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The i3 Validation of SunBay Digital Mathematics

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Disciplines

Education | Educational Technology

Comments

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Abstract

Decades of research have emphasized the need for engaging, accessible ways to help students learn fundamental mathematical concepts. Research also shows that teacher beliefs about teaching and learning have an outsized influence on the quality and effectiveness of curricular interventions. This article reports results of an independent evaluation of the i3 implementation of SunBay Digital Mathematics, a middle-school math intervention, and examines the program's impacts on both student progress and teachers' beliefs about math instruction. Prior studies have demonstrated the efficacy of SunBay Math for students of varied levels of prior achievement. This independent evaluation included a randomized controlled trial in 60 Florida middle schools during the 2015-16 school year and a mixed-methods implementation study. No impact on student achievement was observed overall; however, the evaluation did reveal positive impacts on teachers' classroom practices and beliefs about the use of technology in math instruction. Inadequate implementation of instructional units and lack of impact on teachers for targeted beliefs about math instruction likely contributed to lack of overall program effects.

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The i3 Evaluation of SunBay Math

Decades of research have emphasized the need for engaging, accessible ways to help students learn fundamental mathematical concepts (Hannula et al., 2016; McLeod, 1992; Gentile & Monaco, 1986). Students' interest in math, their attitudes about its importance and utility, and their confidence in their own capacity for mathematical understanding are important predictors of learning (Goldin, 2018; Hoffman, 2010; Hidi & Renninger, 2006; Köller, Baumert, & Schnabel, 2001; Middleton, 1995; Middleton and Toluk, 1999). To address these priorities and understandings, SunBay Digital Mathematics (SunBay Math) was developed by curriculum and teacher professional development experts at SRI International (SRI). SunBay Math is a middle-school math intervention that integrates conceptually rich curriculum materials with technology-based dynamic representations of math concepts. The program consists of classroom-based replacement units that supplant regular math instruction with the goal of improving students' engagement with and understanding of core math concepts. SunBay Math units are designed to support students' learning throughout the school year by deepening their understanding of foundational concepts through exploration and reasoning around dynamic visual representations.

This report presents findings from an independent evaluation of SunBay Math conducted by the Consortium for Policy Research in Education (CPRE) at the University of Pennsylvania. The evaluation focused on the impacts and implementation of SunBay Math in two large Florida school districts during the 2014–15 and 2015–16 school years. The independent evaluation was funded as part of an Investing in Innovation (i3) award and included a cluster-randomized controlled trial (RCT) implemented during the 2015–16 school year and a mixed-methods implementation study spanning both years of the project. The evaluation sought to address the following two questions: 1) How was SunBay Math implemented during the i3 validation project? and 2) What were the impacts of SunBay Math during the i3 validation project? In this report, we address the first question through an examination of fidelity of program implementation during the i3 project. To address the latter question, we examined the program's impacts on teacher beliefs, on classroom opportunities for student learning, and on student achievement in math, as measured by the Florida Standards Assessment.

Prior Evaluations and Supporting Research

SunBay Math pedagogy and technology are partially descended from SimCalc Mathworlds™ (SimCalc), a program developed in the late 1980s by researchers at the University of Massachusetts, Dartmouth. SimCalc was designed to present concepts of proportionality, linearity and rates of change in accessible ways for middle-school students of all backgrounds, including those with disabilities or limited English proficiency (Roschelle, Tatar, Shechtman, & Knudsen, 2008). The name SunBay was introduced in 2009 when the current, fully online version of the program was introduced.

Prior experimental and quasi-experimental research supports the impact of SunBay Math's predecessor, SimCalc, on student achievement. A 2007 study conducted in Texas found positive effects on students' understanding of rate and proportionality, measured as gain scores on a unit test, with a large estimated effect size of 0.84 standard deviations (Roschelle et al., 2007). A second RCT and a quasi-experimental study, also conducted in Texas, revealed smaller but still significant positive effects from SimCalc on students' complex proportional reasoning across varied contexts and diverse student populations (Roschelle et al., 2010).

Like its predecessor SimCalc, SunBay Math is based on theory about how students can best learn math. One key theoretical understanding that underlies the model is that technology-based dynamic representations can deepen students' engagement with content in ways that build conceptual understanding (Bell, Juersivich, Hammond, & Bell, 2012; Kaput, 1992; McKagan et al., 2008; Hollebrands, 2007; Ploetzner & Lowe, 2004; Roschelle, Noss, Blikstein, & Jackiw, 2016; Orrill & Burke, 2013). The literature on dynamic mathematical representations—some of it an outgrowth of SimCalc research (Orrill & Burke, 2013; Vahey, Knudsen, Rafanan, & Lara-Meloy, 2013)—emphasizes the dynamic engagement with math concepts that such programs permit, and suggests that dynamic learning opportunities support student achievement more effectively than traditional, static representations (Bell et al. , 2012; Hoffler & Leutner, 2007). Citing Pea (1987), Zbiek, Heid, Bloom, and Dick (2007) propose that technology can act as a cognitive tool that can help "transcend the limitations of the mind" (p. 1171). Dynamic representation environments, like those in SunBay Math, serve as a cognitive tool by offering students the opportunity to perform visual manipulations that are simultaneously presented in a variety of ways, including more traditional symbolic representations.

While not foundational to SunBay Math, an important and resonant idea is explored in decades of literature on productive struggle (Granberg, 2016; Hiebert et al., 1996; Hiebert & Wearne, 2003; Warshauer, 2015a, 2015b, 2014a, 2014b; Zaslavsky, 2005; Zeybek, 2016). Sometimes conceptualized as productive failure (Kapur, 2014, 2010), productive struggle is defined by Heibert et al. (1996) as "effort [students expend] to make sense of mathematics" (p. 387). They write:

> The struggle. . . comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed. By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated. (p. 388)

Consistent with evidence and theory supporting productive struggle, SunBay Math's instructional model is designed to shift much of the cognitive work of learning from teacher to students, embed supports and opportunities for student problem-solving in every lesson, and allow students to explore concepts on their own before the teacher discusses them in depth. This approach is substantiated by recent research. For instance, based on two RCT studies, Kapur (2016) concluded that "students who engaged in problem-solving before being taught demonstrated significantly greater conceptual understanding and ability to transfer to novel problems than those who were taught first" (p. 1008). Opportunities for student collaboration—another foundational component of the SunBay Math approach—is also utilized to facilitate productive struggle and conceptual understanding (Sengupta-Irving & Agarwal, 2017). Zeybeck (2016) notes that "sharing, explaining, and justifying one's solution. . .provide[s] a classroom context for students to develop their conceptual understanding" (p. 397).

