IS THE MATHEMATICS DIAGNOSTIC TEST A GOOD PREDICTOR OF COLLEGE STUDENTS' PRIOR KNOWLEDGE IN MATH?

by

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WE, THE UNDERSIGNED MEMBERS OF THE COMMITTEE,

HAVE APPROVED THIS DISSERTATION

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ACCEPTED AND APPROVED ON BEHALF

OF WILLIAM HOWARD TAFT UNIVERSITY

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Abstract

This study was conducted at a college in Ontario with a sample of students from the Game Development program for 5 intake years: 2014–2018, with the purpose of determining the validity of the Mathematics Diagnostic Test used to place 1st semester college students directly into an advanced math course (Math 10), or to leave them in the introductory math course (Math 9). Two streams in senior high school were considered, college preparation and university preparation, for a correlation study between students' grades in senior math courses and GPA, and their math diagnostic test scores. Next, a multiple regression analysis was employed to find any significant predictors of students' prior knowledge to the math diagnostic test. Further, a *t*-test for independent groups was used to determine whether students' placement in the two college math courses was valid.

The study found a medium correlation between two senior college math courses (MBF3C and MAP4C) and the math diagnostic test. The multiple regression analysis established that they explained 18.5% of the variability of the diagnostic test. For the students in the university-preparation stream, there was a higher correlation between the senior math courses MCR3U and MHF4U and the math diagnostic test: they explained 44.1% of the variability of the diagnostic test in the regression model, which is a strong size effect. Further, the linear regression showed that the GPA explained 9.0% of the variability of the diagnostic test, which is considered a weak size effect. Finally, the *t*-test for independent samples found that there was not a statistically significant mean difference in Math 10 grades between the two groups: students with scores above 85% in the math diagnostic test placed directly into Math 10, and students with scores below 85% who were directed to Math 9 course first, then into Math 10. Therefore, taking Math 9 before Math 10 helped the weaker and regular knowledge students raise their knowledge to a level similar to the level of knowledge of students going directly into Math 10. The validity of the math diagnostic test to show students' prior knowledge in math was confirmed.

KEYWORDS: math diagnostic test; prior knowledge; student achievement; college students

Dedication

My parents taught us to leave nothing for tomorrow that we can do today. For the most part of my life, I followed that advice. However, during my doctoral degree, my motto seemed to have switched to Scarlett O'Hara's saying from *Gone with the Wind*: "Tomorrow is another day." Be smart and do not follow this last maxim, as your journey becomes much longer than it should be. This is a dedication to all who have the tendency to follow Scarlett's precept.

Acknowledgements

During this educational journey, all the related work kept me away from spending time with my husband, especially during the summertime. For a college teacher, the school period beginning in September and ending in April is very busy with teaching, marking, and other administrative work. Therefore, my summer vacations for the past 6 years were dedicated mostly to my studies, and I wish to thank my husband for putting up with this for such a long period of time. He has been the most supportive person I have ever known, always encouraging me and even pushing me to do more and trusting that I do great work. He even started to cook meals for the both of us, when he never cooked before. My hope is that next summer we could enjoy a regular vacation, further away from any writing—at least for one summer :).

In addition, I wish to dedicate my work to our daughter, who repeatedly sent me words of encouragement and wonder about my lifelong learning habit. I hope her own educational journey is as filled with pursuing new educational endeavors as mine has been so far. There is great beauty in learning and finding new things, even if sometimes it feels overwhelming to keep the pace.

Last but not least, I wish to thank my dissertation chair, Dr. Jillian Skelton, for believing in me and for her continuous support with my dissertation—I would probably not be on this last step from completing my educational degree without her. I thank her for pushing me along the way, but in the same time encouraging me to continue, always with a smile. Dr. Jillian recognized when my patience and drive were wearing thin, but she always had a good word to say or advice to give to keep me going. I thank her for HER patience, during the almost year and a half since the start of my dissertation.

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Chapter 1

The Problem or Issue

There is a difference between doing something because it is tradition and doing something because it is proved to help solve a problem. The issue, I believe, that merits research relates to the use of the mathematics diagnostic test that most of the 1st-year college students, at the college campus where I teach, take at the beginning of their programs. Traditionally, this math diagnostic test has been used to lead students into two directions, as explained in the following two paragraphs.

For most programs, one scope of the diagnostic test has been to identify the students at risk in the 1st semester, considering that students with a cutoff score lower than approximately 30% in this test may fail the first math college course. This lengthens the duration of their program of study and/or causes these students to withdraw from their program altogether. The students identified here as at risk are directed to have extra support offered by their class teacher, another professional teacher from the College Math Center, and/or a peer tutor, in order to have them succeed in their first mathematics course and continue their college studies with no delays.

For a few select programs, such as the Game Development program, the diagnostic test is used to place the students into two different math courses. Students who score very high in this test are directed to a higher-level math course (Math 10), while the ones who score less than a cutoff score remain in the basic, introductory math course (Math 9). Even though some program coordinators and I consider that the student placement into these two courses gives all students what they need academically, appropriate challenge and support, the cutoff score used for placement may be too high or too low such that some students placed in the Math 10 course may struggle in this course, while others placed in the Math 9 course may feel not challenged enough.

This study seeks to determine if tradition is backed up by statistical results in this correlational/prediction study.

Background of the Study

The practice of using students' scores in the math diagnostic test that students take before starting their college learning, with the purpose of identifying students at risk and/or for placing students into different college math courses, is not used only at my college campus but at other colleges in Ontario as well. A study by Cluett et al. (2009) showed that another college uses a math diagnostic test during the summer of the 1st-year intakes in order to direct students at risk to a prebusiness or pretechnology math course in the 1st semester of their programs. For the prehealth program in particular, the study presented an analysis of students' scores in the math diagnostic test in relationship with students' high school math grades and with students' success in the college math course, with the purpose of identifying students at risk of withdrawal from the program.

Although these practices of using the math diagnostic test scores to identify the level of math preparedness of new intake college students are used in some colleges in Ontario, the documented need to identify students' numeracy and mathematics preparedness for college was proposed first by a college mathematics study done between the years 2007 and 2011 by York-Seneca Institute for Mathematics, Science and Technology Education. This study comprised all 24 colleges and 72 district school boards in all regions of the province of Ontario and was funded by the Ministry of Education and the Ministry of Training, Colleges and Universities with the purpose of analyzing and informing the mathematics achievement of 1st-year college students from Ontario colleges in relation with their secondary school mathematics background (Orpwood et al., 2012). However, since that study, no efforts were made at the college where I work to develop “a common numeracy assessment tool to be used by all colleges as part of their admission and placement process for all incoming college students,” as recommended in the study (Orpwood et al., 2012, p. 4). From the three college campuses, only one campus continues to use a math diagnostic test that was developed locally by two math professors 20+ years ago.

An examination of the pertaining research literature was undertaken in this study in order to verify what current research from the past 5 years reveals about the relevant points related to the topic of this study. First, the theoretical frame of this study incorporated the social constructivist theory related to students’ learning of mathematics during their high school, followed by andragogy as a philosophy of teaching and learning of mathematics for students in postsecondary education. As this study looks at students’ marks from all their high school senior math courses, literature research took note of any influence the tracking of students into different academic streams at high school brought to the math knowledge of 1st-year college students. Moreover, this prior math knowledge students bring with them to the college was considered based on three representations: using senior math courses marks from high school, high school student GPA, and scores in the math diagnostic test at the college. Therefore, research into the diagnostic tests in education, particularly in mathematics, was necessary to find any information about correlations and/or predictions with student math knowledge measures. Finally, the current literature was researched for high school measures that predict students’ placement in different math courses in postsecondary education.

Further, this study presented and used the research methodology, design, and analysis necessary to answer the research questions and their related hypotheses. Considering that student numerical data were collected in the form of marks in high school math courses, college math courses, and the diagnostic test, the study was purely quantitative. To answer the questions of the study presented further in the Research Questions section, there were introduced two statistical designs for this study: a correlational/predictive design that verified the extent students' marks in the high school senior math courses and/or students' high school GPA correlated and even predicted students' scores in the math diagnostic test; and another design was quasi-experimental in order to find out if students who took both college math courses increased their math knowledge to a similar level as the students who took only the higher-level Math 10 course. For the analysis section, Pearson and Spearman correlation coefficients and multiple regression were employed and then the student *t*-test for independent samples, for the two designs respectively. This study also presented conclusions from the statistical analysis, stated interpretations, answered the study's questions, and presented discussions for further research.

Statement of the Problem

For the Game Development program at my campus, the practice has been to use the diagnostic test for students' placement into two math courses: a basic, introductory math course (Math 9), and a more advanced, technical math course (Math 10). The students who score very high in their diagnostic tests are exempted from taking the Math 9 course, as it has been considered that they demonstrate a math knowledge high enough to be deemed successful in the next math course, Math 10. This 2-year Game Development program at the college comprises a total of four math courses, and by tradition high-level math students may skip the Math 9 course and complete their diploma with only three math courses.

In addition, the math diagnostic test has been used by other programs at my campus, and the math diagnostic scores have given information to the math teachers and program coordinators in these programs about their students at risk, thus supports in the form of tutoring and monitoring have been put in place for them. The question has always been whether this math diagnostic test we use is a valid test, capable of representing students' prior knowledge in math from high school. Moreover, as we use this math diagnostic test to place students in two different level math courses, another question relates to whether this test places students correctly in math classes for their specific background knowledge in math.

This study looked at the Game Development program for 5 years of intake: Fall 2014 through Fall 2018. Students' marks in their high school senior math classes and their GPA upon their high school graduation were considered for the present study as representing students' prior knowledge in math before entering the college; in addition, the admission process at the college bases students' acceptance into college programs on students' high school prerequisites. Students' scores in their math diagnostic test for these cohorts of students were considered in this study, in order to verify the assumption that they demonstrate students' prior knowledge in math. Students' marks in the two college math courses Math 9 and Math 10 also were collected for the same intake years to see if students' placement in the introductory Math 9 course gave the students the corresponding math knowledge in order to succeed in the Math 10 course mandatory for all students.

Purpose of the Study

The purpose of this correlational-predictive quantitative study was to find to what extent, if any, the students' high school math marks and GPA predicted students' scores in the math diagnostic test. The study verified whether indeed this math diagnostic test used for 20+ years at our college campus can statistically reflect (correlate with) students' prior math knowledge from their previous high school education, so that when used for students' placement in different math courses, it supports all students' success in their 1st-year math courses. The inquiry is related to the traditional belief of the math teachers and program coordinators at my campus that the math diagnostic test is necessary. Can tradition be supported statistically, or does it remain just a practice based on teachers' belief that it gives the students what they need: challenge to high-level math students, increased support to at-risk students, and proper math content to medium-level math students?

Research Questions

As discussed previously, traditionally, the math diagnostic test has been applied to the new intake students in most of the college programs at my campus; the scores students obtain in this diagnostic test are thought (traditionally) to quantify these students' background knowledge in mathematics. Students who score low in the diagnostic test are considered at risk, and supports for their math learning are put in place for them from the beginning of their Math 9 course at the college. Students who score very high in the math diagnostic test are given the opportunity to enroll in a more challenging, higher-level math course, Math 10.

Related to students' background knowledge in math when they first come to the college, the only source that a math teacher at the college can base their knowledge of students' math level, is represented by students' scores in the math diagnostic test, as background academic information of students is confidential and retained by the registrar at the college. The main question was whether the diagnostic test represents a good source for teachers' placing students in three categories: at risk of failure if not given supports, medium knowledge in math, and very high knowledge in math.

This study first looked at answering two main questions related to the possible correlation/prediction between students' prior knowledge in math as given by their high school math courses and their GPA, and their scores in the math diagnostic test:

RQ1: Does students' prior knowledge in math (as perceived from students' marks in different level senior math courses) predict their score in the mathematics diagnostic test?

RQ2: Does students' prior knowledge in math (as perceived from students' GPA from high school) predict their score in the mathematics diagnostic test?

The second direction of this study was represented by the accuracy of placing the students who score very high in their math diagnostic test into a higher-level math course (Math 10), while all the other students with medium math knowledge and who are at risk (with supports in place) take the introductory Math 9 course first, and if successful, they take the Math 10 course and continue their college path. The question related to this matter is articulated below:

RQ3: Is students' placement in the introductory Math 9 course helping them be successful and achieve similar marks in their Math 10 course as the students who are placed directly in Math 10?

Chapter 3 of this study presents these main research questions with eventual subquestions, their related hypotheses (H), and each hypothesis with the null (H_0) and alternative (H_1) hypotheses, for the purposes of applying the corresponding methodology for this quantitative study. The only main research question that has two subquestions is the first one (RQ1), as there are two different senior high school groups: college preparation and university preparation. The two subquestions related to the research question RQ1 ask: (a) if students' marks in college preparation high school math courses are significant predictors of students' scores in the diagnostic math test, and (b) if students' marks in university preparation high school math courses are significant predictors of students' scores in the diagnostic math test.

Importance of the Study

The results of this study will benefit not only the Game Development program at our college campus to which the study is mainly directed, but other college programs that presently accept high-level students' placement in the more advanced Math 10 course instead of the programs' own basic math course, even if the Math 10 course is not listed on their programs' outlines. In case the placement based on the math diagnostic test scores is supported statistically by this study, other programs may introduce specifically this Math 10 course in their outlines as an alternative for the high-level math students. Therefore, what is used at our campus because of tradition, could have the statistical recognition from the results of this study, so more programs would possibly follow this trend, for the benefit of all students. Currently, the medium-level and lower-level students get what they need anyway: appropriate math content difficulty and tutoring. If the math diagnostic test is indeed a valid test to represent students' prior knowledge in math, all the high-level students in math in all technical programs at the college would get what they need as well by taking the Math 10 course: more challenge, and this Math 10 course can offer that.

Definition of Terms

There are several terms that have been used throughout this paper: math background knowledge or prior math knowledge, college-bound senior math courses, university-bound senior math courses, GPA, academic preparedness, college readiness, cut-off scores, diagnostic/placement test, diagnostic/placement scores.

Academic preparedness: The academic preparedness refers to students' level of high school achievement as perceived by students' marks in different level math courses, students' GPA, and the type of secondary school attended. Students who are not on the academic track, from vocational schools, and having lower marks in mathematics courses, are considered displaying significantly poorer math performance in colleges (Derr, 2017; Giersch et al., 2021; Leme et al., 2020).

College preparation senior math courses: Tracking system in Ontario's high schools starts in Grade 9, when students are self-registering in either the "academic" or "applied" streams. Students who are in the applied stream in Grades 9 and 10 must follow the "college preparation" stream courses when in Grades 11 and 12 of high school. There are four math courses in this stream: Foundations for College Mathematics (MBF3C), Functions and Applications (MCF3M), Mathematics for College Technology (MCT4C), and Foundations for College Mathematics (MAP4C; Ontario Ministry of Education, 2007).

College readiness: College readiness refers to how prepared a student is to complete college-level courses successfully without significant tutoring; it is represented by a score above the cut-off score on the placement test (McAdams, 2017; Wilson, 2018). Others define college readiness as having a standard high school diploma and successfully passing 1st-year college-level courses, thus the student continues the regular college courses without remediation (Woods et al., 2018).

Cut-off scores: Cut-off scores are points on the math diagnostic test's score scale that determine the student's level of proficiency in mathematics (Zieky et al., 2006). Cut-off scores are used to place students into the regular Math 9 course or the advanced Math 10 course at the college. A smaller than a fixed, low cut-off score in the diagnostic test is also used to identify the students at risk, who are given extra supports in terms of tutoring.

Diagnostic/Placement test: A diagnostic test represents a pen-and-paper or computerized test that is important to measure the level of students' prior knowledge in mathematics, as students come from secondary schools to the college with a great variation in their mathematics skills (Conforme et al., 2019; Shim et al., 2017). Math teachers and program coordinators use these results to make the best decisions related to placing their students in the most appropriate level math courses and supporting the learning of at-risk students with tutoring and learning skills workshops.

Diagnostic/Placement test scores: Diagnostic test scores indicate students' skill level in mathematics, and based on preestablished cut-off scores, placement scores determine whether students are enrolled in the introductory math course Math 9 (with or without intensive tutoring), or in the advanced math course Math 10 in the 1st semester at the college (Wilson, 2018).

High School GPA: GPA represents the weighted mean of all the marks a student obtains in all courses finished in high school. GPA considers each course with the assigned credit points resulting from the number of hours per week and per school year, and the student's mark at the end of each of these courses (Belfield & Crosta, 2012).

Math background knowledge or prior math knowledge: These two concepts are used interchangeably throughout the study. Prior knowledge represents one of the most powerful cognitive predictors of students' academic success in college (Binder et al., 2019; Derr et al., 2018; Smith et al., 2019). The measures of prior knowledge the student is considered to come with from high school to college programs are math courses in senior high school, high school GPA, and the diagnostic test in math prior to starting college programs.

University preparation senior math courses: Similarly, the students who were tracked in the academic stream in Grades 9 and 10 in high school follow the university preparation stream courses in Grades 11 and 12 of high school. There are four math courses in this stream: Functions (MCR3U), Functions and Applications (MCF3M), Advanced Functions (MHF4U), Calculus and Vectors (MCV4U), and Mathematics of Data Management (MDM4U; Ontario Ministry of Education, 2007). It is important to note here that the math course Functions and Applications (MCF3M) is the only senior math course that can be accessed by both streams: university/college preparation in Grade 11, where M stands for medium (Ontario Ministry of Education, 2007).

Delimitation, Limitations, and Assumptions

The following components are discussed further related to this section: (a) the *delimitations* part, which indicates exceptions and reservations in selecting the sample of students; (b) the *limitations* part, which specifies the boundaries for the college program chosen and how the results of the study may or may not be used for other programs; and (c) the *assumptions* part, which presents any biases that the researcher/teacher (me) may have related to choosing the specific college program and the topic of this study.

Delimitations

The participants in this study are the students admitted into the Game Development program at my college campus with the intakes starting with Fall 2014 and up to Fall 2018, included. Every intake has about 40 students, thus there should be about 200 students who participated in this study, from all students who started this program at the college from its inception to present. The Fall 2018 intake is the last one participating in this study, to ensure students’ academic privacy of their past education is respected while students are still at the college completing their program courses.

From these approximate 200 students, in order to ensure consistency in using the same high school senior math courses related to the predictor variables in this study, only students who had high school records from the Ontario province were considered. Students’ marks in specific math courses at the senior level while enrolled in Ontario high schools and their high school GPAs were considered for this study. Any students who finished their high school in another province (Quebec) as well as the international students were not considered in answering the first two research questions of this study, as their academic records from high school are different from the corresponding records of Ontario domestic students. However, all students who took the diagnostic test were considered to answer the third research question of this study. The students who did not take the diagnostic test were excluded entirely from this study, as the diagnostic test represents a variable when answering all three questions.

Limitations

This quantitative study used a sample of participants from the Game Development program at my college campus, and it did not look at other college programs. The first two research questions investigated these students' prior knowledge in math through specific high school academics and the diagnostic test taken in the first math class at the college; this means there was no connection with what students learned in the specific college programs they registered into; only their prior academics were examined. Therefore, the results of this study could be transferred easily to all students coming to the college, independent on their program of choice at the college. However, one limitation can exist here. The majority of students coming to the Game Development program are males, with very few female students. Therefore, the extrapolation of results might be applied more to male students than to female students. Moreover, the third research question in this study referred to students' marks at the end of two math courses: Math 9 that some students take, and Math 10 that all students in Game Development program take as part of their program. The results of the study related to this last research question were limited to this program, as no other program at the college has the Math

10 course mandated, only as an optional, more advanced math course that students with high scores in the diagnostic test may take.

Assumptions

I was personally not only the math teacher for both Math 9 and Math 10 courses in the Game Development program for all the intake years in this study, but also the invigilator for all math diagnostic tests, and the marker of all these tests. I should not have had any bias marking these diagnostic tests for the students in the 5 intake years, or any bias when teaching and assessing students' work in both college math courses. The last semester considered for Math 10 marks was winter of 2019, at which time I was still doing coursework for the doctorate in Education program, and I did not have a defined theme for my dissertation that could have biased me by any stretch of imagination. The results of this entire study should be generalized to all further Game Development program intakes, and the study's answers to the first two questions should be generalized to all programs at the college. However, I should acknowledge that I am somehow biased in that the diagnostic test in math represents a good indicator of students' prior knowledge in math, and the study's quantitative research results verified whether my long-time-assumption was statistically correct.

Nature of the Research

The topic of this study and its stated research questions led the researcher toward a quantitative methodology to collect specific numerical data, design the processes for analyzing the data, and finally analyze the data. The three research questions in the study refer to students' grades in different high school math courses and 1st-year college math courses, high school GPA levels, and scores in a math diagnostic test, all of which, by their nature, represent numerical data, thus quantitative data. The study's designs to analyze this numerical data are characteristic to the quantitative method of research as well. Considering the research questions of this study, there are two different designs that were used: correlational/predictive and quasi-experimental. The first two research questions looked at finding whether a predictive correlation exists between the predictive variables: marks in the senior high school math classes and the high school GPA respectively; and the criterion variable: scores in the math diagnostic test. The prediction design determined the predictive validity of the measuring instrument: the math diagnostic test.

The third research question of the study is about the importance of taking the introductory Math 9 course for medium-level math knowledge and at-risk students, in order to have these students achieve in the more advanced Math 10 course at a similar level as the students who are placed directly into the Math 10 course. The research design used was quasi-experimental, and a statistical *t*-test of the Math 10 marks for independent groups was applied to the two groups: students who took both Math 9 and Math10 courses, and students who took only the Math 10 course. The statistical analysis related to the third question concluded whether the Math 9 course is necessary for medium and weaker students to succeed further in the more complex technical math course Math 10. The conclusion here may also give validity to the math diagnostic test applied to the new intake students at the college and their placement in these two different math courses using the cut-off score.

Summary of Chapter 1

This chapter introduces the traditional way of placing 1st-year college students from the Game Development program into two college math courses: the introductory Math 9 and the technical Math 10 courses. This placement is based on students’ scores in a math diagnostic test, which presumably represents students’ prior knowledge in math as given by their grades in their senior high school math courses and their high school GPA. The purpose of doing this study can be articulated using the question: Is tradition backed up by statistics for student placement?

Next, there are three research questions introduced in this study, which were developed into subquestions and hypotheses in the next chapters; further they were answered using the quantitative methodology processes for design and analysis of the data collected. Terms used frequently also were defined to eliminate any misunderstanding of their use in the paper. In addition, delimitations, limitations, and assumptions of the study are important to fit this study into the bigger frame of students' math learning at the college where I teach, based on their placement into two college math courses and based on a math diagnostic test. This chapter ends with presenting the structure of this study in the next chapters in conformity with the traditional dissertation.

The following Chapter 2 presents a current literature review related to the topic of this study. The main points presented here were the theoretical framework, including social constructivism and andragogy; student tracking in Ontario's high schools and in other countries; the role of prior knowledge in mathematics; diagnostic tests in education and in mathematics in particular; and high school's measures that predict college placement. Each section was thoroughly researched in order to find what other studies found about the use of the diagnostic test as a measure of students' prior knowledge and further placement in college courses. A look at the gaps observed in the literature review through the studies' results and their further recommendations and/or delimitations have given a basis and a starting point to this study.

The next Chapter 3 introduces the statistical methodology that this study employed in order to answer the three research questions posed earlier. The three research questions were presented fully with their subquestions, hypotheses, and null and alternative hypotheses. Two research designs corresponding to the first two research questions and to the third research question respectively were specified and described. Moreover, assumptions needed to be met in order to apply the corresponding research designs were included toward the end of this chapter.

The following Chapter 4 completed the statistical analysis related to the three research questions and their hypotheses. The SPSS statistical software was employed to perform the research pertaining to correlation using Pearson and Spearman analysis. Then multiple regression analysis looked at predicting math diagnostic test scores given marks in different high school senior math courses and high school GPA as predictor variables. Next the *t*-test was used to verify whether there was a significant difference in students' marks in their Math 10 grade between the two cohorts of students: the ones who went directly to Math 10 and the ones who needed to take the introductory course Math 9 first. A discussion of the statistical results is presented in this chapter as well.

Finally, the last Chapter 5 presents an overall analysis and interpretation of the findings in the Chapter 4 of the study and recommends areas for additional research. This chapter ends with a summary of the important chapter points, with a brief restatement of the problem, literature reviewed, research methodology employed, and restatement of study's purpose and objectives.

Chapter 2

Literature Review

Traditionally, students who are admitted to the programs offered by the college campus where I work as a mathematics professor must complete a math diagnostic test in their first math class, with the exception of students admitted to programs that do not have any math course. Math and science teachers and program coordinators at my campus believe that the math diagnostic test can be used to identify the math background knowledge students bring with them to the college from their senior math courses taken in high school, as students take high school math courses at different academic levels: college preparation or university preparation and finish them with marks ranging from 50% to 100%. This study may statistically justify the two trends we have at our college campus:

1. Placing students in different level college math courses based on students' score in the math diagnostic test, believed to identify students' prior math knowledge from high school (as perceived from senior math courses and/or GPA).
2. Identifying students at risk. Traditionally, students with very low scores in the diagnostic test were either placed in a remedial math course (Math 5), which was eliminated a few years ago because of high attrition, or placed in the introductory Math 9 course and given tutoring support, with the hope that their math achievement (their grades in the more advanced Math 10 course) is increased.

Considering the information above, this study looks at answering the following related research questions:

RQ1: Does students' prior knowledge in math (as perceived from students' marks in different level senior math courses) predict their score in the mathematics diagnostic test?

RQ2: Does students' prior knowledge in math (as perceived from students' GPA from high school) predict their score in the mathematics diagnostic test?

RQ3: Is students' placement in the introductory Math 9 course helping them achieve similar marks in their Math 10 course as the students who are placed directly in Math 10?

This chapter researched the current literature from books and articles on the Internet related to the above questions about students' math knowledge. An initial, generic search of the term "students' achievement in mathematics" through Google Scholar yielded 2.45 million results. To narrow the results, articles within the last 5 years were given preference, with the new count of 165,000 articles. Further on, using key terms related to six major categories corresponding to the topic of this study (and presented next) directed my readings to the compatible literature. From those articles, the references with the most citations were recorded and used in this chapter. Other times, the references posted at the end of a study seemed related to my study as well; thus, the most current (within the last 5 years) were selected and used, while the older ones were searched again through the Google Scholar and other related articles were found. A great number from all these articles did not offer the full text on Google Scholar; therefore, I used the regular Google engine to search for those articles, and indeed most were posted as full text in the latter. In addition, the databases accessed through the William Howard Taft University such as ERIC and ProQuest under the Education theme were used, but not many related articles were found there.

The themes I consider related to this study's topic, researched in great depth in the current peer-reviewed literature from the academic sites mentioned above, are the following: (a) social constructivism theory that frames this study; (b) andragogy, as another theoretical frame of this study; (c) high school tracking of students, which sometimes limits students' higher education opportunities; (d) the role of prior math knowledge and its definition in different studies; (e) diagnostic tests in education in general and in mathematics in particular, and how they predict student achievement; and (f) high school indicators that can predict college placement into regular math courses or developmental ones. These six categories used to organize the related articles are presented in great detail next.

Related Theory: Social Constructivism

Literature shows a perceived deficiency in content knowledge in mathematics and science in 1st-year college and university students, and even if the problem has been acknowledged repeatedly, it did not diminish (Baldwin & Squires, 2019; Binder et al., 2019; Derr et al. 2018; McAdams, 2017; Woods et al., 2018). Baldwin and Squires (2019) stipulated that there could be a disconnection between what the students learned in high school and what they learn in their 1st year at college or university. However, this perceived decline of student knowledge related to content level and of analytical skills has led to a widening gap between secondary and postsecondary education (Baldwin & Squires, 2019). For the past 2 decades, the increasing lack of success in mathematics has started to be seen as a roadblock to student’s achievement and further graduation, especially in technical fields, and educational institutions have begun to identify the need to reevaluate mathematics learning and teaching in order to be competitive on the global market (Baldwin & Squires, 2019; Prendergast et al., 2017).

Additionally, the research has begun to shift focus away from *what* mathematics content was taught to *how* mathematics was taught (Baldwin & Squires, 2019). Freeman et al. (2014) analyzed the data on examination scores and failure rates from 225 studies between 1998 and 2010 and compared student performance in university science, technology, engineering, and mathematics courses using traditional lecturing versus active learning. The results showed that university students under traditional lecturing were 1.5 times more likely to be unsuccessful than under a more active learning approach, and the average examination scores increased by about 6% when using active learning (Freeman et al., 2014). Luitel (2019) also found that teaching using student activities is better for students' understanding of the concepts than teaching using lecture. Students need different activities to learn and cannot learn only by hearing what the teacher says and writes on the board. The concept of activity-based learning alone follows the constructivist approach, when students can learn by using their experience and prior knowledge (Belbase, 2014; Luitel, 2019; Simon, 2017). However, especially in mathematics, activity-based learning is not enough for learning; knowledge is constructed by using prior knowledge and experiences while interacting with peers and teachers, using language to build up their understanding, and taking in knowledge in smaller chunks (Finnegan & Ginty, 2019; Luitel, 2019). As Finnegan and Ginty (2019) summarized, this represents the social constructivist approach to learning, with four key principles: knowledge construction, active learning, social interaction, and scaffolding.

Related to the first two key principles of the social constructivist approach to learning, Bonti et al. (2020) argued that there are two different ways in which mathematics can be learned, depending on the kind of knowledge targeted: the instrumental-mechanical and the relational-conceptual types of knowledge. The instrumental-mechanical knowledge can be grasped by learning rules and applying them in practical situations. The relational-conceptual knowledge refers to an understanding of those rules and their application in different contexts (Bonti et al., 2020). Related to the learning of mathematics, the authors also presented the two major types of knowledge defined by Piaget: physical knowledge, derived directly from acting on objects, and logico-mathematical knowledge, where objects represent the condition that permits the construction of knowledge. Binder et al. (2019) introduced the same types of knowledge when they looked for significant predictors of academic success in a physics course in the 1st year of university, but they named them differently: declarative knowledge (prior knowledge of physics concepts) and procedural knowledge (the ability to apply the rules and solve subject-specific problems). These different types of content knowledge are generally based on Bloom's Taxonomy of cognitive understanding, which includes knowledge, comprehension, application, analysis, synthesis, and evaluation (Ralph, 2015). They also follow Piaget's constructivist learning theory, where learners construct knowledge using prior experiences and understandings through an *active* process of dis-equilibration and reequilibration, which continues the reconstruction of knowledge (Bozkurt, 2017; Ralph, 2015).

Therefore, knowledge is not passively received but *actively* built by the learner. The main role of the teacher is to facilitate and guide, but the student is the one who actively participates in his learning, and teaching and learning activities are more effective and meaningful in this way (Belbase, 2014; Bozkurt, 2017; Luitel, 2019). Luitel (2019) introduced the term Activity Based Instruction, which represents a face-to face interaction among students and their teachers, where students actively engage in the lesson instead of just sitting, listening, practicing on paper, and absorbing the content. Class activities are suitable to the specific school subject and age of students, seamlessly integrated into the curriculum requirements and knowledge content to be learned, and effectively engaging students (Luitel, 2019). Another researcher introduced the term Project Based Learning, where learners construct knowledge using authentic, real-world tasks that require longer periods of time, promote interdisciplinary work, and encourage a comprehensive understanding of content (Ralph, 2015). These real-world activities support student experiences and can transfer to future courses and careers. Ralph (2015) mentioned a previous study that was focused on actively building cognitive knowledge by developing a video game; the result was that 87% of students wanted to continue with more video game development beyond that class assignment. Yet another study generated a theory of learning mathematics concepts and built an instructional design approach that permitted the construction of particular mathematical concepts based on particular prior knowledge (Simon, 2017). This Learning Through Activity instructional approach considers specific prior knowledge and learning targets, meaningful learning activities, planning for tasks designed to elicit the activities, cultural factors and artifacts, and the use of symbols and vocabulary.

