Why Are Spatial Skill and Mathematics Related?

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Abstract

Research has demonstrated strong relations between spatial skill and mathematics across ages and in both typical and atypical populations, suggesting that a significant proportion of variance in mathematics performance can be explained by variance in spatial skill. Why do these relations exist and how do they develop? Studies of dimensionality in the two domains suggest the relation holds across tasks and is not limited to specific spatial or mathematics subskills. Spatial skills might perform several functions in real-time problem solving, but these have not been differentiated empirically. The relation appears to be based on automatic shared processing, as well as strategic recruitment of spatial processes. Developmentally, the relation is consistent in its strength, but may change qualitatively, particularly in response to novel mathematics content.

People who perform well on spatial tasks, such as imagining objects turning, also tend to perform well on mathematics tasks (see Mix & Cheng, 2012, for a review). The space-mathematics connection is evident from infancy to adulthood (See Verdine, Golinkoff, HirshPasek, & Newcombe, 2017), in both typical and atypical populations (e.g., Passolunghi & Mammarella, 2012), and may be causal (e.g., Lowrie, Logan, & Ramful, 2017). Strong, consistent evidence of a cross-domain connection compels the following questions: Why are spatial skill and mathematics related? What underlying structures are indicated? As my colleague and I have argued (Mix & Cheng, 2012), a critical goal for researchers is to work out mechanistic explanations for these results that go beyond a simple reiteration of the relation itself. In this brief review, I outline candidate mechanisms, highlight evidence, and identify areas for ongoing study.

Mechanistic Frameworks

It is straightforward to say spatial skill and mathematics are related. It is much less so to say how and why. A clear understanding of the mechanisms connecting these two domains is needed both to pinpoint the ways they relate developmentally and to leverage the relation clinically and educationally (Mix & Cheng, 2012). This understanding is elusive because not only are multiple mechanisms plausible, but these mechanisms do not necessarily compete since all of them could be engaged for various tasks and various learners. To encompass this complexity, I have organized this review around four key questions that represent different approaches to parsing the theoretical landscape:

1) Is the relation general or specific?
2) How do the two domains relate in real time?
3) Is the relation automatic or strategic?

4) How do the two domains relate across developmental time?

Question 1: Is the Relation General or Specific?

One way to look at potential mechanisms is to determine whether the same relation holds no matter which domain subskills are measured. The dimensionality within each domain has been debated over the years, with some investigators arguing that certain measures tap distinct subskills and other investigators claiming that measures within domains are essentially interchangeable (see Young, Levine, & Mix, 2018, for a review). If the two domains are multidimensional, the space-mathematics connection may unfold at a task-specific level. However, this hypothesis has been difficult to evaluate because most studies have focused on only a few skills at a time, testing, for example, whether visuospatial working memory is related to mathematics.

To address this gap, my colleagues and I measured a range of spatial and mathematics skills in children from three grades (kindergarten, third grade, and sixth grade; Mix et al., 2016; Mix, Levine, Cheng, Young, et al., 2017). Factor analyses revealed distinct factors for space and mathematics onto which each of the domain-specific measures loaded significantly. These factors were highly correlated but separate. A few measures from each domain loaded significantly onto both factors. For example, mental rotation loaded significantly onto both the spatial and mathematics factors in kindergarten, and visuospatial working memory and figure copying loaded significantly onto both factors in sixth grade (when children are about 11 years old). These findings suggest that certain spatial skills may have a particularly strong relation with mathematics and that these specific relations vary with age, a conclusion that received additional support from regressions of the spatial measures against children's mathematics factor scores (Mix et al., 2016). However, when my colleagues and I repeated the study with a second cohort of children (Mix, Levine, Cheng, Young, et al., 2017), the same overall latent structure was replicated but the patterns involving cross loadings were not, suggesting caution in drawing strong conclusions at the level of specific measures. Taken together, the findings indicate that spatial skill and mathematics most likely relate at a general level.

Question 2: How Do the Two Domains Relate in Real Time?

Another way to look at potential mechanisms is to consider cross-domain relations that might occur in the moment of problem solving (i.e., real time) as opposed to relations that might unfold developmentally over months and years. That is, when someone attacks a specific mathematics problem, what possible functions might spatial processing serve? Several possibilities exist (Lourenco, Cheng, & Aulet, 2018; Mix et al., 2016; Newcombe, 2017).