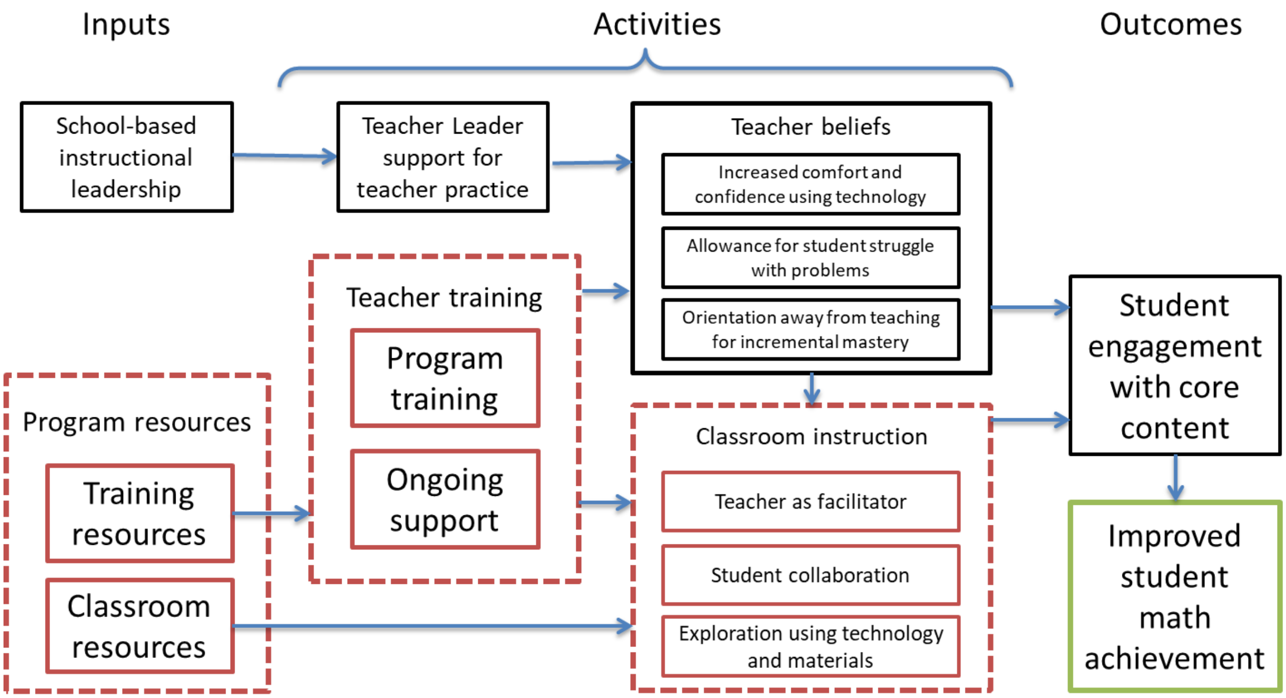
Instruction that supports student understanding and productive struggle often requires significant shifts in beliefs and mindsets on the part of teachers. In traditional mathematics instruction, the teacher and the text are the source of knowledge and the teacher's role is to provide clear explanation and gradual release to allow students to successfully complete a set of practice and application problems. Students are expected to solve procedural tasks, often using known procedures rather than engaging in reasoning, conjecturing, or sense-making (Stigler & Hiebert, 1999). Research on efforts to reform mathematics instruction highlights the importance of teachers' pedagogical content knowledge (and in particular mathematical knowledge for teaching) (Copur-Genctruk, 2015; Hill, Ball, & Schilling, 2008), beliefs about teaching and learning (Lloyd, 1999; Munter, 2014; Stipek, Givvin, Salmon, & MacGyvers, 2001; Wilhelm, 2014), and instructional practices that center on eliciting and responding to student thinking (Cobb & Jackson, 2011; Franke et al., 2007).

A final area of scholarship reflected in SunBay Math's design focuses on the role of curriculum materials in facilitating teacher learning (Davis & Kradjcik, 2005; Remillard, 2005; Ball & Cohen, 1996). In this literature, well-designed math curriculum materials are understood to serve two functions: facilitating student learning and supporting the development of teachers' own understanding of the content.

SunBay Math Program Model

A key goal of SunBay Math's hands-on, exploration-centered approach is "democratizing access to mathematics" (Kaput, 1994) or supporting deep understanding of core mathematics concepts among all students, including those who have traditionally struggled with mathematics (Vahey, Lara-Meloy, & Knudsen, 2009). SunBay Math was designed as a *curricular activity system* in which technology, standards-aligned materials, and teacher professional development are integrated to meet the needs of varied local educational contexts (Vahey et al., 2013). SunBay Math units focus on core concepts in middle-school math (Leinhardt, Zaslavsky, & Stein, 1990): ratios and proportional relationships (Grades 6 and 7); expressions and equations (Grades 6 and 7); and functions and geometry (Grade 8). The units are designed to be taught in place of the regular math program over a two-week period whenever the unit content occurs in a district's pacing guide. Instructional is delivered via a program model that is represented in Figure 1.

Figure 1. SunBay Math Program Model



Moving backwards from the program's goals of increasing students' engagement with and achievement in math, the program model illustrates the SunBay Math model's three key programmatic components: A) *Classroom instruction*: ongoing cycle of activities that include teachers, students, and the curriculum; B) *Teacher training*: professional development and ongoing support for teachers; and C) *Program resources*: inputs for training and classroom instruction.

Classroom Instruction

Every SunBay Math unit is composed of seven to 10 investigations, most of which are designed to be completed in one class period. Daily investigations are oriented around engaging students in SunBay Math's *Predict-Check-Explain* (PCE) cycle (Vahey et al., 2013). In this approach, students first draw on their prior knowledge to make a prediction about a mathematical situation; then check their prediction with the technology; and finally draw on multiple representations to provide a conceptual explanation for why their prediction was correct or incorrect. The designers of SunBay Math present this approach in contrast to the more prevalent “symbols first” approach to traditional mathematics instruction, where mathematical rules are learned in isolation and then applied to situations. SunBay Math provides a narrative framework that allows students to access their intuitive knowledge of familiar situations (e.g., mixing paint or playing sports) through a hands-on and engaging approach to learning that harnesses middle-school students’ natural curiosity and excitement.

