}{3}$; if 9%, add $\frac{1}{2}$; if 5%, deduct $\frac{1}{6}$; if 4%, deduct $\frac{1}{3}$; if 3%, take $\frac{1}{2}$.

The rate for 200 months is 100%; that is, the interest is equal to the principal.

200 months of $100 is	$100.
20 months of $100 is	$10.
2 months of $100 is	$1.
30 days, or 1 month	$0.50.
3 days of $100 is	$.05.
1 day of $100 is	$.01$\frac{2}{3}$.
2 days of $100 is	$.03$\frac{1}{3}$.

EXAMPLES.

1. Find the interest of $625, at 6%, for 8 months.

Rate for 8 mo. is $\frac{4}{100}$.

$$\$6.25 \times \frac{4}{\not{1}\not{0}\not{0}} = \$25.00.$$

For 8 mo. and 20 days, 8$\frac{2}{3}$ mo., rate 4$\frac{1}{3}$%.

$$\$625 \times \frac{4\frac{1}{3}}{100} = \overset{\$6.25}{\$\not{6}\not{2}\not{5}} \times \frac{13}{\underset{3}{\not{3}\not{0}\not{0}}} = \$27.08\tfrac{1}{3}.$$

If months and days are computed separately.

$$\$625 \times \frac{4}{100} = \$25.00$$
$$\$625 \times \frac{1}{300} = \underline{\quad 2.08\tfrac{1}{2}}$$
$$\$27.08\tfrac{1}{3}$$

2. What is the interest of \$650, at 6%, for 1 year 6 months and 24 days?

Reduce to days 1 year	=	360
Reduce to days ½ year	=	180
		24
		564

$$\$650 \times \frac{\overset{94}{\cancel{564}}}{\underset{1000}{\cancel{6000}}} = \$61.10.$$

$\frac{1}{2}$ of $18\frac{24}{30} = 9\frac{12}{30}\% = 9.4\%$.

$$\$650 \times \frac{94}{1000} = \$61.10.$$

At 7%	=	\$61.10	At 8%	=	\$61.10
Add $\frac{1}{6}$	=	$10.18\frac{1}{3}$	Add $\frac{1}{3}$	=	$20.36\frac{2}{3}$
		$\$71.28\frac{1}{3}$			$\$81.46\frac{2}{3}$

3. What is the interest of \$575, at 6%, for 2 years 4 months and 18 days?

2 years	=	720 days.
4 months	=	120 days.
		18 days.
		858 days.

$$\$575 \times \frac{\overset{143}{\cancel{858}}}{\underset{1000}{\cancel{6000}}} = 575 \times \frac{143}{1000} = \$82.225.$$

4. A note due April 1st, 1872, for \$2000, bearing interest at 6%. On the back of the note were the following credits: May 1st, 1873, \$230; June 1st, 1874, \$223.50; July 1st, 1875, \$217. What was due July 1st, 1877, when the note was taken up? *Ans.* \$1904.

First credit	\$230.
Second credit	\$223.50.
Third credit	\$217.

5. A note dated 1st April, 1870, for $200, bearing interest from date at 6%, has the following endorsements on the back of it.

May 1st, 1871, paid $12.
May 1st, 1872, paid $12.
May 1st, 1873, paid $12.

What was due on the note April 1st, 1875, when it was paid?

Int. of $200 for 1 year is	$12	
	5	years.
	$60	
Add Principal,	200	
	$260	
Deduct sum of payments,	36	
April 1st, 1875, balance,	$224	paid.

As the sum of the payments never equaled the interest, no computation need be made until the end, when the sum of the payments must be deducted.

PROB. 1.—In the case of Partial Payments, when the payment is greater than the interest until the time of the payment, the interest is computed until the date of the payment and added to the principal, and from this sum the payment is deducted, and the balance is regarded as the principal of the note.

PROB. 2.—When the payment is less than the interest, no computation is made; but whenever the sum of the payments is greater than the interest until that time, then the interest is to be computed to the date and added to the principal, and from this sum the sum of all the payments is deducted, and the balance regarded as the principal of the note.

These computations are to be repeated until the note is paid.

BANK DISCOUNT.

Bank Discount is reckoned the same as interest on the face of the note and for the time the note is given, plus three days, which are called days of grace, and the note need not be paid until the last day of grace, three days after the time specified in the note.

It is called Discount, because the borrower does not receive the sum specified in the note, but the difference of this sum and the interest.

