Examining the Role of Domain-General Skills in Mathematics Learning and Intervention Response in Kindergarten

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Abstract

Effective early mathematics instruction is critical to support long-term mathematics achievement. Given that student response to typical instruction varies, a range of mathematics interventions have been developed to support foundational mathematics development. However, not all students respond to these interventions. To better understand factors associated with intervention response, the current study explored how domain general cognitive skills were associated with intervention response for 621 kindergarten students with or at risk for mathematics difficulties. Results indicated that although domain general skills were associated with mathematics achievement, there was no evidence of differential response to intervention based on cognitive skills. When examining differences while holding initial mathematics skill constant, there was a non-significant, but potentially important pattern of students with higher domain general skills demonstrating greater mathematics gains as a result of intervention participation. Implications for mathematics intervention and curriculum development, including potentially impactful instructional approaches and cognitive scaffolds are discussed.

Examining Relations Between Domain General Skills, Mathematics Learning, and Intervention Response in Kindergarten

Many U.S. students struggle to attain adequate levels of mathematics proficiency (Kena et al., 2015), and students who are not proficient in mathematics are at risk for poor educational and postsecondary outcomes (Morgan et al., 2011; Ritchie & Bates, 2013; Rivera-Batiz, 1992). Because mathematics trajectories are often established in the primary grades (Geary, 2013; Kohli et al., 2015; Morgan et al., 2009), and tend to remain fairly stable over time (Duncan et al., 2007; Shanley, 2016), attending to and supporting early math achievement is critical. Efforts in recent years have resulted in a number of effective early mathematics interventions (Bryant et al., 2011; Clarke, Doabler, Smolkowski, Baker et al., 2016; Clements et al., 2011), suggesting that early and strategic mathematics intervention programs can improve the academic outcomes for most students with mathematics difficulties (MD: Gersten, Beckman, et al., 2009; Misquitta, 2011; Swanson, 2009). Yet not all students respond to otherwise effective interventions, and intervention effects often fade out over time (Bailey et al., 2017; Fuchs & Vaughn, 2012; Li et al., 2017). Thus, a better understanding of why and how mathematics interventions work for some students and not for others is an important consideration for the provision of effective, lasting interventions.

Domain-General Skills and Early Mathematics Achievement

Domain-general skills are skills that support learning and problem solving across a wide range of domains (Cowan & Powell, 2014; Tricot & Sweller, 2014), and are one potentially important factor associated with mathematics achievement. Evidence about associations among cognitive skills and mathematics outcomes is plentiful (Fuchs et al., 2006; Geary, 2013; Mazzocco & Myers, 2003), and a number of studies have explored the relative importance of domain-general and domain-specific skills in predicting mathematics performance. While the majority of studies have found that domain-specific skills are more important (Östergren & Träff, 2013; Vanbinst & De Smedt, 2016), or at least equally important, in explaining mathematics achievement (e.g., Chu et al., 2016; Hornung et al., 2014; Passolunghi & Lanfranchi, 2012; Vanbinst et al., 2014), there is some evidence to suggest that domain-general skills can predict mathematics achievement in the early grades above and beyond domain-specific skills (Geary, Nicholas et al., 2017). Given these findings, domain-specific skills may help to explain differences in intervention response among students who are at risk for mathematics difficulties.

Over the last twenty years, a variety of domain-general cognitive processes, including nonverbal problem solving, phonological processing, rapid automatized naming, visual-spatial skills, and working memory, have been explored as predictors of mathematics achievement and mathematics difficulties (Fuchs et al., 2005; Geary et al., 1999; Swanson & Jerman, 2006). Results of research studies in this area have found that various constellations of cognitive measures can explain up to half of the variance in number skill and elementary mathematics performance (Fuchs et al., 2006; Passolunghi et al., 2015). Correspondingly, a recent meta-analysis found that mathematics difficulties are associated with deficits in phonological processing, processing speed, working memory, attention, short-term memory, other executive functioning skills, and visual-spatial skills (Peng et al., 2018).

Although much is known about the relations between various cognitive constructs and mathematics achievement, especially within typical developmental trajectories, there remains quite a bit of grey area and nuance to those relations. For example, most studies, including many of those mentioned above, examine static relations between cognitive skills and concurrent mathematics skills. While these studies are valuable in establishing potentially important foundational skills, it remains unclear whether those relations hold when examining associations between cognitive skills and mathematics gains or learning. The lack of established directionality in correlations between cognitive factors and mathematics achievement can give rise to many competing hypotheses. For one, it could be that cognitive skills determine a student's ability to process and acquire new knowledge and skills, to the extent that cognitive skills cause mathematics achievement. In this case, cognitive scores would predict learning gains, and students with lower cognitive abilities would demonstrate reduced response to instruction. Alternatively, one could hypothesize that mathematics learning and instruction boosts cognitive skills, so that improved mathematics achievement leads to improved cognitive abilities. This hypothesis would lead one to expect that cognitive scores would be correlated with concurrent achievement, but would not necessarily be predictive of learning gains.