Each investigation contains a short warm-up activity; a main instructional activity consisting of multiple technology-based tasks; a teacher-facilitated wrap-up discussion intended to make the math explicit and consolidate learning; and practice problems that can be used as classwork or homework. During the main instructional activity, students engage with one of four dynamic web-based tools for generating and manipulating multiple, linked relationships through simulations: 1) manipulating visual representations to learn about variables and symbolic expressions; 2) exploring discrete and continuous representations to learn about ratio and proportionality; 3) manipulating compositions of geometrically and analytically specified transformations to learn foundational geometric concepts; or 4) exploring dynamic, linked representations of linear functions and graphs. By manipulating specific inputs—for instance, adding discrete units of color in a paint mixture to create a new color shade—students can produce linked changes across the multiple representations, such as a graph and a color spectrum. This linked structure allows students to explore underlying mathematical relationships by observing how different inputs produce different responses that are represented in multiple ways. Multiple representations provide different points of access to, or ways of understanding, complex mathematical concepts through a technology-based experience that serves as a focal point for classroom dialogue and shared insights.

Teacher Training

To support teachers’ learning of SunBay Math’s instructional approach in the i3 study, 1.5 days of summer professional development are provided prior to implementing the first SunBay Math unit, and an additional half-day of training prior to implementing any new unit for the first time. Training sessions included time for teachers to engage with the SunBay Math technology’s dynamic mathematical representations, and to explore and reason about the mathematical content, just as their students would do. Time was also provided for teachers to plan the implementation of the unit. During the second year of implementation, the training also focused on strategies for facilitating high levels of cognitive demand through responses to student struggle; strategies for supporting productive classroom discussion and collaboration; targeting tasks to students’ needs; and enrichment activities. Underlying principles—such as the belief that all students can learn math—were also reinforced. The training was delivered to teachers in the two districts by six trained SunBay Math facilitators, who also provided on-site support and coaching to teachers throughout the school year via conferencing, observation, reflection and modeling.

Program Resources

The SunBay Math program model assumes the availability of certain classroom and training resources. Each classroom must include laptops or tablets and internet access. Each unit has a student workbook in which students can read the problems and record their responses for each task. The Teacher Guide includes an overview, suggested timings, tips and questions for each investigation, and includes the corresponding student workbook pages with sample answers. Each Teacher Guide includes introductory sections on the learning progression, standards, key ideas, implementation and STEM connections embedded in the unit.

The i3 Evaluation of SunBay Math

The research detailed in this report was conducted in the context of the i3-supported implementation of SunBay Math in two school districts in Florida during the 2014–15 and 2015–16 school years. The implementation was led by SRI in collaboration with the University of Florida Lastinger Center, with funding from a validation grant from the U.S. Department of Education's Office of Innovation and Improvement's i3 program, and a matching grant from the Helios Educational Foundation. The evaluation detailed here included a cluster- RCT implemented during the 2015–16 school year and a mixed-methods implementation study spanning both years of the project.

Study Participants

Thirty schools in each participating district were selected to participate in the RCT. The sample of schools included a total of 60 of the 72 middle schools. There were eight non-Title I schools in District A and 15 non-Title I schools in District B, all of which were included in the RCT, for a total of 23 schools. To reach the target of 60 participating schools, CPRE randomly selected 22 additional schools from District A and 15 additional schools from District B for a total of 30 from each district. After the study schools were identified, whole schools were randomly assigned to either treatment or business-as-usual control condition within two blocking variables, District and Title I status. Table 1 shows the results of the random selection and assignment process. All study schools remained in their assigned condition throughout the duration of the study. All teachers and students of regular and advanced math in Grades 6 and 7 and regular math in Grade 8 were involved in the study as part of either the treatment or the control group.

Table 1. Random Selection and Assignment of Schools

Random Sample	Non-Title I		Title I		Non-RCT	Total
District A	8		22		9	39
District B	15		15		2	32
Total	23		37		11	71
Random Assignment	Non-Title I Control	Non-Title I Treatment	Title I Control	Title I Treatment		Total
District A	4	4	11	11		30
District B	8	7	7	8		30
Total	12	11	18	19		60

Note: Sampling and Assignment were within District and school Title I status; all Non-RCT schools were Title I.

The two Florida districts that participated in the i3 implementation were selected as the sites for the project in part because they offered an opportunity to assess SunBay Math's impacts with diverse, high-need populations. Table 2 shows demographic information for the student populations in participating districts during the 2015–16 school year. The District A student population comprised 39% African-American and 32% Hispanic students, with 61% eligible for free or reduced-price lunch; the District B student population comprised 32% Hispanic and 28% African-American students, with 59% of students eligible for free or reduced-price lunch.

Table 2: 2015–16 School District Attributes

	District A	District B	Florida
Public school student enrollment by race and Hispanic origin			
White non-Hispanic	22.3%	32.6%	39.5%
Black non-Hispanic	39.4%	28.3%	22.5%
Other race alone non-Hispanic	4.0%	3.9%	3.1%
Two or more races non-Hispanic	2.6%	2.8%	3.4%
Hispanic	31.7%	32.4%	31.5%
Students enrolled in classes for English language learners	11.4%	11.8%	9.8%
Students eligible to participate in free/reduced-price lunch	60.9%	59.1%	58.7%
Students enrolled in exceptional education programs	17.0%	20.6%	19.0%

Source: Division of Accountability, Research and Measurement, PK-12 Education Information Services, Florida Department of Education, Tallahassee, FL

SunBay Math was introduced in Grade 6 (regular and advanced classes) and Grade 7 (regular only) in the 2014–15 pilot year. For full implementation in the 2015–16 school year, Grade 7 (advanced) and Grade 8 (regular only) units were added. Depending on teachers' assignments, they may have implemented some or all the SunBay Math units for either one or both years. Similarly, students in Grades 6 and 7 during the 2014–15 school year may have received one or two years of the intervention. Teachers in the 60 participating schools—both treatment and control—were expected to attend any teacher professional development required by their districts; to follow district pacing guides; and to administer regular district assessments. Teachers in the control schools did not have access to the SunBay Math program until the experiment ended.