Notes in Banks are usually given for a short time, viz., for 30 days, 60 days, or 90 days; and the interest is computed for 33 days, 63 days, or 93 days.

1. The bank discount on a note for $100 at 30 days.

$$\$100 \times \frac{6}{100} \times \frac{33}{\underset{60}{360}} = \frac{11}{20} = \$.55.$$

Borrower gets $100 — $.55 = $99.45.

2. Borrower gets $100—$1.05 = $98.95. $100 for 60 da.

$$\$100 \times \frac{6}{100} \times \frac{63}{\underset{60}{360}} = \frac{21}{20} = \$1.05.$$

3. Borrower gets $100—1.55=$98.45. $100 for 90 days.

$$\$100 \times \frac{6}{100} \times \frac{93}{\underset{60}{360}} = \frac{31}{20} = \$1.55.$$

4. Borrower gets $324—$5.02 = $318.98. $324 for 90 days.

$$\overset{164}{\$324} \times \frac{\overset{31}{93}}{\underset{1000}{6000}} = \$5.022.$$

TRUE DISCOUNT

is the abatement made on a note not yet due, or the difference between the face of the note and a sum of money which, placed at interest, will amount to the face of the note by the time the note is due; thus, the present value of a note for $100, due in one year without interest, when money is worth 6%, is such a sum as will amount to $100 in one year at 6% interest.

Cor.—As $100 now will be worth $106 at the end of the year, so the present worth of money due in one year is $\frac{100}{106}$ of the face of the note due in one year.

Cor.—The ratio to obtain the present worth from the face of the note, has for its numerator 100, and the denominator is 100 increased by the interest for the time and rate.

EXAMPLES.

1. What is the present worth of a note for $324, due in one year, rate of money 6%?

$$\$324 \times \tfrac{100}{106} = \$305.66+\text{, present worth.}$$

$18.34, discount.

2. Due in 2 years 1 month.

$$\text{Rate} = \frac{100}{112\frac{1}{2}} = \frac{200}{225} = \frac{8}{9}.$$

$$9 \text{ months} = \frac{100}{104\frac{1}{2}} = \frac{200}{209}$$

EXCHANGE.

DEF. 1.—***Exchange*** is the system by which payments are made at a distant place by means of Bills of Exchange or Drafts.

2. The person making or signing the Bill or Draft, is called the ***Maker*** or ***Drawer;*** the one to whom it is addressed is the ***Drawee,*** and the person to whom it is ordered to be paid is the ***Payee.***

3. The person in possession of the draft is the ***Holder,*** and if he endorse it, he becomes responsible for the payment, unless otherwise specified.

4. Exchange between the different cities of one's own country is ***Domestic,*** and that with a foreign country is ***Foreign Exchange.***

DOMESTIC EXCHANGE.

PROBLEM.

To find the cost of a draft on Philadelphia or New York when at a premium, and also when at a discount; thus, if the premium is 1%, the draft will cost $\frac{101}{100}$ of its face; if at a discount of 1%, it will cost $\frac{99}{100}$ of its face.

REM.—The face of the draft must be equal to the sum of money that we wish to remit.

EXAMPLES.

1. What is the cost of a draft on Philadelphia for \$500, at $\frac{1}{2}\%$ premium?

$$500 \times \frac{100\frac{1}{2}}{100} = 5\cancel{0}\cancel{0} \times \frac{201}{2\cancel{0}\cancel{0}} = \frac{1005}{2} = 502\tfrac{1}{2}.$$

2. I owe \$3000 in New York, and the premium on a New York draft is $\frac{1}{4}\%$. What must I pay for the draft?

$$\$\overset{15}{\cancel{3}\cancel{0}\cancel{0}\cancel{0}} \times \frac{100\frac{1}{4}}{100} = \frac{401}{4\cancel{0}\cancel{0}} = 15 \times \frac{401}{2} = \$3007.50.$$

3. What is the cost of a draft on New Orleans for \$500, at $\frac{1}{2}\%$ discount?

$$5\cancel{0}\cancel{0} \times \frac{99\frac{1}{2}}{\cancel{1}\cancel{0}\cancel{0}} = 497\tfrac{1}{2}.$$

REM.—The computation may be made as interest; thus, first example, \$500 at 1% is \$5, and $\frac{1}{2}$ of \$5 is \$$2\frac{1}{2}$ = premium; second example, \$3000 at 1% is \$30, at $\frac{1}{4}\%$ it is \$7.50 premium; in the third example, \$500 at $\frac{1}{2}\%$ is \$2.50, the discount.