One is that spatial skill could help people decode mathematics equations or diagrams based on the spatial arrangement of the written symbols. A simple example would be using spatial position to know that 53 is a larger number than 35 because it has five tens instead of three tens. Many mathematics symbols derive meaning from their spatial positions and these relations can become quite complex, such as when interpreting an algebraic equation or tracking one's place in a long division problem. Spatial skill may play a crucial role in helping children decode these symbols.

A second possible function is that spatial skill can help in mapping the meaning of mathematics symbols or problems to their referents (i.e., symbol grounding). For example, when people solve word problems, they might imagine the scene described in a problem as a way to check their understanding. Similarly, problem solvers asked to calculate the area of an irregular shape might create simple pictures or diagrams to represent the problem. Early in development, mental models might be used to ground the meaning of simple symbols such as written numerals or signs for operations. For example, children might imagine two sets coming together to represent the meaning of the plus sign. The ability to create, maintain, and update mental models likely depends on spatial skill.

A third possible function involves using conventional spatial representations such as number lines to solve problems. For example, children might be taught how to solve simple calculation problems by putting their fingers on a number line where the first addend is located, and then counting up the number of spaces for the second addend to arrive at a solution. Many such spatial tools are available to teachers and students, including base-10 blocks, fraction bars, and tens frames (Mix, 2010; Newcombe, 2017). Children may find it easier to understand and implement these tools if they have strong spatial skills.

These three possible functions of spatial skill are not mutually exclusive. Any or all of these functions could be engaged at different developmental stages for different content given different levels of spatial skill or mathematics skill, or even at different levels of processing. To illustrate, consider a study that compared children's arithmetic performance under conditions of either visuospatial or phonological disruption (McKenzie, Bull, & Gray, 2003). The authors reported clear effects of visuospatial disruption in 6- to 7-year-olds, but a mixture of effects involving both forms of disruption in 8- to 9-year-olds. In a separate study involving adults, researchers reported visuospatial disruption for visual arithmetic problems but not auditory problems (Logie, Gihooly, & Wynn, 1994).

Results like these may reflect a complex developmental pattern in which younger children rely on spatial representations to generate mental models and do so for most problems regardless of presentation because they are focused on grounding the meaning of numeric symbols, whereas older children use a combination of verbal and spatial strategies. In adults, the residual spatial effects may be the result of using a specific spatial representation, such as the mental number line, that is not necessarily activated by auditory presentation. Indeed, the potential for different spatial strategies to be used at different stages of processing during the same tasks has complicated attempts to categorize spatial tasks and assess multidimensionality (Mix, Hambrick, Satyam, Burgoyne, & Levine, 2018). Clearly, much remains to be investigated about these complex patterns.

Question 3: Is the Relation Automatic or Strategic?

Another approach to identifying mechanisms is to determine whether space and mathematics are inherently related because they share the same processes, or they become related through strategic recruitment of spatial processing in mathematics tasks. Some have suggested that spatial reasoning is fundamental to abstract thought more generally (Lohman, 1996), and abstract thought would certainly include mathematics. Indicative of automatic shared processing, neuropsychological evidence suggests that the same brain regions are activated for spatial and numerical tasks (e.g., Hubbard, Piazza, Pine, & Dehaene, 2005). However, recall that spatial skill and mathematics form separate factors and are highly, but not completely, correlated (Mix et al., 2016). Furthermore, language proficiency and executive function are also strong predictors of mathematics performance, independent of spatial skill (e.g., Bull & Scerif, 2001; Geary, 2011). Thus, it unlikely that space and mathematics are simply different contexts in which to apply the same processes. Perhaps—as has been claimed for number processing in particular (e.g., Dehaene, Piazza, Pinel, & Cohen, 2003)—mathematical thought is served by spatial, linguistic, and quantitative representational systems, so mathematics may be inherently spatial when its spatial modality is activated, and inherently linguistic when its linguistic modality is activated. Under what conditions are these modalities activated—for example, is modality determined by the mathematics task? Or are there developmental changes in the relative contribution of these systems over time?