Table 3: Teacher Attributes

	Treatment	Control
Average years teaching	12.3	11.8
% Attained Master's degree or higher	32.3%	34.5%
Courses completed in methods of teaching mathematics	2.4	2.4
Courses completed in mathematics	4.0	3.4
Participated in math related coursework at University of Florida this year	11.7%	8.8%
Taken graduate courses on instructional or educational technology	46.3%	50.0%

Note: Data collected in Spring 2016 with 74% response rate

Table 3 provides summary statistics for background characteristics of the teachers in both treatment and control groups in Spring 2016. Across the 30 schools that implemented SunBay Math as part of the i3 project, 342 classroom teachers were trained to implement the program. Overall there were 725 teachers participated in the SunBay Math i3 study in either the treatment or the control group.

Studying Implementation Fidelity

CPRE's evaluation of the implementation of SunBay Math in the i3 project included an analysis of fidelity to the program model. Documenting a program's implementation—including any challenges implementers encountered—can offer useful lessons for future users of this or other similar programs and provides an opportunity to investigate if the intervention being assessed through the RCT was implemented as its developers intended. Much evaluation literature attests that, without this assurance, it is impossible to make definitive assertions about the role of an intervention in producing any observed impacts, despite a strong causal research design (Abry, Hulleman, Rimm-Kaufman, 2015; Gerstner & Finney, 2013; Pas & Bradshaw, 2012; Nelson et al., 2012; Hulleman & Cordray, 2009). CPRE developed a specific set of indicators to assess fidelity of implementation in the key three areas represented in Figure 1, The SunBay Math Program Model: classroom instruction, teacher training, and program resources. Table 4 details the three key components and the indicators CPRE assessed within each.

Table 4. 2015–16 Implementation Fidelity Indicators and Definitions

Key Components & Indicators	Indicator Definitions
Component 1: Classroom Instruction	
SunBay Math units are embedded in regular instruction	Teachers deliver two SunBay Math units per school year to all students in classes during scheduled time for math instruction
Trained teachers deliver SunBay Math lessons	SunBay Math units are implemented by trained teachers
Teachers facilitate student engagement in all unit investigations	Students engage in daily, peer-to-peer and student-to-teacher interaction about lesson content for all required investigations.
Student exploration using technology	Dynamic representations-based activities are used by students (and the simulation run, when applicable)
Student collaboration	Technology-based learning activities are completed in groups of three students or fewer.
Component 2: Teacher Training	
Teacher training prior to teaching SunBay Math	Before teaching a SunBay Math unit for the first time, teachers engage in at least 12 hours of face-to-face PD with a qualified Facilitator
Teacher training prior to teaching additional units	Before teaching each new SunBay Math unit, teachers engage in at least 6 hours of face-to-face PD with a qualified Facilitator.
Ongoing teacher support from SunBay Math Facilitators.	At least once per school year, teachers participate in a meeting with a qualified Facilitator.
Ongoing teacher support for implementing SunBay Math units	During ongoing meetings with teachers, SunBay Math Facilitators provide support related to instructional planning, SunBay Math lesson delivery, and/or math content.
Component 3: Program Resources	
Qualified SunBay Math Facilitator	SunBay Math Facilitators hold masters or doctorates in an education-related field and have at least two years' teaching experience in middle-school mathematics.
Prepared SunBay Math Facilitator	SunBay Math Facilitators have implemented at least four SunBay Math units in the classroom or participated in SunBay Math Training of Trainers by SRI.
Training materials	During training, teachers have access to online program materials and a printed teacher's guide.
Student access to technology	SunBay Math classrooms have adequate technology to permit a 3:1 ratio of students to internet-accessible tablets (or computer).
Student access to materials	Each student has a SunBay Math workbook for each unit being implemented.

To determine fidelity of implementation to the program model, CPRE specified a threshold adherence for adequate implementation for indicators in Table 4. For each indicator, at least 80% of teachers (or facilitators when relevant) needed to report full implementation to be scored adequate. To achieve overall fidelity for key components, all indicators within the component had to meet this 80% threshold. Data for the fidelity analysis were obtained using the following sources:

Training Documentation and Observations: Researchers analyzed training records that contained detailed information about which teachers participated in which training and professional development activities, including the summer and mid-year trainings. In addition, researchers conducted observations during SunBay Math professional development in both districts and with teachers of all three grade levels. Researchers compiled field notes during training session observations, which were analyzed to inform insights about training content, format and pedagogical approaches of the trainers.

Facilitator Logs: Researchers developed and coded activity logs to examine the types and amount of support provided to teachers by SunBay Math Facilitators on site visits to a school. Activity codes included: co-teaching/modeling (i.e., demonstrating SunBay Math instruction in classrooms); planning SunBay Math lessons; meeting administrators and other school staff to discuss SunBay Math; supporting teachers in using SunBay Math technology; and conducting classroom observations of SunBay Math.

Teacher Surveys: Teacher surveys were administered to all teachers of Grades 6, 7 and 8 math at regular or advanced levels, in 60 RCT schools, at three points during the study: the fall of 2014, prior to the implementation of the first SunBay Math unit; the spring of 2015; and the spring of 2016. Response rates across teachers in both treatment and control schools were 74% for fall 2014, 70% for spring 2015, and 76% for spring 2016. Surveys were administered electronically using an online survey management platform.

CPRE collected additional data on program implementation and implementers' experiences; their perceptions of its strengths, weaknesses, and impacts; and key barriers and facilitators of implementation across the two districts. The inquiry drew on data sources such as interviews, classroom observations, student workbooks, and SunBay Math technology usage data. These data were used to inform findings included in current and forthcoming companion reports (*see* Ebby, Fink & Sirinides, 2019).

Studying Impacts

CPRE's evaluation examined the program's impacts in three main areas: 1) Impact on teacher beliefs; 2) Impact on classroom opportunities for student learning; and 3) Impact on student achievement in math.