5. What is the cost of a draft, at 60 days, on New York for \$500, premium $\frac{1}{2}\%$, interest off at 6%?

$$\begin{aligned} \text{Discount} &= \tfrac{63}{60}\% = \tfrac{21}{20}\%. \\ \text{Premium} &= \tfrac{1}{2}\% = \tfrac{10}{20}\%. \\ \text{Discount above premium} &= \tfrac{11}{20}\%. \end{aligned}$$

$$\$\cancel{5}\cancel{0}\cancel{0} \times \frac{11}{\underset{4}{\cancel{2000}}} = \$2.75, \text{ discount.}$$

$$\begin{array}{r} \$500 \\ 2.75 \\ \hline \$497.25 \end{array} \text{, cost of draft.}$$

FOREIGN EXCHANGE.

EXAMPLES.

1. What is the cost in New York of a draft on London for £500, exchange at $4.80 to the £?

$$\$500 \times 4.80 = \$2400.$$

2. What amount of debt in London can be paid with $4000 in New York, rate of exchange as above?

$$\frac{\$4000.0}{\$4.8} = \frac{10000}{12} = £833\ 6s.\ 8d.$$

3. What will a draft on London for £480 15s. 6d. cost in New York, rate as above?

$$\begin{array}{l} 15s.\quad 6d. \\ \underline{12} \\ \frac{186}{240} = \frac{31}{40}. \end{array}$$

$$£480\tfrac{31}{40} \times 4.80 = \$2307.72.$$

4. What will $500 in New York pay in Paris, exchange on London as above, and £1 = 25.2 Francs.

$$\overset{125}{\$\not{5}\not{0}\not{0}} \times \frac{\overset{\overset{21}{\not{6}\not{3}}}{\not{2}\not{5}.\not{2}}}{\underset{\underset{4}{\not{1}\not{2}}}{\not{4}.\not{8}}} = 125 \times 21 = 2625 \text{ Francs.}$$

5. What sum in New York will pay 2625 Francs in Paris?

6. What will $2540 in New York pay in Paris? as above.

7. What sum in New York will pay 52500 Francs in Paris, exchange at 25.2 Francs to the £ and £1 = $4.80?

REM.—As the rates of exchange are not constant, it is not proper to insert a table.

Averaging Accounts.

When sales are made at different times and on different terms, to find a mean time when the whole may be paid without loss to either party; thus,

A merchant sells goods as follows:

January 1st for $100 on 1 month.
January 18th for $200 on 1 month.
February 1st for $300 on 2 months.
February 12th for $250 on 3 months.

No payment has yet been made. In how many days should the whole be paid, in order that there be no loss to either party?

Begin when the first account is due, February 1st.

$$\begin{array}{rcrcl}
\$100 & \times & 0 & = & \\
200 & \times & 17 & = & 3400 \\
300 & \times & 59 & = & 17700 \\
\underline{250} & \times & 100 & = & \underline{25000} \\
850 & & & 850\ ) & 46100\ (\ 54 \text{ days.} \\
 & & & & \underline{4250} \\
 & & & & 3600 \\
 & & & & \underline{3400} \\
 & & & & 200 = \frac{200}{850} = \frac{4}{17}.
\end{array}$$

Due 54 days after February 1st, that is, 27th March.

BANK ACCOUNT.

In bank, accounts are closed at the end of the year, and the balance, which is always in favor of the depositor, is brought to the credit side of the account, as over-drawing is not permitted.

To this balance each deposit is added at the date on which it is made, and from the credit side is subtracted each sum drawn by check at its date.

Each balance, or sum, is multiplied by the number of days from its date until the next transaction, and lastly by the time between the last transaction and the end of the year.

The sum of all these products will be the number of days that one dollar is at interest.

DR. JOHN JOHNSON in acct. with MECHANICS' BANK. CR.

						DA.	PRO.
1877.			1876.				
Jan. 6.	To check.	$200	Dec. 31.	By bal. old acct.	$600	6	3600
" 15.	" "	100			400	4	1600
" 18.	" "	200			700	5	3500
" 29.	" "	350			600	3	1800
					400	7	2800
					1150	4	4600
					800	2	1600
						6)	19,500
					Interest,		3.25

The last sum or difference on the credit side is the Principal.