However, having students engaged in a meaningful activity is not enough for constructing their cognitive knowledge. Researchers found that learning through any class activity, either the Activity Based Instruction, Project Based Learning, Learning Through Activity, or any other is strongly influenced by *collaboration* or *teamwork*, which is the third key principle of social constructivism (Bozkurt, 2017; Luitel, 2019; Ralph, 2015; Simon, 2017). In addition, Ralph (2015) suggested that there is a positive relationship between learning content knowledge and the Project Base Learning in collaborative settings, as students use language to communicate ideas and listen to others' ideas. As Luitel (2019) contended, the social constructivist approach to learning is needed, especially in mathematics, because this is how knowledge is constructed: by frequent interactions among students and their teachers using language to communicate. Luitel (2019) argued that, especially in mathematics, collaborative learning is necessary for independent learning, but the inverse is not true. Learner's knowledge development starts at the social level and goes to the individual's level through a process called "internalization" by Vygotsky (as cited in Simon, 2017, p. 2754). Social constructivism means that both learning, individual and through social interaction, "play pivotal and crucial parts in the learning of mathematics," and the learner is considered "an interactive co-constructor of knowledge" (Bozkurt, 2017, p. 211; Luitel, 2019). Similarly, Bonti et al. (2020) viewed mathematical activity as a social as well as a cognitive phenomenon and specified that depending on the group of learners, math practices can be different.

As Belbase (2014) contended, both teachers and students must take their responsibilities and understand their roles in teaching and learning following a social constructivist model. The role of teachers is to create an interactive environment for their students, so they actively construct knowledge by interacting with their peers and the teacher in the class and out of the class (Belbase, 2014). Baldwin and Squires (2019) presented mathematics instructors at university level who were asked to change the way of teaching the subject from a lecture format to using social constructionist models that included real-life problem exploration and solving. Given the very high enrollment numbers and unprepared instructors for the change in pedagogy, the reality at the university level remained that classes continued to be taught mostly using lecture format. The role of student-learners is viewed as constructors or coconstructors of knowledge together with the teacher and their peers, and not simply as receivers of knowledge (Belbase, 2014). Thus, teachers have more opportunities to facilitate learning in nontraditional classrooms when students work together, named "team-based learning" by Allen et al. (2016, p. 51). Students usually perceive teamwork as valuable for their learning of subject concepts, and as reflecting possible future experiences (Ralph, 2015).

Unfortunately, not all students experience positive group work. Ralph (2015) suggested to look at the structure of groups in technical university classrooms and especially at the ideal number of group members in order to be effective in terms of working on the content and in terms of conflict situations, because collaborative learning refers to both group and individual successes.

A fourth and last key principle of social constructivist approach to learning, as seen by Finnegan and Ginty (2019), is represented by *scaffolding* the content to be learned. Teachers play a central role in students' learning not only through the use of language and identifying students' prior knowledge and how they learn, but by fostering their learning through scaffolding (Bozkurt, 2017). Educationally, scaffolding represents an instructional method where the teacher structures the steps of a mathematical task, models the thinking through and solving, and provides the necessary support until the learner is able to do the task independently (Christmas et al., 2013). Scaffolding the content makes learning easier and more meaningful, manageable, effective, and efficient. Related to scaffolding, an Irish higher educational institute looked at its online platform Moodle, whether it supported teaching and learning from the social constructivist point of view (Finnegan & Ginty, 2019). The study found that Moodle does not promote social constructivism principles in general, but it does facilitate limited conceptual scaffolding. The conclusion was that most teaching and learning occurs in the classroom, and the most common way in which Moodle is used remains the delivery of content and course administration. In other research, Simon (2017) developed an integrated theory of mathematics learning and teaching with four steps through the Learning Through Activity program. The last step was about designing a sequence of tasks in order to bring out the previous activity—this can be seen as scaffolding the learning activity in small, successive chunks, meant to help students build their understanding of the phenomenon or concept.

Related to learning and meaning making of mathematics, Bonti et al. (2020) reported the importance of social interaction and verbal communication, as maintained by Vygotsky in his social constructivist theory. According to Vygotsky, social interaction among learners when they participate in learning activities produces the internalization of the collective meaning making and the development of individual cognition, if students are guided by the teacher or a more skilled peer (as cited in Bozkurt, 2017). As Bozkurt (2017) remarked, internalization is at the basis of the Vygotsky's social constructivist theory. As all learning is social in Vygotsky's theory, in order to understand learners' mental processes when acquiring knowledge, the teacher must look at their social origins (Bonti et al., 2020). In addition to social context, interpersonal communication and interaction through the use of language are necessary for learners' cognitive development, as per Vygotsky's theory (Belbase, 2014; Bozkurt, 2017). Moreover, the development of individual knowledge is promoted by cooperation in the individual's zone of proximal development concept introduced first by Vygotsky (Bozkurt, 2017).

According to Vygotsky, the zone of proximal development represents "the distance between the actual development level, as determined by independent problem solving, and the level of potential development, as determined through problem solving under adult guidance or collaboration of more capable peers" (as cited in Christmas et al., 2013, p. 371). The zone of proximal development is critical for mathematics teaching and learning because the learning can be mediated and scaffolded by the teacher or a knowledgeable peer if they work in learner's zone of proximal development (Bozkurt, 2017). As Christmas et al. (2013) pointed out, the major challenge for a teacher is identifying the level of content to be taught that is desirable to each learner in her classroom. As the zone of proximal development differs from one student to another, students' ability to capture the new content oscillates between boredom and confusion. As Christmas et al. (2013) stipulated, research showed that teachers cannot give the required support in mathematics to each individual student; therefore, students fail at acquiring knowledge despite having the potential to learn.

Cooperation and scaffolding are fundamental concepts of the zone of proximal development concept (Christmas et al., 2013). As Vygotsky argued, with peer collaboration or some teacher support, learners are able to accomplish more mathematical tasks independently than they are able to perform alone (Bozkurt, 2017; Christmas et al., 2013). Therefore, when effectively used, the zone of proximal development concept can improve learners' mathematics achievement.

Related Theory: Andragogy

For many graduates of secondary school, the community college system is the only opportunity to enter higher education because of lower costs and the open-door policy (Acosta et al., 2016). However, a growing number of graduates of high school admitted into colleges and even universities need developmental courses in mathematics and English, designed for students who have weaker academic backgrounds (Acosta et al., 2016; Bahr et al., 2019; Derr, 2017; Morales, 2018; Qin, 2017). A study done by Acosta et al. (2016) at Odessa College looked at 290 students who had completed developmental math courses. The students were considered nontraditional, adult students, with an average of 5.37 years since they finished high school. When considering the age of students, even if the time from completing high school was not a predictor of success in the college math courses, the study suggested that andragogy may be important (Acosta et al., 2016). Andragogy is defined as a teaching and learning philosophy for adults and considers the specific needs of adult learners and their engagement in college classes (Henschke, 2008; Merriam et al., 2007; Odigiri et al., 2020). A key element of andragogy stipulated by Acosta et al. (2016) is that adult students learn to take control of their own learning experience; therefore, they become more self-directed and intrinsically motivated to learn (Mann & Willans, 2020; Merriam et al., 2007; Odigiri et al., 2020). Thus, adult students with lower academic background benefit from a developmental program that focusses on each student and how they use their own different learning experiences to understand the content (Acosta et al., 2016).

Another study considered 10 students taking one or two math classes in the Skills for Tertiary Education Preparatory Studies at CQ University in Australia (Mann & Willans, 2020). The study found that students were able to learn how to become self-directed when their teachers *tailored* teaching to their needs. Data from the interviews were examined to identify how andragogy helped students become self-directed learners when studying mathematics (Mann & Willans, 2020). They concluded that tailoring is the key to ensure that all students understand the importance of learning mathematics in their current education and for their future lives, which increased their motivation to learn. In addition, Ford et al. (2017) indicated that adults improve their learning when teachers personalize their teaching and adapt it to their students' abilities. As Holton et al. (2001) argued, andragogy is an individual learning framework; however, there must exist a collaborative effort between learner and teacher, as teaching is inseparable from the student's learning experiences (Mann & Willans, 2020).

Discussions about how adults learn and what distinguishes adult education from education of children and education in general appeared in the U.S. in 1968 with the work of Knowles (Merriam et al., 2007). As Holton et al. (2001) and Merriam et al. (2007) argued, there does not exist a single, encompassing theory about adult learning, but there are a few models or frameworks that contribute to understanding of adults as learners. Knowles was the first one to popularize the term in the U.S. in 1970s and 1980s (Holton et al., 2001; Merriam et al., 2007).

However, Alexander Kapp authored the term andragogy about a century earlier in 1833 in Germany; also, the first time andragogy was introduced into an U.S. article was in 1926 by Linderman, as Henschke (2008) stipulated.

Henschke (2008) embarked on a lengthy study of about 240 major works published in the English language about andragogy and found that the term andragogy has been used with different understandings related to adult learning. For some, andragogy identifies with adult learning, or designates strategies and methods used to help adults learn. For others, andragogy represents "a theory that guides the scope of both research and practice on how adults learn, how they need to be taught, and elements to be considered when adults learn in various situations and contexts" (p. 2). For even others, Henschke shared, andragogy is seen as a set of tools for teaching adults, and yet for others, andragogy represents a scientific discipline that examines how adults can be brought to "their full degree of humanness" (p. 2).

Knowles introduced the term andragogy in the U.S. as a contrast between the European definition of andragogy, "the art and science of helping adults learn," and pedagogy, "the art and science of helping children learn" (Merriam et al., 2007, p. 84). Knowles proposed six assumptions about the adult learner that constitute the basis of andragogy: (a) when people mature, their dependent personality moves toward a self-directing one; (b) adults accumulate a great deal of experience, which represents their resource for learning; (c) an adult presents a readiness to learn, which is dependent on the developmental tasks they perform in the society; (d) adults are more problem-centered than subject-centered; (e) adults' learning is motivated intrinsically, not extrinsically; and (f) adults need to know why they learn something (Merriam et al., 2007). Merriam et al. stipulated that Knowles' andragogy represents the best-known set of assumptions about how adults learn; it is most popular with educational practitioners, as its assumptions give some “intuitive validity” to the use of andragogical practices with adults (p. 104). Ford et al. (2017) argued that these andragogy assumptions should be viewed along a pedagogical-andragogical continuum, from the child learner to the adult learner. Moreover, Allen and Withey (2017) considered the college classroom as a mixture of ages, experiences, life stages, and different educational expectations and called it “a marriage between pedagogy and andragogy” (p. 51). In practice, as seen by Holton et al. (2001), andragogy perceives the heterogeneity of learners and of learning situations. Merriam et al. (2007) also stipulated that educators must be aware of the prior knowledge their adult learners possess in specific areas and the different experiences they bring to educational institutions. Moreover, Merriam et al. (2007) said that educators must plan programs that assist “novices” to become “experts” in the areas significant to them, and not only “more experienced novices” (p. 405).

As most adults have a much greater prior knowledge and experience than children, reflected in their declarative and procedural knowledge, educators must ensure that this knowledge is connected to learning new materials (Merriam et al., 2007). Also, Ford et al. (2017) indicated that learning is improved when weaknesses are identified and addressed immediately, also part of the andragogy process’s elements identified by Knowles: to evaluate students’ learning and support their specific needs. Related to mathematics, understanding individual students’ problem areas requires detailed and specific diagnostic information (Fox & Artemeva, 2017). Teachers and administrators use this diagnostic information to support students’ learning further in tertiary education, either in the curriculum courses or in developmental ones.

Student Tracking

Allocating students into different classrooms based on prior performance, called *ability tracking* or *ability grouping* in the U.S. (Fu & Mehta, 2018; Leme et al., 2020; Yassin et al., 2015), represents a practice that many countries are using not only in tertiary education but at secondary and primary education as well. Also called *streaming* or *school stratification* in other countries (Roller & Steinberg, 2020; Yassin et al., 2015), the separation of students in classes by ability is a teaching strategy commonly used in schools that allows teachers to focus on a more-narrow range of students and tailor their teaching styles to meet the needs of those specific students. The purpose of streaming is to help both high- and low-performing students achieve their learning goals at an appropriate pace and level. For their low-ability students, teachers can focus on foundational skills, while for their high-ability students, they can spend time on more advanced material that usually is omitted in heterogeneous classrooms (Collins & Gan, 2013).

However, there is a considerable variation in results from empirical work that assessed the effect of ability tracking on the level and distribution of achievement. Most of the time, streaming may benefit students of certain ability levels while hurting others. Roller and Steinberg (2020) measured the long-run effects of early school stratification on individual PISA test scores in Lower Saxony, Germany. They found that when performance-based tracking was preponed from Grade 7 to Grade 5, the performance of low-ability students was reduced and high-ability students was increased, while the average achievement was not changed, as the effects compensated for each other. Fu and Mehta (2018) used data from an early childhood longitudinal study based on more than 20 students from each school in the U.S. from kindergarten through middle school. They found that banning tracking increased the achievement of students with prior scores below the median by 2.2% of a standard deviation in the outcome test score and reduced the achievement of students with prior scores above the median by 4.2% of a standard deviation.

On the other hand, another study done by Collins and Gan (2013) examined all third grade students in the 2003–2004 school year who became fourth graders in 2004–2005 from 135 different schools in Dallas Independent School District. The results uncovered that tracking by previous performance significantly improved students' math and reading scores, suggesting that the net effect of streaming is beneficial for both high- and low-performing students. Another study by Yassin et al. (2015) investigated the effect of streaming on secondary school students' performance in additional mathematics in a secondary school in Brunei. The mean scores in their final examination indicated that the students had improved significantly in their overall performance, where the highest percentage of improvement was for the students who scored a D letter grade in their prior examination. Giersch et al. (2021) also used a longitudinal data set that followed one complete cohort of North Carolina public school students from middle school through high school and into the University of North Carolina system. They found that academic track placement in secondary school had a stronger relationship with students' outcomes in the 1st year of university than any other measure of school performance.

Yet another study used a difference-in-differences cross-country analysis of 20 countries. Leme et al. (2020) found that the choice of academic or general tracking in upper-secondary school systems did not account for differences in either the mean performance or the inequality of students' test scores across the 20 countries studied. However, countries that exhibited high variance of students' test scores (PISA) before entering secondary school showed low variance on the posttest (PIAAC) and vice versa. Thus, decisions of tracking are important, as many schools would implement valuable changes in their educational system without large amounts of additional resources, if a particular sorting mechanism is found to be especially beneficial (Collins & Gan, 2013).

Tracking students by ability continues in the tertiary education as well. The U.S. university and college systems usually require all students to take a version of a math online placement test (ACCUPLACER, COMPASS, ALEKS adaptive test, or other locally developed web-based diagnostic test in mathematics: PERT, ACT Compass placement examination, TSIA, etc.) after admission, in order to assess whether they are capable of completing college-level work. Literature shows that, based on their prior mathematics knowledge as assessed using these examinations, students were directed either into college math courses or into a few developmental math courses (Bahr et al., 2019; Derr, 2017; McAdams, 2017; Qin, 2017; Wilson, 2018). As Van den Broeck et al. (2019) argued, math examinations (such as the Dutch Cognitive Ability Test: CoVaT-CHC) before entering the universities in Flanders, Belgium, as a prior knowledge indicator, were the most predictive for student academic achievement in university.

Most university programs in Canada have their admission requirements based on senior high school academic courses in mathematics, sciences, and English fulfilled with a specified minimum grade and on the high school GPA, both of which have proved to be relevant predictors for academic success in math and science subjects at university. Therefore, there is no need for other examinations of math prior knowledge, as the results of the academic track show a significant relationship with students' university outcomes (Giersch et al., 2021). Giersch et al. (2021) suggested that taking most of the high school math and science courses at the honors level rather than taking them all at the regular level was associated with a 0.3-point increase in the university's 1st-year GPA.

However, the senior high school math and science courses selected by students may be a result of students' and parents' choice of applied or academic streams starting even in Grade 9 or earlier (Ontario Ministry of Education, 2007; Quebec Ministry of Education, 2004). This type of tracking might not divide students by ability at the beginning of their secondary school but rather by area of interest and specialization of students and their parents (Leme et al. 2020). However, during the 4 years of high school tracking, students get oriented toward university, college, or vocational by the admission requirements of specific university and college programs.

Ability tracking is controversial, as some students of certain ability levels benefit from it, while others are affected negatively. The category of students that may suffer the most because of tracking in the public system of education is represented by low-achievement students. Low-ability students generally have a limited knowledge of both conceptual and procedural math knowledge (Derr et al., 2018). Sometimes, college programs with heavy mathematics course loads require the new college students coming from either high school, adult education, or the workforce to take placement exams in mathematics prior to the start of their 1st school year, in order either to direct them to developmental math courses or to inform further teaching and tutoring in the regular math courses (Cluett et al., 2009; Leeds & Mokher, 2019; Parker et al., 2018). The reason behind administering these placement tests is to stream the new students into the math courses corresponding their current level of knowledge, because students placed in courses that are too difficult for them are unlikely to do well (Bahr et al., 2019; Derr, 2017; Qin, 2017). Therefore, accurate placement of students could potentially increase their chances of academic success in their 1st-year college courses.

Fu and Mehta (2018) suggested that in heterogeneous classrooms from kindergarten to Grade 8 in the U.S. schools, peer quality was estimated to be a significant determinant of achievement: keeping all other inputs constant and increasing peer quality by one standard deviation (determined by a Grade 3 examination) caused an average of 20% standard deviation increase in the Grade 8 examination score. In homogeneous classes as a result of tracking students by ability, there is a peer effect, which causes a particular student's achievement to be influenced by the quality of peers in their classroom as well. Related studies indicated that having around better students, low-ability students experience positive achievement results (Collins & Gan, 2013; Giersch et al., 2021; Roller & Steinberg, 2020). A study by Yassin et al. (2015) found that in a higher-level math class in a secondary school in Brunei, lower-ability students scored higher as a result of being among high-ability peers.

On the other hand, Elsner and Isphording (2017) found that independent on how smart a student is, their ordinal rank in a specific homogenous classroom (called "small fish in a big pond") significantly affects their educational outcomes later in life: finishing high school, attending college, or a 4-year university (p. 787). This study suggested the pervasiveness of choosing an academically better secondary school or a higher-level math course: a low-ranked student in that class underinvests in their human capital even if they have a high ability compared to other students of the same age. In addition, the effects of tracking individual students depend not only on students' ability and peer influence but also how early (the school year) it was implemented and on the school climate in which it was implemented (Fu & Mehta, 2018).

Role of Prior Knowledge

One major characteristic of students coming from secondary schools to colleges and universities is represented by a great difference among students' basic skills and knowledge. Especially in science-related fields (such as engineering and computer science), a decreased number of students nowadays are able to finish their 1st school year, which results in high dropout rates and a declining number of students graduating from these fields (Binder et al., 2019). Therefore, an early identification of good predictors of 1st-year college grades can provide proper support to at-risk students by either directing them into developmental courses, offering them in- and out-of-class tutoring, or adjusting course delivery if possible.

Literature showed that one of the most powerful cognitive predictors of academic achievement is prior knowledge (Binder et al., 2019; Derr et al., 2018; Smith et al., 2019). There are a few measures of student prior knowledge that have been considered as being influential to students' academic success in college and university: the academic level of senior classes in mathematics, the domain-specific courses in high school, high school GPA, and the diagnostic test in math prior to starting college or university programs. Studies showed that, based on their prior mathematics knowledge as assessed using some of these measures, and especially the math diagnostic examinations, students went directly into regular curriculum math courses or were directed to noncredit, developmental math courses (Bahr et al., 2019; Derr, 2017; McAdams, 2017; Qin, 2017; Wilson, 2018). Students with a high level of prior knowledge in mathematics and in other domain-specific subjects were found to acquire new knowledge easier, especially in the 1st year of their higher education (Binder et al., 2019; Derr et al., 2018; Rach & Ufer, 2020; Smith et al., 2019).

In a study by Binder et al. (2019), two measures of prior knowledge, high school GPA and diagnostic test scores in domain-specific knowledge (applied to all students at the beginning of their tertiary education), were considered for a correlation analysis. The study found two significant predictors of academic success in the 1st-year university physics course: prior knowledge of physics concepts (declarative knowledge) as well as their ability to apply it and to solve subject-specific problems (procedural knowledge). When tried, high school GPA did not contribute to the analysis significantly if the knowledge types were included (Binder et al., 2019). For the university biology course, there was only one significant predictor of academic success: prior declarative knowledge. Additionally, considering domain-related prior knowledge, Rach and Ufer (2020) investigated which prior mathematical knowledge was necessary for academic success when studying in a mathematics university program. Using a math diagnostic test, they found that both conceptual and procedural knowledge types associated to calculus (thus heavily focused algebra content) significantly correlated with the 1st-year analysis course.

Another study showed that both prior mathematical knowledge and prior programming experience significantly predicted students' success in a computer science program (Smith et al., 2019). An aptitude diagnostic test (PAT) given at the beginning of the program evaluated students' knowledge of fundamental programming concepts and conceptual solution design. In addition, the results from Derr's et al. (2018) study support the existing literature that domain-related prior knowledge plays a dominant role regarding success in engineering. However, neither the secondary school GPA nor the math diagnostic pretest did predict student academic achievement in the 1st year of the engineering program. The study suggested though to have discussions with the students about the pretest scores and math content to raise awareness about the role of prior knowledge in mathematics and the importance of completing the pretest to the best of their ability.

However, mathematics diagnostic test scores, as a measure of students' prior knowledge in mathematics, have been found predictive of 1st-year success in a science university in Malaysia (Shim et al., 2017). Moreover, Morales (2018) presented in his study that preuniversity students in Uruguay made their own decisions to register into math courses at the insufficient-, enough-, or advanced-knowledge levels, and not as recommended by their results in a computerized and adaptive test in mathematics. Future research was suggested in order to find out whether the diagnostic test was a good predictor of student performance in the initial courses of mathematics at the university (Morales, 2018).

As Woods et al. (2018) argued, “Mathematics is a gatekeeper to future educational success” (p. 439). However, many 1st-year students in tertiary education have considerable knowledge gaps especially in mathematics (Derr et al. 2018; Orpwood et al., 2012). Studies present a higher risk to succeed in postsecondary education for students whose math prior knowledge is limited: the ones who were streamed in applied or college preparation courses at the beginning of their secondary school and did not perform well there, and the ones who attended a vocational school (Derr et al., 2018; Leme et al., 2020). Cassells (2018) found that even at-risk students can succeed in university programs given an early warning system, corresponding tutorial classes in the subject, and an adjustment of teaching depending on the level of achievement of students. However, researchers argue, vocational stream is designed to prepare students for entering the labor market following their high school graduation, not for university or college admission (Leme et al., 2020).

Van den Broeck et al. (2019) argued that it is logical the education prior to enrolment at university be the most predictive one for students’ further academic achievement. There are a few academic measures that were studied as means of evaluating student’s readiness for higher education: (a) the following section looks at the predictive value of the diagnostic test applied to the incoming students of colleges and universities; and (b) the next section looks at other cognitive measures of students’ achievement in tertiary education: high school GPA and grades in specific senior high school courses. The aim of a multitude of studies in higher education has been to find a good predictor of students’ success in postsecondary education, as students coming to colleges and universities show a great diversity in their prior knowledge, especially in mathematics. It has been a practice of universities and colleges to apply a postadmission diagnostic test in mathematics to 1st-year students, and based on the cut-off score of the placement test, place them in different mathematics courses. Given students' diversity, students placed in courses that are too difficult for them are unlikely to do well, while those placed in courses that are too easy for them are likely to underperform because of boredom and lack of motivation (Qin, 2017). Therefore, accurate placement of students could potentially increase their chances of academic success in their first college courses and reduce the need for developmental education (Qin, 2017; Wilson, 2018).

Diagnostic Tests in Education

The term "diagnosis" is frequently used in the medical field and defined as a careful, critical study of an illness to determine its nature (Hunter & Brown, 2018; Shim et al., 2017). In education, diagnosis represents an evaluation to find information about students' mastery of relevant prior knowledge and skills toward any subject of learning, strengths and weaknesses, as well as preconceptions or misconceptions about the material (Conforme et al., 2019; Morales, 2018; Shim et al., 2017). Zhan (2020) introduced the term "learning diagnosis" because the diagnostic test objectively quantifies students' current learning status and provides diagnostic feedback (p. 1). It is considered important to administer a diagnostic evaluation usually during the orientation week or 1st few weeks of the semester to define the competencies that the student has at the beginning of the year (Conforme et al., 2019; Orpwood et al., 2012; Shim et al., 2017). Teachers and administrators use these results to make the best decisions related to teaching strategies, teaching the curriculum content, supporting students' learning through tutoring, or directing them to specific remedial courses.

In some cases, some diagnostic tests are not applied at the beginning of the school year: large-scale diagnostic platforms are typically given to students from kindergarten to Grade 12 toward the end of their school years and are used for program evaluation, resource allocation purposes, and, hopefully, to inform instruction in the future (Clark et al., 2018). An example of this diagnostic assessment in the U.S. is represented by the standardized tests in math and reading proficiency every year from third through eighth grade and once in high school, used to make important decisions about students, educators, schools, or districts, most commonly for the purpose of accountability. In Ontario, Canada, the Education Quality and Accountability Office test is applied to students toward the end of their third and sixth grades, with the purpose of increasing the quality of education in Ontario, while also being used to make plans for future improvement, with the results transmitted to all stakeholders. Moreover, high stakes tests such as the SAT, ACT, LSAT, MCAT, and others are used to diagnose students' prior knowledge in different domains for college and university admissions. In Canadian universities, tests such as the LSAT, MCAT, and GMAT are used for specific universities' admission, beside secondary school's academic performance and other requirements. Once admitted, students select their majors at entry and begin their degree programs for their 1st year.

The diagnostic evaluation is a pen-and-paper or computerized adaptive (or not) test that aims to measure the level of cognitive development that students have achieved in basic skills. Students come from secondary schools to colleges and universities with a great variation in their prior knowledge in mathematics. Mathematics mastery and mathematical literacy ability have a great importance to students' future educational success because they support the learning and mastery in science fields (Pertiwi et al., 2019; Woods et al., 2018). Furthermore, mathematics is essential because the majority of courses require the use of mathematical concepts in their learning, and it is very important for educators to detect students' basic knowledge before teaching them the new content (Shim et al., 2017; Woods et al., 2018; Zhan, 2020).

The 1st-year success in university and college is directly linked with students' attrition and retention, and significant numbers of students drop out of their 1st-year undergraduate programs (Fox & Artemeva, 2017). Early detection of students' mathematical weakness allows educators to conduct intervention or remedial instruction to assist weak students to cope better in the regular, curriculum courses (Derr, 2017; Shim et al., 2017). If the diagnostic test results indicate that a student may be at risk, the student is directed to meet with an academic counsellor, and a plan is put in place for subsequent academic support. Early intervention and support for at-risk students have been determined to make a meaningful difference in students' ultimate academic success. Pertiwi's et al. study (2019) showed that specific learning models coupled with pre- and postdiagnostic assessment increased the achievement of students' mathematical literacy ability. Cassells (2018) found that students who were at the highest risk of failure benefited the most from the application of an early diagnostic assessment, corresponding tutorial classes, and an adjustment of teaching by altering the approach to content, assessment, and learning to benefit the student. Conforme et al. (2019) argued that when diagnostic test results show that some students need an academic reinforcement, teachers develop instructional strategies according to the reality, interest, and needs of those students.

As a postadmission evaluation of students' prior knowledge to predict students' academic success in university and college, the math diagnostic examination has been used; however, with controversial results. A few studies found a strong positive correlation between mathematics diagnostic test results and students' mathematics achievement in the 1st year of university. Shim et al. (2017) found that students' prior knowledge in mathematics affected their performance in the 1st year of university in Malaysia. Derr et al. (2018) noticed that poor values in any of students' achievement-related variables, such as the diagnostic test, represent a risk factor regarding tertiary achievement. Derr (2017) also argued that the most consistent predictor of students' academic achievement in the 1st year of a technical university in Germany was the students' results in a web-based diagnostic pretest in mathematics, which was also found significantly related to students' final GPA. Morales (2018) remarked that a mathematics computer adaptive test taken prior to the beginning of the university in Uruguay determined the level of performance of students. However, as students chose their own direction into different levels of university courses despite the test feedback, the study was not able to show whether the computer adaptive test was a good predictor of students' performance in the initial courses of mathematics at the university. Another study done at a college in Ontario, Canada, wanted to verify math teachers' perception that the math diagnostic test was a good predictor of incoming students' success in the PreHealth Sciences program, as failure in the program was often a result of failure in the 1st semester mathematics course (Cluett et al., 2009). They found that math diagnostic scores lower than 40% were indicators of student failure in the 1st semester mathematics course, while scores above 70% in diagnostic were indicators of withdrawal from the 1st semester mathematics course and from the program altogether.

However, the prior knowledge level of a student may be over- or underestimated by the diagnostic test score, and making inferences about the knowledge and skills associated with the score may not accurately describe students' knowledge and skills (Walker, 2017). McAdams (2017) reflected that expecting diagnostic exams to predict student success is a common misconception, as these are achievement tests that measure a student's basic academic skills level at the time of testing. Moreover, Van den Broeck et al. (2019) contended that the predictive value of the diagnostic test was “disappointing,” and urged to make adjustments to the composition of the test (p. 1005). Other studies found also that the mathematics diagnostic test alone was not a clear predictor of students’ performance in the 1st year of university (Cluett et al., 2009; Leeds & Mokher, 2019; McAdams, 2017; Wilson, 2018). Walker (2017) mentioned the need for extra information about the students’ academic achievement before the test, such as their math course grades in high school or their high school GPA, in order to characterize students’ academic knowledge and predict academic success. In addition, a study by Scott-Clayton (2012) found that diagnostic exams are more predictive of who is likely to do well in college-level coursework than of who is likely to fail.

Considering the opportunity of evaluating students’ knowledge fast and classifying them immediately at different levels of performance, the math diagnostic test at the beginning of the postsecondary school aims to rank-order test takers along a continuum in a given domain, skill, or on a global ability, above or below a cut-off score, Toprak and Cakir (2018) argued. This provides guidance to the administration about two academic trajectories for students in the 1st year of colleges or universities: to take regular mathematics courses or to start with developmental courses in mathematics (Morales, 2018; Toprak & Cakir, 2018). For this reason, the diagnostic test has been also called a placement test. As McAdams (2017) specified, placement testing is common in community colleges in the U.S.; it started as a college response to an increase in the number of students with poor performance on standardized tests. Students who score below a cut-off score in the diagnostic test are placed in a series of developmental courses before they can register into the college-level courses for the program. If a student achieves a score above the cut-off score on the placement test, that student is considered college ready. However, there are many assumptions underlying this simple placement rule: how the results are interpreted, how the cut-off scores are established, and what student abilities a college believes are needed for college-level courses (Belfield & Crosta, 2012). Logue et al. (2016) found that currently about 68% of 1st-year students in U.S. community colleges and about 40% of 1st-year students in universities take at least one developmental course in mathematics, reading, and writing.

Research conducted by Scott-Clayton (2012) found that following the placement test, a large number of students, who could have passed regular curriculum courses, were incorrectly placed in remedial courses. The problem is that most students who take remedial courses before they can enter their regular postsecondary classes never complete them (Logue et al., 2016; Logue et al., 2019; Wilson, 2018). For this reason, taking developmental classes is considered to be “the largest single academic block to college student success” (Logue, 2019, p. 308). Leeds and Mokher (2019) suggested to research the cut-off scores at which students are considered either college ready or needing developmental courses.