Teacher beliefs

Teacher beliefs and orientations towards technology, mathematics, and student capacity to engage in high-level reasoning and exploration, are important factors in the way curriculum and technological tools are used during lesson enactment. It has long been recognized that teacher beliefs play an important role in the implementation of instructional reform efforts (Spillane, 1999; Wilson, 1990). A key assumption of the SunBay Math program model is that SunBay Math training and the process of implementing the units facilitates changes in teachers' beliefs. Specifically, the model anticipates growth in teachers' comfort and confidence using technology for instruction; in their tolerance for and understanding of the importance of student struggle as a part of the learning process; and in their recognition of mathematics learning as a process of building deep conceptual understandings, rather than incrementally mastering specific skills. Along with the fidelity information discussed above, the teacher surveys were designed to collect data on teacher beliefs about math instruction, technology use, and how students learn. More specifically, surveys included scales pertaining to the following:

Comfort and confidence using technology for instruction. To use technology as a learning tool, teachers must effectively integrate technology in instructional practices. The first of these areas of anticipated teacher growth is a construct that represents teachers' comfort with integrating technology into math curricula and instruction. Through SunBay Math training and experience interacting with the program's technology, teachers should experience growth in this area. For the independent evaluation, Comfort and Confidence using Technology for Instruction (CCTI) was assessed via a scale adapted from the Technological Knowledge scale (Zelkowski, Gleason, Cox, & Bismarck, 2013; Schmidt, Baran, Thompson, Mishra, Koehler, & Shin, 2009), conceptualized as part of Technological Pedagogical Content Knowledge (TPCK). TPCK is a framework for the types of flexible knowledge needed to successfully integrate technology use into teaching. Items on this scale asks teachers to rate their agreement with statements like "Using technology increases student engagement and interest in learning mathematics" and "I have the knowledge and skills I need to use technology effectively for math instruction."

Allowance for student struggle as a part of the learning process. The second dimension of teacher beliefs that is theorized to be impacted by SunBay Math, is that students should be allowed to struggle as a part of the learning process measured by the Teacher Allowance for Student Struggle with Problems (TASSP) scale (developed by Clark et al., 2014). Teachers with higher scores on the TASSP scale are more likely than those with low scores to agree, for example, with the statement "teachers should not necessarily answer students' questions immediately but rather let students do the work of figuring out how to solve many mathematics problems without being told what to do" (Clark et al., 2014). The TASSP scale is an appropriate indicator of teachers' alignment with the approach underlying the SunBay Math instructional model, which is designed to engage students with problem-solving experiences that require high levels of thinking. SunBay Math is based on the understanding that high-cognitive-demand activities produce increased learning. It is expected that students engaged with SunBay Math activities may experience some cognitive conflict as they build conceptual understanding. By measuring teachers' willingness to accept some degree of struggle as a part of the learning process, TASSP scores provide an indicator of teachers' openness to this fundamental assumption of the program model, and to the process of supporting students as they try different approaches and work to build their own understandings in response to problems.

Recognition of mathematics learning as a process of building deep conceptual understandings. Finally, CPRE assessed the extent to which teachers in both treatment and control conditions expressed an understanding of math learning as a process of building deep and connected conceptual understandings, rather than a process of mastering isolated facts, skills and procedures. This was measured via the Teacher Modeling for Incremental Mastery (TMIM) scale (Clark et al., 2014). While agreement with items on the TMIM scale does not necessarily imply a belief that teaching for meaning or conceptual understanding is unimportant, teachers who embrace an instructional approach that emphasizes procedural understanding are more likely to agree with statements like: "I like my students to master basic mathematical operations before they tackle complex problems." Teachers who score higher on the TMIM measure generally hold the belief that students learn math best by being shown how to use procedures and algorithms, that is, when teachers first demonstrate problem-solving approaches and students repeatedly mimic those approaches through practice. This perspective, which stands in opposition to the approach espoused by SunBay Math, sees mathematical learning as a sequential process that relies on students' mastery of basic skills prior to engaging with complex problem-solving. Teachers with this pedagogical orientation typically believe that instruction should be front-loaded with facts, skills and procedures.

Once data from these scales were collected, impacts were estimated using a regression model comparing responses from teachers in the treatment and control groups. The model included a treatment status indicator and a random effect for schools to account for the cluster randomized design in which whole schools were assigned to condition.

Classroom opportunities for student learning

The teacher surveys also included questions about specific classroom practices during the main portion of a typical math lesson. The additional questions were specifically designed to assess the extent to which teachers' reports of classroom practices reflected three key ideas embodied in the SunBay Math program model: 1) that teachers support student learning by acting as facilitators of students' self-directed exploration of content; 2) that student collaboration is supported by classroom arrangements encouraging discussion; and 3) that students use technology to learn through exploration. Data collection occurred post intervention in both treatment and control teachers in the spring of 2016.

The survey asked teaches about their approach to instruction during the main portion of a typical math lesson. Teachers were also asked about the arrangement of student desks at the beginning and end of the year. The survey also includes a series of questions about students' use of technology in the classroom to learn mathematics. Impacts were then estimated using a mixed-effects logistic regression model to estimate the change in odds of each practice reported by teachers in the treatment group, as compared to the control group. The model included a treatment status indicator and a random effect for schools to account for the cluster randomized design in which whole schools were assigned to condition.

Student achievement

To assess the impact of SunBay Math on student achievement, CPRE analyzed end-of-year Florida Standards Assessment (FSA) in Mathematics scores for students of Grades 6, 7 and 8 enrolled in the districts during the 2015–16 school year. FSA scores were provided by partner districts as standard scores for mathematics achievement along with student math course codes. The FSA is a new set of computer-based assessments developed by the American Institutes for Research with the goal of aligning with the Florida State Standards. It was first implemented as the Florida end-of-year assessment for Grades 3 through 10 in 2015, one year prior to estimation of impacts for this study. Total math is measured via standard norm-referenced scales which have reported internal consistency (Cronbach's alpha) of .92 to .95 for Grades 6, 7 and 8.

Impacts were estimated using a multilevel model that nested students (Level 1) within teacher/course (Level 2) within school (Level 3). The model included a school-level main effect for treatment status along with school and student covariates. At the school level, fixed effects were included for district and Title I status. Student grade level was included as a fixed effect along with prior student achievement scores on the state math test. The non-independence of students within classrooms, and of classrooms within schools, was specified using random effects associated with classroom and school which were both assumed to be normally distributed.

Findings

In the following section, we report the findings of our mixed-methods i3 evaluation of SunBay Math. Evaluation findings pertain to both the impacts of the program and its implementation and are presented in response to the guiding research questions for the study. First, we share the findings related to implementation fidelity. Then, we present findings on SunBay Math's impact on teachers' beliefs and instruction. Finally, we discuss SunBay Math's impacts on student learning.