Domain-General Skills and Intervention Response

In addition to gaps in the literature relating to the direction of relations between cognitive skills and mathematics achievement, knowledge about how cognitive skills are related to early mathematics achievement and intervention response for students with or at risk for MD also remains somewhat unclear (Knops et al., 2017). Because there is compelling evidence that domain-general cognitive skills provide important information about mathematics difficulty (Kaufmann et al., 2013), identifying factors associated with intervention response is critical to increasing our understanding of mathematics development, and to improving intervention utility for all learners.

While research on factors associated with mathematics achievement remains somewhat scant, there is a research base in reading that may help to inform questions relating to the role of domain-general skills in intervention response. Meta-analytic research and literature syntheses in the field of reading suggest that domain-specific skills may have more utility than domain-general skills and intervention screening (Burns et al., 2016; Stuebing et al., 2015), and findings associated with cognitively focused interventions are mixed at best (Kearns & Fuchs, 2003). Small meta-analytic effect sizes for domain-general skills when compared to domain-specific skills in reading may suggest that domain-general skills have limited utility as a lever for improving intervention response in mathematics, but research in the domain of mathematics is needed to explore this hypothesis. In addition, nonzero effect sizes for domain-general skills were present in all of the above-mentioned meta-analyses, and efforts to identify skills that can improve intervention response for the small population of non-responders may necessitate a closer look at domain-general skills.

Thus, a more thorough investigation of relations between essential domain-general constructs, mathematics achievement, mathematics instruction or intervention, and mathematics difficulties is needed. Given their associations with mathematics performance, the following domain-general skills were explored. Known relations between these constructs and mathematics achievement gains or learning, and differential response to a mathematics intervention for students with mathematics difficulties, are discussed below.

Fluid reasoning. Fluid reasoning skills are one aspect of fluid intelligence, and are closely associated with learning outcomes (Voelkle et al., 2006; Watkins et al., 2007). Whereas fluid intelligence encompasses the use of many kinds of mental operations (i.e., drawing inferences, comprehending implications, and transforming information; McGrew, 2009) to solve novel problems (Primi et al., 2010), fluid reasoning requires one to find relations and identify patterns (Kaufman, 2014). Fluid reasoning is closely related to working memory, especially when performance is timed (Kaufman, 2014); but in untimed learning situations, fluid reasoning is a distinct construct that has been associated with math performance (Chuderski, 2013; Floyd et al., 2003; Taub et al., 2008). Fluid reasoning has been linked to future mathematics achievement across ages 6-21 (Green et al., 2017), and examinations of cognitive profiles for at-risk students revealed lower scores on measures of fluid reasoning commensurate with lower performance on measures of mathematics reasoning (Proctor et al., 2005.

Visual-spatial skills. Visual-spatial skills are associated with mathematics development across a wide range of ages (Mix & Cheng, 2012), including early elementary-aged children (Cornu et al., 2017). Research conducted with young children suggests that visual-spatial activities and early mathematics tasks have similar foundations (Verdine, Golinkoff et al., 2014). For example, pattern construction, spatial assembly, and other spatial visualization and visual motor integration tasks have been linked to early mathematics achievement (Cameron et al., 2012; Verdine, Irwin et al., 2014), student understanding of cardinality (Ansari et al., 2003), and early arithmetic skills (Cornu et al., 2017; Gunderson et al., 2012; Zhang et al., 2014). It is hypothesized that the visual-spatial ability to generate and manipulate mental representations of number and mathematics problems supports one's ability to efficiently solve equations and find missing terms (Cornu et al., 2017), and dual task studies conducted with early elementary-aged children suggest there is significant overlap between visual-spatial processing and arithmetic performance (McKenzie et al., 2003). Additionally, learners who generate spatial schematic diagrams to solve mathematics problems have higher word problem solving scores and perform better on measures of spatial ability (Hegarty & Kozhevnikov, 1999), and visual-spatial working memory deficits are associated with poor arithmetic skills in early elementary aged children (McLean & Hitch, 1999).