Logue et al. (2019) stated that all traditional prerequisite remediation was either made optional, or abolished recently in multiple states and institutions of higher education in the U.S. Therefore, other strategies are recommended for improving the pass rate of college students: streamlining remedial material so students focus on topics needed for their other coursework, condensing the remedial work, and/or combining remedial material with topics within other courses. Related to the contextualization of developmental math content into the content of other courses, a study suggested that introducing the developmental math content into a first semester course, that is, Introduction to Sociology, had a positive impact on some students’ understanding of the math topics from a concurrent course: Elementary Algebra (Parker et al., 2018). However, the researchers raised awareness about the small number of participants enrolled concurrently in the two courses, which limits the generalization of the study. Another study showed that a corequisite mathematics course Stat-WS was effective at increasing students' success over time (Logue et al., 2019). Following the academic performance of the students who had been assessed in need of remedial elementary algebra but were registered in the statistics course instead, the result was that students were as good as, or even better than, the students within traditional remediation courses. Significantly more students who took Stat-WS than did not graduated the 3-year program (Logue et al., 2019).

In Canada, in technical and professional university and college programs students begin the specified curriculum courses immediately. However, some higher education institutions apply a postadmission math diagnostic test so that students who score low on the diagnostic test either register for extra tutoring in their curriculum math course or take a preparatory math course in the 1st year concurrently to the regular math courses (Fox & Artemeva, 2017). Understanding individual students' strengths and weaknesses in mathematics skills requires detailed and specific diagnostic information. Fox and Artemeva (2017) contended that the criteria used to develop a meaningful diagnostic test have to be specific to the target domain (i.e., the undergraduate engineering in their study), rather than a generic, one-size-fits-all approach. Pertiwi et al. (2019) specified that appropriate assessments to identify students' difficulties more specifically need to be further explored. Van den Broeck et al. (2019) mentioned that although the cognitive part of the diagnostic test applied to 1st-year university students in Germany was found to have no predictive value, the diagnostic test was not useless. In order to increase the predictive value of the diagnostic test, they proposed that adaptations be made, because it could be a useful tool for students who want to have an idea about their possible future study success, thus feedback for students may be given.

In order to ensure a good predictability of students' academic success in tertiary education, other studies looked at the composition of the diagnostic test, not only at the aggregated total test scores (Javidanmehr & Sarab, 2017; Toprak & Cakir, 2018). One study distinguished between the diagnostic test and the cognitive diagnostic assessment (CDA), where CDA combines the cognitive psychology of learning with advanced statistics for scoring, in order to provide reliable diagnostic classifications for the skills required in specific fields (Toprak & Cakir, 2018). The term "cognitive" in CDA refers to assessments that rely on cognitive models, while the "diagnostic" focus of CDA is to obtain "fine-grained and detailed information" about the mastery status and performance of the 1st-year university students (p. 248). In addition, another study contended that the cognitive diagnostic assessment is constructed such that students' cognitive subskills are explicitly defined and targeted in the test construction phase, in line with the institutions' instructional goals (Javidanmehr & Sarab, 2017). CDA is concerned with understanding the response patterns related to different cognitive processes, skills, and knowledge structures that help answer the questions correctly (Toprak & Cakir, 2018). Moreover, the study by Zhang (2018) revealed that the cognitive diagnostic assessment can effectively report learners' progress in an algebra university course for both content knowledge and thinking skills.

Another view about the diagnostic examination was introduced by Hunter and Brown (2018). They highlighted that learners often do not understand the need for diagnostic assessment and do not see the value of it. Moreover, a test may create anxiety for students, especially for those who had negative experiences with tests in their prior education. Hunter and Brown (2018) remarked that how assessments are conducted is more important to both learners and teachers than the specific assessment tool. They advised to avoid the term assessment and also to implement it at an appropriate time, but not during the initial meeting with the students.

High School Measures That Predict College Placement

A growing number of undergraduate students worldwide lack basic mathematical skills and are not ready for the demands of a technical university or college program, Derr (2017) and McAdams (2017) argued. College readiness was defined as having a standard high school diploma and successfully passing 1st-year college-level courses, so the student continues their regular college courses without remediation (Woods et al., 2018). However, the high school education does not prepare students with the basic academic skills to be successful in college-level courses (McAdams, 2017; Woods et al., 2018). In addition, the mathematics concepts in the first mathematics course in college challenge many students, and failure in the college program is often linked with failure in the 1st-semester mathematics course (Cluett et al., 2009; Derr et al., 2018; Giersch et al., 2021). Still, Derr et al. (2018) and Van den Broeck et al. (2019) contented that the level of academic education prior to enrolment at university is the most predictive one for further academic achievement. The matter of students' prior academic education refers to students' level of high school achievement and the type of secondary school attended. Students from vocational schools, not in the academic track, and showing poor achievement in general and especially in mathematics courses, are considered displaying significantly poorer math performance in colleges (Derr, 2017; Giersch et al., 2021; Leme et al., 2020)

Related to student success in colleges and universities, the limited research to date has indicated that measures of high school achievement (GPA, math and science courses) are more accurate in recommending how students be placed in postsecondary courses (either remedial or curriculum) than the cut-off scores based on postadmission diagnostic tests, as they result in fewer students being under- or overplaced in mathematics courses. A study from a state college in Florida found that approximately a quarter of students were misplaced in math courses because of the cut-off score of the placement test (Leeds & Mokher, 2019). In their study, the researchers argued that switching from test-based placement to high school GPA-based placement may have smaller effects on misplacement. Another study found that high school GPA had the highest predictive value for students' academic success at a community college, while the diagnostic test scores predicted student success only when students were placed in courses with academic support, such as tutoring (Qin, 2017). However, for some college students, a placement test may be the only mechanism to determine appropriate college courses, considering that graded achievement and tested achievement still show a moderate correlation generally (Woods et al., 2018). These students may not have high school transcripts, or they are not considered being current, especially if students are returning to school after many years of being away from the education field.

A few studies found that students' high school GPA alone was a strong predictor of students' academic success in university. A study involving five Flemish universities found that students' prior GPA was a strong predictor of the 1st semester, 2nd semester, and final GPA in university, while the beginning diagnostic test did not show any correlation with students' achievement in university (Van den Broeck et al., 2019). Another study involving students at a technical university in Germany also found that prior achievement measured by secondary school GPA was a valid predictor of final GPA and of student retention (Derr, 2017). Yet, another study by Bahr et al. (2019) found that cumulative high school GPA was the most consistently useful predictor of performance on different levels of math coursework at community colleges in California. Similarly, a study involving the statewide community college system in the city of New York found that high school GPA correlated significantly with college credit accumulation and also had a strong correlation with college GPA: students' college GPA were approximately 0.6 units below their high school GPA (Belfield & Crosta, 2012). They also determined that the placement test scores were positively, but weakly, associated with college GPA.

Some other studies found that another measure of high school achievement was predictive of students' success in their 1st year of postsecondary and beyond: grades in selected knowledge high school courses (Binder et al., 2019; Cluett et al., 2009; Smith et al., 2019). A study comprising students from the Pre-Health Sciences program from a college in the Ontario, Canada, found that some senior high school math courses (MAP4C) correlated strongly with students' GPA at the end of their program, while the math diagnostic test alone was not a clear predictor of students' success in the Pre-Health Sciences mathematics curriculum (Cluett et al., 2009). Another study found that students' knowledge of physics concepts from high school and their ability to apply them predicted achievement in the 1st year in science-related programs at university (Binder et al., 2019). Yet another study identified the significance of prior mathematical and programming experience in high school in predicting students' success in computer science programs (Smith et al., 2019). Moreover, Woods et al. (2018) observed that students with higher levels of high school preparation were predicted to pass postsecondary courses at higher rates: 54.4% of students who earned the college Algebra 2 course credit passed an Algebra 1 course, but 66.7% of students with advanced math coursework from the secondary school passed the Algebra 2 course.

However, McAdams (2017) and Scott-Clayton (2012) advised that only one assessment method could not represent the best predictor of students' success. The use of "multiple measures" is more effective at measuring an individual's college readiness and future college success than separate measures such as the individual placement test score, the high school GPA, or grades in selected high school courses (Wilson, 2018, p. 92). Woods et al. (2018) also recommended the use of multiple indicators, including course grades and noncognitive factors to predict students' success in college. Moreover, a study at a community college found that using multiple measures (diagnostic test, high school algebra course grades, and high school GPA) for student placement was more predictive of student success than using either variable by itself (Qin, 2017). Even though the cumulative high school GPA was found to be the most consistent predictor of performance at community colleges in California, a combination of students' high school GPA and passing specific secondary mathematics courses was considered more useful for signaling competency across all mathematics skills needed in the mathematics curriculum at the college (Bahr et al., 2019).

Studies also showed that current methods of placing first-time community college students into developmental courses based only on the diagnostic test as a measure of placement resulted in a large number of misplaced students, and some students did not really need to complete the remedial courses to be successful in college (Leeds & Mokher, 2019; McAdams, 2017). The placement problem mostly relates to the students at or somewhere near the cut-off score of the diagnostic test (McAdams, 2017; Wilson, 2018). Researchers urged the placement advisers to consider multiple measures when assessing students' readiness for curriculum courses at the college. Using multiple measures was determined to place more accurately students into appropriate courses, so that there were more students successfully completing their assigned courses (McAdams, 2017; Scott-Clayton, 2012; Wilson, 2018). A study by Scott-Clayton (2012) suggested that using multiple measures to make placement decisions could reduce severe misplacements by about 15%. Related to that, researchers advised that additional studies involving multiple measures are needed to determine which multiple measures show the most accurate placement for 1st-year postsecondary students (Bahr et al., 2019; Wilson, 2018).

Summary of Chapter 2

Considering the perceived need of the researcher to justify statistically the two trends at the college to: (a) consider the diagnostic test as a good information about students' prior knowledge in math, and (b) place students into different difficulty college math courses as a result of the students' scores in the diagnostic test; and the contradictory results of related studies presented in the above literature research, this study looks to answer the following three questions:

RQ1: Do students' marks in their senior high school math courses predict students' scores on the diagnostic math test? Students' tracking in their senior high school courses into college preparation and university preparation must be considered here.

RQ2: Do high school student GPAs predict students' scores on the diagnostic math test?

RQ3: Considering the students' placement into two college math courses (Math 9 and Math 10) based on their diagnostic test scores, is there a significant difference (in their final scores in Math 10) between the students who entered directly Math 10 and the students who took Math 9 prior to taking Math 10?

The six themes researched in the academic literature and presented in this chapter are regarded by the researcher as necessary in order to find answers to the above questions. In order to frame the teaching and learning of mathematics, two theories were researched in literature:

social constructivism and andragogy. Related to the social constructivism theory, current research found a shift in focus from what mathematics content is taught to how mathematics is taught: have students engaged in meaningful activities while working collaboratively (Bozkurt, 2017; Luitel, 2019; Ralph, 2015; Simon, 2017). However, to my knowledge, students' past learning experiences from high school might not have been framed in social constructivism, as students come to the college with an increasing lack of prior math knowledge, which impedes their college success and graduation.

All students who enter the technical programs at college are older than 18 years old, and some of them are more mature, having worked a few years after they graduated from high school. Therefore, another theory may also frame the teaching and learning of mathematics and science at college: andragogy. Knowles was the first one to introduce andragogy in the U.S. and defined it as the teaching and learning philosophy for adults (Henschke, 2008; Merriam et al., 2007; Odigiri et al., 2020).

Another theme related to researching literature relates to the effect of ability tracking from high school on the level and distribution of mathematics achievement in postsecondary education. The present study looks at 1st-year college students who come from two different high school academic levels: the college preparation and the university preparation. Decisions of tracking are important in postsecondary, as students come to the college from different high school streams, and they continue to be streamed by ability in their 1st year at the college into different math courses. Literature research shows that most of the time streaming may benefit some students but can hurt others (Giersch et al., 2021; Leme et al., 2020; Yassin et al., 2015).

Yet another theme that needed literature research was the concept of prior math knowledge. The present study was set to investigate which measure, if any—the marks in students' high school senior math courses, their high school GPA, or their scores in a postadmission math diagnostic test—can be considered to reflect students' prior math knowledge. Currently, students' scores in the math diagnostic test at the college account for streaming them into the introductory or into the advanced math courses in their 1st postsecondary year. The matter that this study wants to answer relates to this math diagnostic test as a measure of students' prior knowledge.

The fifth theme presented above looks closer at the use of diagnostic tests in education and in particular the mathematics diagnostic tests and the decisions related to place students in different level college math courses based on their diagnostic scores. The results of the present study for my college are important mainly for increasing students' retention in the 2-year Game Development program at my college campus. Ford et al. (2017) indicated that learning is improved when weaknesses are identified and addressed immediately. The present study verified whether placing weaker students (with lower scores in the math diagnostic) into the introductory math course raised their math knowledge to a level needed to perform in the more advanced math course similarly to the stronger students.

The last theme researched in literature looks at high school indicators that can predict college placement into different math courses. Literature research to date indicates that measures of high school achievement (GPA, math and science courses) are more accurate in recommending how students are placed in postsecondary courses than the scores in a postadmission diagnostic test (Belfield & Crosta., 2012; Derr, 2017; Leeds & Mokher, 2019; Qin, 2017; Van den Broeck et al., 2019). Considering the impossibility of acquiring students' high school credentials from the registrar at the college on a yearly basis because of perceived teacher-student conflict of interest and lack of academic privacy, the present study tried to verify if scores in the math diagnostic test can be used successfully for placement purposes in two different level math courses, as being the only measure available at every student intake.

The next chapter, Chapter 3: Methodology, presents the statistical steps necessary to answer this study's research questions. The chapter starts with the research questions and related hypotheses, including the null and the alternative hypotheses for each hypothesis. Next, the method and study designs employed are presented in great detail, and assumptions for each design are stated. Details about the measuring instrument used, data collection and data analysis and procedures are exposed, and finally information about the next steps of the study is introduced.

Chapter 3

The Research Methodology

This chapter is divided into seven sections that detail the main features of the research process: the methodology of the study; the statistical design of the study; the research questions, hypotheses, and description of the statistical methods used to analyze them; the survey instrument used with its validity and reliability; the protocol for data collection and analysis; the sample and population employed for this study; and a summary of this chapter while introducing the next chapter of this study.

Restatement of Research Questions

This chapter presents the three research questions and their corresponding hypotheses (H), each with their null (H_0) and alternative (H_1) hypotheses:

RQ1: Do students' marks in their senior high school math courses predict students' scores on the diagnostic math test? There are two different senior high school groups: college preparation and university preparation.

H1: Are students' marks in college preparation high school math courses significant predictors of students' scores in the diagnostic math test?

H_0: College preparation math course marks do not correlate significantly with the diagnostic test scores.

H_1: College preparation math course marks correlate significantly with the diagnostic test scores.

H2: Are students' marks in university preparation high school math courses significant predictors of the diagnostic math test?

H_0: University preparation math course marks do not correlate significantly with the diagnostic test scores.

H_1: University preparation math course marks correlate significantly with the diagnostic test scores.

RQ2: Do high school student GPAs predict students' scores on the diagnostic math test?

H3: Is the high school GPA a significant predictor of the diagnostic math test scores?

H_0: Students' high school GPAs do not correlate significantly with students' diagnostic test scores.

H_1: Students' high school GPAs correlate significantly with students' diagnostic test scores.

RQ3: Considering the students placement into two college math courses (Math 9 and Math 10) based on their diagnostic test scores, is there a significant difference (in their final scores in Math 10) between the students who entered directly Math 10 and the students who took Math 9 prior to taking Math 10?

H4: Students who take Math 9 prior to Math 10 finish their Math 10 course with similar marks as the students who are directed to Math 10 only.

H_0: Students who take Math 9 prior to Math 10 finish their Math 10 course with no significant difference in marks than the students who are directed to Math 10 only.

H_1: Students who take Math 9 prior to Math 10 finish their Math 10 course with significantly different marks than the students who are directed to Math 10 only.

Research Design

As mentioned previously, the tradition at my college is to use the math diagnostic test as a measure of students' prior math knowledge, in order to identify at-risk students and to place

1st-year students into two different level math courses. The purpose of this study is to verify this tradition statistically, with the scope of validating the math diagnostic test and continue and expand its uses in the programs at my college. This section articulates and describes the methodology and the research designs of this study.

Description of the Methodology

This study, as all educational research, requires that some kind of data is collected and analyzed using a specific methodology, so that this process can lead to a conclusion, and the information gathered can be interpreted properly. Depending on the research question(s) posted in each study, the types of data collected, and the method to analyze the data, there are two distinctive methods used in statistics: quantitative and qualitative approaches.

At the basis, a quantitative approach means that numerical data is collected and analyzed in order to describe, explain, predict, or control a specific phenomenon of interest (Gay et al., 2009). At the inception of a quantitative study, the researcher must state hypotheses to investigate and procedures to collect the data and analyze thereafter. The advantage of having a quantitative study relies in the power of numbers that can measure a specific event, investigate relations, and draw conclusions about the initially stated hypotheses of the phenomenon under scrutiny. Conclusions from a quantitative study may be generalized to other studies given similar conditions related to sample and population, independent and dependent variables, and design and analysis of the study, while looking at the limitations of the study as well.

Although it emerged slowly during the last few decades, the qualitative approach to research looks at the collection and analysis of nonnumerical data, such as narratives, visual, historical, ethnical, and other particular phenomena of interest. In qualitative studies, all content is positioned in a particular context, and different people may have different interpretations of the respective phenomenon. This makes the interpretation relevant for that specific aspect and context but very different from other contexts (Gay et al., 2009).

Considering the topic of this study and its stated questions and hypotheses, this research used the quantitative methodology for collecting the necessary numerical data, designing the processes for analyzing the data, and finally analyzing the data. The three questions in the study refer to student scores and marks in different courses and tests, which by their nature represent numerical data, thus quantitative data. The study designs (specified next) to analyze the data are characteristic to the quantitative method of research as well. The conclusions drawn toward the end of this quantitative study may give legitimacy to the practice of administering math diagnostic tests to the new intake students at the college and may validate the placement of the students in different math courses.

Design of the Study

Considering the three research questions to be answered by this study, there were two different designs that were used in the study: correlational/ predictive and quasi-experimental. A correlational study is said to describe an existing condition, so that the data collected will determine whether, and to what degree, a relationship exists between two or more numerical variables (Gay et al., 2009). Depending on the calculated value of a correlational coefficient called Pearson coefficient *r*, the found relationship can be used to make predictions if this coefficient is big enough (a decimal close to 1.00 usually, or to -1.00). The independent variables are called predictor variables in this case, and the dependent variable is called the criterion variable. If this correlational coefficient is close to zero for a variable, it means that the specific variable does not predict the criterion variable, so the variable must be dropped from further examination.

The quasi-experimental design represents one form of the experimental design in which the two groups cannot have students assigned randomly as in a true experimental design, but the groups already exist. This is the case with the two groups of college students assigned to the two math classes, Math 9 and directly to Math 10, based on their scores in the diagnostic test. As the quasi-experimental design compared the two groups by using students' marks in the same math course, Math 10, the groups are considered as equivalent as possible, which strengthens the study and reduces validity threats related to the interactions between selection, maturation, history, and testing. The quasi-experimental design controls for most of the threats to internal and external validity of the study (Gay et al., 2009).

The first question looks at finding if a predictive correlation exists between the predictive variables, marks in the senior high school math classes, and the criterion variable, scores in the math diagnostic test. If students' grades in their different level (college vs. university preparation) high school math courses correlate strongly with diagnostic test scores, then these variables are called predictor variables for the criterion variable: scores in the diagnostic test. The prediction design determined the predictive validity of the measuring instrument: the diagnostic test.

When the correlational testing shows several predictor variables correlating moderately well with the criterion variable, then a multiple correlation (regression) is applied that usually shows an even higher correlation coefficient. Students' marks from multiple math courses in high school generally predict the scores in their diagnostic test better than marks from individual math courses (Bahr et al., 2019; Qin, 2017). The data collected and analyzed using a correlational/predictive design, such as the multiple regression, measure the predictive validity of the diagnostic test—this design was used in answering the first question of the study. Similarly, the second question of the study was using the correlational/predictive design, where the predictor variable was represented by the students GPA in high school, and the criterion variable was students' scores in their math diagnostic test.

The third question of the study asks if taking an introductory math course (Math 9) before going into the technical math course (Math10) raised students' math knowledge from a lower level of prior math knowledge to the level of becoming as successful as the students with an initial higher level of prior math knowledge, who entered directly the Math 10 course. The statistical design necessary to use here is the quasi-experimental study, where the grades at the end of Math 10 course from the two groups (students with both Math 9 and Math10 courses and students with only Math 10 course) are applied to a statistical *t*-test for independent groups. If no significant difference is found to exist between the two sets of grades, then the study could conclude that taking the introductory math course helps the group of weaker students in math raise their math knowledge to a level similar to the other group of students who are exempted from Math 9; therefore, the conclusion is that the Math 9 course is necessary for them to succeed further in the more complex technical math course Math 10.

Survey Instrument

The statistical instrument used in this study is represented by a math diagnostic test that is applied to all students who are accepted into any college program that has a math class in the 1st semester. The test is administered to the students in their first math class in their 1st semester by their math class teacher and is marked by that math teacher. The students are given 1 full hour, they must work individually, and they are allowed a scientific, nongraphical calculator.

The purpose of this diagnostic test for some programs, such as the Game Development program, is twofold: (a) the test is used for student's placement into their regular math course for the program or into a higher-level math course, depending on their scores below or above a cut-off score of 85% respectively; (b) students' low scores in this test raise a student-at-risk flag for the corresponding math teachers, so these students are given more supports for their learning: teacher, professional, and peer tutoring. It has to be mentioned that not all programs that have math courses will move students to the higher-level math course Math 10, only the technical programs, because the Math 10 course is designed as a technical math course. Students in the business programs, for example, take only the math courses designed for their program; however, teachers must know their students at risk, which are categorized at my campus by a lower than 30% score in the math diagnostic test.

The math diagnostic test was created from the collaboration between a mathematics and physics teacher who taught in the Game Development and General Arts and Science programs at the college and another math teacher who taught math in all other programs and in the General Arts and Science program. The inception and use of the earlier version of the test started a few decades ago (about 20+ years ago, as my colleagues at the college mentioned; the two teachers who created it are longtime retirees). The earlier version of the diagnostic test, I was told, contained more dry, algebra examples and not many real-life word problems.

The latter version of the test appeared about 10 years ago, before my hiring at the college, from a need to adapt the test to the changes in the program curricula and the student math knowledge needed at the time related to problem-solving skills. Some pure arithmetic and algebra questions were kept the same in the latter version of the test, while other dry, arithmetic examples were transformed into relatively simple word problems, with one or two steps involved in solving them. The test has always contained 32 questions from arithmetic, algebra, linear and quadratic functions and graphs, 2D and 3D geometry, and trigonometry of right triangles. This latter version has been administered to all 1st-year students since then, in most programs at my college campus. An exemplar is attached in the Appendix A of this dissertation study.

Validity of the Instrument

Validity of a measuring instrument (as the math diagnostic test) refers to the degree the instrument measures what it is supposed to measure, so the teacher/researcher can interpret its scores appropriately. Validity represents the most important characteristic of a test, since without validity, data interpretation is either inappropriate or without meaning (Gay et al., 2009). This math diagnostic test being used in all programs that have a math course at our college campus is applied to every student intake with different purposes. First, for all programs, this diagnostic test has informed the programs' coordinators and the math teachers about their students at risk, so tutoring and other supports were put in place for them. Second, for a few programs, mostly technical, the diagnostic test has been used for student placement directly into a more advanced math course. Finally, for the Game Development program that has more advanced math courses in subsequent semesters, the diagnostic test has given the higher-level math students the opportunity to skip the introductory math course, so that these students would graduate with three math courses instead of four. The diagnostic test was updated about 10 years ago to follow the programs' and math curricula changes to include simple work problems together with pure arithmetic and algebra questions. The total number of questions and the total score has been the same for the two versions.

By answering the first two questions of this study (RQ1 and RQ2), the research may establish the predictive validity of the current math diagnostic test: if students' prior math knowledge, as seen by their marks in the high school senior math courses or their high school GPA, can predict students' scores in the math diagnostic test. In this way, math teachers and program coordinators are aware of and address their students at risk or a math course exemption by using students' scores in the diagnostic test, without the need to ask the registrar for students' high school credentials, which can be seen as introducing a lack of academic privacy for the students in many ways. The registrar does not approve the release of student prior academic credentials unless strong reasons are presented.

Answering the third question of this study (RQ3) may give substance to the use of this math diagnostic test in the Game Development program at the college: as a placement test to place students in different level college math courses using students' math abilities, as demonstrated by their scores in this diagnostic test (above or below a cut-off score). The administration of this math diagnostic/placement test at our college campus for all new intake students (provided their program has at least one math course) has been the tradition without any statistical support, only based on math teachers' and coordinators' experience and belief of being a good tool for both teachers and students. For teachers, separating the strong students into the more challenging course produces more homogenous classes that are better to plan for and conduct and better for taking care of the weaker students. For students, the diagnostic/placement test tries to give the weaker students what they need: more support and guidance, and to the stronger students: the needed challenge, while recognizing their solid prior math knowledge by exempting them from taking the basic math course.

Reliability of the Instrument

The purpose of any test, as a measuring instrument of a student abilities, is to provide valuable data to the teacher and researcher, thus the measuring instrument used to collect this data must be both valid and reliable. A test's reliability represents the degree the test consistently measures the same thing. A high reliability means that the test measures consistently the same thing, correct or wrong (Gay et al., 2009). While validity is about the appropriateness of a test, reliability is about the consistency of the test scores. It is important to mention here that a test that is valid is always reliable, measuring consistently what it is supposed to measure, but a test that is reliable is not always valid because it might measure the wrong thing (Gay et al., 2009).

A test's reliability is expressed numerically as a reliability coefficient found by using correlation. A coefficient value close to 1.00 means high reliability of the test, with minimum measurement error, while a lower, close to zero coefficient, will indicate a nonreliable test and also low validity for the test. For the diagnostic test used in this study, there was no possibility to calculate this reliability coefficient, thus the reliability, as the diagnostic test data saved by the teacher-researcher contains only total scores for every student, and not students' answers to the individual test questions (32 questions). Therefore, whether there is a consistency of scores in answering the individual questions cannot be answered from these secondary data. The test's reliability is just connected to the validity of the test, as a test that is valid is always reliable, as defined above.

Data Collection

The data that were collected and used in this study represent only secondary data, which corresponds to 5 years of intake at the college, between Fall 2014 and Fall 2018. Thus, data collection used cluster sampling, where the five clusters of students corresponded to the 5 years of intake mentioned above. The study considered these 5 years and not others from the following two reasons: First, the teacher/researcher started working at the college only in 2013; therefore, the 1st year of getting through all content taught in the two math classes was exempted from the study. Thus, the Fall of 2014 was the 1st year for this study's data collection. Second, the last year of intake considered was the Fall of 2018 to ensure that all students graduated from their 2-

year program, and even the ones who might have an extended time in the program have had the opportunity to graduate by the time data collection and analysis began. In this way the use of secondary data did not interfere or influence students' academics while still in college.

The college stipulates the minimum requirement of high school academics for acceptance in the Game Development program at the college to be one Grade 11 college preparation math course. However, most students coming to this program graduate from high school with more than one Grade 11 math course at different levels (college preparation [C], university preparation [U], or college/university preparation [M], the last one being considered by teachers as in fact college preparation level). In addition, some students may have even completed Grade 12 math courses at these different levels. With this in mind, first secondary data collected were represented by students' grades in their high school senior math courses, called a predictor variable for the purposes of answering the first question in the study. The few Grade 11 math courses students may have graduated from are MBF3C (Mathematics of Personal Finance), MCF3M (Functions and Applications), and/or MCR3U (Relations and Functions), and the few Grade 12 math courses students may have finished are MAP4C (Foundations for College Mathematics), MCT4C (Mathematics for College Technology), MDM4U (Mathematics and Data Management), MHF4U (Advanced Relations and Functions), MCB4U (Advanced Functions and Introduction to Calculus), and/or MCV4U (Calculus and Vectors; Ontario Ministry of Education, 2007).

Second, students' GPA at their graduation from high school should represent another part of the secondary data collected from the registrar at the college to represent another predictor variable for the study in order to answer the second research question. However, because the college registrar does not collect students' GPA from high school and only individual marks for all senior high school courses, a Senior Average Grade was calculated and used instead of the traditional GPA. This information is mentioned when answering the research question RQ2, as it may influence the results of the statistical analysis.

Third, math diagnostic test scores obtained from the students from their first math class at the college were collected from the researcher. The teacher, who is the researcher, printed all tests, supervised all the test taking, scored all tests, and archived all the information in a separate folder on the college computer from her office. This folder is password protected and contains all students' marks in their diagnostic tests since her hiring at the college, for each year intake of students in this program and in other programs at the college. These data were used as a criterion variable for investigating the first two questions of the study, and as the basis for student placement in the two math courses: Math 9 and Math 10 for the purposes of answering the third question of the study. Finally, the students' marks in the two math courses: Math 9 and Math 10 recorded by the registrar at the college for the Fall 2014 through Fall 2018 intakes represented the last secondary data used in this study, where students' grades in the Math 10 course were used to answer the third question of the study, with its related hypothesis.

All students in the Game Development program under study had the same teacher who supervised and scored their math diagnostic tests and taught the two college math courses Math 9 and Math 10. This represents a strong point of the study, as there was not much variability in the teaching of the two math courses, Math 9 and Math 10, from the point of view of personality, preparedness, experience, teaching style, methods of evaluating students, and expectations for her students; there was only one teacher: the researcher. As the Game Development program is a 2-year program at the college, and the last intake in the study was set to be Fall 2018, all the students had finished their program by the time this collection of data and analysis of the data commenced—thus, no ethical issues related to teacher-student conflict of interests, or related to students' right of their academic privacy while still studying at the college existed.

The secondary data collected from the registrar at the college and from my diagnostic tests folder from teacher/researcher's office computer were analyzed using the two different quantitative designs mentioned above, correlational/predictive and quasi-experimental, in trying to answer the three questions set up in this study. The first two questions determined any correlation that was predictive between (a) the predictive variables: the high school students' marks in senior math courses or their GPA respectively, and (b) the criterion variable: scores in their diagnostic test at the college. Related to data analysis procedures, simple correlations were investigated and multiple regression was used to establish the predictability between these variables. By answering these two questions, the predictive validity of the diagnostic test was observed as well.

The third question looks at the importance of offering the Math 9 course to weaker students in math for increasing their math knowledge to the level of the stronger math students as measured by these students' marks at the end of their Math 10 course. The statistical design used was quasi-experimental, and the data analysis procedure that was implemented was the *t*-test for independent groups. The two independent groups were represented by (a) the cohort of students taking Math 9 prior to taking the Math 10 course, and (b) the cohort of students registered directly into Math 10 after taking the diagnostic test. The dependent variable was represented by the students' marks at the end of their Math 10 course, for the two groups of students. Data analysis results were explained and presented in tables and figures specific to the individual and multiple procedures applied in this study, and posted in the Chapter 4: Data Analysis, and also listed at the beginning of this dissertation paper under List of Tables and List of Figures.

Comments, conclusions, and discussions related to the results of these analyses are part of the Chapter 5: Conclusions and Discussion of the dissertation paper.