An alternative to the notion of shared processing is that spatial processes are recruited purposely as a problem-solving strategy or leveraged by teachers as a pedagogical tool. Even without instruction, adults often use visuospatial representations to solve mathematics problems (e.g., Seron, Pesenti, Noel, DeLoche, & Cornet, 1992), providing support for this basic notion. Furthermore, not all learners use spatial representations in the same way, suggesting strategic choices rather than automatic shared processing. For example, sixth-grade boys who used schematic spatial representations of mathematics problems outperformed those who used more detailed, pictorial spatial representations (Hegarty & Kozhenivkov, 1999). Gifted students are also more likely than students with mathematics learning disabilities to use schematic drawings, and using these drawings correlated strongly with problem-solving scores (van Garderen, 2006). Relatedly, in another study, children with low spatial ability were more likely to use detailed pictures than schematic images (van Garderen, 2006). These examples illustrate that even when learners operate in the spatial modality, the way spatial processes are recruited and applied varies, with more advanced learners using less literal models.

Taken together, the evidence is consistent with both automatic shared processing and effortful, strategic use of spatial representations. It is likely that both types of engagement come into play across a range of mathematics problems and differences among learners, but it remains to be seen under what conditions and why.

Question 4: How Do the Two Domains Relate Across Developmental Time?

Up to this point, we have considered relations that might unfold as people solve a particular mathematics problem. However, those relations occur within the larger context of developmental change, which takes place over months or years. The prospect of longer-term interactions between spatial skill and mathematics raises questions such as whether spatial skill is used differently at different ages, or whether improvement in one domain leads to improvement in the other as children learn and acquire new skills. For example, one possibility is that spatial skill is related to mathematics in adolescence only when children deal with very abstract, complex mathematical problems. An alternative possibility is that spatial skill is related to mathematics only in early childhood, when children map newly acquired numeric symbols to their concrete referents.

However, the evidence argues against such developmental shifts. My colleagues and I (Mix et al., 2016; Mix, Levine, Cheng, Young, et al., 2017) did not find a change in the overall correlation between spatial skill and mathematics from kindergarten to sixth grade. The correlation was equally strong in all three grades, suggesting a lack of quantitative change. One could argue the shifts in cross-domain loadings we reported in our work (Mix et al., 2016; e.g., mental rotation in kindergarten, and visuospatial working memory, and figure copying in sixth grade) may reflect qualitative change in the relation between spatial skill and mathematics; these effects were fragile and not always replicated.

An intriguing hypothesis, and one that may explain the failure of the cross-loadings to replicate, is that spatial skill is engaged more often for novel content (Uttal & Cohen, 2012). For example, a child who is unfamiliar with addition and subtraction may rely more on spatial skill to ground unfamiliar symbols, whereas children who have mastered these operations may rely less on spatial skill to solve these problems or may use spatial skill differently (e.g., to support symbol decoding). This cycle of engaging spatial skill differentially could be repeated every time new mathematics content is learned, yielding a developmental landscape of specific quantitative and qualitative changes driven by learners' mastery of specific mathematics content. As evidence, in one study (Lowrie & Kay, 2001), sixth-grade students were more likely to generate spatial representations while solving novel or difficult mathematics problems than they were while solving familiar, routine problems. My colleagues and I (Mix et al., 2016) also found that performance on a spatial visualization measure (i.e., the Block Design subtest of the Wechsler Intelligence Scale for Children) correlated significantly with mathematics scores for novel content only, whereas figure copying correlated significantly with familiar content. The novelty of content may also explain why, for teenagers and adults, the space-mathematics connection is evident on advanced mathematics skills, such as solving complex equations (e.g., Wei, Yuan, Chen, & Zhou, 2012), as well as on quite low-level mathematics tasks among preschool children (Verdine et al., 2014). If the same pattern of engaging spatial skill repeats itself whenever new mathematics content is encountered, determining whether and how spatial skills are engaged probably depends more on the learner's level of mastery than on age alone.

A second developmental question is whether improvement in one domain leads to improvement in the other. One approach has been to improve spatial skill through training and then test for improvement in mathematics. Although not all such attempts at training have succeeded (e.g., Hawes et al. 2015; Xu & LeFevre, 2016), some have (Cheng & Mix, 2014; Lowrie et al., 2017; Mix, Levine, & Cheng, 2017; Schmitt, Korucu, Napoli, Bryant, & Purpura, 2018). For example, in one study (Mix, Levine & Cheng, 2017), training to improve mental rotation and spatial visualization in first- and sixth-grade students led to significant improvement in mathematics scores compared to a control group that was trained on something else. Researchers have also provided deliberate instruction in using spatial strategies to positive effect. For example, in one study (van Garderen, 2007), researchers taught three eighth-grade students with mathematics learning disabilities to use schematic diagrams while solving word problems. After training, the use of diagrams to solve word problems increased significantly, and students' test scores improved from an average of 37% at pretest to an average of 79% following training. We should extend this type of research to larger samples and wider age ranges and mathematics content.