Phonological skills. In addition to the somewhat intuitive connections among fluid reasoning, visual-spatial skills, and mathematics performance, phonological skills are also important factors in mathematics development (LeFevre et al., 2010; Purpura et al., 2017). In fact, phonological processing has been associated with mathematics performance in the elementary grades (Hecht et al., 2001), and reading-related skills are reliable correlates of mathematics difficulties (Mazzocco & Myers, 2003). Phonological skills have been linked to tasks that require the memory and retrieval of number words and spoken numerals, including counting, retrieving numeral names, and math facts (Geary, 1993; Krajewski & Schneider, 2009; Logie & Baddeley, 1987). Phonological processing is associated with mathematics activities that involve verbal codes (Simmons et al., 2008), including fact fluency and number word identification (Fuchs et al., 2005; Fuchs et al., 2006). While visual-spatial skills are often highly associated with mathematics performance for younger children, as they support the visualization of numerical quantities (Gunderson et al., 2012), phonological skills are typically associated with mathematics achievement in older children (Holmes et al., 2008; Krajewski & Schneider, 2009).

Current Study

Given these well-documented associations, and the limited value of associations that do not account for prior performance or intervention instruction, calls for new research in the fields of mathematics development and numerical cognition have emphasized undertaking studies that can help researchers and practitioners better understand and identify relations between cognitive skills and mathematics achievement, and can elucidate whether and how students with MD employ cognitive skills differently than their typically achieving peers when learning mathematics and performing mathematical tasks (Alcock et al., 2016; Knops et al., 2017). Knowledge of how cognitive skills are related to intervention response for at-risk students can provide information about relations between domain-general skills and specific instructional approaches and intervention scaffolds. These findings can then inform future efforts to develop and implement multi-faceted, targeted, and maximally effective interventions. Thus, the current study sought to examine relations between domain-general cognitive skills and mathematics achievement in the context of a large-scale, randomized control trial of a kindergarten mathematics intervention. The following research questions and associated hypotheses were explored:

1. To what extent is performance on mathematics measures associated with phonological memory, visual-spatial skills, and fluid reasoning skills for kindergarten students at risk for math difficulties?

Because of the ubiquitous nature of relations between domain-general cognitive skills and mathematics achievement (Geary, Berch et al., 2017), it was hypothesized that mathematics performance would be associated with concurrent domain-general skills.

2. To what extent are phonological memory, visual-spatial skills, and fluid reasoning skills associated with gains in mathematics skills for kindergarten students at risk for math difficulties?

In light of the strong research base indicating relations between cognitive skills and mathematics achievement, it was hypothesized that gains in mathematics would also be associated with initial cognitive skills. Specifically, because phonological memory is related to the encoding and processing of number words and numerals and processes that involve them (Simmons et al., 2008), it was hypothesized that phonological memory would be associated with gains on curriculum-based measures, given the numeral-based nature of brief curriculum-based measures. Similarly, because short-term phonological memory has been shown to be an influential predictor of word problem solving (Andersson, 2007), it was also hypothesized that phonological memory scores would be related to gains on measures that assess problem solving. Based on the stable relation between visual-spatial ability and later mathematics achievement (Cheng & Mix, 2014), it was hypothesized that performance on a measure of visual-spatial skills would also be related to gains on measures of mathematics achievement. Similarly, recent findings suggest that fluid reasoning is predictive of long-term mathematics achievement (Green et al., 2017), so it was hypothesized that fluid reasoning would be associated with mathematics gains for study participants. It was also expected, however, that these relations would be weaker, given the influence of initial mathematics skill on mathematics gains.

3. How do relations among phonological memory, visual-spatial skills, fluid reasoning skills, and mathematics gains differ based on intervention receipt?

Because domain-general skills are associated with mathematics achievement, especially in the early grades (i.e., Geary, Nicholas et al., 2017), it was hypothesized that kindergarten students at risk for MD with more robust cognitive skills may experience larger gains in mathematics achievement in response to an efficacious intervention. It is also possible, however, that students with less developed domain-general skills may demonstrate comparable gains in mathematics achievement in response to an evidence-based mathematics intervention, if that intervention is able to mitigate domain-general deficits that impact response to typical instruction. Ultimately, because specific cognitive training was not included, and because there were no targeted domain-general adaptations to the intervention curriculum, it was hypothesized that the mathematics intervention would be consistently effective for students across the range of domain-general skills, and that there would not be evidence of differential response to the intervention based on cognitive skill.

4. After matching on initial mathematics skill, to what extent do students with high and low cognitive skills demonstrate differential mathematics gains as a result of intervention participation?

Given the relations between domain-specific skills and academic performance, along with the expectation that domain-general skills would not predict intervention response, it was hypothesized that domain-general skills may in fact be related to intervention response and gains in mathematics achievement when students are matched on initial mathematics skill. We hypothesized that students with higher domain-general skills would receive an added boost from the Tier 2 intervention. In other words, it was hypothesized that students with low initial mathematics skills, but robust cognitive skills, would demonstrate greater gains in mathematics achievement as a result of intervention participation when compared to their peers with similar mathematics and domain-general skills who did not receive intervention.