Preparation of Data

As specified above, the three research questions in this study were answered using two different quantitative designs: correlational/predictive and quasi-experimental. For completing the data analysis required in Chapter 4, this study used the SPSS statistical software, version 25. The first two questions of the study and their related hypotheses asked to identify a predictive correlation between the predictor variables: students' marks in their senior math courses at high school and/or students' high school GPA, and the criterion variable: students' scores in their math diagnostic test in the first math class at the college. The predictor variables were collected from the registrar at the college for the 5-year intakes Fall 2014 through Fall 2018, and the criterion variable was released from the teacher/researcher's office computer. Once these data were collected and the variables paired, the study used Pearson correlation to determine the strength and direction of a linear relationship between these two continuous variables: independent/predictor variables and dependent/criterion variables. In order to identify if Pearson correlation can be used with this data, a number of five assumptions must be met, as described below.

Assumptions needed to be met for using Pearson correlation analysis are:

1. Data collected must be continuous: all data are numerical and continuous, as they are represented by grades, test scores, and GPA values.
2. Identified variables must be paired: both variables: predictor and criterion are paired. The correlation is done at this step between predictor/independent variable and the criterion variable.

3. The pairs of data are checked for a linear relation by using a visual method: a scatter plot.
4. The scatter plot must show no significant outliers: once the data are collected and presented in scatter plots, any possible significant outlier is identified, and decisions about eliminating it from the data or keeping it further for analysis are made.
5. The data must show bivariate normality: a test for normality (Shapiro-Wilk) will be used in SPSS, and *p*-value will be compared with the cutoff value of 0.05. In case *p*-value is smaller than 0.05, which means the test of normality failed, data needs to be analyzed using a nonparametric test of correlation; therefore, the Spearman coefficient is calculated and used further to interpret the results, not the Pearson coefficient.

Once a correlation between these variables is identified, to find if a prediction can be made between predictor variables and the criterion variable is represented by the multiple regression analysis. This analysis must meet eight assumptions, from which five are similar to those related to the above Pearson correlation design—for this reason, answering the first research question was done by using multiple regression analysis only.

Assumptions needed to be met for using multiple regression analysis are:

1. The criterion variable must be continuous: it is continuous, as the variable is represented by test scores.
2. The predictor variables must be continuous: both course grades and GPA represent continuous variables.
3. The data must show independence of observations (residuals): using the SPSS software, the independence of observations is checked using the Durbin-Watson

statistic.

4. There has to exist a linear relationship between each predictor variable and the criterion variable, and between all predictor variables and the criterion variable. Visual methods to determine linearity relate to scatter plots, as employed above for establishing a correlation relation.

5. The residuals are equal for all values of the predictor variable: the variances along the line of best fit from the scatter plots at point 4 are checked to remain similar along the lines of best fit.

6. Data must not show multicollinearity: no two or more predictor variables should be highly correlated to each other.

7. Data must not have significant outliers: as discussed above, the scatter plots and SPSS analysis will show any significant outliers, if any, that will be dealt with when data analysis begins in Chapter 4 of the dissertation.

8. Check that the residuals are approximately normally distributed: either a histogram with superimposed normal curve or a Normal Q-Q Plot of residuals is employed.

After running the multiple regression procedure and testing that the data meet the assumptions of a multiple regression (or a linear regression for RQ2), a number of tables were generated that contain all the information needed to report and interpret the results of the regressions, which are part of Chapters 4 and 5 of this dissertation paper respectively.

The last question of this study was related to the quasi-experimental statistical design to check that there was no significant difference between students' marks in the Math 10 course for the two groups of students: the ones who took Math 9 prior to Math 10, and the ones who only took the Math 10 course. The test used for these two independent variables was a statistical *t*-test for independent samples, with six assumptions that needed to be considered. These assumptions relate to the choice of the study design and the characteristics of the collected data.

Assumptions needed to be met for using *t*-test for independent samples analysis are:

1. There is only one dependent variable measured at the continuous level: students' marks in the Math 10 course.
2. There is one independent variable that consists of two categorical, independent groups: the two independent groups are the two cohorts of students: the students who took both math courses Math 9 and Math 10, and the students who took only Math 10.
3. There exists independence of observations: there is no relationship between the math marks in each group of the independent variable or between the groups. This is true indeed, as there are different participants in each group.
4. There should be no significant outliers in the two groups of independent variables in terms of the dependent variable: marks at the end of the Math 10 course. For verifying this assumption, box plots for the two sets of data (Math 10 marks) are drawn, and outliers identified; the decision of keeping or eliminating them is dealt with when data are analyzed.
5. The dependent variable should be approximately normally distributed for each group of the independent variable. It is considered that an approximately normal curve for each of the two groups would be showing valid information for the *t*-test if the two distributions have a similar skewness (either moderately positive, or moderately negative). This testing uses the Shapiro-Wilk test of normality. In case this test fails to show an approximate normality of the dependent variable for the two independent

variables, the recommended *t*-test for independent variables is nonparametric, such as the Mann-Whitney test.

6. The variance of the student grades in Math 10 is equal in each group of independent variables. Depending on the values of variances in the two groups, the researcher choses the most appropriate *t*-test for independent samples: for equal variances or unequal variances.

After running the independent-samples *t*-test procedure (or the alternative, nonparametric *t*-test if the assumption of normality fails for the dependent variable in the two groups) using the SPSS software, there were a number of tables that contained all the information needed to report the results of the *t*-test. At this point in data analysis, the researcher must have all tables and values necessary to start reporting the results, interpret them, and answer the three research questions of this study.

Population and Sample

The participants at this study were a sample of students who were admitted into the Game Development program at the college between the years 2014 and 2018, from the full population of all students who entered this program since its inception at the college. Considering that each intake into this program consists of about 40 students, the sample for the 5 years considered in this study added up to about 200 students. The majority of students admitted into the program are domestic, coming from either Ontario’s high schools, or from Quebec. However, every year there is a small number of international students as well. For the purposes of answering the first two questions of this study, as the data related to prior math knowledge are represented by students’ marks in their high school senior math courses and/or high school GPA, some students from the total of about 200 did not have these credentials available with the registrar at the college: the students from Quebec and the international students. Therefore, these students were not considered for answering the first two research questions of the study. In addition, the students who did not take the math diagnostic test were eliminated from the study entirely, as the diagnostic test scores were used as variables for all questions in the study. For the purposes of answering the third question, all students who had taken the diagnostic test and took the Math 9 and/or Math 10 courses in the 1st year were considered, even if they were students from Quebec or international students.

The student population in this program is generally 18 years old, having just finished their high school; however, there is a very small number of students every intake year who worked for a few years after their high school graduation. All these students were considered in this study, given that marks for their high school senior math courses and high school GPAs were available from the registrar at the college. As for gender, the Game Development program attracts mostly male students with the rare occasions when one or two female students enter the program.

Summary of Chapter 3

This study is thought to give the researcher the confirmation that the current practice at the college is worthy to be continued: for some technical programs, testing students' prior math knowledge (perceived as their marks in the high school senior math courses and/or GPA) with the purpose of placing them in different level college math courses; and for all college programs, identifying students at risk. As specified in the research of related literature, there is a perceived deficiency in math and science content knowledge for the 1st-year college and university students (Baldwin & Squires, 2019; Binder et al., 2019; Derr et al., 2018; McAdams, 2017; Woods et al., 2018). Therefore, placing weaker students in a basic math course in the 1st semester of their postsecondary education should support their learning of basic math concepts and prepare them for the following math courses.

In order to answer the three questions of this study posted at the beginning of this chapter and their related hypotheses, first, the researcher collected student data from 5 consecutive year intakes related to students' marks in math courses at high school and college, students' high school GPA, and students' scores in their math diagnostic test, thus numerical data. Second, the data collected were analyzed using two quantitative designs: correlational/predictive and quasi-experimental. These data analyses for these designs required the use of the SPSS software for the following tests: Pearson and Spearman correlational testing depending on meeting or not the assumption of normality of data, multiple (linear) regression analysis for identifying the stronger predictor variables and verifying the predictive validity of the math diagnostic test, and finally the statistical *t*-test for independent samples for testing the importance of taking a basic college math course upon entering the college in order to increase weaker students' math knowledge for further success in math, as described in this chapter. The data analysis mentioned here is part of the following Chapter 4 of the dissertation, where results were reported in tables and figures.

Chapter 4

Results and Discussion

The purpose of this chapter is to share the results of the statistical analysis performed for each research question of this study related to the use of a math diagnostic test in the first math class at the college as a way to identify students' prior knowledge in math for their further placement in two different college math courses. The secondary data used are represented by numerical continuous data gathered from the registrar at the college (grades from senior secondary and tertiary education) and from the teacher-researcher's archived data (scores from the math diagnostic test) of the college students in the Game Development program from the intake years 2014 through 2018. The data have been analyzed using the IBM SPSS (Version 26) statistical software, and the statistical designs employed consisted of correlation/prediction for the first two research questions and a student *t*-test for independent samples for the third research question. Data were analyzed using a level of significance α of 0.05, which means that the confidence level the researcher has that the results from the sample can be expanded to the whole population the sample is coming from is 95%.

This chapter is divided into two main sections corresponding to (a) results from the statistical analysis presented in figure and table format and with final statements for hypothesis testing and (b) a discussion of the results. Each main section of this chapter includes three subsections related to the three research questions of this study (RQ1, RQ2, and RQ3) and their hypotheses. The null and the alternative hypotheses for each research question are stated in complete and in symbolic forms at the beginning of each subsection, and the results and conclusions from each statistical analysis related to accepting or rejecting the null hypotheses and their meaning are presented. In addition, each research question is analyzed by looking first at meeting the assumptions of the corresponding statistical design, then performing the related analysis. Finally, Chapter 5 is introduced.

Results Presented in Figure and Table Format

This section presents the findings relative to the three research questions of this study communicated separately for each of them in three subsections. Each subsection, in turn, introduces the research question and its null and alternative hypotheses in complete and in symbolic forms, the data sample used, any assumptions that needed to be met before performing the corresponding study design, the results of the study design applied, and whether the results are statistically significant relative to the null hypothesis tested.

Research Question RQ1

This section looks at answering the first research question (RQ1) and its corresponding hypotheses (H1 and H2), each with their null (H_0) and alternative (H_1) hypotheses, where ρ represents the correlation coefficient for the population:

RQ1: Do students' marks in their senior high school math courses predict students' scores on the diagnostic math test? There are two different senior high school groups: college preparation and university preparation.

H1: Are students' marks in college preparation high school math courses significant predictors of students' scores in the diagnostic math test?

H_0: College preparation math course marks do not correlate significantly with the diagnostic test scores ($\rho = 0$).

H_1: College preparation math course marks correlate significantly with the diagnostic test scores ($\rho \neq 0$).

H2: Are students' marks in university preparation high school math courses significant predictors of the diagnostic math test?

H_0: University preparation math course marks do not correlate significantly with the diagnostic test scores ($\rho = 0$).

H_1: University preparation math course marks correlate significantly with the diagnostic test scores ($\rho \neq 0$).

In order to answer this research question, two numerical variables were necessary: students' grades from high school in either college preparation (H1) or university preparation (H2) and students' scores in their math diagnostic test upon starting their college program. The diagnostic test scores, as the dependent/criterion variable, were archived in the researcher's office computer. On the other hand, the independent/predictor variables, students' grades in their high school math courses, were retrieved from the college registrar.

The sample size used in each of these hypotheses depended on the number of students for whom the needed variables (grades in high school math courses and scores in the math diagnostic test) existed in the college registrar's archive and researcher's archived secondary data, respectively. Therefore, from the 202 students who started this college program in the Fall 2014 through Fall 2018 cohorts, only 71 students were considered for the hypothesis H1 (college preparation stream in high school) and 31 students were considered for the hypothesis H2 (university preparation stream in high school). The difference number (100 students) did not have marks in their high school Grade 11 or 12 math courses archived with the college registrar, and/or did not take the diagnostic test upon their start of their first math course at the college.

The statistical analysis design employed was the standard multiple regression used to predict a continuous dependent variable based on multiple independent variables. The standard multiple regression also looked to determine the overall fit (variance explained) of the model and the relative contribution of each of the predictors to the total variance explained. As this research question RQ1 has two hypotheses H1 and H2 related to students' grades in the college respectively university preparation, this statistical analysis was done for each hypothesis.

Hypothesis H1: Multiple Regression Design

The hypothesis H1 related to the research question RQ1 asked the following: Are students' marks in college preparation high school math courses significant predictors of students' scores in the diagnostic math test?

At the college preparation level, students had a choice of six math courses in their Grades 11 and 12: MAP4C, MCT4C, MEL4E, MBF3C, MCF3M, and MEL3E. As not all students who finished their high school in the college preparation stream took all six math courses, and some students may have taken only one course, all courses had missing marks. Only the most used math courses with the least missing values are considered for statistical analysis, as imputing values where they are missing may introduce errors in analysis, and the more imputed values, the greater the errors introduced. The following three procedures were performed first:

1. The Expectation Maximization (EM) method was used to find the percent of missing values for each of the six math courses from high school, as presented in Table 1. Two courses MBF3C (with 58 values out of 71 students) and MAP4C (with 54 values out of 71) were considered further as they had the least missing values, thus the least imputed values added.

Table 1

EM Method

Univariate Statistics							
				Missing		No. of Extremes[a,b]	
	N	Mean	Std. Deviation	Count	Percent	Low	High
MEL3E	1	57.00	.	70	98.6	.	.
MCF3M	15	60.87	13.378	56	78.9	1	0
MBF3C	58	71.62	12.476	13	18.3	0	0
MEL4E	2	68.50	16.263	69	97.2	0	0
MCT4C	2	47.00	41.012	69	97.2	0	0
MAP4C	54	71.13	12.544	17	23.9	0	0
Diagnostic test	71	54.413	17.4630	0	.0	0	0

a. Number of cases outside the range (Q1 - 1.5*IQR, Q3 + 1.5*IQR).

b. indicates that the inter-quartile range (IQR) is zero.

2. Further, the imputation procedure was applied only for three columns that were used further: MBF3C, MAP4C, and Diagnostic test, in order to handle the missing data: the randomness of the imputed data for the two math courses MAP4C and MBF3C was completely insignificant, as given by the significance coefficient $p = 0.317 > 0.05$, as presented in Table 2. The imputation was performed, and the statistical analysis continued.

Table 2

Imputation Procedure

EM Means[a]		
MBF3C	MAP4C	Diagnostic test
72.23	70.81	54.413

a. Little’s MCAR test: Chi-Square = 5.892, DF = 5, Sig. = .317

3. The standard multiple regression procedure was run for the three columns (the two math courses and the diagnostic test scores), and five new variables were asked for: the unstandardized predicted values (PRE_1), studentized residuals (SRE_1), studentized deleted residuals (SDR_1), Cook's distance values (COO_1) and leverage values (LEV_1).

Upon checking the SDR_1 column for outliers (values outside the interval [-3, 3]), no values were outside this interval; therefore, all observations (rows) were considered further.

Verifying Assumptions

The standard multiple regression applied for these two independent variables and the dependent variable was checked for eight assumptions, as presented next.

1. There is one dependent variable that is measured at the continuous level.

Indeed, the dependent/criterion variable is numerical and continuous, represented by students' scores in their math diagnostic test taken at the beginning of their college program. This assumption is met.

2. There are two or more independent variables that are measured either at the continuous or nominal level.

Indeed, the independent variables are measured at the continuous level, represented by students' grades in math courses they chose to take in their senior high school at the college preparation level. Only two courses were considered for the analysis: MAP4C and MBF3C, as they were taken by most students. This assumption is met.

3. There should be independence of observations (independence of residuals).

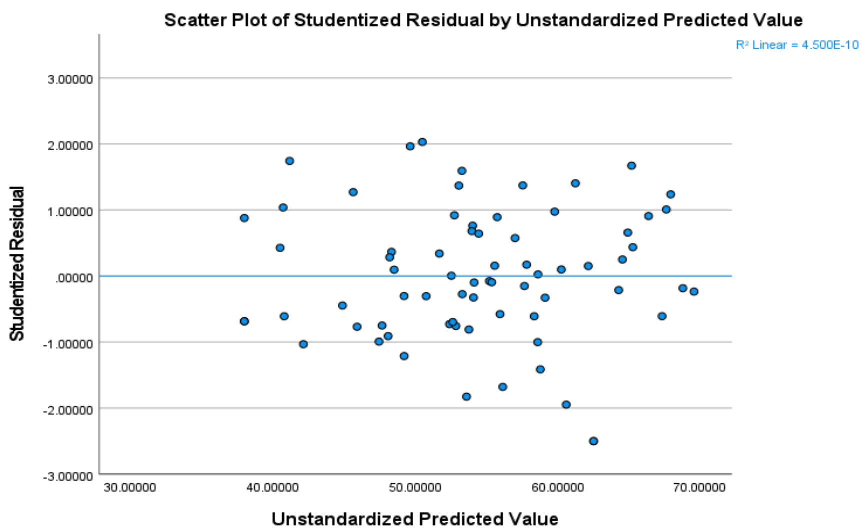
Independence of observations was not necessary to be checked by the Durbin-Watson test, as all rows of data represent different students who were entered in an arbitrary order and not having a time order, so it is highly unlikely that the 71 rows/students will be related. Therefore, there is independence of observations. This assumption is met.

4. There needs to be a linear relationship between (a) the dependent variable and the independent variables collectively and (b) the dependent variable and each of the independent variables.

To establish if a linear relationship exists between (a) the dependent and independent variables collectively was achieved by plotting a scatterplot of the studentized residuals against the unstandardized predicted values. As the residuals follow a horizontal band, the relationship between the dependent variable and the independent variables collectively is likely to be linear, as seen in Figure 1.

Figure 1

Scatter Plot of Studentized Residual by Unstandardized Predicted Value



To establish if a linear relationship exists between (b) the dependent variable and each of the independent variables, the partial regression plots between each independent variable and the dependent variable were performed. As remarked visually, there is an approximately linear relationship between the diagnostic test and each of the two math courses considered (see Figure 2 and Figure 3).

Figure 2

Scatter Plot of Diagnostic Test by MAP4C

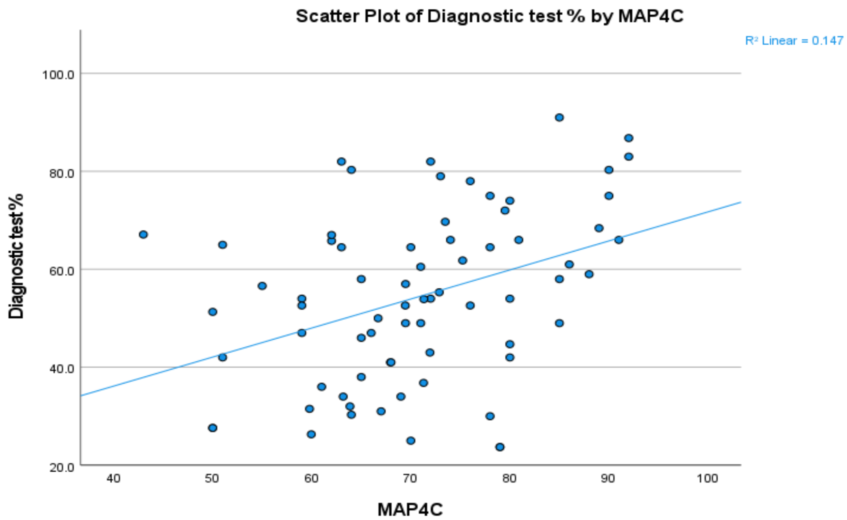
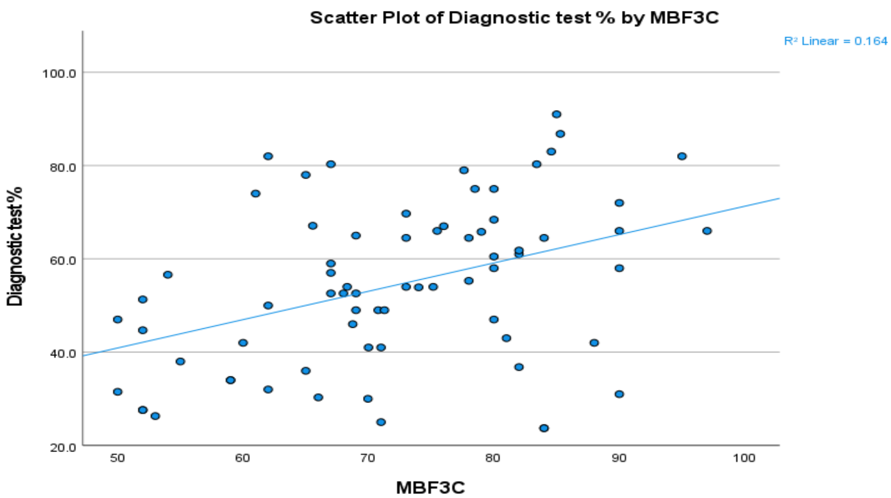


Figure 3

Scatter Plot of Diagnostic Test by MBF3C

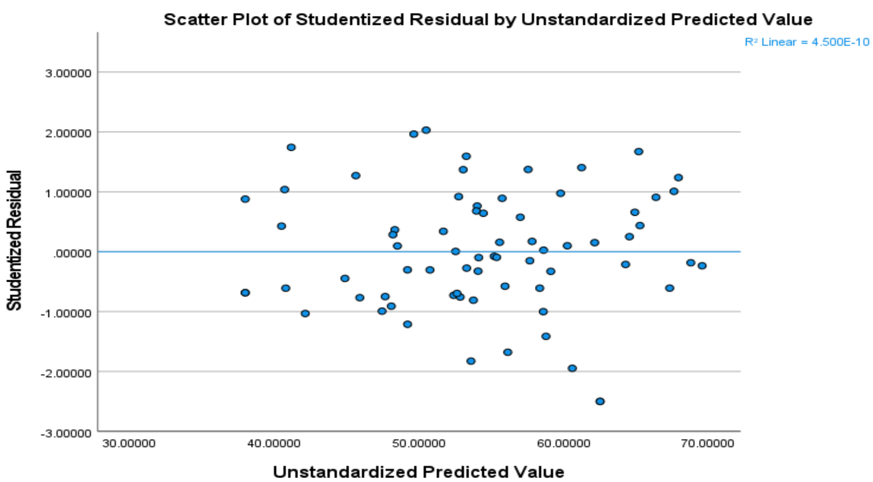


5. Data needs to show homoscedasticity of residuals (equal error variances).

The assumption of homoscedasticity is that the residuals are equal for all values of the predicted dependent variable (the variances along the line of best fit remain similar along the line). To check for heteroscedasticity, the plot created to check linearity in the previous assumption between the dependent and independent variables collectively is interpreted further. As shown in Figure 4, the spread of the residuals does not increase or decrease when moving across the predicted values (they are approximately constantly spread). Therefore, the data show homoscedasticity of residuals, as assessed by visual inspection of the plot of studentized residuals versus unstandardized predicted values.

Figure 4

Scatter Plot of Studentized Residuals by Unstandardized Predicted Value



6. Data must not show multicollinearity.

Multicollinearity occurs when there are two or more independent variables that are highly correlated with each other. This leads to problems in understanding which independent variable contributes to the variance explained in the dependent variable. There are two stages to identify multicollinearity: inspection of correlation coefficients and Tolerance/VIF values. From Table 3 it seems there are no correlations between MBF3C and MAP4C as the correlation coefficient between them is 0.496 < 0.7 (limit specified in Laerd Statistics, n.d.).

Table 3

Pearson Correlation Coefficients

Correlations

		Diagnostic test %	MAP4C	MBF3C
Pearson Correlation	Diagnostic test %	1.000	.384	.406
	MAP4C	.384	1.000	.496
	MBF3C	.406	.496	1.000
Sig. (1-tailed)	Diagnostic test %	.	<.001	<.001
	MAP4C	.000	.	.000
	MBF3C	.000	.000	.
N	Diagnostic test %	71	71	71
	MAP4C	71	71	71
	MBF3C	71	71	71

When looking at either Tolerance or VIF (last two columns in Table 4), the Tolerance values are greater than 0.1 for both math courses, which give VIF values smaller than 10 (VIF value is 1.326 < 10); therefore, this second stage does not show collinearity of independent variables. This assumption is met.

Table 4

Coefficients for MAP4C and MBF3C

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95.0% Confidence Interval for B Lower Bound	95.0% Confidence Interval for B Upper Bound	Correlations Zero-order	Correlations Partial	Correlations Part	Collinearity Statistics Tolerance	Collinearity Statistics VIF
1	(Constant)	-2.980	13.706		-.217	.829	-30.329	24.369					
	MAP4C	.374	.192	.242	1.948	.056	-.009	.757	.384	.230	.210	.754	1.326
	MBF3C	.428	.186	.286	2.299	.025	.056	.799	.406	.269	.248	.754	1.326

a. Dependent Variable: Diagnostic test %

7. There should be no significant outliers, high leverage points, or highly influential points.

Outliers, leverage, and influential points are different terms that represent unusual observations in the data set when performing a multiple regression analysis, specifically for the regression line. The first unusual point (outliers) was considered for the initial data as corresponding to values of SDR_1 outside the interval (-3, 3). No outliers were identified initially, and no Casewise Diagnostic table was produced during the multiple regression run.

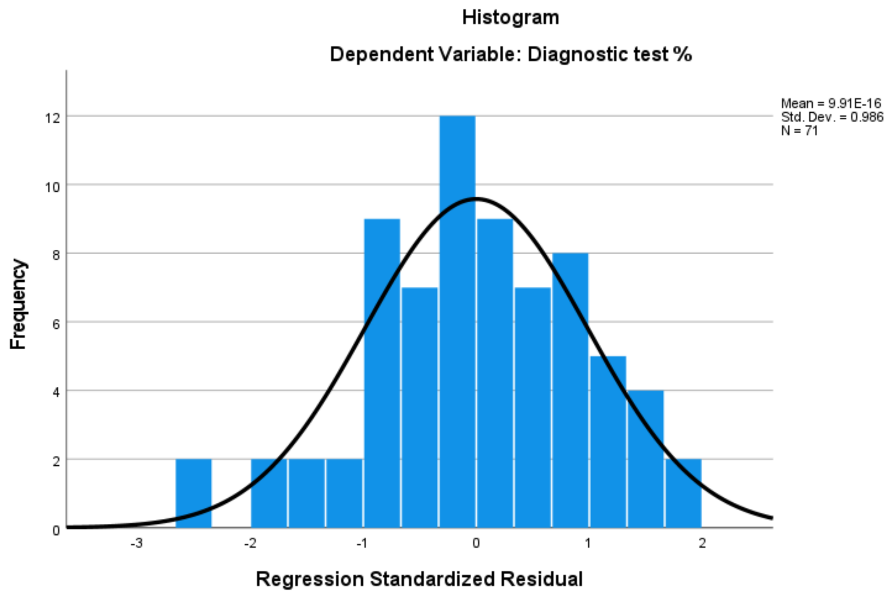
To determine whether any rows show high leverage, a look at all LEV_1 values produced in SPSS showed that all are smaller than 0.13748, thus smaller than 0.2; therefore, leverage values are all safe, by the general rule of thumb (Huber, 1981, cited in LaerdStatistics.com). When looking for *highly influential points*, Cook's Distance (COO_1) is a measure of influence, and as a rule of thumb, COO_1 values above 1 should be investigated (Huber, 1981, cited in Laerd Statistics, n.d.). The column COO_1 in SPSS showed all values under 0.12344. Therefore, there are no unusual points. This assumption is met.

8. The residuals (errors) are approximately normally distributed.

In order to run inferential statistics and determine statistical significance, the errors (residuals) in prediction need to be normally distributed. A common method used to check for the assumption of normality of the residuals employs a histogram with superimposed normal curve and a P-P Plot. The histogram in Figure 5 was produced automatically when selecting Plots in the Linear Regression in SPSS.

Figure 5

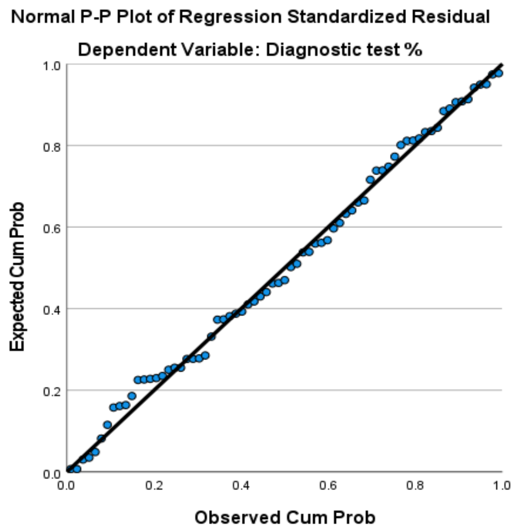
Histogram of Standardized Residuals



The histogram in Figure 5 shows that the standardized residuals appear to be approximately normally distributed, where the mean is 0 and standard deviation very close to 1 (0.986). To confirm the normal distribution, a look at the P-P Plot in Figure 6 shows that although the points are not aligned perfectly along the diagonal line, they are close enough to indicate that the residuals are close enough to normal for the analysis to proceed. The assumption of approximate normality is met.

Figure 6

Normal P-P Plot



Standard Multiple Regression

As the eight assumptions were met, running the standard multiple regression produced a few tables that were used to interpret and report the results, presented in three stages.

First, it was determined whether the multiple regression model was a good fit for the data. There were a number of statistics used to determine this: the multiple correlation coefficient, the proportion of variance explained, and the statistical significance of the overall model. Table 5 shows these three coefficients. First, the multiple correlation coefficient *R* represents the Pearson correlation coefficient between the scores predicted by the regression model (the predicted scores, PRE_1) and the actual values of the dependent variable (Diagnostic scores). The *R* value of 0.457 indicated a moderate-strength level of correlation.

Table 5

Model Summary

Model Summary[b]

Model	*R*	R Square	Adjusted *R* Square	Std. Error of the Estimate
1	.457[a]	.209	.185	15.7616

a. Predictors: (Constant), MBF3C, MAP4C

b. Dependent Variable: Diagnostic test %

However, as the multiple correlation coefficient *R* is not a common measure used to assess how good the model fits the data, the coefficient of determination R^2 was assesses next. This coefficient represents the proportion of variance in the dependent variable that is explained by the independent variables. The value of R^2 of 0.209 shows that the independent variables explained 20.9% of the variability of the dependent variable (Diagnostic test) in the regression model. However, R^2 is based on the sample, and another measure called adjusted R^2 corrects for the population. The adjusted R^2 is 18.5%, which is considered by Cohen (1988, as cited in Laerd Statistics, n.d.) as a medium-size effect.

Finally, the statistical significance of the overall model is presented in the Sig. column of the Table 6.

Table 6

ANOVA Table

ANOVA[a]						
Model		Sum of Squares	*df*	Mean Square	*F*	Sig.
1	Regression	4453.755	2	2226.878	8.964	<.001[b]
	Residual	16893.204	68	248.429		
	Total	21346.959	70			

a. Dependent Variable: Diagnostic test %

b. Predictors: (Constant), MBF3C, MAP4C

As $p < 0.001$ satisfies $p < 0.05$, and the *F* statistic value is $F(2, 68) = 8.964$, there is a statistically significant result, which means that the independent variables model is a statistically significant fit to the data.

Null Hypothesis

The null hypothesis of the research question RQ1 for the college preparatory stream of students written in symbolic form is H_0: $\rho = 0$. As $R = 0.457$ is statistically significant because *p*-value < 0.5 (in the ANOVA Table 6 above, $p < 0.001$), then the null hypothesis is rejected and the alternative hypothesis is accepted: college preparation math course marks correlate significantly with the diagnostic test scores. This result also showed that at least one regression (slope) coefficient (except the y-intercept) is statistically significantly different than zero.