Another question for researchers is whether these training effects have an upper limit. Such limits have been reported in research on the relation of intelligence to creativity (Jauk, Benedek, Dunst, & Neubauer, 2013). Segmented regression analyses revealed that for certain measures, there was a strong positive relation up to a particular threshold, after which additional IQ points did not improve creativity scores. The same pattern may hold between spatial skill and mathematics, that is, higher spatial scores may be associated with higher mathematics scores up to a point, beyond which there is no significant relation—a hypothesis that was partially borne out in a recent dissertation (Freer, 2017). Another interesting question is whether these training effects are bidirectional. Given the correlation between domains, it is just as likely for spatial skills to improve from mathematics training as the reverse (Lourenco et al., 2018). Although such bidirectional effects have been reported occasionally (e.g., Geer, Quinn, & Ganley, 2018), they have not been studied extensively; this may be a promising direction for research.

A second approach to asking whether more optimal spatial skills lead to improvement in mathematics has been to track children's performance longitudinally. Some studies have focused on adolescents and adults, demonstrating, for example, that spatial skill in high school is a significant predictor of subsequent college and career success in science, technology, engineering, and mathematics (Li & Geary, 2013; Wai, Lubinski, & Benbow, 2009). Other studies have focused on children, sometimes starting at very young ages. For example, in one study (Lauer & Lourenco, 2016), infants' ability to discriminate mirror images from nonmirror images at 6 and 13 months predicted variation in the children's mathematics performance at 4 years. Similar results have been reported for preschoolers (i.e., spatial skills at 3 years predicted mathematics performance at 5 years; Verdine et al., 2017), as well as for older children (i.e., spatial skills at 5 years predicted mathematics performance at 7 years; Gilligan, Flouri, & Farran, 2017).

Longitudinal studies have also revealed less direct pathways between spatial skill and mathematics. In one study, spatial skill was related to development of an accurate linear number line representation at 6 years, and this representation in turn mediated the relation between spatial skill at 5 years and mathematics performance at 8 years (Gunderson, Ramirez, Levine, & Beilock, 2012). Similarly, the significant relation between spatial skill in kindergarten and mathematics achievement in first grade was mediated by knowledge of the counting sequence (Zhang et al., 2014). These examples suggest that spatial skill could exert its influence via children first mapping mathematical symbols onto a spatial representation and subsequently using that representation in mathematics tasks.

In summary, the relation between space and mathematics is strong and stable across development, but qualitative changes in the specific functions and processes that are engaged may exist. These qualitative changes may be part of a cyclic pattern in which spatial skill is engaged differently when mathematics content is unfamiliar than after it has been mastered. Both training and longitudinal studies have demonstrated effects of spatial skill on mathematics outcomes, suggesting causality, but it remains to be seen whether these relations are either bidirectional or subject to threshold limits.

Conclusions

Interest in the relation between spatial skill and mathematics has increased, and recent research has added significantly to what we know. However, many questions remain. For example, the engagement of various spatial functions and the complex ways mathematics and spatial skill might relate in real-time problem solving are poorly understood. We have more to discover about the role of spatial processing for novel versus familiar mathematics problems. Additional training studies may be helpful in this regard to compare the effects of pure spatial training to training that is integrated with mathematics content. Training studies could also probe for bidirectional effects (e.g., does mathematics training automatically improve spatial skills?), threshold effects, and effects that equalize individual differences. It is important to address these questions, not only for what they can reveal about the nature of the relation between spatial skill and mathematics, but also because of the potential to leverage this connection to improve student learning.