		$800
Interest	=	3.25
Total balance	=	$803.25

The account is only rendered for one month.

Alligation.

Alligation is the mixing of different qualities of grain, groceries, liquors, etc., either to obtain an average price of the mixture, which process is termed ***Alligation Medial***, or to make a mixture of several kinds that shall have a certain value, and this is called ***Alligation Alternate.***

ALLIGATION MEDIAL.

EXAMPLES.

1. Mix together 10 lbs. of tea at 40 cents a lb., 12 lbs. at 50 cts. a lb., 5 lbs. at 60 cts. per lb., and 3 lbs. at $1.00 per lb.; what is the mixture worth per lb.?

lbs.		*cts.*		*cts.*	
10	×	40	=	400	
12	×	50	=	600	
5	×	60	=	300	
3	×	100	=	300	
30 lbs.			=	1600	cts.
1 lb.			=	$53\frac{1}{3}$	cts.

2. Mix together 12 gallons of wine at 50 cents a gal., 10 gallons at 60 cents, 5 gallons at 80 cents, and 3 gallons of water; what is the mixture worth per gallon?

gal.		cts.		cts.
12	×	50	=	600
10	×	60	=	600
5	×	80	=	400
3	×	0	=	0
30 gal.			=	1600 cts.
1 gal.			=	$53\frac{1}{3}$ cts. per gal.

3. Mix 10 gallons of wine worth $2 a gallon, 8 gallons at $1.50, 16 gallons at $1.25, and 4 gallons of water; what is one gallon worth ?

10	×	2	=	$20
8	×	1.50	=	12
16	×	1.25	=	20
6	×	0	=	0
40 gal.			=	$52
1 gal.			=	$1.30

6. Mix 50 lbs. coffee worth 14 cents, 40 lbs. at 20 cts., 100 lbs. at 15 cts., and 30 lbs. at 10 cts.; what is a lb. of the mixture worth? *Ans.* 15 cts.

7. A farmer has 10 pigs worth $3 each, 12 worth $4 each, and 8 large ones worth $9 each; what is the average worth? *Ans.* $5 each.

ALLIGATION ALTERNATE.

PROBLEM.

What relative quantity of each must be taken of two kinds of sugar, the one worth 5 cts. per lb. and the other 9 cts., in order that the mixture be worth 6 cts. per lb.?

6	5	1	3		=	15
	9	$\frac{1}{3}$	1		=	9
			4	lb.	=	24
			1	lb.	=	6

By making 1 lb. at 5 cents worth 6 cents, there is a gain of 1 cent; and in 1 lb. at 9 cents made at 6 cents, there is a loss of 3 cents; hence, in $\frac{1}{3}$ of a lb. at 9 cents there is a loss of 1 cent; therefore, in mixing 1 lb. at 5 cents with $\frac{1}{3}$ of a lb. at 9 cents, the mixture is worth 6 cents, and the gain and loss are equal. By reducing the 1 and $\frac{1}{3}$ to a common denominator and multiplying both by it, they become integral numbers, and the relative quantity of each is the same.

Cor. 1.—It matters not how many different qualities are to be mixed, they must be mixed by pairs, one of the values less and the other greater than the mean value.

Cor. 2.—If there are odd numbers to be mixed, or more above or below the mean value, one of those having the smaller number must be mixed with two or more of those having the greater number on the other side of the mean.

EXAMPLES.

1. Mix coffee at 15 cts., 17, 19, and 22 cts. per lb., so that the mixture is worth 18 cts.

18	15	$\frac{1}{3}$	1
	19	1	3

18	17	1	4
	22	$\frac{1}{4}$	1

18	15	$\frac{1}{3}$	4
	22	$\frac{1}{4}$	3

18	17	1
	19	1

18	15	$\frac{1}{3}$	~~1~~	4
	17	1	~~4~~	1
	19	1	~~3~~	1
	22	$\frac{1}{4}$	~~1~~	3

18	15	$\frac{1}{3}$	1
	17	1	4
	19	1	3
	22	$\frac{1}{4}$	1

The sum of these will also prove a correct mixture; that is,

5	at	15	=	75
5	at	17	=	85
4	at	19	=	76
4	at	22	=	88
			18)	324
				18

COR.—If any two values, one greater and the other less, be connected together, and the difference between each one and the mean value be placed opposite the one with which it is connected, the quantity opposite each value will be its relative quantity.