The Regression Line

The second stage to interpret and report the results of the standard multiple regression is to understand the coefficients of the regression model in order to establish a linear relationship between the dependent variable and the independent variables. The regression equation for this data sample can be expressed in the following form:

Predicted Diagnostic = b_0 + (b_1 x MAP4C) + (b_2 x MBF3C),

where b_0 is the y-intercept (a constant) and b_1 and b_2 are the slope coefficients (one for each predictor variable). By substituting the values for b_0 through b_2, the Diagnostic test value can be predicted for any values of these two independent variables. The values of these coefficients are given in Table 7: $b_0 = -2.980$, $b_1 = 0.374$, and $b_2 = 0.428$.

Table 7

Coefficients, Slope Coefficients

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B		Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Lower Bound	Upper Bound	Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	-2.980	13.706		-.217	.829	-30.329	24.369					
	MAP4C	.374	.192	.242	1.948	.056	-.009	.757	.384	.230	.210	.754	1.326
	MBF3C	.428	.186	.286	2.299	.025	.056	.799	.406	.269	.248	.754	1.326

a. Dependent Variable: Diagnostic test %

I can also define a range of plausible values for the slope coefficients at the 95% confidence, as shown in the middle two columns of the Table 7. For example, b_1 coefficient is between -0.009 and 0.757, and b_2 coefficient is between 0.056 and 0.799. Looking at the significance level (Sig. column) we can see that p-value = 0.025 < 0.05 for the independent variable MBF3C, which means that the slope coefficient b_1 is statistically significant, thus there is a linear relationship in the population for this course. On the other hand, the value for b_2 coefficient for MAP4C is not statistically significant: p-value = 0.056 > 0.05.

Hypothesis H2: Multiple Regression Design

The hypothesis H2 related to the research question RQ1 asks the following: Are students' marks in university preparation high school math courses significant predictors of students' scores in the diagnostic math test? At the university preparation level, students had a choice of six math courses in their Grades 11 and 12: MHF4U, MCV4U, MDM4U, MCB4U, MCF3M, and MCR3U. Some students took one, two, or three courses from this list, and some courses were a more frequent choice than others. As such, similarly to the procedure performed when looking at the hypothesis H1, imputation of values where they were missing was done to the most frequent math courses students took in their Grades 11 and 12. The following three procedures were performed first:

1. The EM method used to find the percent of missing values for each of the six math courses from high school found that two courses MHF4U (with 19 values out of 31 students) and MCR3U (with 25 values out of 31) had fewer missing values, and they were considered further for the statistical analysis, as shown in Table 8.

Table 8

EM Method

Univariate Statistics							
				Missing		No. of Extremes[a]	
	N	Mean	Std. Deviation	Count	Percent	Low	High
MHF4U	19	72.47	12.527	12	38.7	1	0
MCV4U	9	70.56	17.436	22	71.0	1	0
MCB4U	1	92.00	.	30	96.8	.	.
MDM4U	7	69.71	13.035	24	77.4	0	0
MCF3M	7	79.57	11.502	24	77.4	0	0
MCR3U	25	71.20	15.443	6	19.4	1	0
Diagnostic test	31	77.965	14.4044	0	.0	0	0

a. Number of cases outside the range (Q1 - 1.5*IQR, Q3 + 1.5*IQR).

2. The imputation procedure was applied for these two independent variables and the dependent one in order to handle the missing data for these two math courses: MCR3U and MHF4U; the randomness of the imputed data is completely

insignificant, as given by the significance coefficient $p = 0.053 > 0.05$ presented in Table 9.

Table 9

Imputation Procedure

EM Means[a,b]		
MHF4U	MCR3U	Diagnostic test
65.94	70.48	78.930

a. Little's MCAR test: Chi-Square = 10.907, DF = 5, Sig. = .053

b. The EM algorithm failed to converge in 25 iterations.

3. The standard multiple regression procedure was run and five new variables were asked for: the unstandardized predicted values (PRE_1), studentized residuals (SRE_1), studentized deleted residuals (SDR_1), Cook's distance values (COO_1), and leverage values (LEV_1). Upon checking the SDR_1 column for outliers (values outside the interval [-3, 3]), one row was deleted: row 21. Everything was run again without this one row (30 values now, from the 31 values I started with). No other SDR_1 value was outside the interval (-3, 3); hence, no more outliers. Therefore, the two math courses considered further as independent variables in the standard multiple regression were MHF4U (with 18 values out of 30 students) and MCR3U (with 24 values out of 30), as presented in Table 10.

Table 10

EM Method

Univariate Statistics

	N	Mean	Std. Deviation	Missing		No. of Extremes[a]	
				Count	Percent	Low	High
MHF4U	18	72.61	12.876	12	40.0	1	0
MCR3U	24	71.00	15.742	6	20.0	1	0
Diagnostic test	30	78.930	13.5922	0	.0	1	0

a. Number of cases outside the range (Q1 - 1.5*IQR, Q3 + 1.5*IQR).

Verifying Assumptions

The standard multiple regression applied for these two independent variables and the dependent variable was checked for eight assumptions, as presented below:

1. There is one dependent variable that is measured at the continuous level.

Indeed, the dependent/criterion variable is numerical and continuous, represented by students’ scores in their math diagnostic test taken at the beginning of their college program. This assumption is met.

2. There are two or more independent variables that are measured either at the continuous or nominal level.

Indeed, the independent variables are measured at the continuous level, represented by students’ grades in the math courses they chose to take in their senior high school at the university preparation level. Only two courses were considered for the analysis: MHF4U and MCR3U, as they were taken by most students. This assumption is met.

3. There should be independence of observations (independence of residuals).

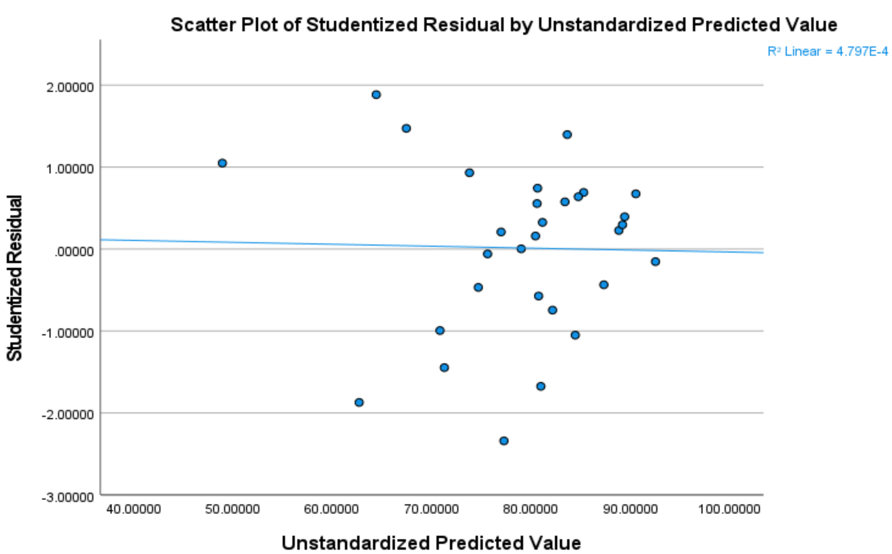
Independence of observations was not necessary to be checked, as all rows of data represent different students who were imputed in the data set in an arbitrary order, not dependent on time, thus it is highly unlikely that the 30 rows/students will be related. Therefore, no Durbin Watson test was necessary. This assumption is met.

4. There needs to be a linear relationship between (a) the dependent variable and the independent variables collectively and (b) the dependent variable and each of the independent variables.

To establish if a linear relationship exists between (a) the dependent and independent variables collectively, a scatterplot of the studentized residuals against the unstandardized predicted values was plotted. As the residuals follow an approximately horizontal band, the relationship between the dependent variable and the independent variables collectively is likely to be linear (see Figure 7).

Figure 7

Scatter Plot of Studentized Residuals by Unstandardized Predicted Value



To establish if a linear relationship exists between (b) the dependent variable and each of the independent variables, the partial regression plots between each independent variable and the dependent variable were plotted; they show an approximately linear relationship (see Figure 8 and Figure 9).

Figure 8

Scatter Plot of Diagnostic Test by MHF4U

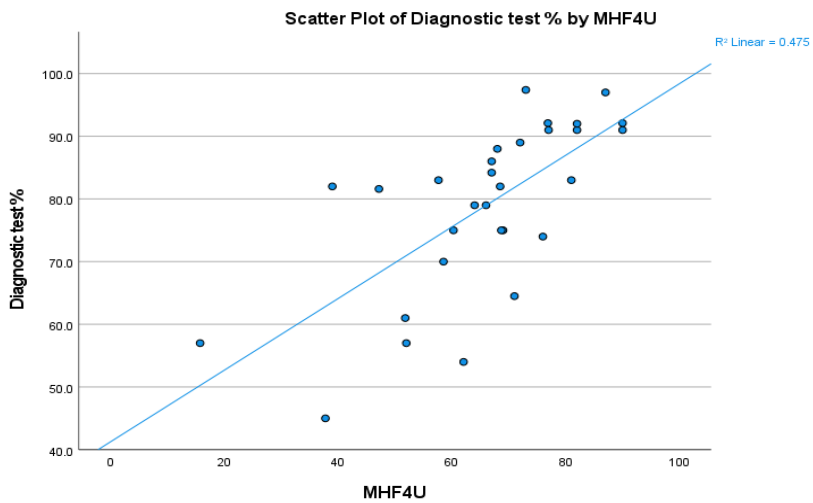
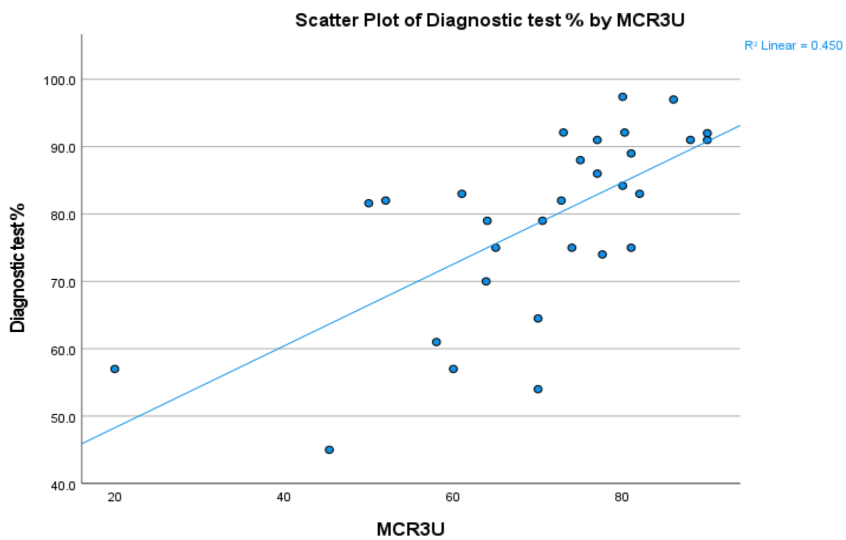


Figure 9

Scatter Plot of Diagnostic Test by MCR3U

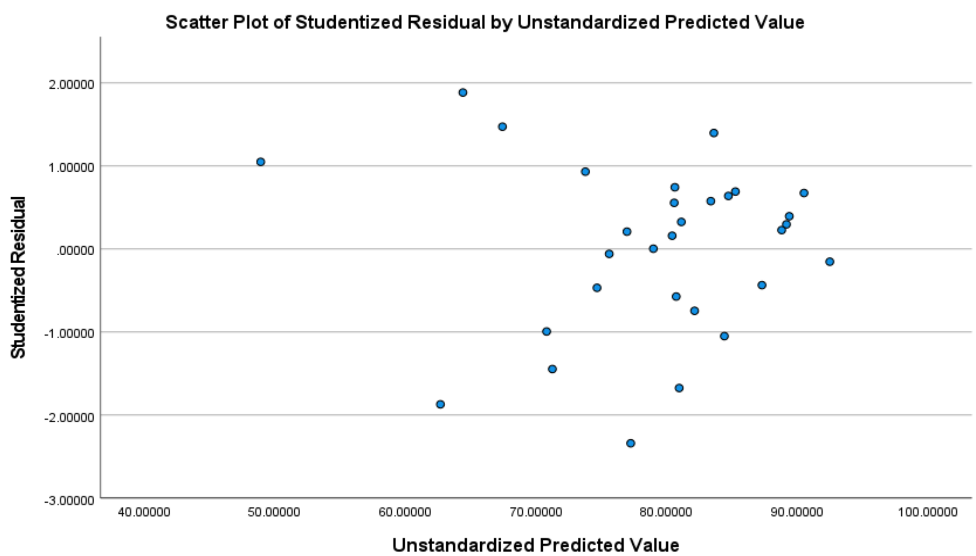


5. Data needs to show homoscedasticity of residuals (equal error variances).

To check for heteroscedasticity, the plot created to check linearity in the previous assumption between the dependent and independent variables collectively shows that the spread of the residuals does not increase or decrease when moving across the predicted values (they are approximately constantly spread). Therefore, the data show homoscedasticity of residuals, as assessed by visual inspection of the plot in Figure 10.

Figure 10

Scatter Plot of Studentized Residuals by Unstandardized Predicted Value



6. Data must not show multicollinearity.

The two stages to identifying multicollinearity are using an inspection of the correlation coefficients and the Tolerance/VIF values. From Table 11, it seems there are no correlations between MCR3U and MHF4U as the correlation coefficient between them is 0.629 < 0.7 (Laerd Statistics, n.d.).

Table 11

Pearson Correlation Coefficients

Correlations

		Diagnostic test %	MCR3U	MHF4U
Pearson Correlation	Diagnostic test %	1.000	.656	.814
	MCR3U	.656	1.000	.629
	MHF4U	.814	.629	1.000
Sig. (1-tailed)	Diagnostic test %	.	<.001	<.001
	MCR3U	.000	.	.000
	MHF4U	.000	.000	.
N	Diagnostic test %	30	30	30
	MCR3U	30	30	30
	MHF4U	30	30	30

When looking at either Tolerance or VIF (last two columns in Table 12), the Tolerance value is greater than 0.1, which gives a VIF value smaller than 10 (VIF value is 1.654 < 10); therefore, this second stage does not show collinearity of independent variables.

Table 12

Coefficients for MCR3U and MHF4U

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95.0% Confidence Interval for B Lower Bound	95.0% Confidence Interval for B Upper Bound	Correlations Zero-order	Correlations Partial	Correlations Part	Collinearity Statistics Tolerance	Collinearity Statistics VIF
1	(Constant)	8.273	10.247		.807	.428	-12.978	29.525					
	MCR3U	.205	.129	.239	1.585	.127	-.063	.473	.656	.320	.186	.605	1.654
	MHF4U	.800	.182	.663	4.391	<.001	.422	1.177	.814	.683	.516	.605	1.654

a. Dependent Variable: Diagnostic test %

7. There should be no significant outliers, high leverage points or highly influential points.

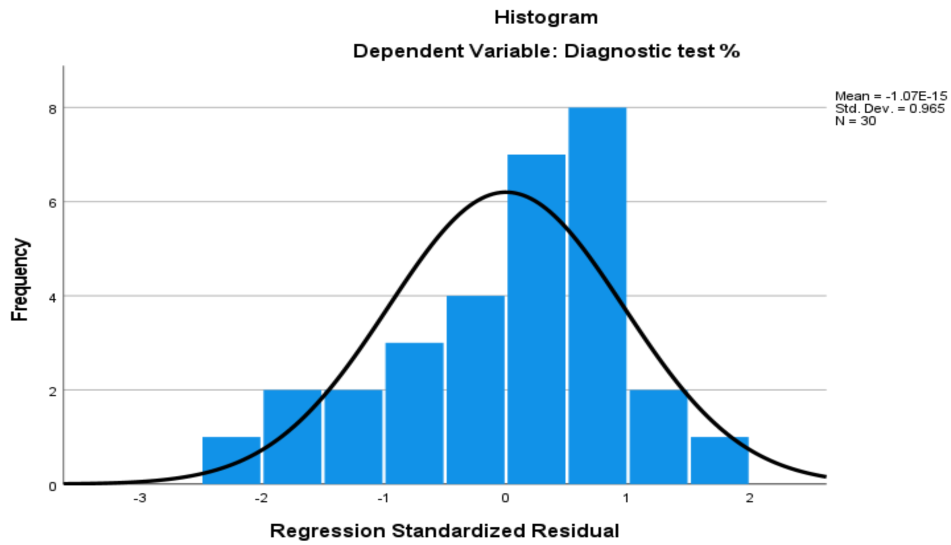
The first unusual point, outliers, was looked at initially and one outlier was deleted from the data. For the second unusual point, high leverage, one general rule of thumb is to consider leverage values (LEV_1) less than 0.2 as safe, 0.2 to less than 0.5 as risky, and values of 0.5 and above as dangerous (Huber, 1981, cited in Laerd Statistics, n.d.). A look at the LEV_1 column of the data shows that 28 values are under 0.2; however, there are two values that are greater than 0.2: one is 0.39268 (case 22) and one is just above 0.5: 0.50149 (case 28). When looking for highly influential points, Cook's Distance (COO_1) is a measure of influence, and as a rule of thumb, COO_1 values above 1 should be investigated. The column COO_1 in SPSS shows all values under 0.27242, including the ones for cases 22 and 28: 0.27242 and 0.05963 respectively. Therefore, I can consider that there are no unusual points, and I recorded the cases 22 and 28 that have higher leverage, but they do not lead to high influence on the regression line.

8. The residuals (errors) are approximately normally distributed.

A common method to check for the assumption of normality of the residuals is assessing visually a histogram with superimposed normal curve and a P-P Plot. The histogram in Figure 11 was produced automatically when selecting Plots in the Linear Regression:

Figure 11

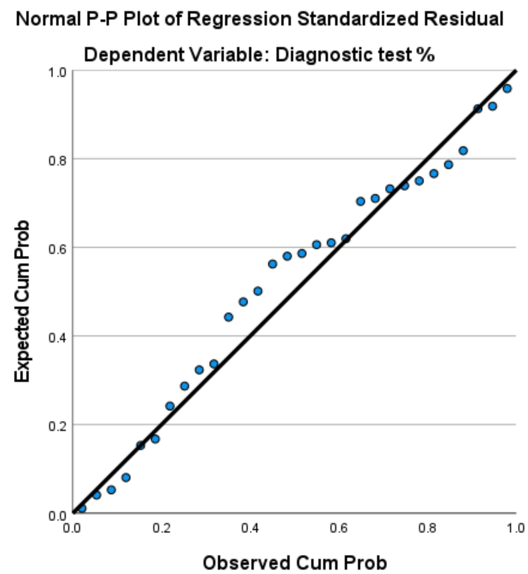
Histogram of Standardized Residuals



The histogram in Figure 11 shows that the standardized residuals appear to be slightly negatively skewed, but this assumption asks for an approximately normal distribution, which can be assumed from the values of the mean and standard deviation: the mean is 0 and standard deviation very close to 1 (0.965). To confirm the normal distribution, a look at the P-P Plot (see Figure 12) shows that although the points (residuals) are not aligned perfectly along the diagonal line, they are close enough. The assumption of approximate normality is met.

Figure 12

Normal P-P Plot



Standard Multiple Regression

As the eight assumptions were met, the standard multiple regression run produced a few tables that are used to interpret and report the results. First, to determine if the multiple regression model is a good fit for the data, a look at the multiple correlation coefficient from Table 13 shows that this coefficient $R = 0.692$. This value 0.692 indicates a high moderate-strength level of association. Further, the coefficient of determination $R^2 = 0.479$ shows that the independent variables explain 47.9% of the variability of the dependent variable (Diagnostic test) in the regression model. As R^2 is based on the sample, another measure called adjusted R^2 that corrects for population is 44.1%, which is considered by Cohen (1988, as cited in Laerd Statistics, n.d.) as a high medium size effect.

Table 13

Model Summary

Model Summary[b]				
Model	*R*	R Square	Adjusted *R* Square	Std. Error of the Estimate
1	.692[a]	.479	.441	10.1639

a. Predictors: (Constant), MCR3U, MHF4U

b. Dependent Variable: Diagnostic test %

Finally, the statistical significance of the overall model is presented in the Sig. column of Table 14. As $p < 0.001$ satisfies $p < 0.05$, and the *F* statistic value is $F(2, 27) = 12.431$, the independent variables model is a statistically significant fit to the data.

Table 14

ANOVA

ANOVA[a]						
Model		Sum of Squares	*df*	Mean Square	*F*	Sig.
1	Regression	2568.437	2	1284.219	12.431	<.001[b]
	Residual	2789.246	27	103.305		
	Total	5357.683	29			

a. Dependent Variable: Diagnostic test %

b. Predictors: (Constant), MCR3U, MHF4U

Null Hypothesis

The null hypothesis of the research question RQ1 for the university preparatory stream in high school written in symbolic form is H_0: $\rho = 0$. As $R = 0.692$ is statistically significant because *p*-value < 0.5 (in the ANOVA Table 14, $p < 0.001$), then the null hypothesis is rejected and the alternative hypothesis is accepted: university preparation math course marks correlate significantly with the diagnostic test scores. This result also showed that at least one regression coefficient (except b_0) is statistically significantly different than zero.

The Regression Line

The second part in interpreting and reporting the results of the standard multiple regression is to understand the coefficients of the regression model and establish a linear relationship between the dependent variable and the independent variables. The regression equation for this data sample can be expressed in the following form:

Predicted Diagnostic = b_0 + (b_1 x MHF4U) + (b_2 x MCR3U),

where b_0 is the y-intercept (a constant) and b_1 and b_2 are the slope coefficients (one for each variable). By substituting the values for b_0 through $b_{2,}$ the Diagnostic test value can be predicted for any values of these two independent variables. The values of these coefficients are in Table 15: $b_0 = 8.273$, $b_1 = 0.800$, and $b_2 = 0.205$.

Table 15

Coefficients for MCR3U and MHF4U

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients			95.0% Confidence Interval for B		Correlations			Collinearity Statistics	
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound	Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	8.273	10.247		.807	.428	-12.978	29.525					
	MCR3U	.205	.129	.239	1.585	.127	-.063	.473	.656	.320	.186	.605	1.654
	MHF4U	.800	.182	.663	4.391	<.001	.422	1.177	.814	.683	.516	.605	1.654

a. Dependent Variable: Diagnostic test %

The 95% confidence interval for the slope coefficient of each independent variable also is given in the middle two columns of the Coefficients table. For example, b_1 coefficient is between 0.422 and 1.177, and b_2 coefficient is between -0.063 and 0.473. Looking at the significance level (Sig. column) we can see that p-value < 0.001 for the independent variable MHF4U, which means that the slope coefficient b_1 is statistically significant, thus there is a linear relationship in the population for this course. On the other hand, the value for b_2 coefficient for MCR3U is not statistically significant: p-value = 0.127 > 0.05.

Research Question 2

This section looks at answering the second research question (RQ2) and its corresponding hypothesis (H3) with its null (H_0) and alternative (H_1) hypotheses written in complete and symbolic forms:

RQ2: Do high school student GPAs predict students' scores on the diagnostic math test?

H3: Is the high school GPA a significant predictor of the diagnostic math test scores?

H_0: Students' high school GPAs do not correlate significantly with students' diagnostic test scores ($\rho = 0$).

H_1: Students' high school GPAs correlate significantly with students' diagnostic test scores ($\rho \neq 0$).

In order to answer this research question, two numerical variables were necessary: students' GPA from high school, and students' scores in their math diagnostic test upon starting their college program. As the students' high school GPAs were not available at the college registrar, only students' senior grades for all courses taken in their Grades 11 and 12, a Senior Grade Average was able to be calculated. This average was considered as the independent/predictor variable instead of the GPA for answering this second research question.

The sample size used in this hypothesis H3 depended on the number of students for whom the needed variables (GPA or the Senior Grade Average from high school, and the scores in the math diagnostic test) existed in the college registrar's archive and researcher's archived secondary data, respectively. Therefore, from the 202 students who started this college program in the Fall 2014 through Fall 2018 cohorts, only 106 students were considered. The difference number (96 students) did not have the Senior Grade Average from their high school, and/or did not take the diagnostic test upon their start of their first math course at the college.

The statistical analysis design employed in order to determine the strength and direction of a linear relationship between the two variables: independent variable (the Senior Grade Average) and dependent variable (Diagnostic test scores) is correlation using Pearson or Spearman coefficients. Further, the statistical design used to determine if a prediction can be made between the independent/predictor variable and the dependent/criterion variable is linear regression. Both statistical designs were needed in order to answer the hypothesis H3: Is the high school GPA a significant predictor of the diagnostic math test scores?

Hypothesis H3: Correlation Design

Verifying Assumptions

The five assumptions to be met in order to identify if Pearson correlation can be used with this data are:

1. Data collected must be continuous.

Indeed, all data are numerical and continuous, as the independent variable is represented by the average of all senior high school grades, and the dependent variables is represented by students' scores in their math diagnostic test taken at the beginning of their college program. This assumption is met.

2. Identified variables must be paired.

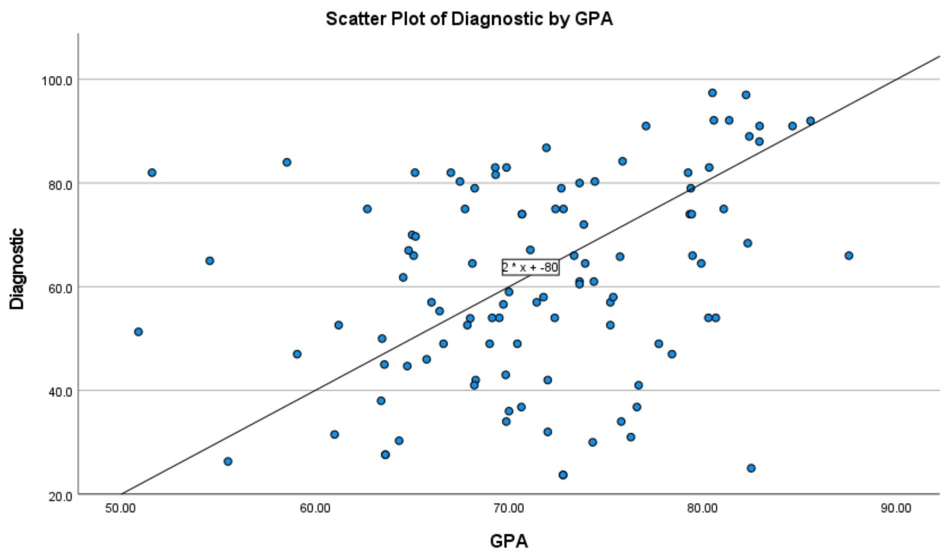
Indeed, both variables are paired for each student. This assumption is met.

3. There needs to be a linear relationship between the two variables.

A scatter plot was used as a visual method, and a line of best fit was drawn. The positive slope shows a positive correlation between the two variables: the Diagnostic test scores and the students' GPA (see Figure 13).

Figure 13

Scatter Plot of Diagnostic Test by GPA



4. The scatter plot must show no significant outliers.

Table 16 shows descriptive statistics corresponding to the two variables (Diagnostic score and GPA).

Table 16

Descriptives for Diagnostic Test and GPA

Descriptives				
			Statistic	Std. Error
Diagnostic	Mean		61.264	1.8872
	95% Confidence Interval for Mean	Lower Bound	57.522	
		Upper Bound	65.006	
	5% Trimmed Mean		61.442	
	Median		61.400	
	Variance		377.527	
	Std. Deviation		19.4301	
	Minimum		23.7	
	Maximum		97.4	
	Range		73.7	
	Interquartile Range		32.0	
	Skewness		-.136	.235
	Kurtosis		-.914	.465

(continued)

			Statistic	Std. Error
GPA	Mean		71.5743	.71050
	95% Confidence Interval for Mean	Lower Bound	70.1656	
		Upper Bound	72.9831	
	5% Trimmed Mean		71.8005	
	Median		71.6050	
	Variance		53.510	
	Std. Deviation		7.31504	
	Minimum		50.88	
	Maximum		87.54	
	Range		36.66	
	Interquartile Range		9.72	
	Skewness		-.310	.235
	Kurtosis		.153	.465

Using the means, standard deviations, and the minimum/maximum values for the two variables, the extreme *z*-scores were calculated (minimum and maximum) for each variable using the formula $z_{score} = \frac{x - mean}{st.dev.}$, where *x* represents the minimum and maximum values from the two variables. The following extreme values for *z*-scores (Table 17) show that there are no outliers for any of the two variables, as the extreme *z*-scores for the data are in the interval (-3, 3). This assumption is met.

Table 17

Z-Score Limits

	z-score (min)	*z*-score (max)
Diagnostic:	-1.93	1.86
GPA:	-2.83	2.18

5. The data must show bivariate normality.

A test for normality (Shapiro-Wilk) was performed in SPSS, and the superior Wilk value for the diagnostic test was 0.023, which is below the 0.05 threshold, as seen in Table 18. This means that the test of normality for the diagnostic scores failed.

Table 18

Test for Normality Shapiro-Wilk

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	*df*	Sig.	Statistic	*df*	Sig.
Diagnostic	.084	106	.066	.972	106	.023
GPA	.060	106	.200*	.985	106	.259

*. This is a lower bound of the true significance.

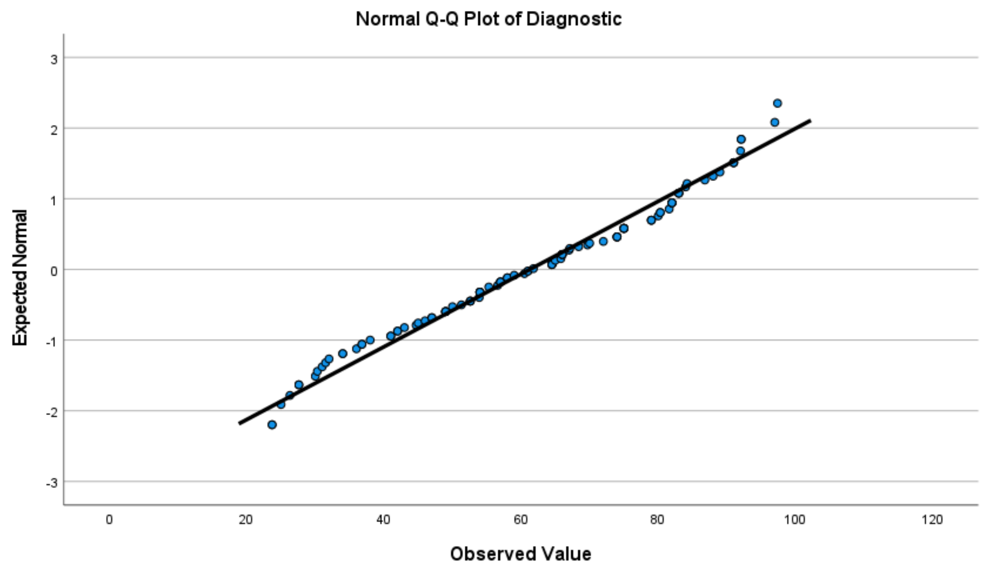
a. Lilliefors Significance Correction

Even if the test shows normality for the GPA variable, it seems that Pearson correlation cannot be used; therefore, a nonparametric test of correlation such as the Spearman test was used to calculate the coefficient *r* and used further to interpret the results, not the Pearson coefficient.

However, Laerd Statistics (n.d.) raises awareness about larger sample sizes (above 50 cases) that can lead to a statistically significant result (where data are nonnormal) even when data are normal. For larger sample sizes, as it is in this case (sample size 106), a graphical interpretation of such data such as the Normal Q-Q Plot is preferred (see Figure 14). The diagnostic scores in this case that do not show normality by the Shapiro-Wilk test.