Method

The efficacy of the Roots intervention was examined in a randomized controlled trial that utilized a partially-nested design, with students nested with interventionists and interventionists nested within classrooms. This study analyzed data collected in a four-year efficacy trial funded by the Institute of Education Science. The 10-12 lowest performing students from each participating kindergarten classroom were randomly assigned within their classroom to one of three conditions: (1) a Roots instructional group with a 2:1 student-teacher ratio, (2) a Roots instructional group with a 5:1 student-teacher ratio, or (3) a no-treatment control condition. Students randomly assigned to the two treatment groups received the Roots intervention in addition to district-approved core mathematics instruction. Students in the control condition received district-approved core mathematics instruction only. Across four cohorts of participants (two cohorts in Oregon and two cohorts in Massachusetts), 255 Roots intervention groups were conducted. Student-level mathematics achievement data were collected during the kindergarten year at the intervention's pretest (T1) and post-test (T2) times, and at a follow-up (T3) approximately six months into the students' first-grade year.

Participants

Participants in the current study were drawn from cohorts 2 (i.e., Massachusetts, year 1) and 3 (i.e., Oregon, year 2) of the larger efficacy study described above. For more information about the four-cohort sample, see Clarke et al., 2020.

Schools. This study took place in kindergarten classrooms located in six school districts in Oregon and in the metropolitan area of Boston, MA. Of the six participating districts, two were located in the Boston area, and nine schools participated from these two districts. Across the 9 Boston area schools, 3-17% of students were African American; 1-7% were Asian; 45-83% were Hispanic; 0-1% were Native American; 9-85% were White; 0-<1% were Native Hawaiian, Pacific Islander; and 1-7% were Multi-Race, Non-Hispanic. Additionally, 8-24% of students had disabilities, 0-27% of students were English language learners, and 22-86% of students received free or reduced lunch.

Fourteen elementary schools from four Oregon school districts also participated in the study. Within the 14 schools, 0%-12% of students were American Indian or Native Alaskan, 0%-16% were Asian, 0%-9% were Black, 0%-74% were Hispanic, 0%-2% were Native Hawaiian or Pacific Islander, 19%-92% were White, and 0%-15% were more than one race. Eight percent to 25% of students received special education services, 5%-69% were English language learners, and 17%-87% were eligible for free or reduced lunch.

Classrooms. A total of 76 kindergarten classrooms (*n* = 37 in Oregon and *n* = 39 in Massachusetts) from the 23 schools described above participated in the current study. All classrooms provided five days per week of core mathematics instruction in English and had an average of 23.8 students (*SD* = 5.7). Participating classrooms were taught by 62 certified kindergarten teachers. All teachers were female, and the majority (88%) of teachers were White. Teachers had an average of 15.6 years (*SD* = 8.68) of teaching experience and 9.5 years (*SD* = 7.29) of kindergarten teaching experience.

Interventionists. Roots intervention groups were taught by interventionists, who were instructional assistants either employed by participating school districts or hired specifically for the intervention study. Most interventionists (i.e., 94%) were female, and 60% of interventionists had a bachelor's degree or higher. Ninety-two percent had prior experience with providing small-group instruction, and 58% had taken at least one college-level course in algebra. Seventy-five percent of interventionists were White, 10% of interventionists were

Hispanic, and 2% identified as another ethnicity. All interventionists participated in two five-hour professional development workshops delivered by project staff, and also received between two and four coaching visits from Roots coaches during intervention implementation.

Criteria for participation. In each participating classroom, all students with parental consent were screened in the late fall of their kindergarten year. The screening process included the Assessing Student Proficiency in Early Number Sense (ASPENS; Clarke et al., 2011) and the Number Sense Brief (NSB - later published as the Number Sense Screener: Jordan et al., 2012). Students were eligible for the Roots intervention if they received an NSB score of 20 or less and an ASPENS composite score in the strategic or intensive ranges. These thresholds were employed based on research indicating that students with similar scores demonstrated an increased likelihood of future mathematics difficulties (Clarke et al., 2011; Jordan et al., 2010). After being determined as eligible for the Roots intervention, students' NSB and ASPENS scores were separately converted into standard scores and then combined to form an overall composite score for each at-risk student. All data management and processing were conducted by an independent evaluator. Composite scores within each classroom were then rank-ordered, and the 10 Roots-eligible students with the lowest composite scores were randomly assigned to one of three conditions: (a) a 2:1 Roots group, (b) a 5:1 Roots group, or (c) a no-treatment control condition. Of the 76 classrooms, 30 did not meet the random assignment requirement of 10 Roots-eligible students. In these instances, a cross-class grouping procedure was applied, in which students from more than one classroom per school were combined and then rank-ordered based on their overall composite scores. The randomization procedure generated a total of 64 2:1 Roots groups and 63 5:1 Roots groups.