REM.—The relative quantity of each is not absolute, as any two opposite quantities may be mixed, and, after being mixed differently, these mixtures may be added and the relative quantities changed.

2. A grocer has spices at 18, 24, 36, and 42 cts. per lb., of which he wishes to make a mixture worth 32 cents per lb.

32	18	10	=	180	
	24	4	=	96	
	36	8	=	288	
	42	14	=	588	
		36		) 1152	(32 cts.
				108	
				72	
				72	

3. A merchant has five qualities of liquors, at the following prices per gallon, viz., $1.25, $1.45, $1.60, $1.80,

and \$1.90; and he has an order for liquor at \$1.50 per gallon; at the rate of \$1.90 he has but 40 gallons, all of which he wishes to put in the mixture; how much must be taken of each of the other kinds?

150	\$1.25	10 + 40 =	$50 \times \frac{8}{5} = 80$
	1.45	30	$30 \times \frac{8}{5} = 48$
	1.60	25	$25 \times \frac{8}{5} = 40$
	1.80	5	$5 \times \frac{8}{5} = 8$
	1.90	25	$25 \times \frac{8}{5} = 40$

$$25 \times \tfrac{8}{5} = 40.$$

4. A farmer has six kinds of wheat of different values per bushel, viz., \$1.00, \$1.05, \$1.10, \$1.20, \$1.35, and \$1.40; he makes a sale at \$1.15 per bushel; how many bushels of each kind must he take?

115	\$1.00	25	= 2500
	1.05	20	= 2100
	1.10	5	= 550
	1.20	5	= 600
	1.35	10	= 1350
	1.40	15	= 2100
		80	) 92.00
			1.15, Proof.

NOTE.—Different connections will produce different results; the only principle involved is that of the two quantities mixed, one must be less and the other greater than the mean.

Involution and Evolution.

Involution and ***Evolution*** correspond very nearly to multiplication and division. Involution consists in the multiplication of the same number by itself, or of the same factor employed two or more times to produce the product; whilst Evolution restores the common factors from the product.

INVOLUTION

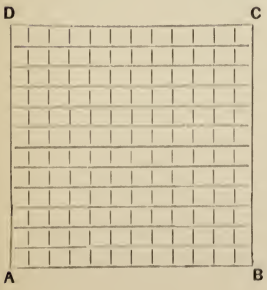
is called the raising of powers; thus. $2 \times 2 = 4$, is called the second power of 2, and is expressed by 2^2.

$2 \times 2 \times 2 = 8 =$ 3d power of 2, and is expressed by 2^3.

$3 \times 3 = 9 =$ 2d power of 3, and is expressed by 3^2.

$3 \times 3 \times 3 = 27 =$ 3d power of 3, and is expressed by 3^3.

THEOREM.



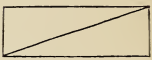
The area of a square is equal to the square of its side.

As there are 12 inches in the length of this square, and for every inch in breadth there will be 12 sq. in., thus, $12 \times 1 = 12$ sq. in.; $12 \times 2 = 24$ sq. in.; $12 \times 5 = 60$ sq. in.; and $12 \times 12 = 144$.

The area of the square is equal to the square of its side.

Cor. 1.—If, instead of taking the whole side for its breadth, a part only is taken, as 1 in., 2 in., 3 in., or 5 in., the figure is a rectangle, and its area is the product of its length and breadth.

Cor. 2.—If a diagonal is drawn to the rectangle, it divides it into two equal triangles.



Cor. 3.—It is proved by a simple geometrical demonstration that the area of any parallelogram is also equal to the product of its base and perpendicular altitude; therefore the area of any triangle is equal to one-half the product of its base and perpendicular altitude.

Rem. 1.—A plumb-line which extends directly toward the centre of the earth is apparently perpendicular to the surface of the earth, but really perpendicular to a plane which is a tangent to the surface of the earth.

Rem. 2.—The floor of a house is either tangent to the surface of the earth, or is parallel to another plane which is tangent to the surface of the earth.*

Rem. 3.—Planes holding these positions are called horizontal planes, and planes which are everywhere equally distant are parallel.

EVOLUTION,

or the extracting of roots, is exactly the reverse of involution, and is expressed by the radical sign $\sqrt{}$ for square root, and $\sqrt[3]{}$ for cube root.