Figure 14

Normal Q-Q Plot



Upon reviewing the Normal Q-Q Plot for the Diagnostic scores, the values appear to be normally distributed, therefore the high number of the sample size (106) in the data set may have given here a false positive. Thus, a calculation of both the Pearson and Spearman coefficients is employed next. Table 19 and Table 20 show the Pearson coefficient and the Spearman coefficient at a level of significance of 0.01 with a two-tailed p-value ≤ 0.001:

Table 19

Pearson Coefficients

Correlations			
		Diagnostic	GPA
Diagnostic	Pearson Correlation	1	.314**
	Sig. (2-tailed)		.001
	N	106	106
GPA	Pearson Correlation	.314**	1
	Sig. (2-tailed)	.001	
	N	106	106

**. Correlation is significant at the 0.01 level (2-tailed).

Table 20

Spearman Coefficients

Correlations				
			Diagnostic	GPA
Spearman's rho	Diagnostic	Correlation Coefficient	1.000	.324**
		Sig. (2-tailed)	.	<.001
		N	106	106
	GPA	Correlation Coefficient	.324**	1.000
		Sig. (2-tailed)	<.001	.
		N	106	106

**. Correlation is significant at the 0.01 level (2-tailed).

As can be seen, both the Pearson correlation and the Spearman's rho were significant at the $p \leq 0.001$ level, with values of Pearson $r = 0.314$ and Spearman rho = 0.324.

Null Hypothesis

Considering the symbolic form of the null and alternative hypotheses:

H_0: $\rho = 0$ (the correlation coefficient is significantly zero).

H_1: $\rho \neq 0$ (the correlation coefficient is significantly different than zero).

the null hypothesis is rejected as $R = 0.324 \neq 0$ for p-value ≤ 0.001, thus the alternative hypothesis must be true. Thus, there is a medium positive significant correlation between the high school GPA and the diagnostic math test scores students take in their first math class at the college.

Hypothesis H3: Linear Regression

However, there is a second part needed to answer the research question RQ2 related to prediction: Can students' GPA from high school predict their scores in the math diagnostic test? There are seven assumptions to be met in order to identify if there is a linear regression for this data, from which five are similar to those related to the above Pearson correlation design. After these assumptions were met, a simple linear regression equation was introduced and used to make predictions about the dependent variable (diagnostic test scores) based on the independent variable (GPA scores).

Verifying Assumptions

1. There is one dependent variable that is measured at the continuous level.

The dependent/criterion variable is continuous, as it is represented by the math diagnostic test scores. This assumption is met.

2. There is one independent variable that is measured at the continuous level.

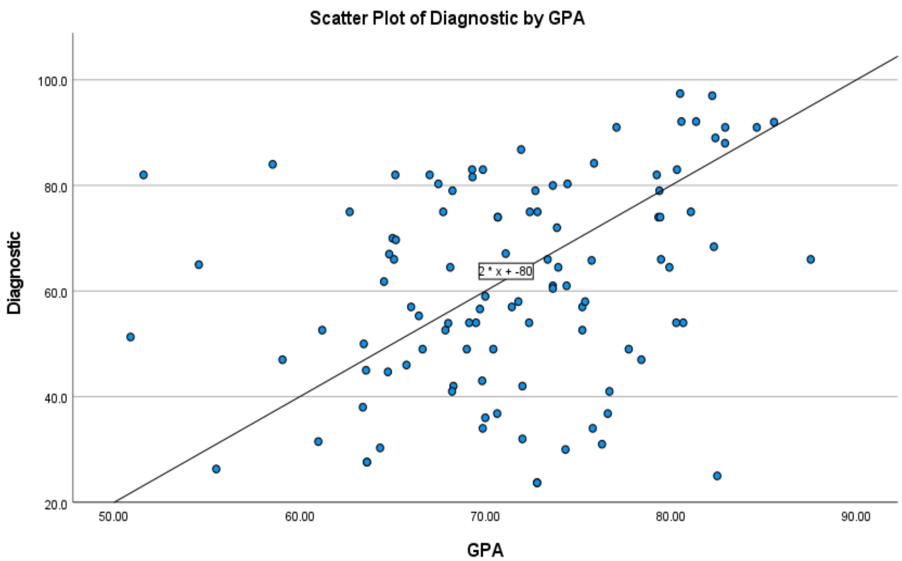
The GPA represented by the Senior Average Grades is the independent/predictor variable measured at the continuous level. This assumption is met.

3. There has to exist a linear relationship between the predictor variable and the criterion variable.

From the scatter plot drawn at the correlation design in Figure 15, it seems there is a positive linear correlation between the GPA and the Diagnostic scores in math. This assumption is met.

Figure 15

Scatter Plot of Diagnostic Test by GPA



4. The data must show independence of observations (residuals).

Independence of observations was not necessary to be checked, as all rows of data represent different students who were entered in an arbitrary order, so it is highly unlikely that the 106 rows/students will be related. No Durbin Watson test was necessary. This assumption is met.

5. Data must not have significant outliers.

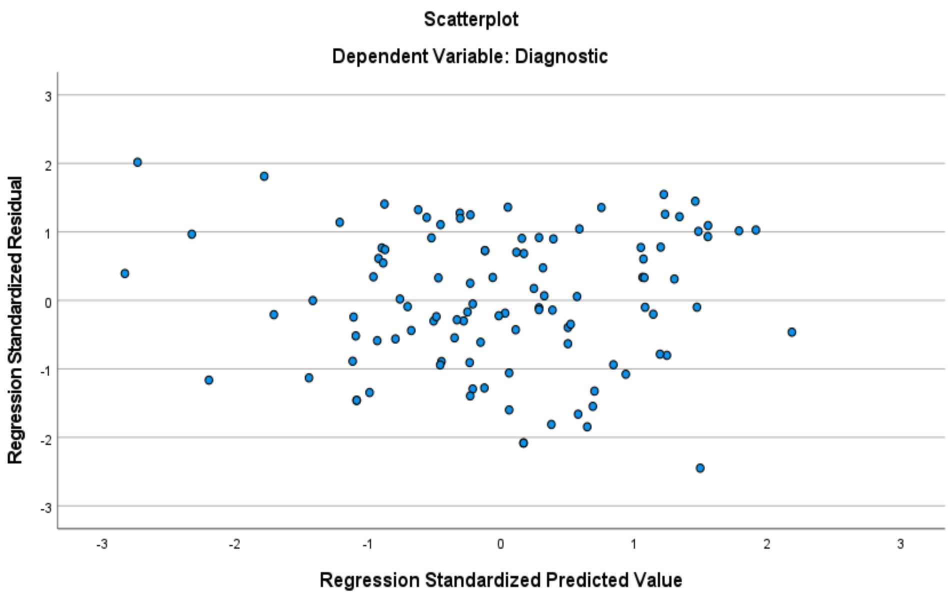
As discussed above at correlation, from the Descriptives table, the extreme values for *z*-scores were calculated and as they were in the interval (-3, 3), the conclusion was drawn that there are no outliers for any of the two variables. When running the Casewise Diagnostic on SPSS, no table called Casewise Diagnostics that contains case numbers (outliers) was produced. This assumption is met.

6. The data needs to show homoscedasticity (assumption of equal error variances).

This assumption of equal error variances was checked by inspection of a plot of the standardized residuals against the standardized predicted values. When running the previous procedure, Figure 16 was generated using standardized residuals.

Figure 16

Scatterplot for Diagnostic Test



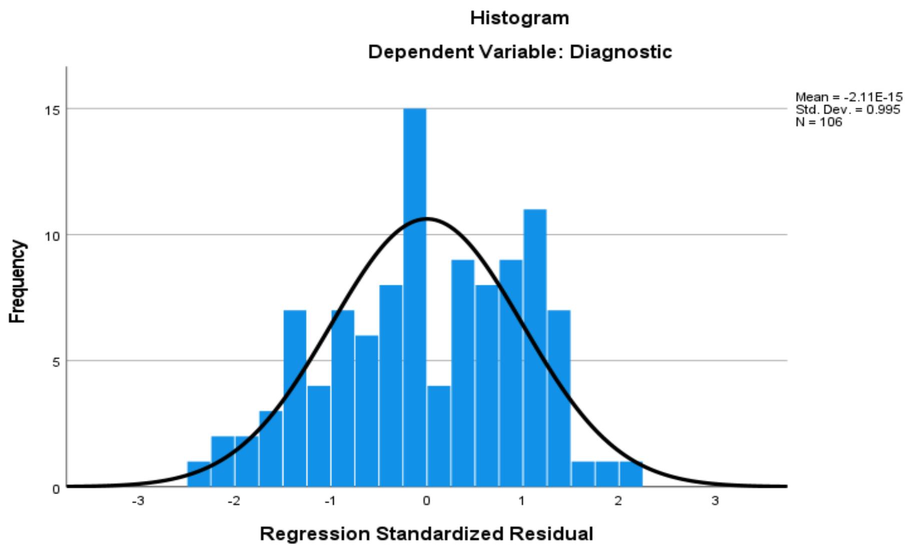
By visual inspection, the residuals appear randomly scattered with no trend or pattern. On this basis, it would appear that the assumption of homoscedasticity has been met.

7. Check that the residuals are approximately normally distributed.

The method employed to check whether the residuals are normally distributed used a histogram with superimposed normal curve and the Normal P-P Plot. The histogram in Figure 17 shows visually that the standardized residuals appear to be approximately normally distributed, and the mean and standard deviation for the dependent variable are approximately 0 and 1 respectively.

Figure 17

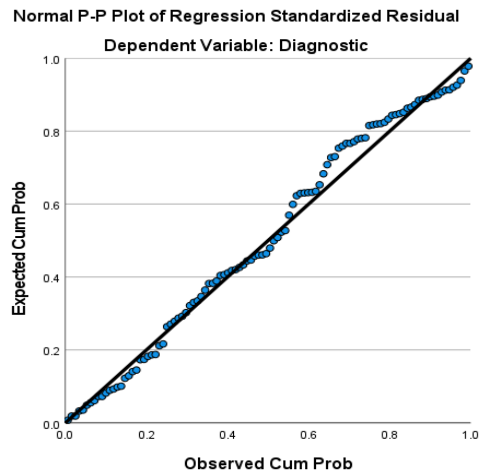
Histogram for Diagnostic Test



To confirm the normality based on the visual inspection of the above histogram, the Normal P-P Plot that was produced in SPSS, and presented in Figure 18. Although the points are not aligned perfectly along the diagonal line, they are close enough to indicate that the residuals are approximately normally distributed. Therefore, the assumption of approximate normality is met.

Figure 18

Normal P-P Plot



The Regression Line

As the seven assumptions for linear regression between the predictor variable (GPA) and the criterion variable (diagnostic test scores) are met, the linear regression equation written for the sample is $y = b_0 + b_1 x$, where:

y = the dependent variable (diagnostic test scores)

x = the independent variable (GPA marks)

b_1 = the slope parameter of the regression line

b_0 = the y intercept

When interpreting and reporting the results from the linear regression, first we determine whether the linear regression model is a good fit for the data. Table 21 presents information on the proportion of variance explained. As there is only one independent variable for this research question, R represents the multiple correlation coefficient, which is the Pearson correlation coefficient between the dependent variable and the independent variable. As $R = 0.314$, this indicates a moderately medium correlation.

Table 21

Model Summary

Model Summary[b]					
Model	*R*	*R* Square	Adjusted *R* Square	Std. Error of the Estimate	Durbin-Watson
1	.314[a]	.099	.090	18.5360	1.789

a. Predictors: (Constant), GPA

b. Dependent Variable: Diagnostic

The R^2 value (*R* Square), $R^2 = 0.099 = 9.9\%$ represents the percentage of variance in the dependent variable (diagnostic test scores) that can be explained by the independent variable (GPA marks). However, R^2 is based on the sample and is a positively biased estimate of the proportion of the variance of the dependent variable accounted for by the regression model (it is too large), which leads us to the adjusted R^2 statistic. The adjusted R^2 value = 0.09 = 9.0% corrects positive bias to provide a value that would be expected in the population. Adjusted R^2 is also an estimate of the effect size, and 9.0%, indicates a moderately medium effect size, according to Cohen's classification (Laerd Statistics, n.d.). Finally, the ANOVA table (see Table 22) shows whether the regression model results in a statistically significant prediction of the dependent variable, diagnostic test scores.

Table 22

ANOVA

ANOVA[a]						
Model		Sum of Squares	*df*	Mean Square	*F*	Sig.
1	Regression	3907.612	1	3907.612	11.373	.001[b]
	Residual	35732.711	104	343.584		
	Total	39640.324	105			

a. Dependent Variable: Diagnostic

b. Predictors: (Constant), GPA

The values of the last two columns from Table 22 indicate that the regression model is statistically significant, $F(1, 104) = 11.37$, $p = .001 < 0.05$, thus there is a statistically significant linear relationship between the two variables. Therefore, the linear regression model is a good fit for the data.

Next, using the coefficients of the regression model, the regression equation for this research question is expressed as:

Predicted diagnostic = $b_0 + (b_1 \text{ x GPA})$

By substituting the values for b_0 and $b_{1,}$ I can predict the diagnostic test score given the GPA from high school. The intercept ($b_0 = 1.574$) is not usually of much interest, and it is not statistically significant ($p = 0.930 > 0.05$), meaning that it is not different from zero. Of greater interest is the slope coefficient $b_1 = 0.834$, which is statistically significant (p-value = 0.001 < 0.5) and shows that there is a predicted increase in the diagnostic test score of 0.834 for every extra one mark of the GPA from high school.

Table 23

Prediction Coefficients

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95.0% Confidence Interval for B Lower Bound	95.0% Confidence Interval for B Upper Bound
1	(Constant)	1.574	17.791		.088	.930	-33.706	36.854
	GPA	.834	.247	.314	3.372	.001	.344	1.324

a. Dependent Variable: Diagnostic

It can also be defined a range of plausible values for the slope coefficient b_1 with 95% confidence: between 0.344 and 1.324.

Research Question RQ3

This section looks at answering the third research question (RQ3) and its corresponding hypothesis (H4) with its null (H_0) and alternative (H_1) hypotheses:

RQ3: Considering the students placement into two college math courses (Math 9 and Math 10) based on their diagnostic test scores, is there a significant difference (in their final scores in Math 10) between the students who entered directly Math 10 and the students who took Math 9 prior to taking Math 10?

H4: Students who take Math 9 prior to Math 10 finish their Math 10 course with similar marks as the students who are directed to Math 10 only.

H_0: Students who take Math 9 prior to Math 10 finish their Math 10 course with no significant difference in marks than the students who are directed to Math 10 only (H_0: $\mu = 0$).

H_1: Students who take Math 9 prior to Math 10 finish their Math 10 course with significantly different marks than the students who are directed to Math 10 only (H_1: $\mu \neq 0$).

The sample size used in this hypothesis depends on the number of students for whom the needed variables (diagnostic test scores and grades in the Math 10 course from the two groups) existed in the college registrar's archive and researcher's archived secondary data, respectively. Therefore, from the 202 students who started this college program in the Fall 2014 through Fall 2018 cohorts, only 54 students were considered for the group who went through the two math courses, Math 9 and Math 10, and 31 students were considered for the group who went directly to Math 10. The difference number (117 students) either did not take the diagnostic test (even if they took the two college math courses), or took the diagnostic test but withdrew at some point during their 1st college year (failed Math 9 and did not continue into Math 10 or withdrew while in Math 9 or in Math 10).

Hypothesis H4: The *t*-Test Design

In order to determine if a difference exists between the means of two independent groups (Math 9 & 10 and Math 10 only) on the continuous dependent variable (Math 10 grades), an independent-samples *t*-test was used. Specifically, this test determined whether the difference between these two groups was statistically significant.

Verifying Assumptions

There were six assumptions that needed to be considered, which relate to the choice of the study design (*t*-test for independent samples) and the characteristics of the collected data. The six assumptions are presented below:

1. There is only one dependent variable measured at the continuous level.

Indeed, all students' marks in the Math 10 course is a variable at the continuous level. This assumption is met.

2. There is one independent variable that consists of two categorical independent groups.

Indeed, the two independent groups are the two cohorts of students: the students who took both courses Math 9 and Math 10, and the students who took only Math 10. This assumption is met.

3. There exists independence of observations.

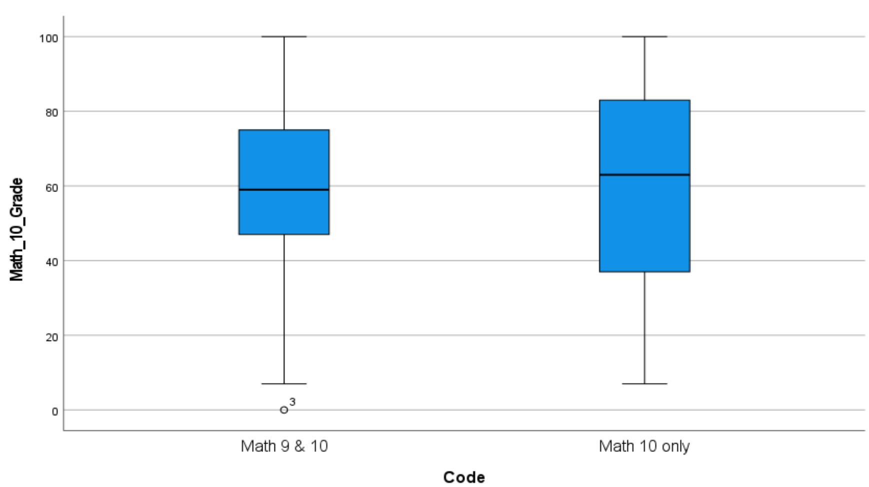
Indeed, there is no relationship between the math marks for Math 10 in each group of the independent variable or between the groups, as there are different students in each group. This assumption is met.

4. There should be no significant outliers in the two groups of independent variable in terms of the dependent variable: marks at the end of the Math 10 course.

For verifying this assumption, box plots for the two sets of data (Math 10 marks) are drawn in Figure 19.

Figure 19

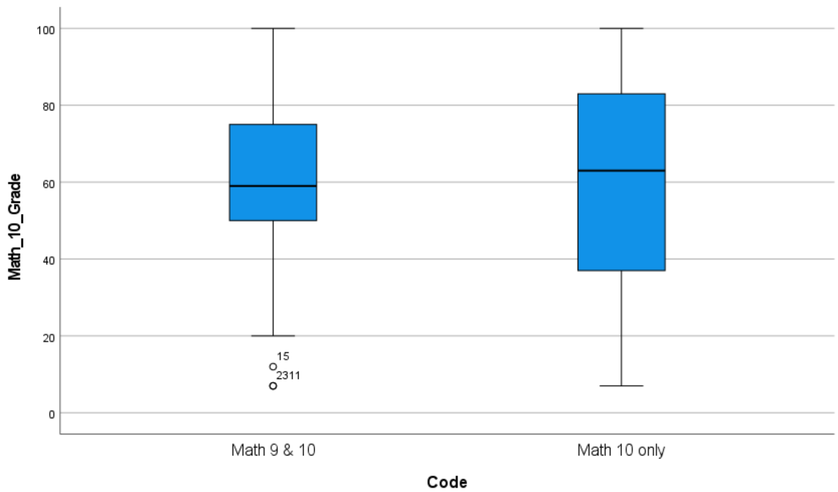
Box Plot 1 for Math 10 Grades for the Two Groups



There is one outlier in the Math 9 and 10 group, data point 3, which is eliminated and the data is run again as presented in Figure 20.

Figure 20

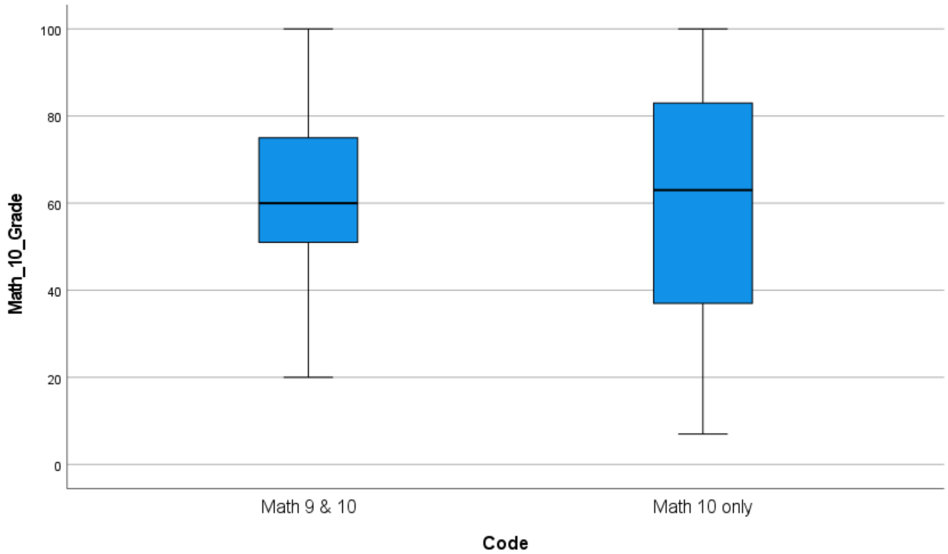
Box Plot 2 for Math 10 Grades for the Two Groups



There are three other outliers, data points 11, 15, and 23, which were removed as well. The outlier analysis is rerun and this time there are no outliers (see Figure 21). This data were used further to verify the next assumptions and for the *t*-test for independent samples. This assumption is met.

Figure 21

Box Plot 3 For Math 10 Grades for the Two Groups



5. The dependent variable should be approximately normally distributed for each group of the independent variable.

The descriptive statistics in Table 24 correspond to the dependent variable (Math 10 marks) for the two independent groups (Math 9 and 10) and Math 10.

Table 24

Descriptives for Math 10 Grades for the Two Groups

Descriptives

	Code			Statistic	Std. Error
Math_10_Grade	Math 9 & 10	Mean		62.62	2.820
		95% Confidence Interval for Mean	Lower Bound	56.95	
			Upper Bound	68.29	
		5% Trimmed Mean		62.79	
		Median		60.00	
		Variance		397.547	
		Std. Deviation		19.939	
		Minimum		20	
		Maximum		100	
		Range		80	
		Interquartile Range		25	
		Skewness		.061	.337
		Kurtosis		-.547	.662

(continued)

	Code			Statistic	Std. Error
Math_10_Grade	Math 10 only	Mean		60.61	4.976
		95% Confidence Interval for Mean	Lower Bound	50.45	
			Upper Bound	70.77	
		5% Trimmed Mean		61.32	
		Median		63.00	
		Variance		767.512	
		Std. Deviation		27.704	
		Minimum		7	
		Maximum		100	
		Range		93	
		Interquartile Range		46	
		Skewness		-.459	.421
		Kurtosis		-.956	.821

This testing uses the Shapiro-Wilk test of normality, presented in Table 25. The Shapiro-Wilk values for Math 10 grade were 0.426 and 0.058 for both groups, which are above the 0.05 threshold, which means that it exists an approximate normality of the dependent variable (Math 10 grades) for the two independent groups. This assumption is met.

Table 25

Test of Normality Shapiro-Wilk

Tests of Normality

	Code	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	*df*	Sig.	Statistic	*df*	Sig.
Math_10_Grade	Math 9 & 10	.074	50	.200*	.977	50	.426
	Math 10 only	.136	31	.149	.934	31	.058

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

6. There should be homogeneity of variances for the two independent groups (the variance is equal in each group of your independent variable).

As sample sizes for the two independent groups are quite different, the independent-samples *t*-test is sensitive to the violation of this assumption. The Levene's test of equality of variances is performed to test if groups' variances are different in the population. If p-value > 0.05, then the population variance of both groups is equal, thus the assumption of homogeneity of variances is met.

First, the SPSS program generated a Group Statistics as in Table 26 containing some useful descriptive statistics for the two groups—Math 9 and 10 and Math 10 only—which shows some important characteristics for the Math 10 grades for the two groups of students: sample size, mean, standard deviation and standard error of the mean.

Table 26

Statistics for the Two Groups

Group Statistics					
	Code	*N*	Mean	Std. Deviation	Std. Error Mean
Math_10_Grade	Math 9 & 10	50	62.62	19.939	2.820
	Math 10 only	31	60.61	27.704	4.976

Table 26 already shows that the standard deviations, thus the variances, for the two groups are quite different. At this step, the table informs that there were 50 students in the Math 9 and 10 group and 31 in the Math 10 only group. It seems the mean of grades for the Math 9 and 10 group (62.62) was greater than the mean of grades for the Math 10 only group (60.61). However, the variation of grades was greater for the Math 10 only group than for the Math 9 and 10 group.

Then the magnitude (size) of the mean difference between the means of the two independent groups was determined to find whether this mean difference is statistically significant. The assumption of equal variances was used, so that I correctly report the results of the *t*-test for independent groups. From the Table 25, the variances of the two groups are quite different: the variance of the Math 10 only group (767.512) is about twice the variance of the Math 9 and 10 group (397.547). These estimates from the two samples in this study may differ because of sampling variability. Levene's Test for Equality of Variances is used to test formally whether these variances are different in the population. The results of this test are presented in the first section (first two columns of numbers) of the Independent Samples Test in Table 27.

Table 27

Levene's Test for Equality of Variances

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means							
						Significance		Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
		F	Sig.	t	df	One-Sided p	Two-Sided p			Lower	Upper
Math_10_Grade	Equal variances assumed	1.537	.219	-.373	83	.355	.710	-2.150	5.761	-13.607	9.308
	Equal variances not assumed			-.360	56.094	.360	.720	-2.150	5.972	-14.113	9.813

This test gave a *p*-value = 0.219 > 0.05; therefore, the population variance of both groups is considered significantly the same, indicating that the assumption of homogeneity of variances is met.

The* t*-Test for Independent Samples

Once the six assumptions to applying the *t*-test for equal variances are met, the above Independent Samples Test with the first row of equal variances assumed was used to interpret the results from the standard *t*-test that uses pooled variances in its calculations and requires no modification to the degrees of freedom. From this first row, the last four columns show the mean difference between the two groups and provide a measure of the likely range of this mean difference. The mean difference between these two group means is -2.15, which means that indeed the grades mean of Math 9 and 10 group is greater than the mean of grades for the Math 10 only group by 2.15%. From Table 27, I can report a measure of variability of the mean difference, using the standard error of the mean difference, which is 5.76, or using the 95% confidence interval, which is (-13.61, 9.31). Therefore, with 95% confidence, I can say that the true mean difference lies somewhere between -13.61 and 9.31.

Finally, to determine whether the *t*-test for independent samples is statistically significant (if the mean difference is statistically significant), the middle portion of the Independent Samples

Test table is used, where p-value = 0.710 > 0.05; therefore, there is not a statistically significant mean difference in Math 10 grades between the two groups, thus we accept the null hypothesis H_0: $\mu = 0$: Students who take Math 9 prior to Math 10 finish their Math 10 course with no significant difference in marks compared to the students who are directed to Math 10 only.

A measure of the practical significance of the result is calculated next, called effect size and given by a value d of Cohen's test. This value is calculated using the pooled variance from the variances of the two groups when there is homogeneity of variances, as it is in this t-test for independent samples. Table 28 obtained in SPSS gives Cohen's coefficient d = 0.087, which is interpreted in the next section.

Table 28

Independent Samples Effect Sizes

Independent Samples Effect Sizes

				95% Confidence Interval	
		Standardizer[a]	Point Estimate	Lower	Upper
Math_10_Grade	Cohen's d	23.196	.087	-.362	.535
	Hedges' correction	23.419	.086	-.359	.529
	Glass's delta	27.704	.072	-.377	.520

a. The denominator used in estimating the effect sizes.

Cohen's d uses the pooled standard deviation.

Hedges' correction uses the pooled standard deviation, plus a correction factor.

Glass's delta uses the sample standard deviation of the control group.

Discussion of the Results

From the above section with the results from the study designs related to the three research questions and their associated hypotheses, a series of tables and figures was created above in SPSS that helped with verifying the assumptions for each study design and answered the hypotheses. A discussion of the results from the statistical analysis is presented below, corresponding to each research question of this study. References to certain tables and figures from the Result section are done without posting them again.

RQ1 Hypothesis 1

Hypothesis 1 of research question 1 asked whether students' marks in college preparation high school math courses are significant predictors of students' scores in the diagnostic math test. Using a sample of 71 students from the college preparation stream in high school that entered the Game Development program at our college during the intakes from Fall 2014 to Fall 2018, first, the eight assumptions for the standard multiple regression analysis were verified and all were met. Next, the standard multiple regression was applied to only two predictor variables: two math courses from Grade 11 and 12 high school in the college preparation stream (MBF3C and MAP4C) that had the greatest enrollment, in order to minimize the number of imputing values, as not all students took them or took both of them. The statistical analysis resulted in conclusions related to: (a) rejecting the null hypothesis, (b) finding the level of prediction of the criterion variable, (c) finding the regression line, and (d) examples of predicting the criterion variable.

1. First, the null hypothesis for the college preparatory stream of students, H_0: $\rho = 0$ was rejected, as the multiple regression coefficient $R = 0.457$ (for the two predictor variables) was found statistically significant and different than zero. Therefore, the alternative hypothesis was accepted: college preparation math course marks correlate significantly with the diagnostic test scores.
2. Second, the independent variables model, including the two independent high school math courses at college preparation level MBF3C and MAP4C, was found to be a

statistically significant fit to the data. The value of adjusted R^2 of 0.185 showed that these two math courses from high school explained 18.5% of the variability of the dependent variable (Diagnostic test) in the regression model, which is considered to be a medium size effect (Laerd Statistics, n.d.).

3. The regression line used to make predictions of the dependent variable based on values of the independent variable was determined to be:

   Predicted Diagnostic = b_0 + (b_1 x MAP4C) + (b_2 x MBF3C).

   The values of these coefficients were $b_0 = -2.980$, $b_1 = 0.374$, and $b_2 = 0.428$. The only one coefficient that was statistically significant was b_2, while the constant b_0 and the coefficient b_1 were not statistically significant.

4. In order to make predictions for the Diagnostic test scores given the grades in these two high school math courses, grade values of these two courses may be introduced in the predicted equation:

   Predicted Diagnostic = -2.980 + (0.428 x MBF3C) + (0.374 x MAP4C)

For example, if MAP4C = 80% and MBF3C = 80%; then the predicted diagnostic score in the math test is 61.18%.

Therefore, these slope coefficients represent the change in the dependent variable for a one-unit change in the independent variables. As such, an increase of 1% in the MBF3C mark is associated with an increase of 0.428 in Diagnostic test score, and an increase of 1% in the MAP4C mark is associated with an increase of 0.374 in the Diagnostic test score. It can be seen that the MBF3C mark has a slightly greater importance in getting a higher score in the diagnostic test than the other course MAP4C.

RQ1 Hypothesis 2

Hypothesis 2 of research question 1 asked whether students' marks in university preparation high school math courses are significant predictors of students' scores in the diagnostic math test. Using a sample of 31 students from the university preparation stream in high school during the intakes from Fall 2014 to Fall 2018 into the Game Development college program, the eight assumptions to use the standard multiple regression analysis were verified and met. Then, the standard multiple regression was applied to only two predictor variables: two math courses from Grade 11 and 12 high school in the university stream (MCR3U and MHF4U) that had the greatest enrollment, so that the number of imputing values was minimal, as not all students took them or took both of them. After the statistical analysis was performed, the results were analyzed and conclusions are specified next related to: (a) rejecting the null hypothesis, (b) finding the level of prediction of the criterion variable, (c) finding the regression line, and (d) examples of predicting the criterion variable.