Students. In all, 1,807 kindergarten students were screened for intervention study eligibility in the late fall of 2013. Of these students, 621 met eligibility criteria and were randomly assigned within classrooms to the 2:1 Roots group condition (n = 129), the 5:1 Roots group condition (n = 312), or the no-treatment control condition (n = 180). As with previous analyses (Clarke, Doabler, Smolkowski, Kurtz Nelson et al., 2016), the current study combined students in the two-student and five-student Roots conditions in order to assess the effects of Roots intervention as compared to the control condition. Demographic information for all Roots-eligible students is presented in Table 1.

Intervention

Roots is a Tier II mathematics program designed to build students' proficiency in whole number mathematics. The Roots intervention was delivered in 20-minute small group sessions 5 days per week for approximately 10 weeks. Instruction for all students began in the late fall and ended in the spring, and this start date was selected to provide students with the opportunity to respond to initial core mathematics instruction, and therefore to minimize the identification of typically achieving students. Roots was delivered in addition to students' core mathematics instruction, and was scheduled at times that did not conflict with core instruction in mathematics.

Roots instruction is aligned with the Common Core State Standards for mathematics (Common Core State Standards - Math, 2010), and with recommendations from expert panels to focus on whole-number concepts and skills (Gersten, Beckman et al., 2009). Specifically, Roots instruction emphasizes concepts from the Counting and Cardinality and Operations and Algebraic Thinking domains of the CCSS-M. The Roots instructional approach is drawn from principles of explicit and systematic mathematics instruction (Coyne et al., 2011; Gersten, Chard et al., 2009), including explicit teacher modeling, deliberate practice, visual representations of mathematics, and academic feedback. Roots also includes frequent opportunities for students to verbalize their mathematical thinking and discuss problem-solving methods. More information about the Roots intervention is provided in Clarke, Doabler, Smolkowski, Kurtz Nelson et al. (2016).

Control Condition

Core mathematics instruction served as the control condition in this study, as both treatment and control students continued to receive their daily core mathematics instruction. Treatment students received Roots instruction in addition to core mathematics instruction. Features of the control condition were documented via teacher surveys and direct observations of classroom instruction. Teachers used a variety of published mathematics curricula during their core mathematics instruction, including Scott Foresman, enVisionmath, Houghton Mifflin, and Everyday Mathematics.

Teacher-led instruction was the primary instructional delivery format occurring in 84.7% of all observations. Other instructional methods such as peer learning (4.2%), independent student learning (5.9%), and math centers (3.4%) were observed less frequently. Slightly over half of all observations (55.1%) noted a focus on operations and algebraic thinking as the primary mathematics domain, while 27.1% of observations documented a primary focus on counting. Observations of classroom instruction indicated that Roots materials were not used during core mathematics instruction, and there was no evidence of treatment diffusion.

Fidelity of Implementation

Fidelity of implementation was measured via direct observations by trained research staff, with each Roots group observed three times during the course of the intervention. Observers rated the extent to which the interventionist (a) met the lesson's instructional objectives (including administration of embedded assessments), (b) followed the provided teacher scripting, and (c) used the prescribed mathematics models for that lesson, on a 4-point scale (4 = all, 3 = most, 2 = some, 1 = none). Observers also recorded whether the interventionist taught the number of activities prescribed in the lesson. Across the complete four-cohort study sample, interventionists were observed to meet instructional objectives (*M* = 3.49, *SD* = 0.69), follow scripting (*M* = 3.31, *SD* = 0.75), and use prescribed models (*M* = 3.61, *SD* = 0.64). Interventionists also taught the majority of prescribed activities (*M* = 4.14 out of 5 activities per lesson, *SD* = 0.77).

Measures

Trained research staff administered five measures of mathematics achievement and four subtests from neuropsychology batteries intended to assess domain-general cognitive skills. The domain-general measures were only administered at pretest, while all other measures were administered to participating students at pre- and post-test. Raw scores were used in all analyses unless otherwise noted.

Roots Assessment of Early Numeracy Skills (RAENS; Doabler et al., 2012). RAENS is a researcher-developed, individually administered instrument (32 items) that assesses counting and cardinality, number operations, and the base-10 system. RAENS' predictive validity ranges from .68 to .83 with widely used measures of mathematics achievement including the Test of Early Mathematics Ability – Third Edition (TEMA; Ginsburg & Baroody, 2003) and the Number Sense Brief (NSB; Jordan et al., 2012). Inter-rater scoring agreement was reported at 100% (Clarke et al., 2014), and internal consistency was high: Cronbach's alpha = .91 (Clarke, Doabler, Smolkowski, Baker et al., 2016).

Oral Counting – Early Numeracy Curriculum-Based Measurement (Clarke & Shinn, 2004). Oral Counting is a fluency-based measure that requires students to orally count in English for one minute. Concurrent and predictive validities range from .46 to .72.