As we have seen in square measure that the area of the

---

* A line touching a circle, or a plane touching a sphere at but one point is called a *tangent*.

surface is found by the product of two equal sides, so in evolution we have given the surface to find the side.

As the product is the result of two equal factors, it appears to be the natural solution to factor this product; thus,

PROBLEMS.

$4 = 2 \times 2$; therefore 2 is the square root of 4.
$9 = 3 \times 3$; therefore 3 is the square root of 9.
$16 = 2 \times 2 \times 2 \times 2 = 4 \times 4$; ∴ 4 is the sq. root of 16.

$25 = 5 \times 5.$	$169 = 13 \times 13.$
$36 = 6 \times 6.$	$196 = 14 \times 14.$
$49 = 7 \times 7.$	$225 = 15 \times 15.$
$64 = 8 \times 8.$	$256 = 16 \times 16.$
$81 = 9 \times 9.$	$289 = 17 \times 17.$
$100 = 10 \times 10.$	$324 = 18 \times 18.$
$121 = 11 \times 11.$	$361 = 19 \times 19.$
$144 = 12 \times 12.$	$400 = 20 \times 20.$

REM.—Above is given all the square numbers to 400.

COR.—For any other numbers below 400 only an approximate root can be obtained.

$2 \times 2 \times 2 = 8.$	2 is the cube root of 8.
$3 \times 3 \times 3 = 27.$	3 is the cube root of 27.
$4 \times 4 \times 4 = 64.$	4 is the cube root of 64, etc.

$5 \times 5 \times 5 = 125.$	$9 \times 9 \times 9 = 729.$
$6 \times 6 \times 6 = 216.$	$10 \times 10 \times 10 = 1000.$
$7 \times 7 \times 7 = 343.$	$11 \times 11 \times 11 = 1331.$
$8 \times 8 \times 8 = 512.$	$12 \times 12 \times 12 = 1728.$

REM.—When the given number is not a power of an integer, the root may still be approximated by trial with decimals. Another method is generally adopted; thus,

You can readily see the number of figures the root is composed of.

$1 \times 1 = 1$	$9 \times 9 = 81$
$11 \times 11 = 1{,}21$	$99 \times 99 = 98{,}01$
$111 \times 111 = 1{,}23{,}21$	$999 \times 999 = 99{,}80{,}01$

An additional figure in the root produces two in the power. When the left-hand figure in the root is small, there will be an odd number of figures in the power; but when the left-hand figure is more than 3, there will always be twice as many figures in the power as are in the root.

In raising a number consisting of two figures to the second power; as,

$$\begin{array}{r} 12 \\ \underline{12} \\ 144 \end{array}$$

$$\begin{array}{r} 10 \times 10 = 100 \\ 2 \times 10 \times 2 = 40 \\ 2 \times 2 = \underline{\ \ 4} \\ 144 \end{array}$$

144 is composed of the square of 10, and of twice $10+2$ multiplied by 2; twice $10+2$ must therefore be the second divisor; hence,

$$\begin{array}{l} 10\ )\ 144\ (\ 10 \\ \quad\ \ \ \underline{100} \\ 22\ )\ 44\ (\ \underline{\ 2} \\ \quad\ \ \ \underline{44}\ \ 12\text{, root.} \end{array}$$

When there are three figures in the root; as,

```
  123
  123
  ---
  369
 246
123
-----
15129
```

$$100 \times 100 = 10000$$
$$200 + 20 \times 20 = 4400$$
$$(200 + 40 + 3)\,3 = 729$$

```
100 ) 15129 ( 100          1,51,29 ( 123
      10000                1
220 )  5129 (  20       22 ) 51
       4400                  44
243 )   729 (   3      243 ) 729
        729   ---              729
              123
```

Rem.—The result of the last process, without the zeros, is the same as of those illustrated with the zeros.

Cor.—To extract the square root of a number, point it off in periods of two figures, beginning with units. Place in the root the largest figure, the square of which is not greater, but is either equal to, or less than the left-hand period of the power. Square the root obtained, and subtract the square from the left-hand period; bring down the next period, and for a divisor double the root found, regarding it as tens, and try how often it will go into the power brought down; place this figure in the root, and add it to the divisor, which completes the divisor; then divide as before, and bring down the next period; again doubling the root, regarding it as tens, and continue the same process.

REM.—As each figure in the root is placed to the right of the previous figure, each one holds the place of tens to the one following it.