Fidelity of SunBay Math i3 Implementation

CPRE's application of the fidelity analysis described above revealed that SunBay Math was implemented with fidelity to the program model for teacher training and program resources. However, inadequate adherence was observed for the implementation of the classroom instruction, the program area most directly related to student outcomes. Table 5 presents the results of CPRE's analysis of implementation fidelity in the RCT districts.

Table 5: Fidelity of Implementation: Key Components and Indicators

Key Components and Indicators	Adherence
Component 1: Classroom instruction	
SunBay Math units are embedded in regular instruction	80.3%
Trained teachers deliver SunBay Math lessons	91.8%
Teachers facilitate student engagement in all unit investigations	*67.8%*
Student exploration using technology	91.1%
Student collaboration	97.4%
Component 2: Teacher training	
Teacher training prior to teaching SunBay Math	95.1%
Teacher training prior to teaching additional units	79.8%
Ongoing teacher support from SunBay Math Facilitators	87.7%
Ongoing teacher support for implementing SunBay Math units	100.0%
Component 3: Program resources	
Qualified SunBay Math Facilitator	100.0%
Prepared SunBay Math Facilitator	80.0%
Training materials	97.0%
Student access to technology	90.3%
Student access to materials	97.0%

Inadequate fidelity to classroom instruction

The delivery of SunBay Math units is a key component of the program model, encompassing classroom activities most proximate to the intended outcomes of increased student engagement and achievement in math. However, teachers reported inadequate adherence to an important aspect of the model: while students are expected to complete all investigations in each SunBay Math unit, 20% of teachers reported they did not implement all investigations. Moreover, while daily peer-to-peer and student-to-teacher interaction about lesson content is a fundamental part of the SunBay Math model, based on teachers' self-report on the survey, only 67.8% of teachers reported enactment of all required investigation activities for all SunBay Math lessons in the units. District and program records offer supporting evidence of inconsistencies in the duration and timing of SunBay Math implemented in this i3 study (see Appendix). Deviation from this aspect of the program model is detrimental since the duration and sequencing of SunBay Math units allow teachers and students have time to work through all of the investigations in a unit and ensure key mathematical concepts are reinforced throughout the school year.

Positively, teachers did report that SunBay Math technology-based dynamic mathematical representation activities were used in lessons during the implementation of

SunBay Math units, and that activities were almost always done in groups of two or three students, as the program model requires.

Good fidelity to teacher training and program resources

Across the two-year implementation, key project participants and partners were engaged in developing, refining, and delivering training. Training was tiered, with training-of-the-trainers at the top level. At four separate points during the project, SunBay Math Facilitators received formal training from SRI. At the second tier of the training model, Facilitators delivered professional development and ongoing support to implementing classroom teachers. While summer and mid-year trainings were required for all teachers implementing SunBay Math, ongoing support from SunBay Math Facilitators was provided upon request. Although not all teachers took advantage of this support, Facilitator logs reveal many teachers did request and receive direct support.

Regarding fidelity of teacher training, nearly all SunBay Math teachers (98.2%) reported having received training from a qualified instructor, presented here in Table 6.

Table 6: Percentage of Teachers who Completed Training Prior to Implementing Unit

	2014–15	2015–16[1]	Combined
Individual Units			
True Colors Murals	99.5%	87.9%	97.3%
Little X Games	98.4%	59.0%	88.8%
Managing the Soccer Team	98.5%	76.8%	94.4%
3D Design Studio	99.0%	65.9%	90.0%
Sand Circle App		92.2%	92.2%
Transformation Nation		82.5%	82.5%
Multiple Units			
Received training on at least one unit			98.2%
All relevant units based on teaching assignment			79.8%

Note: n = 342; [1] 2015–16 percentages include any carry-over teachers from prior year that did not complete required unit training even if they implemented that unit in 2014–15.

Training records indicate that 80% of teachers received the full dosage of content training required by the program model prior to implementing every new SunBay Math unit. Of those who did not receive the full amount, most received some on-site Facilitator support. In the 15 District B SunBay Math schools, for instance, Facilitators logged a total of 390 hours of support over the term of the project, for an average of 26 hours per school. Overall, there were 266 visits to schools across both districts in the second year of the study. Table 7 presents Facilitators' reported activities during school visits, represented as percentages of total visits. Nearly 80% were for planning or co-teaching; the remainder were to support technology, to conduct observations, and to meet with administrators.

Table 7. Facilitators' Ongoing Support Activities, as Percentage of Total Hours

Co-teaching/modeling (i.e., demonstrating instruction in classrooms)	45.9%
Planning SunBay Math lessons	32.7%
Meeting administrators (and other staff) to discuss SunBay Math	7.9%
Supporting teachers in using SunBay Math technology	7.1%
Conducting classroom observations of SunBay Math lessons	4.5%
Other	1.9%

Note: 266 total Facilitator visits

We also observed strong adherence to the program model in Program resources. Nearly all teachers reported having access to online materials and printed teacher's guides during training. Classroom teachers reported that they had the teacher guides and student materials needed to implement SunBay Math units, and regular access to enough internet-enabled computers or tablets to permit students to work in groups of two or three.

Impact of SunBay Math on Teachers' Beliefs

In keeping with the program model, we assessed the impact of SunBay Math on teachers' beliefs about math instruction and use of technology. To do so we analyzed treatment and control teachers' responses to three dimensions of teacher beliefs expected to mediate some of the intervention impacts: 1) teachers' comfort and confidence using technology for instruction, as measured by the CCTI scale; 2) their tolerance for and understanding of the importance of student struggle as a part of the learning process, as measured by the TASSP scale; and 3) their recognition of mathematics learning as a process of building deep conceptual understandings, rather than incrementally mastering specific skills, as measured by the TMIM scale.