1. First, the null hypothesis for the university preparatory stream of students, H_0: $\rho = 0$ was rejected, as the multiple regression coefficient $R = 0.692$ (for the two predictor variables) was found statistically significant and different than zero. Thus, the alternative hypothesis was accepted: university preparation math course marks correlate significantly with the diagnostic test scores.
2. Second, the independent variables model, including the two independent high school math courses at university preparation level MCR3U and MHF4U, was found to be a statistically significant fit to the data. The value of adjusted R^2 of 0.441 showed that these two math courses from high school explained 44.1% of the variability of the

dependent variable (Diagnostic test) in the regression model, which is considered to be a strong size effect (Laerd Statistics, n.d.).

3. The regression line used to make predictions of the dependent variable based on values of the independent variable was determined to be:

Predicted Diagnostic = b_0 + (b_1 x MHF4U) + (b_2 x MCR3U),

The values of these coefficients were $b_0 = 8.273$, $b_1 = 0.800$, and $b_2 = 0.205$. The only one coefficient that was statistically significant was b_1, while the constant b_0 and the coefficient b_2 were not statistically significant.

4. In order to make predictions for the Diagnostic test scores given the grades in these two high school math courses, grade values of these two courses may be introduced in the predicted equation:

Predicted Diagnostic = 8.273 + (0.800 x MHF4U) + (0.205 x MCR3U).

One similar example would be if MHF4U = 80% and MCR3U = 80%; then the predicted diagnostic score is 88.7%.

Therefore, these slope coefficients represent the change in the dependent variable for a one-unit change in the independent variables. As such, an increase of 1% in the MHF4U mark is associated with an increase of 0.8 in Diagnostic test score, and an increase of 1% in the MCR3U mark is associated with an increase of 0.205 in the Diagnostic test score. It can be seen that the MHF4U mark has a greater importance in getting a higher score in the diagnostic test than the other course MCR3U.

RQ2 Hypothesis 3

Hypothesis 3 of research question 2 asked whether students' high school GPA are significant predictors of students' scores in the diagnostic math test. However, students' high school GPA was not available at the college registrar because high schools in Ontario do not calculate their students' GPA at the end of their secondary education, but they send the high school academic credentials to the colleges, so they can make better use of the information they need, as specified by the registrar. Consequently, the registrar was able to provide students' senior grades for all courses taken in their Grades 11 and 12, so that a Senior Grade Average was able to be calculated. This average was considered as the independent/predictor variable instead of the GPA for answering this second research question. Literature also showed that some tertiary educational institutions consider the GPA from all 4 years of secondary education as a measure of prior knowledge (Bahr et al., 2019; Belfield & Crosta, 2012; Van den Broeck et al., 2019), while others look only at the senior GPA (Derr, 2017).

Using a sample of 106 students that entered the Game Development program at my college during the intakes from Fall 2014 to Fall 2018 for which both variables existed (GPA/SGA and the math diagnostic scores), first, five assumptions were checked for the correlation design. It was found that Pearson correlation could not be used as the data failed the bivariate normality test; thus, the Spearman coefficient was calculated to be 0.324. Further, the Coefficients table indicated that both Pearson and Spearman coefficients were statistically significant to justify the correlation between the two variables (GPA and math diagnostic scores).

Next, the simple linear regression was performed after meeting the applied seven assumptions, in order to check if a prediction can be made between the predictor variable: GPA and the criterion variable: diagnostic test scores. The statistical analysis resulted in conclusions related to: (a) rejecting the null hypothesis, (b) finding the level of prediction of the criterion variable, (c) finding the regression line, and (d) examples of predicting the criterion variable.

1. First, the null hypothesis H_0: $\rho = 0$ was rejected, as the correlation coefficient R = 0.314 was found statistically significant and different than zero. Therefore, the alternative hypothesis was accepted: the high school GPA (in this study the Senior Grade Average was possible to be used) correlated significantly with the diagnostic test scores.

2. Second, the independent variables model was found to be a statistically significant fit to the data. The value of adjusted R^2 of 9.0% indicated a moderate effect size of the variability of the dependent variable (Diagnostic test); therefore, a medium weak statistically significant prediction of the dependent variable in the regression model according to Laerd Statistics (n.d.).

3. The regression line used to make predictions of the dependent variable based on values of the independent variable was determined to be:

Predicted diagnostic = b_0 + (b_1 x GPA)

The values of these coefficients were b_0 = 1.574 and b_1 = 0.834, from which the slope coefficient b_1 was statistically significant. By finding the linear regression equation and the fact that the slope coefficient is significant, the third hypothesis H3 was demonstrated to be supported: GPA grades are significant predictors of the diagnostic math test scores.

4. In order to make predictions for the Diagnostic test scores given the GPA, the linear regression equation becomes:

$$\text{Predicted Diagnostic} = 1.574 + (0.834 \times GPA).$$

Thus, an increase of 1% in the high school GPA mark is associated with an increase of 0.834% in Diagnostic test score. SPSS can be used to make these predictions for any value of the independent variable (GPA): I present here two examples on predicting the diagnostic test score when given GPA values of 60% and 80% just by using the regression equation. The predicted diagnostic test scores are 51.61% and 68.29% respectively.

RQ3 Hypothesis 4

Hypothesis 4 of research question 3 asked whether the students who take Math 9 prior to Math 10 finish their Math 10 course with similar marks as the students who are directed to Math 10 only. The sample size depended on the number of students who went into these two different groups and completed them, for which a diagnostic test score existed. Therefore, from all the students who started this college program in the Fall 2014 through Fall 2018 cohorts, only 54 students were considered for the group who went through the two math courses: Math 9 and Math 10 and 31 students were considered for the group who went directly to Math 10.

The *t*-test for independent samples was this study's design, and the six assumptions stated in Laerd Statistics (n.d.) were verified and met. Levene's Test for Equality of Variances was used to test formally whether these variances were different in the population, and it was found that the population variance of both groups was considered significantly the same; therefore, the *t*-test for equal variances was performed in SPSS. The statistical analysis resulted in conclusions related to: (a) finding the mean difference between the two groups, (b) accepting the null hypothesis after performing the *t*-test, and (c) concluding the results of the *t*-test for independent samples.

1. First, the mean difference between these two group means was -2.15, which meant that the grades mean of Math 9 and 10 group was greater than the mean of grades for the Math 10 only group by 2.15%., and the 95% confidence interval for the mean difference was between -13.61 and 9.31.

2. Second, the null hypothesis H_0: $\mu = 0$ was accepted, as there was not a statistically significant mean difference in Math 10 grades between the two groups.
3. Finally, accepting the null hypothesis means that students who take Math 9 prior to Math 10 finish their Math 10 course with no significant difference in marks compared to the students who are directed to Math 10 only.

Therefore, the last hypothesis, H4 is true, which answers the last research question RQ3: Considering the students placement into two college math courses (Math 9 and Math 10) based on their diagnostic test scores, there is no significant difference (in their final scores in Math 10) between the students who entered directly Math 10 and the students who took Math 9 prior to taking Math 10. Therefore, taking Math 9 before going to Math 10 helps the weaker and regular knowledge students raise their knowledge prior to taking the Math 10 course to a level similar to the level of knowledge of students going directly to Math 10.

In current years in statistics, not only statistical significance has been reported for tests, but also the practical significance of the result of the test. A measure of the practical significance of the result is called effect size and given by a value *d* of Cohen's test. It was found that the effect size value $d = 0.087 < 0.2$, which is considered by Cohen to show small practical significance of the results of this *t*-test for independent samples. This means that statistically the Math 10 grades are significantly the same between the two groups (Math 9 and 10 and Math 10 only), but one can look at the practical significance of this result as well: small practical significance. However, Laerd Statistics (n.d.) warns about one major weaknesses of using effect sizes: there are not comprehensive guidelines for interpreting the strength of an effect size, and that the importance of the effect size is subject-specific.

Lead to Chapter 5

This Chapter 4 started with introducing the three research questions of this study and their hypotheses. The corresponding statistical analysis for each research question was presented, starting with verifying the assumptions of each statistical design. The first research question, RQ1, asked about the predictive value of the math courses students took in their senior high school split in two different streams: college-preparation and university-preparation. Once the eight assumptions for the standard multiple regression were verified and the multiple regression was performed, it was found that the most common math courses taken in high school can predict students' scores in the math diagnostic test at the college, with a medium and strong level of prediction for college- and university-preparation streams, respectively. The regression lines' equations were presented, and some examples were calculated to show the predicted values.

Further, the second research question, RQ2, related to the prediction of scores in the math diagnostic scores given students GPA from high school (it was used the Senior Grade Average instead, as the college registrar did not have the GPA to supply to the researcher). The correlation design was performed first after its five assumptions were verified. Then the seven assumptions for the linear regression were verified and met, and the linear regression was performed in SPSS. A line of regression was defined and used to predict further diagnostic test scores from students' academic information from their senior high school years.

Finally, the last research questions, RQ3, was presented with its hypothesis. After verifying the corresponding six assumptions, the independent *t*-test was performed. It was found that students' grades in the Math 10 course was statistically the same for the students who were directed to the Math 10 course only and the students who took the introductory college course

Math 9 prior to going into the Math 10 course. Chapter 5 presents an overall analysis and interpretation of the findings of this study and recommend areas for additional research.

Chapter 5

Conclusions and Recommendations

This chapter is divided into three sections: Conclusions, Recommendations, and Summary. The Conclusion's section highlights the main results of this study for each of the three research questions asked initially and their related hypotheses. An interpretation of these results in the context of literature researched and any shortcomings encountered during the statistical analysis is presented as well. Recommendations for future research are also articulated as a result of the findings of this study and its limitations.

Conclusions

This study started from the perceived need to find whether the math diagnostic test that 1st-year college students take can be considered to identify their strengths and weaknesses in math, and, therefore, pointing to their math background knowledge. Researched literature revealed that prior knowledge represents one of the most powerful cognitive predictors of academic performance (Binder et al., 2019; Derr et al., 2018; Smith et al., 2019). Studies showed that a few measures of student prior knowledge are considered influential to students' academic success in college and university: the academic level of senior classes in mathematics, the domain-specific courses in high school, high school GPA, and the diagnostic test in math prior to starting college or university programs (Bahr et al., 2019; Derr, 2017; McAdams, 2017; Qin, 2017; Wilson, 2018).

The research questions of this study were formulated with one target in mind: to assess the validity of the diagnostic test in math, as this test is the only measure of students' prior knowledge in math that the teachers at the college may have and use. Placing students in different math courses according to difficulty has always been considered to give the students what they need: support or challenge. Early detection of students' mathematical weakness, just at the beginning of their college programs, allows educators to intervene and assist the weak students (Derr, 2017; Shim et al., 2017). Challenging strong students in math keeps them away from boredom and program withdraw, and takes their knowledge further in math, not backward (Cluett et al., 2009).

The statistical analysis performed in this study was twofold. First, the study was set to find if the diagnostic test in math represents a good measure of students' prior knowledge in math, as grades from high school or students' GPA are not available to college teachers in general. Second, placing the weaker and general knowledge students (with a score less than 85% in the math diagnostic test) into an introductory math course was thought to raise their math knowledge to a similar level to the students who are strong in math (scoring above 85% in the diagnostic test). The results from the statistical analyses of this study did find significant correlations between students' academic performance in high school and the diagnostic test in math, and no significant difference between the students who took both math courses and the ones who were directed to take only the higher-level math course, as interpreted below.

Research Question 1

The first research question asked whether students' marks in their senior high school math courses predicted students' scores in the diagnostic math test. Literature showed that one of the most powerful cognitive predictors of academic achievement is prior knowledge measured by the academic level of senior classes in mathematics (Binder et al., 2019; Derr et al., 2018; Smith et al., 2019). Considering there are two streams in secondary high school, college preparation and university preparation streams, hypothesis H1 and H2 inquired whether the grades in college preparation and university preparation high school math courses, respectively, were significant predictors of students' scores in the diagnostic math test. The study found that specific senior math courses in senior grades correlated significantly with the diagnostic test in math. For the two senior math courses with the greatest enrollment in the college preparation stream, MBF3C and MAP4C, the multiple regression found that they explained 18.5% of the variability of the diagnostic test in the multiple regression model, which is considered to be a medium size prediction effect (Laerd Statistics, n.d.).

Moreover, for the students in the university preparation stream taking the math courses MCR3U and MHF4U, there was a higher correlation to the math diagnostic test: these two math courses explained 44.1% of the variability of the diagnostic test in the regression model, which is considered to be a strong size effect (Laerd Statistics, n.d.). Therefore, the academic background of high school students from the university stream as opposed to the college stream predicts better their scores in the diagnostic test at the college. In addition, the regression equation from the university stream contains coefficients greater than the ones corresponding to the college stream, which leads to higher scores in the diagnostic test for students from the university stream than from the college stream. In the calculated examples related to the predicted regression equations from hypotheses H1 and H2 in Chapter 4, students with an 80% score in each university level math courses from high school scored 27.5% higher in the diagnostic test than the students with similar grades in their college-level math courses (88.7% prediction as opposed to 61.18%).

This conclusion related to the analyzed data from the 5 years' intake into the Game Development program at the college offers important information. It seems that the math diagnostic test marks reflect better students' prior knowledge in math (senior math courses grades) for the students coming from the university preparation stream at high school than from the college preparation stream. Moreover, students coming from the university stream have higher predicted values in their diagnostic test scores than students coming from the college stream given similar high school math grades. Considering the means of math diagnostic scores between the two groups of students, aside from using the regression prediction equations, students from the university preparation stream achieved an average score of 77.965 in their math diagnostic compared to 54.413 average score of students from the college preparation stream. This result is in accordance with what Derr et al. (2018) and Van den Broeck et al. (2019) contented: that the level of academic education prior to enrolment in higher education is the most predictive for further academic achievement.

However, the statistical analysis related to this research question RQ1 has a few shortcomings as a result of (a) the available data to have both the independent and dependent variables, and (b) students' choice of high school courses. First of all, from the 202 students who started the Game Development program in the Fall 2014 through Fall 2018 intakes, only 71 students were considered for the hypothesis H1 (college preparation stream in high school) and 31 students were considered for the hypothesis H2 (university preparation stream in high school). The 100 students left out could not be considered, as they missed one variable: either the independent variable: marks in their high school Grade 11 or 12, or the dependent variable: no written math diagnostic test upon entering their college program.

Second, in both the college preparation and the university preparation streams, students have a choice of six math courses in their Grades 11 and 12 for each stream. As some students may have taken only one math course while others took the most four math courses in their senior high school years, the secondary data used in this analysis had missing marks. Therefore, only the most populated math courses were used, not all six high school courses; thus, the number of students used in this statistical analysis was diminished further.

Moreover, the statistical imputing of missing values was performed only for two math courses taken by most students (MBF3C and MAP4C for the college stream and MCR3U and MHF4U for the university stream). This imputing of missing values may have introduced errors in all calculations, as they were created by the SPSS software. There were 29 imputed values for either the MBF3C or the MAP4C courses for the 71 students considered at the college preparation stream, which means that only 42 students took both of these math courses. Similarly, there were eight imputed values for either the MCR3U or the MHF4U courses for the 31 students considered at the university preparation stream, which means that only 23 students took both of these math courses.

Research Question 2

The second research question inquired whether students' high school GPA predicted students' scores on the diagnostic math test. For this study, the high school GPA was not part of the collected data by the college registrar from high schools, only the students' grades for all senior courses were available, such that this study uses the high school Senior Average Grade (referred to as GPA though) instead of the high school GPA. Literature showed that GPA represents another measure of student prior knowledge that has been considered influential to students' academic success further in college and university (Binder et al., 2019; Derr et al., 2018; Smith et al., 2019).

The statistical analysis for this research question, RQ2, in the present study showed a weak significant correlation between students' GPA and the math diagnostic scores and produced a linear regression equation to predict students' scores in the diagnostic test based on their GPA. However, the regression line for predicting the math diagnostic test scores as a result of the high school GPA had a significant slope coefficient (0.834) greater than the slope coefficients for the senior math courses at both university and college preparation streams that were statistically significant in the corresponding linear regression equations (0.800 for MHF4U and 0.428 for MBF3C). Even if the slope coefficient for GPA had a greater value than the slope coefficients for senior math courses, the adjusted coefficient of determination R^2 for GPA (9%) explained the proportion of variance in the diagnostic test at a much lower level of prediction than the adjusted coefficients for the senior math courses at high school (44.1% for the university preparation courses and 18.5% for the college preparation courses), different than what prior studies found. The difference in this correlation level may result from using the Senior Average Grade and not the real GPA at high school, as most of past research showed that high school GPA had the highest predictive value for students' academic success in tertiary education (Bahr et al., 2019; Derr, 2017; Qin, 2017; Van den Broeck et al., 2019).

Similar to the first research question, the statistical analysis performed for this research question, RQ2, had a shortcoming related to students' numbers. From the 202 students who began this college program in the 5 years considered, only 106 students' data were examined. The difference number (96 students) did not have the Senior Grade Average from their high school, and/or did not take the diagnostic test when they entered the college program. In addition, the college registrar does not require high schools to send students' GPA, but it collects students' grades from all courses at the senior level and from selected courses from Grades 9 and 10. Because of this reason, the Senior Grade Average from all courses from Grades 11 and 12 was used for the statistical analysis instead of the GPA that the researched literature mentions as a measure of students' prior knowledge.

Research Question 3

Research question RQ3 wanted to verify that taking an introductory math course, Math 9, at the college would raise weaker students' marks in the next course (Math 10) to a level similar to the one scored by the stronger college students placed directly into the Math 10 course. The placement into these two college courses has been directed by students' scores either below or above 85% in their math diagnostic test for their placement first into Math 9 or directly into Math 10 courses respectively. As literature demonstrated, one major characteristic of college and university students is represented by a great difference among students' basic skills and knowledge, especially in mathematics (Baldwin & Squires, 2019; Prendergast et al., 2017). As Game Development program is a science-related field, mathematics prior knowledge is key to almost all program courses; literature research shows that an early identification of this knowledge can provide proper support to at-risk students, such as directing them to noncredit developmental math courses (Bahr et al., 2019; Derr, 2017; McAdams, 2017; Qin, 2017; Wilson, 2018). However, at my college, the introductory Math 9 course is a regular curriculum math course, and only students with a high level of math knowledge can bypass it, while regular- and lower-background knowledge students are required to take it in order to advance into the next course: Math 10.

The statistical analysis related to answering this last research question, RQ3, was the *t*-test for independent samples applied to the two groups of students: students who took Math 9 before taking Math 10, and students who were directed to Math 10 as a result of their high score in the math diagnostic test (higher than 85%). The test found that there was not a statistically significant mean difference in Math 10 grades between the two groups, which shows that taking Math 9 before going to Math 10 helps the weaker- and regular-knowledge students raise their knowledge to a level similar to the level of knowledge of students going directly to Math 10. Thus, offering the Math 9 course to the regular and weaker students should continue at the college. Placing higher-knowledge students directly into the Math 10 course also should continue, as current research found that more challenge may benefit students of high ability levels to get more advanced content and avoid boredom (Cluett et al., 2009; Fu & Mehta, 2018; Roller & Steinberg, 2020; Yassin et al., 2015).

Similar to the other research questions, for answering this research question RQ3, the number of students who participated was only 54 students for the group that went through the two math courses: Math 9 and Math 10, and 31 students were considered for the group that went directly to Math 10. Therefore, 85 out of the 202 students who started this college program in the Fall 2014 through Fall 2018 cohorts participated in this analysis, which is a fairly small number in order to draw a definite conclusion from these 5 years and also expand it to the whole program, which is a shortcoming of this study. The difference number (117 students) could not be used in the statistical analysis, as they either did not take the diagnostic test (even if they took the two college math courses), or they took the diagnostic test but withdrew from the program during their 1st college year (failed Math 9 and did not continue into Math 10 or withdrew while in Math 9 or in Math 10). In addition, there were 18 students who took the two math courses (Math 9 and Math 10), but their placement in Math 9 was a result of a lack of the diagnostic test, not to a score lower than 85% in it. Therefore, they could not be used in the analysis, as they might have skewed the results.

Recommendations for Further Studies

This section includes suggestions for further research related to the three questions that guided this study: RQ1—how well students' background knowledge in math as represented by their senior math courses in high school predict their scores in the math diagnostic test; RQ2—how well students' GPA from high school predicts students' scores in the math diagnostic test; and RQ3—whether taking the introductory college math course has helped students' further in the more advanced, technical math course. Ideas for further research are directed by the data available and analyzed, or related to the data and the study's questions and found to be lacking in the literature.

Research Question 1

Research question RQ1 looked at the predictability of students' marks in senior math courses from high school to their scores in the math diagnostic test that 1st-year college students take in their first math class. Because students in high school choose from six different math courses in each of the two streams, college preparation and university preparation, and that the program analyzed, Game Development has a maximum of 40 students in each intake year, the sample size to work with in this study was small. Further research should look at answering the same research question related to senior high school math courses but the data should be collected for a longer period of time, not only from 5 years of intakes. Moreover, similar future research should be conducted with other groups of students for which the class size every year is much bigger (65 to 70 students), such as the 1-year PreHealth program at the college. In this way, there will be the opportunity to compare the programs at the college from the point of view of observing which program shows a greater predictive value of students' prior knowledge into the math diagnostic test at the college.

In addition, the existent student data was analyzed separately for the two hypotheses H1 and H2 related to the research question RQ1 in relationship with correlations and predictions on the math diagnostic test. The two different streams of students, college preparation versus university preparation, could be further used to look at students' retention and attrition after the 1st year of college and the end of the 2-year Game program. This analysis would have implications on the college's setting the students' academic requirements when applying to this college program.

A study by Smith et al. (2019) recommended further research into the importance of prior knowledge in course design and delivery at a large university in the North American Pacific Northwest and mentioned that more studies exploring its role should be conducted using a standardized test as a means of evaluation. The present study used the diagnostic test to evaluate students' prior knowledge in order to place students in the two college math courses, so that students are given the math content and the supports needed corresponding to their background knowledge. However, this present study was conducted on a sample of 102 students at a single institution. The findings may be different if a greater sample size is used, multiple classes are taught by different instructors, or different educational institutions are involved.

While Shim et al. (2017) found that diagnostic testing is a valuable tool for educators to detect students' strength and weakness in mathematics prior to entering a university in Malaysia, they also suggested to have it applied at the beginning of courses in university. For future research, they proposed creating a plan to determine weaknesses mathematics students might have prior to their university math courses in order to implement effective intervention strategies to ensure academic success. The present study answers this need brought up by Shim and the team: my college has a plan in place to help weaker students in math by diverse tutoring, learning workshops, and individual accommodations.

Research Question 2

Research question RQ2 looked at high school students' GPA as a measure of prior knowledge and its predictive value to students' scores in the math diagnostic test. What this study did not define as a separate research question was to consider the research questions RQ1 and RQ2 together and follow other studies' recommendations to take multiple measures as students' prior knowledge: math courses in senior years and also GPA from high school (McAdams, 2017; Scott-Clayton, 2012; Walker, 2017; Wilson, 2018). A further study should observe the predictive values of all these measures and remark the measure with a higher coefficient of prediction from all measures. Moreover, a further study could look at the predictability of students' high school GPA into students' college GPA, as Qin (2017) recommended.

In addition, this study looked only at academic secondary data pertaining to high school and 1st-year college math courses in general. A future study should also collect primary data directly from students by using questionnaires to indicate students' interests, self-concept, study motives, learning behaviors, and others. In addition, data related to students' age, gender, social status, family information, culture, language, and other self-identifying details might give more information about the kind of students attracted by the Game Development program at the college and their academic performance. Similarly, a study by Rach and Ufer (2020) also recommended future research in a university in Germany in order to clarify the interaction among variables such as general school achievement, interest, self-concept, study motives, learning behaviors, and others that are relevant to students' success.

Research Question 3

Research question RQ3 looked at how appropriate the placement of the 1st-year college students enrolled in the Game Development program was into the two college math courses Math 9 and Math 10. The statistical analysis of this research question found that taking a Math 9 introductory course helped weaker and average knowledge students achieve similarly in the Math 10 course as the students who were placed directly into the Math 10 course. However, this study did not look at the marginal students, the ones who scored very close to the cut-off score of 85%, to see whether their placement was indeed appropriate to their math skills. A future study should look only at students who scored within this area to check whether the cut-off score should be set higher or lower for this diagnostic test.

Another research recommendation for the future would be to look at the ratio of students coming from the college preparation stream versus the university preparation stream who are placed in the two college math courses. It would be helpful to see the percentage of students who come from the college stream and go directly to the more difficult math course versus the percentage of students who come from the university stream and do not qualify directly for Math 10. A look at these students' scores in the math diagnostic test and whether they are around the 85% cut-off score might give an indication of the correct or incorrect set up of this cut-off value.

In addition, this study was not set to explore the academic performance of the at-risk students, which are the students who score usually less than 30% in the math diagnostic test. These students are given extra supports such as peer and teacher tutoring, accommodations in terms of test length and use of extra technology, and learning skills workshops. How these students succeed in Math 9 and beyond as a result of these supports should be analyzed in a future study. Similarly, a study by Cassells (2018) focused on identifying at-risk students and recommended further research into how offering peer tutors and volunteers as an early education foundation might improve the academic performance of weaker students from University of Pretoria in South Africa.

Summary

The math diagnostic test has been used by many programs at my college campus to give general information to the math teachers and program coordinators about their students at risk, so that supports in the form of tutoring, workshops, and accommodations are set up for them. For the Game Developing program especially, this math diagnostic test is also used to place stronger math students directly into a technical math course, while skipping the introductory college math course that the weaker and average students in math are required to take. The question that started this study was whether the math diagnostic test the college uses is a valid test, capable of representing students' prior knowledge in math from high school, so that this test places students correctly in college math classes based on their specific background knowledge in math.

This study looked at the Game Development program for 5 years of intake: Fall 2014 through Fall 2018. Students' marks in their high school senior math courses and their Senior Grade Average of all courses (replacing high school GPA that was not available) were considered as representing students' prior knowledge in math before entering the college. Students' scores in their math diagnostic test for these cohorts of students were examined in this study to verify the assumption that they show students' prior knowledge in math, as students' academic credentials from high school are not available to teachers and coordinators on a general basis. Students' marks in the two college math courses Math 9 and Math 10 also were used to verify whether students' placement in the introductory Math 9 course raised these students' math knowledge to a level similar to that of students who were directed only to the Math 10 course.

This research contains five chapters that presented the pertinent information to answer three research questions:

RQ1: Do students' marks in their senior high school math courses predict students' scores on the diagnostic math test? There are two different senior high school streams: college preparation and university preparation.

RQ2: Do high school student GPAs predict students' scores on the diagnostic math test?

RQ3: Considering the students placement into two college math courses (Math 9 and Math 10) based on their diagnostic test scores, is there a significant difference (in their final scores in Math 10) between the students who entered directly Math 10 and the students who took Math 9 prior to taking Math 10?

The study was organized on the following five chapters:

1. Chapter 1: Introduction.

This chapter introduced the study's purpose, background, importance, and limitations.

2. Chapter 2: Literature Research.

Current studies from the past 5 years relevant to the following points were researched: the social constructivist theory, andragogy as a philosophy of teaching and learning of mathematics, the influence of streaming students into different academic streams in high school, the prior math knowledge students bring with them to the college, math diagnostic tests definitions and use, and high school measures that predict students' placement in postsecondary education.

3. Chapter 3: The Research Methodology.

Considering that student numerical data were collected in the form of marks in high school math courses, college math courses, and the diagnostic test, the study methodology was quantitative. To answer the three research questions mentioned above, two designs of the study were employed: a correlational/predictive design to verify that students' marks in the high school senior math courses and/or students' high school GPA correlate and eventually predict students' scores in the math diagnostic test; and the quasi-experimental design in order to find whether students who take the introductory college math courses increase their math knowledge to a similar level as the students who are directed only to the higher level Math 10 course.

4. Chapter 4: Results and Discussion.

This chapter required the use of the SPSS statistical software to answer all three research questions and their related hypotheses. Pearson and Spearman correlations, linear and multiple regression analyses, and students *t*-test for independent samples were presented in great detail, with tables, graphs, and discussion of the results.

5. Chapter 5: Conclusions and Recommendations.

This last chapter presents the conclusions of the research with their implications, limitations, and recommendations for future research.

The purpose of this study was to find to what extent the students' high school math marks and GPAs predict students' scores in the math diagnostic test. The study found that specific senior math courses in senior grades correlate significantly with the diagnostic test in math and predict students' scores in the math diagnostic test at a medium level for the college preparation–stream students and at a strong level for the university preparation–stream students. In addition, it was found a weaker predictive value for the math diagnostic test due to GPA compared to individual math courses from senior high school, which may have been influenced negatively by using the Senior Grade Average instead of the real senior GPA. Finally, the *t*-test found that taking Math 9 before going to Math 10 helps the weaker- and regular-knowledge students raise their knowledge to a level similar to the level of knowledge of students going directly to Math

10. Thus, offering the Math 9 course to the regular and weaker students should continue at the college.

The results of this study prove that the math diagnostic test is a valid test to show students’ math background knowledge before starting their Game Development program. That there was not a significant difference between students’ grades in Math 10 from the two groups, the students who took both Math 9 and Math 10 and the students who took only Math 10, demonstrates that the diagnostic test placed the students correctly into the two groups using the cut-off score of 85%. However, a further study should look at the students who scored around the 85% for eventual misplacement.

Indeed, “Mathematics is a gatekeeper to future educational success,” and by using the math diagnostic test to stream students in postsecondary education, all students benefit (Woods et al., 2018, p. 439). The weaker and regular students are directed to the introductory, basic math course, where they are given needed supports in order to raise their math knowledge to a level necessary to succeed in further math courses. The stronger students in math are offered the challenge they need by directing them to the more advanced math course without the need to take the basic math course.

REFERENCES

Acosta, D., North, T., & Avella, J. (2016). Impact of delivery modality, student GPA, and time-lapse since high school on successful completion of college-level math after taking developmental math. *Current Issues in Education, 19*(1), 1–14. http://cie.asu.edu/ojs/index.php/cieatasu/article/view/1518

Allen, P., Withey, P., Lawton, D., & Aquino, C. T. (2016). Andragogical teaching methods to enhance nontraditional student classroom engagement. *I-Manager's Journal of Educational Technology, 13*(2), 47–59. files.eric.ed.gov/fulltext/EJ1131815.pdf

Allen, P., & Withey, P. (2017). The student customer phenomenon. *Journal of Higher Education Theory and Practice, 17*(3), 45–56. https://articlegateway.com/index.php/JHETP/article/view/1552/1482

Bahr, P. R., Fagioli, L. P., Hetts, J., Hayward, C., Willett, T., Lamoree, D., & Baker, R. B. (2019). Improving placement accuracy in California's community colleges using multiple measures of high school achievement. *Community College Review, 47*(2), 178–218. https://journals.sagepub.com/doi/full/10.1177/0091552119840705

Baldwin, S., & Squires, V. (2019). Changing trends and emerging themes: Teaching and learning in post-secondary mathematics classrooms. *The Mathematics Enthusiast, 16*(1), 47–74. https://scholarworks.umt.edu/tme/vol16/iss1/5

Belbase, S. (2014). Radical versus social constructivism: An epistemological-pedagogical dilemma. *International Journal of Contemporary Educational Research, 1*(2), 98–112. http://ijcer.net/tr/download/article-file/147931

Belfield, C. R., & Crosta, P. M. (2012). *Predicting success in college: The importance of placement tests and high school transcripts* [Working paper No. 42]. Community College Research Center Teachers College, Columbia University.