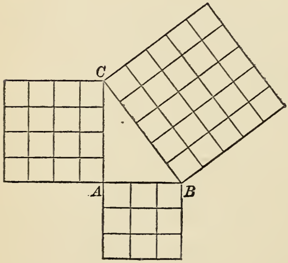
There is also a method similar to this for the extraction of cube root, which I shall not insert in this volume.

EXAMPLES.

1. Extract the square root of 441. *Ans.* 21.
2. Extract the square root of 9801. *Ans.* 99.
3. Extract the square root of 15129. *Ans.* 123.
4. Extract the square root of 103041. *Ans.* 321.

REM.—A fraction is squared when multiplied by itself; therefore the root is extracted by extracting the root of both terms; for, as $\frac{4}{5}\times\frac{4}{5}=\frac{16}{25}$, $\sqrt{\frac{16}{25}}=\frac{4}{5}$; $\sqrt{\frac{25}{36}}=\frac{5}{6}$; and $\sqrt{\frac{36}{49}}=\frac{6}{7}$, etc.

By a proposition in Geometry, it is proved that the square described on the hypothenuse of a right-angled triangle is equivalent to the sum of the squares described on the base and perpendicular; thus,



REM.—Two sides of a square are at right angles, so also the two sides of a rectangle; as the sides of a house, the two sides forming a corner of the house are at right angles; the two sides of the carpenter's square.

In this figure, the base is regarded as 3 inches, the perpendicular 4, and the hypothenuse 5; and

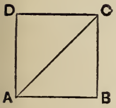
$[(3\times 3)=9] + [(4\times 4)=16] = 25$

and $5\times 5 = 25.$

Cor.—Any two sides of a right-angled triangle being given, the third side can be found; for if the base and perpendicular are given, add their squares, and the square root of the sum will be the hypothenuse; if the hypothenuse and one side are given, square both, subtract their squares, and the square root of the difference will be the other side.

Rem.—The figure ABC is exemplified by the wall of a house, which is always at right angles with the surface of the earth, and a ladder at the distance of the base from the foot of the wall, reaching the wall at the height of the perpendicular. AB is the base, AC the perpendicular, and BC the hypothenuse.

EXAMPLES.



1. What is the length of the diagonal of a square, each side of which is 9 feet.

Rem.—AC is the diagonal corresponding to the hypothenuse of a right-angled triangle.

```
9 × 9 =  81
9 × 9 =  81
        ----
        162 ( 12.72+, Diagonal.
        1
        ----
   22 ) 62
        44
        -----
  247 ) 1800
        1729
        ------
 2542 ) 7100
        5084
        ----
        2016
```

COR. 1.—As the angles B and D are both right angles, hence equal, if the one is placed on the other they will coincide; and as the sides AB, AD, DC, and BC, are equal, these sides will fall upon each other and coincide; and AC is common to the two triangles ABC and ADC; therefore the two triangles are equal, and each is one-half the square.

COR. 2.—A triangle is one-half a square or rectangle, and the surface of a square or rectangle is equal to the product of its two sides, or of the base and perpendicular; therefore the surface of a triangle is one-half the product of its base and altitude.

SCHO.—A triangle that is not right-angled, is one-half a parallelogram; and it is proved in Geometry that a rectangle and a parallelogram of the same base and altitude are equivalent; hence the above holds good for any triangle.

2. A ladder 15 feet long, placed 9 feet from the foot of the wall of a house, reaches the window of the second story of the house; how high is the window?

$$15 \times 15 = 225$$
$$9 \times 9 = \underline{81}$$
$$\sqrt{144} = 12 \text{ feet.}$$

3. How many square feet in the floor or ceiling of a room 20 ft. long and 15 ft. wide?

$$20 \times 15 = 300 \text{ sq. ft.}$$

4. How many square feet in the wall of a room 20 ft. long and 9 ft. high? *Ans.* 180 sq. ft.

5. How many square feet in a wall 15 ft. wide and 9 ft. high? *Ans.* 135 sq. ft.

6. How many square feet in the four walls of a room 20 ft. long, 15 ft. wide, and 9 ft. high? *Ans.* 630 sq. ft.

7. How many square feet of plastering, including the ceiling? *Ans.* 930 sq. ft.

8. How many square feet in a triangle whose base is 15 ft. and altitude 8 ft.?

$$\tfrac{1}{2}(15 \times 8) = 60 \text{ sq. ft.}$$

9. How many cubic feet in a cubical block, the side of each square face being 5 feet?

$$5 \times 5 \times 5 = 125 \text{ cu. ft.}$$

10. How many cubic feet in the room above, 20 by 15 and 9 high?

REM.—The area of a circle is found by squaring the diameter and multiplying the square by .7854.—"Geometry."