Table 8. Intent to Treat Impacts on Teacher Beliefs (with standard errors)

	CCTI			TASSP			TMIM		
Scale									
Number of items	10			6			7		
Range	1–4			1–6			1–6		
Mean	3.14			3.65			4.05		
Standard Deviation	0.50			0.69			0.75		
Reliability	0.863			0.662			0.813		
Fixed Effects									
Intercept	2.98	(0.07)	**	3.48	(0.06)	**	4.22	(0.07)	**
SunBay Math	0.21	(0.08)	*	0.18	(0.06)	*	-0.04	(0.08)	
District 1	0.05	(0.08)		0.03	(0.06)		-0.08	(0.08)	
Title I	0.09	(0.08)		0.14	(0.06)		-0.24	(0.08)	
Random Effects									
School	0.00	(0.01)		0.00	(0.01)		0.02	(0.02)	
Residual	0.24	(0.03)	**	0.47	(0.03)	**	0.53	(0.04)	**
LS Means									
Treatment	3.25	(0.05)		3.74	(0.05)		4.03	(0.06)	
Control	3.04	(0.05)		3.56	(0.05)		4.07	(0.06)	
Difference	0.21	(0.08)	*	0.18	(0.08)	*	-0.04	(0.08)	
Standardized Treatment Effects									
Cohen's D	**0.42***			**0.26***			**-0.06**		

Notes: n = 463 (treatment = 233, control = 230); Spring 2016

Our analyses of teachers' teacher beliefs revealed positive effects in two of the three scales. SunBay Math produced a gain in CCTI equal to 0.29 standard deviations, and a gain in TASSP of 0.26 standard deviations. No group difference was found for TMIM. Thus, we find that SunBay Math was effective in increasing teachers' comfort and confidence for technology integration and teachers' allowance for student struggle. This study offers no evidence that the intervention decreased teachers' beliefs that students learn math best when teachers first demonstrate problem-solving approaches and students repeatedly mimic through practice.

Impact of SunBay Math on Classroom Opportunities for Student Learning

Understanding the impact of SunBay Math on implementing teachers' instruction was a key goal of the evaluation. We first examined teachers' report of instructional practices during a typical math lesson. Table 9 presents descriptive differences in prevalence of general instructional components for teachers in the treatment and control groups.

Table 9: Teacher Reports of Classroom Practices during a Typical Math Lesson

	Control	Treatment	
How often did you typically do the following?			
Go over the homework from the previous class session	4.25	4.19	
Have students complete warm-up questions	4.06	4.58	*
Have students work on problems in groups of 2 or 3	3.36	4.46	*
Have students complete a wrap-up session	2.96	3.93	*
Have whole class discussion on wrap-up	2.74	3.72	*
Assign problem-solving questions for homework	4.29	3.81	*
Have students begin the homework/problem-solving in class	3.22	3.30	
Administer and collect an exit slip at the end of class	2.25	2.39	

Notes: n = 463 (treatment = 233, control = 230); * p<.001 difference sample means. Spring 2016 teacher survey; all items used the 5-point Likert scale (1 = Rarely/Never; 2 = Sometimes; 3 = About half the time; 4 = Usually; 5 = Always;). N = 469 (213 Control, 256 Treatment); overall response rate = 76%

We find that SunBay Math teachers reported significantly higher rates of using warm-up questions, small group work, and wrap-up activities that included class discussion. Control teachers were more likely to report assigning problem-solving questions for homework.

Additionally, we investigated the extent to which teachers reported altering their classroom practices in areas directly related to the program's theory of change, reflected in Figure 1. To understand impacts in this area, we collected and analyzed survey responses about reported classroom practices from teachers in both treatment and control groups regarding: 1) lesson structure and the role of teachers as facilitators; 2) classroom arrangement in clusters for student collaboration; and 3) student technology use for learning through exploration. Table 10 presents percentages in all three areas. Statistics for lesson structure, classroom arrangement, and student technology compare survey response options separately for treatment and control.

Table 10. Group Percentages on Reported Classroom Practices

	Treatment	Control
Lesson structure		
Solely teacher directed	4.7%	62.6%
Mostly teacher directed	16.3%	14.5%
Mix whole class and student groups	54.6%	16.9%
Mostly students working in groups	24.4%	6.0%
Classroom arrangement		
Desks in clusters	53.5%	35.8%
Desks in rows/columns or stadium	46.5%	64.2%
Student technology use		
No student tech use	3.5%	37.7%
Individual students	5.9%	56.5%
Pairs or trios	90.6%	4.7%
Four or more	0.0%	1.1%

Note: Columns sum to 100% for each classroom practice

Tests of significance for the differences in reported classroom practices are reporting in Table 11. Because of the randomized design, significant differences between groups in all three areas offers strong evidence of program effectiveness. Treatment effects are presented as both odds ratios and risk ratios (with 95% confidence interval).

Table 11. Intent-to-Treat Impacts on Reported Classroom Practice

	Students work in groups		Desk arranged in clusters		Technology used in pairs/trios	
Proportion within treatment	79.1%		53.5%		90.6%	
Proportion within control	22.9%		35.8%		4.7%	
Fixed effects						
Intercept	-0.90	(0.42)*	-1.25	(0.42)*	-3.14	(0.75)**
SunBay Math	2.62	(0.46)**	1.08	(0.43)*	6.00	(0.82)**
District 1	-0.51	(0.46)	-0.42	(0.43)	-1.06	(0.75)
Title I	-0.14	(0.46)	1.36	(0.43)*	0.83	(0.75)
Random effects						
School	0.67	(0.51)	0.67	(0.46)	0.56	(0.80)
Treatment Effects (with 95% CI)						
Odds Ratio	**13.7**	(5.5, 34.0)	**2.9**	(1.2, 6.9)	**364.5**	(71.5, ∞)
Risk Ratio	**3.5**	(2.2, 5.5)	**1.5**	(1.1, 2.2)	**19.6**	(8.1, 47.6)

Note: * p < 0.05, ** p < 0.001; Data collected in Spring 2016 post treatment; Regression coefficients and odds ratios estimated using logistic regression; risk ratios were estimated by log-binomial regression.

As reported in Tables 10 and 11, there were clear differences between treatment and control teacher reports of classroom practices that facilitate opportunities for student collaboration. SunBay Math teachers were significantly more likely than control teachers to report having students work in groups or a mix of whole class and small groups during the main portion of a typical math lesson, (79.1% vs. 22.9%). Control school teachers more often reported that the main portion of a typical math lesson was mostly or entirely teacher directed.

Most teachers (63.2%) reported changing the arrangement of desks for SunBay Math units. Of those that changed desk arrangements, over a quarter of the teachers (27.3%) decided to keep the new desk arrangement after completing the SunBay Math unit. By the end of the 2015–16 school year, significant differences in classroom arrangement of desks were found that are attributable to the program (61.3% vs. 38.7%). SunBay Math classrooms were nearly twice as likely to report arranging student seats in clusters instead of rows/columns or stadium seating.