ASPENS (Clarke et al., 2011). ASPENS is a set of three curriculum-based measures that assess early number sense proficiencies, including number identification, magnitude comparison, and missing number. Test-retest reliabilities of kindergarten ASPENS measures are in the moderate to high range (.74 to .85). Predictive validity of fall scores on the kindergarten ASPENS measures with spring scores on the TerraNova 3 is reported as ranging from .45 to .52.

Number Sense Brief (NSB; Jordan et al., 2012). NSB is an individually administered measure with 33 items that assess counting knowledge and principles, number recognition, number comparisons, nonverbal calculation, story problems, and number combinations. Authors report a coefficient alpha of .84 at the beginning of first grade.

Test of Early Mathematics Ability – Third Edition (TEMA; Ginsburg & Baroody, 2003). The TEMA is a norm-referenced, individually administered measure of beginning mathematics ability. The TEMA assesses whole number understanding, including counting, and basic calculations. The test authors report alternate-form reliability of .97, and test-retest reliability ranges from .82 to .93. Concurrent validity with other criterion measures of mathematics is reported as ranging from .54 to .91.

Wechsler Preschool and Primary Scale of Intelligence - Third Edition (WPPSI-III) Subtests: Matrix Reasoning and Block Design (Wechsler, 2012). The WPPSI-III is a comprehensive measure of general cognitive skills normed for children ages 2:6 - 7:7. The complete battery includes 13 subtests, two of which were administered in the current study. One subtest was administered to assess fluid reasoning, a construct that encompasses inductive reasoning, conceptual thinking, and classification ability, and the other subtest assessed visual-spatial processing, a construct that involves organizing visual information, attending to visual detail, and integrating visual and motor functions (Wechsler, 2012).

WPPSI-III Matrix Reasoning (WPPSI MR). WPPSI MR is an untimed fluid reasoning subtest that evaluates the ability to recognize and complete patterns. The subtest includes two sample items, and all following items are completed without feedback. The subtest includes 26 total items containing incomplete matrix patterns that the examinee completes by choosing from five possible choices. Split-half reliability for MR ranges from $r = .88$ to .90 and test-retest reliability is .86 for examinees aged 5:6-7:7. Correlations between WPPSI MR subtest scores and other similar measures of matrices and fluid reasoning range from $r = .48$ to .49.

WPPSI-III Block Design (WPPSI BD). WPPSI BD is a subtest with 17 items that evaluates one's ability to understand and replicate two- and three-dimensional representations. The initial designs are three-dimensional, modeled by the examiner, and the examinee recreates them in three-dimensional space. Later items present a two-dimensional design that is replicated in three-dimensional space. Individuals receive 2 points for designs that are correctly created within the allotted time limit. Some items allow partial credit for designs that are correctly completed on a second attempt. Completion time and error analysis are recorded for each item. Split-half reliability for BD ranges from $r = .84$ to .86, and test-retest reliability is .83 for examinees aged 5:6-7:7. Correlations between WPPSI BD subtest scores and other similar measures of design recall and visual-spatial skills range from $r = .51$ to .58.

Comprehensive Test of Phonological Processing (CTOPP) Subtests: Memory for Digits and Nonword Repetition (Wagner et al., 1999). The CTOPP is a comprehensive assessment of phonological skills, consisting of 13 subtests, that assesses phonological awareness, phonological memory, and rapid naming. Two subtests were administered in the current study, and scores from these subtests - Memory for Digits and Nonword Repetition - combine to form a Phonological Memory composite score (i.e., CTOPP PMC).

CTOPP Memory for Digits. CTOPP Memory for Digits is a 21-item subtest that measures the extent to which an individual can repeat a series of numbers ranging in length from two to eight digits, and is administered to detect the phonological loop of short-term memory. The examinee listens to a series of audio-recorded numbers, presented at a rate of 2 per second, and is asked to repeat the numbers for each set in the same order in which they were heard. The total score is the number of correct responses up to the ceiling (i.e., the individual misses three test items in a row). Feedback is only given for the practice items. Internal consistency for Memory for Digits ranges from $\alpha = .78$ to .81, and test-retest reliability is .74 for examinees aged 5-7. Correlations between CTOPP Memory for Digits scores and other measures of phonological knowledge and reading range from $r = .32$ to .49.

CTOPP Nonword Repetition. CTOPP Nonword Repetition is an 18-item subtest that measures the extent to which an individual can repeat nonwords that range in length from 3 to 15 sounds, and is included as a measure for detecting the efficiency of the phonological loop. The examinee listens to audio-recordings of made-up words, and must verbally repeat them back. The total score is the number of correct test items up to the ceiling (i.e., the individual misses three items in a row). Feedback is only given for the practice items. Internal consistency for Nonword Repetition is $\alpha = .80$, and test-retest reliability is .68 for examinees aged 5-7. Correlations between CTOPP Nonword Repetition scores and other measures of phonological knowledge and reading range from $r = .19$ to .41, and nonword repetition tasks are associated with phonological recall and recognition (Adlof & Patten, 2017), and with measures of phonological awareness (Clark et al., 2012).