11. What is the number of square feet in a circle whose diameter is 4 ft.?

$$4 \times 4 \times .7854 = 12.5664 \text{ sq. ft.}$$

12. How many cubic feet in a cylinder whose circular base is 2 feet in diameter and its altitude 3 feet?

$$2 \times 2 \times .7854 \times 3 =$$

13. How many cubic feet in a pyramid whose base is 3 feet square and altitude 9 feet?

$$\tfrac{1}{3}(3 \times 3 \times 9) = 27 \text{ cu. ft.}$$

14. How many cubic feet in a cone whose circular base is 3 feet in diameter and altitude 9 feet?

$$\tfrac{1}{3}(3 \times 3 \times .7854 \times 9).$$

NOTE.—The volume of a pyramid and also of a cone is the area of the base multiplied by one-third the altitude.

Series of Common Differences.

A ***Series of Common Differences,*** usually termed Arithmetical Progression, is one in which the difference of any two consecutive terms, taken in order, is the same; as, in the series 1, 2, 3, 4, 5, etc., the first term is 1, and the common difference is 1; and it is an increasing series as each successive term is greater than the preceding one.

1, 3, 5, 7, etc., is an increasing series, and the common difference is 2.

2, 5, 8, 11, etc., is an increasing series, and the common difference is 3.

12, 10, 8, 6, 4, 2, etc., is a decreasing series, and the common difference is 2.

THEOREM I.

The sum of a series of equal differences is equal to one-half the product of the number of terms and the sum of the first and last terms.

As, the sum of six terms of the series, 1, 2, 3, 4, etc., is

1	2	3	4	5	6
6	5	4	3	2	1
7	7	7	7	7	7

Sum of 6 terms of both series is

$$6 \times 7 = 42.$$

Sum of 6 terms of one series is

$$\tfrac{1}{2}(6 \times 7) = 21.$$

The sum of 8 terms of the series:

2	5	8	11	14	17	20	23
23	20	17	14	11	8	5	2
25	25	25	25	25	25	25	25

The sum of 2 series is

$$8 \times 25 = 200$$

The sum of 1 series is

$$\tfrac{1}{2}(8 \times 25) = 100$$

Therefore the sum of a series of common differences is one-half the product of the number of terms, multiplied by the sum of the first and last terms.

THEOREM II.

Any term of an increasing series is equal to the first term plus the common difference taken as many times as the number of the terms less one; and any term of a decreasing series is equal to the first term minus the quantity that is to be added above.

The series may be written

	1	2	3	4	5	
	a,	$a+d$,	$a+2d$,	$a+3d$,	$a+4d$,	etc.,
and	a,	$a-d$,	$a-2d$,	$a-3d$,	$a-4d$,	etc.,

in which a represents the first term and d the common difference, and in each term d is once less than the number of terms, either plus or minus; hence any term of a series of equal differences, is equal to the first term plus or minus the common difference taken as many times as the number of the term less one.

Let n represent the number of terms, and any term may be regarded as the last term, and let l represent the last term; thus,

$$l = a + (n - 1)\,d;$$

and $$l = a - (n - 1)\,d.$$

EXAMPLES.

1. What is the sum of 25 terms of the series 1, 2, 3, 4, 5, etc.?

$$S = \tfrac{25}{2}\,(1 + 25) = 325.$$

2. What is the 10th term of the series 1, 3, 5, etc.?

$$\text{10th term} = l = a + (n - 1)\,d.$$
$$l = 1 + 18 = 19.$$

3. What is the sum of the 10 terms of the series?

$$S = \tfrac{10}{2}\,(1 + 19) = 100.$$

4. Find the 6th term of the series 12, 10, 8, etc.

$$\text{6th} = l = a - (n - 1)\,d.$$
$$l = 12 - 10 = 2.$$

5. Find the sum of six terms of the series.

$$\tfrac{6}{2}\,(12 + 2) = 42.$$

6. Find the sum of 100 terms of the series 1, 2, 3, 4, etc.

$$l = 100.$$
$$S = \tfrac{100}{2}\,(1 + 100) = 5050.$$