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References

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology*, *19*, 273-293. doi: 10.1207/S15326942DN1903_3

Cheng, Y.-L. & Mix, K. S. (2014). Spatial training improves children's mathematics ability. *Journal of Cognition and Development, 15*, 2-11. doi: 10.1080/15248372.2012.725186.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, *20*, 487-506. doi: 10.1080/02643290244000239

Freer, D. (2017). *The threshold hypothesis applied to spatial skill and mathematics*. Doctoral dissertation. Department of Educational Psychology and Educational Technology, Michigan State University, East Lansing, MI. Retrieved March 4, 2019, from https://doi.org/10.25335/M5ZC5F

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology*, *47*, 1539-1552. doi: 10.1037/a0025510

Geer, E. A., Quinn, J. M., & Ganley, C. M. (2018). Relations between spatial skills and math performance in elementary school children: A longitudinal investigation. *Developmental Psychology*, *55*, 637-652. doi: 10.1037/dev0000649

Gilligan, K. A., Flouri, E., & Farran, E. K. (2017). The contribution of spatial ability to mathematics achievement in middle childhood. *Journal of Experimental Child Psychology*, *163*, 107-125, doi.org/10.1016/j.jecp.2017.04.016

Gunderson, E. A., Ramirez, G., Levine, S. C., & Beilock, S. L. (2012). The role of parents and teachers in the development of gender-related math attitudes. *Sex Roles, 3*, 153-166. doi: 10.1007/s11199-011-9996-2

Hawes, Z., Moss, J., Caswell, B., & Poliszczuk, D. (2015). Effects of mental rotation training on children's spatial and mathematical performance: A randomized controlled study. *Trends in Neuroscience and Education, 4*, 60-68. doi.org/10.1016/j.tine.2015.05.001

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *Journal of Educational Psychology*, *91*, 684-689. doi: 10.1037/0022-0663.91.4.684

Hubbard, E. M., Piazza, M., Pine, P., & Dehaene, S. (2005). Interactions between numbers and space in parietal cortex. *Nature Reviews Neuroscience*, *6*, 435-448. doi: 10.1038/nrn1684

Jauk, E., Benedek, M., Dunst, B., & Neubauer, A. C. (2013). The relationship between intelligence and creativity: New support for the threshold hypothesis by means of empirical breakpoint detection. *Intelligence*, *41*, 212-221. doi: 10.1016/j.intell.2013.03.003

Lauer, J. E., & Lourenco, S. F. (2016). Spatial processing in infancy predicts both spatial and mathematical aptitude in childhood. *Psychological Science, 27*, 1291–1298. doi.org/10.1177/0956797616655977

Li, Y., & Geary, D. C. (2013). Developmental gains in visuospatial memory predict gains in mathematics achievement. *PLoS One, 8*, e70160.

Logie, R. H., Gilhooly, K. J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory & Cognition*, *22*, 395-410. doi: 10.3758/BF03200866

Lohman, D. F. (1996). Spatial ability and g. In I. Dennis & P. Tapsfield (Eds.), *Human abilities: Their nature and measurement* (pp. 97-116). Mahwah, NJ: Erlbaum.

Lourenco, S. F., Cheung, C.-N., & Aulet, L. S. (2018). Is visuospatial reasoning related to early mathematical development? A critical review. In A. Henik,&W. Fias (Eds.), *Heterogeneity of function in numerical cognition* (pp. 177-210). New York, NY: Academic Press.

doi.org/10.1016/B978-0-12-811529-9.00010-8

Lowrie, T., & Kay, R. (2001). Relationship between visual and nonvisual solution methods and difficulty in elementary mathematics. *The Journal of Educational Research*, *94*, 248-255. doi: 10.1080/00220670109598758

Lowrie, T., Logan, T., & Ramful, A. (2017). Visuospatial training improves elementary students' mathematics performance. *British Journal of Educational Psychology, 87*, 170-186. doi: https://doi.org/10.1111/bjep.12142

McKenzie, B., Bull, R., & Gray, C. (2003). The effects of phonological and visual-spatial interference on children's arithmetical performance. *Educational and Child Psychology*, *20*, 93-108.

Mix, K. S. (2010). Spatial tools for mathematical thought. In K. S. Mix, L.B. Smith, & M. Gasser (Eds.), *The spatial foundations of language and cognition* (pp. 41-66). New York, NY: Oxford University Press.

Mix, K. S., & Cheng, Y. L. (2012). The relation between space and math: Developmental and educational implications. In J. B. Benson (Ed.), *Advances in child development and behavior* (Vol. 42, pp. 197-243). New York, NY: Elsevier. doi: 10.1016/B978-0-12394388

Mix, K. S., Hambrick, D. Z., Satyam, V. R., Burgoyne, A., & Levine, S. C. (2018). The latent structure of spatial skill: A test of the 2x2 typology. *Cognition, 180*, 268-278. doi: 10.1016/j.cognition.2018.07.012

Mix, K. S., Levine, S. C., & Cheng, Y.-L. (2017, October). *Effects of spatial training on elementary mathematics.* Paper presented in N. Jordan & C. Barbieri (Chairs), Usable knowledge for improving mathematics learning: Bridging research in cognition and development with educational practice in diverse contexts. Symposium presented at the biennial meeting of the Cognitive Development Society, Portland, OR.