Binder, T., Sandmann, A., Sures, B., Friege, G., Theyssen, H., & Schmiemann, P. (2019). Assessing prior knowledge types as predictors of academic achievement in the introductory phase of biology and physics study programmes using logistic regression. *International Journal of STEM Education, 6*(33), 1–14. https://stemeducationjournal.springeropen.com/articles/10.1186/s40594-019-0189-9

Bonti, E., Kamari, A., Kougioumtzis, G., Theofilidis, A., & Sofologi, M. (2020). Different theoretical perspectives on specific learning difficulties in mathematics. Implications for special educational intervention and for everyday school practice: An overview study. *International Journal of Education and Research, 8*(2), 107–118. https://www.ijern.com/journal/2020/February-2020/11.pdf

Bozkurt, G. (2017). Social constructivism: Does it succeed in reconciling individual cognition with social teaching and Learning practices in mathematics? *Journal of Education and Practice, 8*(3), 210–218. https://files.eric.ed.gov/fulltext/EJ1131532.pdf

Cassells, L. (2018). The effectiveness of early identification of 'at risk' students in higher education institutions. *Assessment and Evaluation in Higher Education, 43*(4), 515–526. https://repository.up.ac.za/bitstream/handle/2263/62832/Cassells_Effectiveness_2018.pdf?sequence=1&isAllowed=y

Christmas, D., Kudzai, C., & Josiah, M. (2013). Vygotsky's zone of proximal development theory: What are its implications for mathematical teaching? *Greener Journal of Social Sciences, 3*(7), 371–377. https://pdfs.semanticscholar.org/1331/9ed7d61cd5786f062211a2bd34c8f488d01c.pdf

Clark, A., Karvonen, M., Romine, R., & Kingston, N. (2018, April). *Teacher use of score reports for instructional decision-making: Preliminary findings* [Paper presentation]. The 2018 annual meeting of the National Council on Measurement in Education, New York, NY, United States. https://aai.ku.edu/sites/aai.ku.edu/files/docs/conference/NCME_2018_Score_Report_Use.pdf

Cluett, S., Jarosh, J., & Roscoe, W. (2009). Correlating student mathematics diagnostic testing and high school mathematics grades with performance in the Fanshawe College Pre-Health Science program. *Faculty and Staff Publication,* 1–41. http://first.fanshawec.ca/cri_facultystaffpublications/5?utm_source=first.fanshawec.ca%2Fcri_facultystaffpublications%2F5&utm_medium=PDF&utm_campaign=PDFCoverPages

Collins, C. A., & Gan, L. (2013). Does sorting students improve scores? An analysis of class composition. *National Bureau of Economic Research* [Working paper No. 18848]. https://www.nber.org/papers/w18848

Conforme, D., Romero, A., Romero, D., & Laz, E. (2019). Application of diagnostic assessment on beginning school year. *International Research Journal of Management, IT & Social Sciences*, *6*(5), 53–59. https://doi.org/10.21744/irjmis.v6n5.701

Derr, K. (2017). Identifying consistent variables in a heterogeneous data-set: Evaluation of a web-based pre-course in mathematics. *Electronic Journal of E-Learning*, *15*(1), 82–93. https://files.eric.ed.gov/fulltext/EJ1140127.pdf

Derr, K., Hubl, R., & Ahmed, M. Z. (2018). Prior knowledge in mathematics and study success in engineering: informational value of learner data collected from a web-based pre-course. *European Journal of Engineering Education*, *43*(6), 911–926. https://www.tandfonline.com/doi/pdf/10.1080/03043797.2018.1462765?needAccess=true

Elsner, B., & Isphording, I. E. (2017). A big fish in a small pond: Ability rank and human capital investment. *Journal of Labor Economics*, *35*(3), 787–828. http://ftp.iza.org/dp9121.pdf

Finnegan, M., & Ginty, C. (2019). Moodle and social constructivism: Is Moodle being used as constructed? A case study analysis of Moodle use in teaching and learning in an Irish higher educational institute. *All Ireland Journal of Teaching and Learning in Higher Education*, *11*(1), 1–21. https://pdfs.semanticscholar.org/c2b8/39a001bb7bc647a27b826c4ae47514076792.pdf?_ga=2.32305726.1458684089.1594777399-1455199937.1594777399

Ford, C., McNally, D., & Ford, K. (2017). Using design-based research in higher education innovation. *Online Learning*, *21*(3), 50–67. https://files.eric.ed.gov/fulltext/EJ1154158.pdf

Fox, J., & Artemeva, N. (2017). From diagnosis toward academic support: Developing a disciplinary, ESP-based writing task and rubric to identify the needs of entering undergraduate engineering students. *English for Specific Purposes Today*, *5*(2), 148–171. https://doi.org/10.18485/esptoday.2017.5.2.2

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences Early Edition, 111*(23), 8410–8415. www.pnas.org/cgi/doi/10.1073/pnas.1319030111

Fu, C., & Mehta, N. (2018). Ability tracking, school and parental effort, and student achievement: A structural model and estimation. *Journal of Labor Economics*, *36*(4), 923–979. https://www.ssc.wisc.edu/~cfu/tracking_paper1.pdf

Gay, L. R., Mills, G. E., & Airasian P. (2009). *Educational research. Competencies for analysis and applications* (9th. ed.). Pearson.

Giersch, J., Bottia, M. C., Stearns, E., Mickelson, R. A., & Moller, S. (2021). The predictive role of school performance indicators on students' college achievement. *Educational Policy*, *35*(7), 1085–1115. https://journals.sagepub.com/doi/pdf/10.1177/0895904819857827

Henschke, J. A. (2008). A Global Perspective on Andragogy: An Update. In M. Boucouvalas (Ed.), *Proceedings of the Commission on International Adult Education [CIAE] pre-conference, American Association for Adult and Continuing Education [AAACE] conference* (Vol. 1, pp. 43–94). www.umsl.edu/~henschkej/articles/added-02-10/11.pdf

Holton, E. F., Awanson, R. A., & Naquin, S. S. (2001). Andragogy in practice: Clarifying the andragogical model of adult learning. *Performance Improvement Quarterly*, *14*(1), 118–143. http://browningmedportfolio.weebly.com/uploads/2/3/8/1/23811553/clarifying_the_a_model.pdf

Hunter, J., & Brown, A. (2018). A fresh look at initial and diagnostic assessment in community education provision. *Teaching in Lifelong Learning*, *8*(2), 9–24. https://search.informit.com.au/documentSummary;dn=657153335150784;res=IELHSS

Javidanmehr, Z., & Sarab, M. R. A. (2017). Cognitive diagnostic assessment: Issues and considerations. *International Journal of Language Testing*, *7*(2), 73–98. https://ijlte-1.ov2.com/content/ijlte_1_ov2_com/wp-content_138/uploads/2019/07/453-2017-7-2.pdf

Laerd Statistics (n.d.). *Friedman Test in SPSS*. https://statistics.laerd.com/spss-tutorials/friedman-test-using-spss-statistics.php

Leeds, D. M., & Mokher, C. G. (2019). Improving indicators of college readiness: Methods for optimally placing students into multiple levels of postsecondary coursework. *Educational Evaluation and Policy Analysis, 42*(1), 87–109. https://journals.sagepub.com/doi/abs/10.3102/0162373719885648

Leme, A. C., Escardibul, J. O., Nunes, L. C., Reis, A. B., & Seabra, C. (2020). The effect of a specialized versus general upper-secondary school curriculum on students' performance and inequality. A difference-in-differences cross-country analysis. *Applied Economics, 52*(39), 4317–4331. https://www.tandfonline.com/doi/abs/10.1080/00036846.2020.1734183

Logue, A.W., Douglas, D., & Watanabe-Rose, M. (2016). Should students assessed as needing remedial mathematics take college-level quantitative courses instead? A randomized controlled trial. *Educational Evaluation and Policy Analysis, 38*(3), 578–598. https://journals.sagepub.com/doi/pdf/10.3102/0162373716649056

Logue, A. W., Douglas, D., & Watanabe-Rose, M. (2019). Corequisite mathematics remediation: Results over time and in different contexts. *Educational Evaluation and Policy Analysis, 41*(3), 294–315. https://journals.sagepub.com/doi/pdf/10.3102/0162373719848777

Luitel, L. (2019). Collaborative and independent learning in mathematics through activity-based instruction. *Academic Journal of Mathematics Education, 2*(1), 43–53. https://www.researchgate.net/publication/340600325_Collaborative_and_independent_learning_in_Mathematics_through_activity-based_instruction

Mann, G., & Willans, J. (2020). “Monkey see, monkey do, that’s not going to actually teach you”: Becoming a self-directed learner in enabling mathematics units. *Student Success, 11*(1), 55–65. https://doi.org/10.5204/ssj.v11i1.1462

McAdams, C. F. (2017). *Developmental education faculty perspective of the Texas success initiative assessment as a placement exam* (Publication No. 10756384) [Doctoral dissertation, Sam Houston State University]. ProQuest Dissertations and Theses Global.

Merriam, S. B., Caffarella, R. S., & Baumgartner, L. M. (2007). *Learning in adulthood. A comprehensive guide* (3rd. ed.). John Wiley & Sons.

Morales, P. R. (2018). Computerized adaptive test to guide curricular trajectories in mathematics. *International Journal of Innovation and Research in Educational Sciences, 5*(6), 633–636. http://www.ijires.org/administrator/components/com_jresearch/files/publications/IJIRES_1424_FINAL.pdf

Odigiri, A. M., Ozoji, B. E., Onuh, J., Akpata, O., & Dung, M. D. (2020). Pedagogy and andragogy: Meeting point in concepts and missing link in the practice of science, technology, engineering and mathematics (STEM) teacher preparation in Nigeria. *Journal of the Nigerian Academy of Education, 15*(2), 234–247. https://www.naejournal.org.ng/pdfarticles/JonaedVol_15_No2Art19.pdf

Ontario Ministry of Education. (2007). T*he Ontario curriculum, Grades 11 and 12: Mathematics*. http://www.edu.gov.on.ca/eng/curriculum/secondary/math.html

Orpwood, G., Schollen, L., Leek, G., Marinelli-Henriques, P., & Assiri, H. (2012). *College mathematics project 2011. Final report for the Ontario Ministry of Education and the Ontario Ministry of Training, Colleges and Universities*. York-Seneca Institute for Mathematics, Science and Technology Education.

Parker, S., Traver, A. E., & Cornick, J. (2018). Contextualizing developmental math content into introduction to sociology in community colleges. *Teaching Sociology, 46*(1), 25–33. https://journals.sagepub.com/doi/pdf/10.1177/0092055X17714853

Pertiwi, R. D., Kartono, K., & Wardono, W. (2019). Students' mathematical literacy ability in learning using Rally Coach-Schoology model with diagnostic assessment. *Unnes Journal of Mathematics Education Research, 8*(2), 236–247. http://journal.unnes.ac.id/sju/index.php/ujmer

Prendergast, M., Faulkner, F., Breen, C., & Carr, M. (2017). Mind the gap: An initial analysis of the transition of a second level curriculum reform to higher education. *Teaching Mathematics and Its Applications, 36*(4), 1–15. https://www.researchgate.net/publication/312303969_Mind_the_gap_An_initial_analysis_of_the_transition_of_a_second_level_curriculum_reformto_higher_education

Qin, L. D. (May, 2017). *Assessing the relationship between multiple measures placement and student academic success at a community college* (Publication No. 102559250 [Doctoral dissertation, Johnson & Wales University]. ProQuest Dissertations and Theses Global.

Quebec Ministry of Education. (2004). *Québec education program secondary school education, cycle one*. http://www.education.gouv.qc.ca/fileadmin/site_web/documents/PFEQ/qepsecfirstcycle.pdf

Rach, S., & Ufer, S. (2020). Which prior mathematical knowledge is necessary for study success in the university study entrance phase? Results on a new model of knowledge levels based on a reanalysis of data from existing studies. *International Journal of Research in Undergraduate Mathematics Education, 6*, 375–403. https://doi.org/10.1007/s40753-020-00112-x

Ralph, R. A. (2015). Post-secondary project-based learning in science, technology, engineering, and mathematics. *Journal of Technology and Science Education*, *6*(1), 26–35. https://www.redalyc.org/articulo.oa?id=331144428004

Roller, M., & Steinberg, D. (2020). The distributional effects of early school stratification—non-parametric evidence from Germany. *European Economic Review, 125*(C), 1–46. https://doi.org/10.1016/j.euroecorev.2020.103422

Scott-Clayton, J. (2012). *Do high-stakes placement exams predict college success?* [Working Paper No. 41]. Community College Research Center. http://ccrc.tc.columbia.edu/media/k2/attachments/high-stakes-predict-success.pdf

Shim, G., Shakawi, A., & Azizan, F. (2017). Relationship between students' diagnostic assessment and achievement in a pre-university mathematics course. *Journal of Education and Learning*, *6*(4), 364–371. http://doi.org/10.5539/jel.v6n4p364

Simon, M. (2017). *Learning through activity: A developing integrated theory of mathematics learning and teaching* [Research paper]. Proceedings of Conference of European Research in Mathematics Education 10, Dublin, Ireland. https://hal.archives-ouvertes.fr/hal-01948860/document

Smith, D. H, Hao, Q., Jagodzinski, F., Liu, Y., & Gupta, V. (2019). *Quantifying the effects of prior knowledge in entry-level programming courses* [Conference session]. CompEd '19: Proceedings of the ACM Conference on Global Computing Education, Chengdu, Sichuan, China. https://dl.acm.org/doi/pdf/10.1145/3300115.3309503

Toprak, T. E., & Cakir, A. (2018). Where the rivers merge: Cognitive diagnostic approaches to educational assessment. *Journal of Theoretical Educational Science, 11*(2), 244–260. https://dergipark.org.tr/en/download/article-file/437867

Van den Broeck, L., De Laet, T., Lacante, M., Pinxten, M., Van Soom, C., & Langie, G. (2019). Predicting the academic achievement of students bridging to engineering: the role of academic background variables and diagnostic testing. *Journal of Further and Higher Education, 43*(7), 989–1007.

Walker, A. A. (2017). Why education practitioners and stakeholders should care about person fit in educational assessments. *Harvard Educational Review, 87*(3), 426–453. https://cr20y7ujo-mp03-y-https-search-proquest-com.proxy.lirn.net/education/docview/1942146028/61B174194D8A4D01PQ/1?accountid=158450

Wilson, C., Jr. (2018). *Beyond placement testing: improving student placement in community colleges* (Publication No. 10973204) [Doctoral dissertation, University of Maryland University College]. ProQuest Dissertations and Theses Global.

Woods, C. S., Park, T., Hu, S., & Bertrand Jones, T. (2018). How high school coursework predicts introductory college-level course success. *Community College Review, 46*(2), 176–196. https://cr20y84g1-mp03-y-https-search-proquest-com.proxy.lirn.net/education/docview/2017308017/720F63F3E6444947PQ/1?accountid=158450

Yassin, N. H. M., Shahrill, M., Jaidin, J. H., & Harun, H. Z. H (2015). The effects of streaming on secondary school students' achievements in additional mathematics. *European Journal of Social Sciences, 46*(2), 148–158. http://www.europeanjournalofsocialsciences.com/

Zhan, P. (2020). Longitudinal learning diagnosis: Minireview and future research directions. *Frontiers in Psychology, 11*(1185), 1–4. https://doi:10.3389/fpsyg.2020.01185

Zhang, Z. (2018). Designing cognitively diagnostic assessment for algebraic content knowledge and thinking skills. *International Education Studies*, *11*(2), 106–117. https://doi.org/10.5539/ies.v11n2p106

Zieky, M., Perie, M., & Livingston, S. (2006). *A primer on setting cut scores on tests of educational achievement.* ETS. https://www.ets.org/research/policy_research_reports/publications/publication/2006/dbkw

APPENDIX A

Mathematics Diagnostic Assessment

An Ontario College

<u>Our Vision</u>: Rooted in our communities, we will be a globally recognized college delivering innovative learning opportunities and preparing career-ready graduates to be leaders in their fields.
<u>Our Mission</u>: We are dedicated to student success, academic excellence, and leadership in our communities.

MATHEMATICS DIAGNOSTIC ASSESSMENT
Fall 2018

Student Name: ____

Student Number: ______________________________________

Program: ____

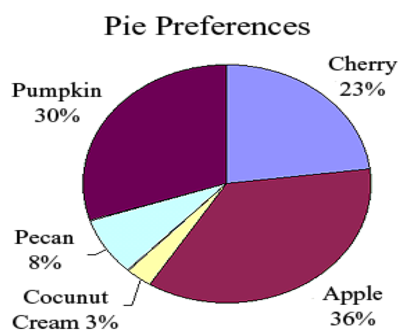
Date: _________________________

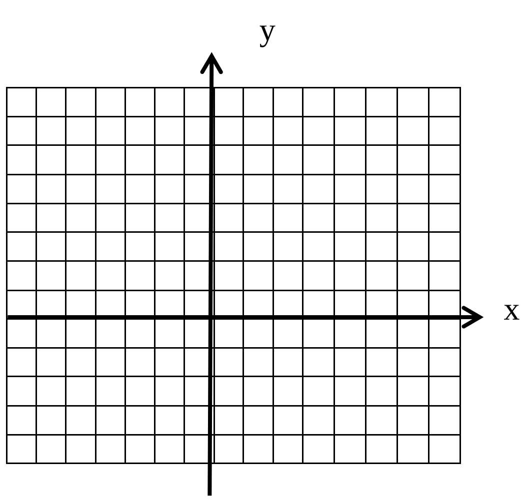
The math diagnostic assessment consists of 32 questions. You have **<u>ONE hour</u>** to answer all questions to the best of your abilities. Please write all your algebraic steps and write your **<u>final answer in the square shaded box</u>** beside the question.
This diagnostic assessment is simply a guide for your program coordinator and mathematics teachers to support your mathematics learning from the beginning of your classes. This assessment might also place you in the most suitable mathematics course for you and your program. Please attempt all questions to the best of your ability.

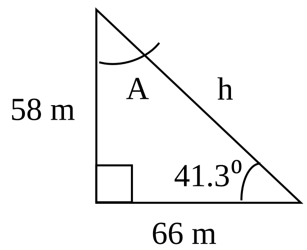
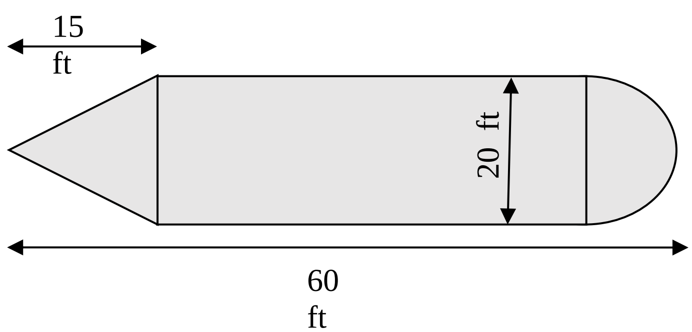
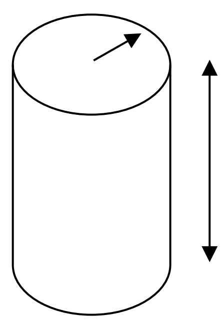
<u>Notes:</u>

- Scientific and non-programmable calculators are permitted for this test. Cell phones/ipads/etc. are not permitted.
- Respect others who are writing with you, please turn off any mobile devices, and exit quietly when you are finished the test.
- This booklet should have a total of six pages.
- **<u>Page 6</u>** contains useful formulas and conversion units.
- Extra paper for calculations is available.
- Pen or pencil is acceptable.

Solve the following:	**Answers**
1) Calculate: $(-5) - (-16) \div (-4) =$	1)
2) What is the average (mean) of the following five exact numbers? 13, 17, 4, 26, 31	2)
3) It takes 8 Litres of paint to cover 6 walls in a building. How many Litres are needed to cover 12 walls?	3)
4) Tom has $2,846 in his bank account. If he deposits now $350 and later $189, and then withdraws $296, how much money is still left in his bank account?	4)
5) A group of 15 students collect money for a present. Each of them give $5.50. How **much more money** is needed, if the present costs $105.60?	5)
6) Evaluate: $320 \times (1 + 0.15 \times \frac{31}{365}) =$	6)
7) A car is driving with 66.5 km/h. How many kilometers did it drive in 4 hours?	7)
8) A city has 28,000 people, but only $\frac{1}{5}$ of them are working now. How many working people is that?	8)
9) A box of biscuits is selling for $3.50. How many can be bought with $98?	9)
10) Human bones contain $\frac{1}{4}$ water, $\frac{3}{8}$ living tissue, and the rest minerals. Thus, the fraction of minerals is: $1 - \frac{1}{4} - \frac{3}{8}$. Find this fraction.	10)

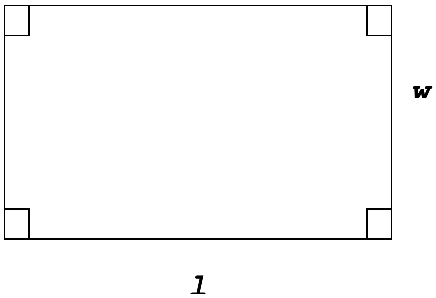
11) U.S. workers work an average of $46\frac{3}{4}$ hours per week, while Canadian workers work $38\frac{3}{8}$ hours. How many more hours per week do U.S. workers work?	11)
12) Write 71% as: a) a fraction b) a decimal	12) a) b)
13) In a recent year, about 80 out of 100 students had completed four years of high school. Write this as a fraction in simplified form.	13)
14) Solve for x: $4x - 7 = 13$	14)
15) What is the area and the perimeter of the following rectangular garden? (Formulas on last page)  w=14 L=17	15) Area = Perimeter =
16) Considering the *pie preferences* of 250 people, how many people prefer pumpkin pie?  	16)
17) Calculate $(-2)^3 \times 4 + 5(-2)^2 \div (-4) =$	17)
18) What is the total price of a car priced for $4,155.00 when the sale tax added is 7%?	18)
19) You have $50 and want to buy a shirt priced at $35. If the tax added to the price is 13%, how much money do you have left after you buy the shirt?	19)
20) Convert 7.0 inches into centimeters. (Conversion table on last page)	20)

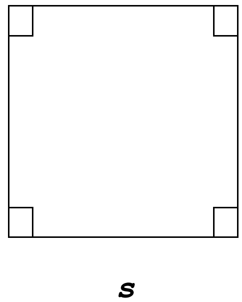
21) A comfortable room temperature is 68^0 Fahrenheit. How many degrees Celsius is that? (Hint: formula needed is on page 6)	21)
22) The population of Toronto is about $5,150,000$. Write this in scientific form.	22)
23) Calculate: $2\frac{4}{5} \times 1\frac{1}{2} =$	23)
24) Simplify: $2(4x - 3) - (x - 3) + 3$	24)
25) Simplify: $\frac{9x^3y^2}{27x^2y^4}$	25)
26) If $x = -2$, what is the value of $\frac{x^2-1}{x-1}$?	26)
27) Solve for x: $2x - 3(x - 4) + 2(x - 2) = 0$	27)
28) Solve for x: $x^2 - 3x + 2 = 0$	28)
29) Graph the following line $y = x + 2$. At what points does the line intersect the axes *x* and *y*? Write their coordinates.  	29) $X_{intercept}$= $Y_{intercept}$=

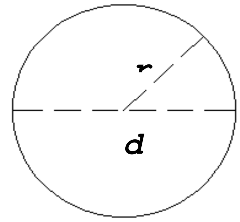
30) A garden is in the shape of a right triangle. Find angle *A* and the distance *h*.  	30) h = <A =
31) Calculate the total area of the shape below: (Geometry formulas on last page)  	31)
32) Find the volume of a cylinder with radius 5.0 cm and height 20 cm. (Geometry formulas on last page) 	32)

Geometry Formulas:

Rectangle **Square** **Circle**



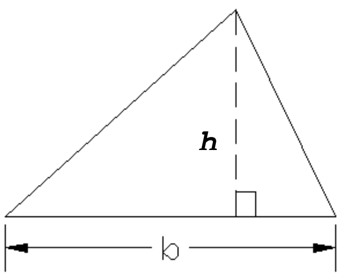


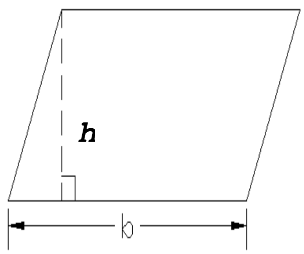


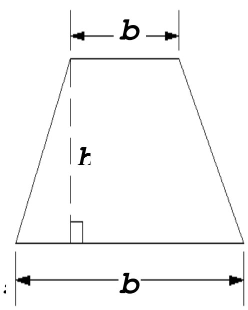
$P = 2(l + w)$ $P = 4s$ $C = 2\pi r = \pi d$

$A = lw$ $A = s^2$ $A = \pi r^2$

Triangle **Parallelogram** **Trapezoid**

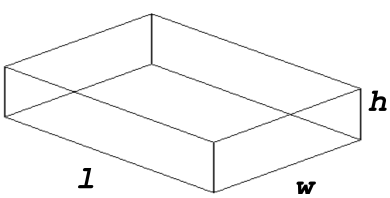


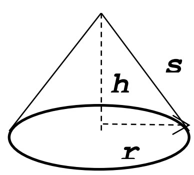


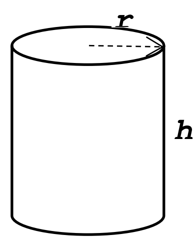


$A = \frac{1}{2}bh$ $A = bh$ A :

Rectangular Prism Cone Cylinder







$A = 2(lw + lh + wh)$ $A = \pi rs + \pi r^2$ $A = 2\pi rh + 2\pi r^2$

$V = lwh$ $V = \frac{1}{3}\pi r^2 h$ $V = \pi r^2 h$

Conversion Tables and Constants:

1 kilometer (km) = 0.62 mile (mi)
1 meter (m) = 3.28 feet (ft)
1 inch (in) = 2.54 centimeters (cm)

Quadratic formula:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Temperature change formula:

$$^0C = \frac{5}{9} \times \left({^0F} - 32 \right)$$

APPENDIX B

Consent Letter

Date: April 29, 2021

Dear Student,

My name is Liliana Simion, and I am a doctoral student at William Howard Taft University. I am asking you to agree to participate in a research study related to your program: Game Development, at our college campus.

I am very interested in verifying statistically if the mathematics diagnostic test you all take at the beginning of your college math course reflects your math knowledge from your secondary school. Moreover, as you all know, after taking the math diagnostic test some of you were directed to the Math 10 course only, while others continued in the Math 9 course before going to Math 10. This research study will look at how well this math diagnostic test was able to read your prior math knowledge to send some of you to the Math 9 course first in order to be successful further in Math 10.

Enclosed is a copy of the Informed Consent Document, which I would like you to read. If you have questions please let me know by coming to my office, room 3220 during my office hours, so everyone is clear about the purpose of this research study.

At your earliest convenience, please let me know if you wish to participate in this study by signing the consent form, which you should either bring it to my office or send it scanned to my email address: lsimion@_______.

Your participation to this research is optional. Please either read and sign the Informed Consent Document consenting to your participation, or let me know that you are declining this offer to be involved in this voluntary study.

Sincerely,

Liliana Simion

Address: Our College, room 3220
E-mail address: lsimion@_______
Telephone #: Our College, ext. 1320

INFORMED CONSENT DOCUMENT

Research topic: IS THE MATHEMATICS DIAGNOSTIC TEST A GOOD PREDICTOR OF COLLEGE STUDENTS' PRIOR KNOWLEDGE IN MATH?

Dear Student,

Please read this consent document carefully and ask as many questions as you like before you decide whether you want to participate in this research study. You are free to ask questions at any time before, during, or after your participation in this research.

I am inviting you to take part in a research study designed to investigate the validity of the math diagnostic test students take at the beginning of their mathematics class at the college. Is this test showing students' prior knowledge in math? Moreover, is this test placing students in the right math course for their prior knowledge in order to be successful in the technical math course Math 10, which is the stepping stone for further math courses?

In addition to being your math teacher for all math courses in your college program, I am also a doctoral student at William Howard Taft University conducting this study. You as a student were selected as a possible participant because you were enrolled in and graduated from the Game Development program at the college, where this study is set to investigate the validity of the math diagnostic test.

Background Information

As you know, some students score high enough in the math diagnostic test to be going directly to Math 10, while other students who score under 85% continue in the Math 9 course and then go to the next math course, Math 10. This research has two goals: first, the study will examine statistically how well your marks from your senior math courses at high school and GPA, as measures of prior math knowledge, correlate and even predict your score in the math diagnostic test. Secondly, your mark from the Math 10 course will be used to find if students directed first to Math 9 have similar marks in their Math 10 course as the students who were directed to Math 10 only. This last question is important to see if taking the introductory math course helps students succeed further in the next math course.

Procedures

The first part of my proposed study would involve asking the Registrar at the College for your grades in your senior high school math courses and GPA, as measures of your prior knowledge in math. Previous research revealed that prior knowledge represents one of the most powerful cognitive predictors of academic performance further in education (Binder et al., 2019; Derr et al., 2018; Smith et al., 2019).
Secondly, the proposed study will employ the use of a statistical program, SPSS, in order to check for correlations and predictions between your high school academic credentials and the

math diagnostic scores that I collect on my office computer for all years of study and programs at our campus. Previous research considered important to administer a diagnostic evaluation usually during the orientation week of the semester to define the academic competencies that the student has at the beginning of their tertiary education (Conforme et al., 2019; Orpwood et al., 2012; Shim et al., 2017). Teachers and administrators use these results to make the best decisions related to teaching strategies, teaching the curriculum content, supporting students' learning through tutoring, or directing them to specific courses.

As literature demonstrated, one major characteristic of college and university students is represented by a great difference among students' basic skills and knowledge, especially in mathematics (Baldwin & Squires, 2019; Prendergast et al., 2017). As Game Development program is a science-related field, mathematics' prior knowledge is key to almost all program courses; literature research shows that an early identification of this knowledge can provide proper support to all students (Bahr et al., 2019; Derr, 2017; McAdams, 2017; Qin, 2017; Wilson, 2018). At our campus students who score less than a cut-off score in the math diagnostic test are required to continue in their Math 9 course, while students who score higher than the cut-off score are placed into the technical Math 10 course directly. Therefore, the third purpose of this study into verify whether placing regular and at-risk students first into the Math 9 course helps them succeed further in the technical math course Math 10 at a similar level as the students who were placed directly into Math 10.

If you would like more details or a copy of the study results after the study is completed, please email me at lsimion@______.

Confidentiality

The academic records used in this study would be considered private information. It is important for you to know that I will not use any names when I do the statistical analysis and when write the research document at any steps and chapters of it. Your identity will not be revealed at any time during this study or after the study. The results are completely confidential.

May I withdraw from the study?

Your participation in this research is completely voluntary. Your decision whether to participate would not affect your current or future relationship with the teacher-researcher, as you already graduated from your college program. You do not have to agree to your participation in this study. Even if you begin your participation, you may withdraw from the study at any time. If you do not agree to participate, or if you decided to withdraw your participation, you will not lose any rights, benefits, or services that you would otherwise be eligible to receive.

Will there be any compensation for participating?

Participation in this study is completely voluntary. You will not receive monetary compensation for participation. The personal benefit of your participation in this study is to know that we as teachers at the college in your program are interested to see whether what we do at the college for our students is benefitting our students' learning. Your placement into the first two math courses for the program you completed was directed by your score in the math diagnostic test, and we would like to see if this placement supported your learning in math.

You may ask any questions you have by contacting the researcher by telephone at extension 1320 or by e-mail at lsimion@_______.

Statement of Consent

(Please initial each of the three below statements that apply to your approval signature):

____ I have read the information herein.

____ I have asked questions and received answers.

____ I have received a copy of this form and consent to participate in this study.

____ ______________

Signature of student Date

____

Please print your name

Candidate/Researcher Statement

All information contained herein is accurate. I have provided the participant with a copy of this form.

______Liliana Simion_________ __April 29, 2020__

Candidate/Researcher Date