In terms of student use of any technology during instruction, SunBay Math teachers reported higher rates than control teachers (97.5% vs. 65.3%). Of the few SunBay Math teachers that reported not providing opportunities for students to use technology in class, all but one teacher cited limited access to technology or technology problems as the reason. The ways in which students used technology also differed greatly between treatment and control classrooms. A large majority of SunBay Math teachers reported having students use technology in pairs or trios (90.6%) during math instruction, while the majority of control school teachers reported students using technology individually (56.5%).

The Impact of SunBay Math on Student Achievement

To assess the impact of the SunBay Math program on the achievement of middle-school students during the 2015–16 school year, we used a multilevel model to regress student FSA scores on an indicator of treatment status and a set of school and student covariates. To test baseline equivalence between treatment and control groups, we used the same model but replaced the outcome with student's prior year test scores. Table 12 provides the fixed and random effects regression coefficients and standard error for both models.

Table 12: Baseline Equivalence and ITT Program Impact

Dependent Variable	Baseline Equivalence 2015 Math FSA SS			ITT Program Impact 2016 Math FSA SS		
Fixed Effects						
Intercept	323.83	(1.73)	**	80.41	(1.21)	**
SunBay Math	-0.88	(1.77)		-1.03	(0.65)	
District 1	4.54	(1.77)	*	1.75	(0.66)	*
Title I	-12.23	(1.77)	**	-4.26	(0.66)	**
2015 Math FSA SS	--			0.78	(0.00)	**
Grade 6	-2.19	(0.54)	**	-1.06	(0.34)	**
Grade 7 *(reference)*	--			--		
Grade 8	-3.88	(0.54)	**	3.04	(0.34)	**
Random Effects						
School *(n = 60)*	36.78	(9.06)	**	3.15	(1.26)	*
Teacher *(n = 629)*	91.98	(6.10)	**	29.74	(1.96)	**
Residual *(n = 45,235)*	318.09	(2.11)	**	141.31	(0.95)	**

Notes: Grade 7 reference category

In both models we find all predictors are significant except treatment status. The non-significant effect in 2015 indicates that the random assignment process was effective in creating equivalent groups. Further, the lack of significance for treatment status on 2016 scores provides evidence that SunBay Math resulted in no difference (positive or negative) in the average student FSA scores. We conducted exploratory analyses of 2016 data to determine if subgroups based on combinations of student and/or school attributes produced significant main effects. The consistent result of those analyses was a non-significant effect of treatment on FSA scores.

This lack of evidence that the intervention impacted student math achievement may be partially related to the assessment itself. The 2014-2015 school year—the first year of the i3 project—was a time of change and uncertainty for both districts. After abandoning its original plan to adopt the Partnership for Assessment of Readiness for College and Career (PARCC) assessment—which SRI regarded as a good measure of SunBay Math content—the state of Florida introduced the FSA during the first year of the project. New state math standards were introduced the same year. It is possible that these simultaneous transitions and/or the FSA's poor alignment to the intervention complicated our ability to detect effects on student achievement.

Summary of Findings

CPRE's evaluation of the i3 implementation of SunBay Math yielded key findings for implementation fidelity, impacts on teacher beliefs, impacts on classroom opportunities for learning, and impacts on student achievement in math. For implementation fidelity, we found mixed results. While the teacher training and program resources components of the program model were adequately implemented across the two school districts, we found inadequate fidelity to the classroom lesson component. More specifically, we found that many teachers did not fully implement the units and/or investigations. Many students in the i3 implementation did not receive the SunBay Math intervention as designed. It is therefore reasonable to hypothesize that we might see different results were the program implemented fully. Given inadequate fidelity, it is not possible to conclude that the lack of impacts on student achievement suggests the program is ineffective. We can only say that SunBay Math, as implemented by the teachers in this study, did not produce impacts on students' math scores on the Florida Assessment. A companion report, *Enactment of Lessons from a Technology-Based Curriculum: The Role of Instructional Practices in Students' Opportunity to Learn* (Ebby, Fink & Sirinides, 2019), provides an in-depth look at the gaps in unit and lesson implementation we observed, both in terms of *how* teachers deviated from the program model and *why* they did so. Here, however, we offer fidelity findings as a potentially important explanation for the lack of impacts we observed on student math achievement.

We did, however, observe impacts in some areas that may point to the potential for student impacts in future research on SunBay Math. Specifically, we find that SunBay Math teachers are on average reporting that they are facilitating more opportunities in class for student collaboration as demonstrated by higher rates of: students working in groups for the main portion of the lesson; students' desks arranged in clusters to enable collaboration; and students using technology collaboratively during math class. Furthermore, we found that implementing the program positively impacted teachers' tolerance of student struggle as part of the math-learning process, and their comfort using technology for instruction.

Teacher beliefs and orientations towards technology, mathematics, and student capacity are important factors in the way curriculum and technological tools are used during lesson enactment. Indeed, SunBay Math was designed with the understanding that successful implementation of the program requires a fundamental pedagogical shift for many teachers and so is based on a curricular activity system in which technology, standards-aligned materials, and teacher professional development are integrated to meet the needs of varied local educational contexts (Vahey, Knudsen, Rafanan, & Lara-Meloy, 2013). This evaluation found that SunBay Math provided opportunities for teachers to engage students in mathematical practices through technology-based inquiry as they explore cognitively demanding mathematical concepts. Ultimately, this study offers a rigorous examination of both the successes and challenges in implementing a curricular intervention that combines a number of research-supported components, while it also raises questions about how to achieve strong implementation among teachers.

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Appendix

SunBay Math units are implemented over a two-week period whenever the unit content occurs in a district's pacing guide. The duration and timing of SunBay Math units are intended to ensure that teachers and students have time to work through all of the investigations in a unit, and that the key concepts SunBay Math emphasizes are reinforced throughout the school year. District and program records documenting the timing of SunBay Math use in schools indicate that adherence to program guidelines were not consistently implemented in this i3 study. Figure 2 shows the number of days from the start of the 2015–16 school year, in which each SunBay Math Unit was implemented.

Figure 2. SunBay Math unit implementation by district and grade