Statistical Analysis

Main effects. Before examining the four research questions pertaining to relations between domain-general cognitive skills and intervention response, we first confirmed that the intervention effects for the specific sample in the current study were consistent with prior studies. We assessed intervention effects on each of the primary outcomes, with a mixed model (multilevel) time-by-condition analysis (Murray, 1998) designed to account for students partially nested within small groups (Baldwin et al., 2011; Bauer et al., 2008).

Because an analysis of intervention effects themselves is not the focus of the present study, we refer readers to Clarke, Doabler, Smolkowski, Kurtz Nelson et al. (2016) for a complete description of the analysis approach.

Correlations. After examining intervention effects, research questions 1 and 2 required tests of associations among cognitive measures and both pretest mathematics measures and gains on mathematics measures across time. Pearson product-moment correlations were calculated for these analyses. Correlation size was prioritized over statistical significance due to the large sample size and likelihood of statistical significance.

Interactions. For research questions 3 and 4, tests of interactions of cognitive variables with condition effects were generated. To test whether intervention effects differed by students' domain-general skills or, equivalently, whether the association between cognitive variables and mathematics gains depended on participation in the Roots intervention, an interaction was added within the models used to test for intervention effects. The main-effects models estimated effects for Time, Condition, and Time × Condition, where the Time parameter estimated gains on mathematics outcomes in the control group and the Time × Condition interaction estimated the difference between intervention and control students on gains on mathematics outcomes. To examine the influence of phonological memory, for instance, we added the phonological memory composite score to the model, as well as its interaction with each of the other three terms. Statistical support for interaction effect implies that the association between cognitive variables and gains in mathematics depends on participating in the Roots intervention.

It is noteworthy that research questions 3 and 4 differ in two important respects. For research question 3, each continuous cognitive variable was added into the models for the full sample. For research question 4, the level of baseline mathematics performance across groups of students with higher versus lower cognitive function on each measure was held constant.

To do so, each of the three cognitive variables was dichotomized at the median, so the models for research question 4 use a dichotomous rather than continuous moderator for measures of phonological memory, visual-spatial skills, and fluid reasoning.

Matching. Next, an exact matching procedure using the TEMA was applied to match all students who scored above the median with students who scored below the median on each measure. For each score on the TEMA, the same number of students in each group were selected, as defined by the cognitive measures. For example, if the low-phonological memory group contained four students with a TEMA score of 7 and the high-phonological memory group contained five students with a TEMA score of 7, the matching procedure would randomly select four of the five students from the high-phonological memory group. This process was repeated for each TEMA score. The entire procedure was then repeated for the other domain-general variables, creating three matched samples, one for each of the cognitive measures.

The matching procedure removed any correlation between cognitive-group membership and the TEMA. In the full sample, the TEMA was positively correlated with the group variables for the WPPSI BD (r = .14), WPPSI MR (r = .23), and CTOPP PMC (r = .35). After matching, those three correlations decreased to zero, but the matching procedure reduced the sample size from approximately 600 students, depending on the cognitive measure, to 412 for the phonological memory groups, 464 for visual-spatial skills groups, and 406 for fluid reasoning groups.

Model estimation. We fit the statistical models to our data using SAS PROC MIXED version 14.2 (SAS Institute, 2016) with full-information maximum likelihood estimation to minimize the potential for bias due to missing data (Allison, 2009; Graham, 2009). To account for missing data, the Time × Condition and growth models were fit with all available data on mathematics measures, but not with domain-general measures (1.1% to 2.5% missing).

Reporting. The American Statistical Association (Wasserstein & Lazar, 2016; Wasserstein et al., 2019) has strongly urged researchers to abstain from using bright-line rules for claims of "statistical significance" such as $p < .05$ or other metrics (e.g., effect sizes > 0.25). We reported *p*. We then estimated model probabilities (*w*), described next, to characterize the strength of evidence for the alternative hypothesis over the null hypothesis. We used tables to summarize parameter estimates with standard errors (see Tables A1-A7 in the Appendix for detailed results of all analyses). *P* values, defined as a measure of incompatibility between the observed data and all assumptions of the statistical model including the null hypothesis, H_0, are difficult to interpret (Wasserstein & Lazar, 2016). They provide information about neither which assumptions are incorrect nor the importance of the association (Greenland et al., 2016). We have therefore complemented *p* values with a model probability (also called Akaike weights; Akaike, 1973) that describes the strength of evidence for the hypothesis of an effect (Burnham et al., 2011).