Mix, K. S., Levine, S. C., Cheng, Y., Young, C., Hambrick, D. Z., Ping, R., & Konstantopolous, S. (2016). Separate but correlated: The latent structure of space and mathematics across development. *Journal of Experimental Psychology: General, 145*, 1206-1227. doi: 10.1037/xge0000182

Mix, K. S., Levine, S. C., Cheng, Y.-L., Young, C. J., Hambrick, D. Z., & Konstantopoulos, S. (2017). The latent structure of spatial skills and mathematics: Further evidence from wave 2. *Journal of Cognition and Development, 4,* 465-492. doi: 10.1037/xge0000182

Newcombe, N. (2017). *Harnessing spatial thinking to support STEM learning*. Organisation for Economic Co-operation and Development, Education Working Papers, No. 161. Paris, France: OECD Publishing. http://dx.doi.org/10.1787/7d5dcae6-en

Passolunghi, M. C., & Mammarella, I. C. (2012). Selective spatial working memory impairment in a group of children with mathematics learning disabilities and poor problem-solving skills. *Journal of Learning Disabilities*, *45*, 341-350. doi: 10.1177/0022219411400746

Schmitt, S. A., Korucu, I., Napoli, A. R., Bryant, L. M., & Purpura, D. J. (2018). Using block play to enhance preschool children's mathematics and executive functioning: A randomized controlled trial. *Early Childhood Research Quarterly*, *44*, 181-191. doi: 10.1016/j.ecresq.2018.04.006

Seron, X., Pesenti, M., Noel, M. P., Deloche, G., & Cornet, J. A. (1992). Images of numbers, or "When 98 is upper left and 6 sky blue." *Cognition*, *44*, 159-196. doi: 10.1016/0010-0277(92)90053-K

Uttal, D. H., & Cohen, C. A. (2012). Spatial thinking and STEM education: When, why, and how? In *Psychology of learning and motivation* (Vol. 57, pp. 147-181). Cambridge, MA: Academic Press. doi: 10.1016/B978-0-12-39429307.00004-2

Van Garderen, D. (2006). Spatial visualization, visual imagery, and mathematical problem solving of students with varying abilities. *Journal of Learning Disabilities*, *39*(6), 496-506. doi: 10.1177/00222194060390060201

van Garderen, D. (2007). Teaching students with LD to use diagrams to solve mathematical word problems. *Journal of Learning Disabilities, 40,* 540-553.

Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. S. (2017). Links between spatial and mathematical skills across the preschool years. *Monographs of the Society for Research in Child Development, 82*. doi.org/10.1111/mono.12280

Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., Newcombe, N. S., Filipowicz, A. T., & Chang, A. (2014). Deconstructing building blocks: Preschoolers' spatial assembly performance relates to early mathematical skills. *Child Development, 85*, 1062-1076. doi: 10.1111/cdev.12165

Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology*, *101*, 817-835. doi: 10.1037/a0016127

Wei, W., Yuan, H., Chen, C., & Zhou, X. (2012). Cognitive correlates of performance in advanced mathematics. *British Journal of Educational Psychology*, *82*, 157-181. doi: 10.1111/j.2044-8279.2011.02049.x

Xu, C., & LeFevre, J.-A. (2016). Training young children on sequential relations among numbers and spatial decomposition: Differential transfer to number line and mental transformation tasks. *Developmental Psychology ,52*, 854-866. dx.doi.org/10.1037/dev0000124

Young, C. J., Levine, S. C., & Mix, K. S. (2018). What processes underlie the relation between spatial skill and mathematics? In K. S. Mix & M.Battista (Eds.), *Spatial visualization in mathematics* (pp. 117-148). New York, NY: Springer. doi: 10.1007/978-3-319-98767-5

Zhang, X., Koponen, T., Räsänen, P., Aunola, K., Lerkkanen, M. K., & Nurmi, J. E. (2014). Linguistic and spatial skills predict early arithmetic development via counting sequence knowledge. *Child Development*, *85*, 1091-1107. doi.org/10.1111/cdev.12173