The model probability is based on the AICc, a second-order, small-sample bias correction to the AIC (Akaike, 1973; Anderson, 2008). The probability or weight, *w*, can be interpreted as the probability that the same model would be selected with a "replicate data set from the same system" (Burnham et al., 2011, p. 30). We defined a model for each of two hypotheses - the hypothesis of an interaction effect between a domain general skill and condition (H_A), and the hypothesis of no interaction effect (H_0) - and reported the model probability, *w*, for the model with the interaction (H_A). With only two models, the model probability for H_0 (no interaction) is $1 - w$. For example, if $w = .75$, it suggests that the probability of H_A is .75 while the probability of H_0 is .25. The model for H_A is estimated to have an approximately 75% chance of being the better-fitting model, given the data and the two models. Equivalently, the model for H_A is three times as likely as the model for H_0. We interpret model probabilities as a continuous level of evidence for an interaction and avoid using a particular probability level as a cutoff for "significant."

Results

Main Effects

Previous research indicated condition differences favoring the Roots intervention with subsets of the sample used in the present study (Clarke et al., 2017; Doabler et al., 2016), but, as noted above, main effects for the specific sample include in the present study had not been tested previously. Therefore, condition differences in gains on the RAENS, oral counting, ASPENS composite, NSB total raw score, and TEMA measures were examined, and statistically significant main effects were found for all but the oral counting variable (see Appendix Table A1). These results are consistent with analyses that include some of the sample utilized in the present study (Clarke et al., 2017; Doabler et al., 2016) as well as replications with other samples (Clarke, Doabler, Smolkowski, Baker et al., 2016). Given the consistency of results across analyses, condition effects are not extensively discussed herein.

Associations Between Domain General Skills and Pretest Mathematics

To test the extent to which performance on pretest mathematics measures was associated with phonological memory, spatial reasoning, and fluid reasoning skills, each baseline math measure was correlated with CTOPP PMC, WPPSI BD, and WPPSI MR scores. The cognitive variables were modestly associated with the mathematics outcomes. In Table 2, correlations ranged from .33 to .39 (11% to 15% overlapping variance) for PMC, .07 to .19 (0% to 4%) for BD, and .13 to .27 (2% to 7%) for MR. For all but one correlation, p_{BH} (*p* after the Benjamini-Hochberg correction) was less than .05.

Associations Between Domain-General Skills and Gains in Mathematics

To test associations among gains in mathematics and domain-general measures, each of the cognitive measures was correlated with gain scores (i.e., pretest to post-test) on each mathematics measure. The correlation between PMC and NSB was .14 (2% overlapping variance; $p_{BH} = .0225$). See Table 2 for details. The remaining domain-general measures correlated poorly with gains in mathematics, with correlations ranging from -.03 to .09, a finding that implies 0% to 1% overlapping variance ($p_{BH} < .05$).

Moderation Effects

Next, the extent to which the association between domain-general skills and mathematics achievement differed by receipt of the Roots intervention was examined. The interaction between each domain-general skill and condition tests whether the association between domain-general measures and mathematics gains differ by condition. This is equivalent to a test of whether domain-general skills moderate intervention effects on gains in mathematics skill. The tests for moderation by domain-general skills produced no statistically significant interaction effects ($p > .10$). Similarly, model probabilities, *w*s, ranged from .26 to .55, indicating that models without interaction terms produced a better fit to the data in most cases (see Appendix Tables A2-A4). Thus, participating in the Roots intervention did not change associations among domain-general skills and gains in mathematics; equivalently, domain-general skills did not change the relationship between the Roots intervention and mathematics outcomes.

Moderation Effects After Matching on Initial Mathematics Skill

As discussed above, domain-general skills and mathematics achievement were correlated at baseline, implying that the test of the influence of intervention receipt on the associations between cognitive measures and gains in mathematics may depend, in part, on those correlations between domain-general skills and baseline mathematics performance. Hence, level of mathematics performance at pretest was held constant while again testing the extent to which associations between domain-general skills and mathematics achievement differed based on receipt of the Roots intervention.

This question differs from the third question in two ways. First, students were split at the median into high and low levels of domain-specific skills rather than investigating domain-specific abilities on a continuous scale. Second, initial mathematics skill was held constant across the two domain-general skill groups with an exact matching procedure. With the matched-pairs sample, tests of moderation for the TEMA were examined. The TEMA was used as a mathematics outcome measure for these analyses because the TEMA was also used to match cases across the high- and low-performance groups for each domain-general measure.

The association between high-phonological memory versus low-phonological memory group membership and gains on the TEMA was larger for students who received Roots than for students in the comparison condition, but the interaction was not statistically significant ($p = .1005$). Model probability indicated that the interaction model was slightly more likely to fit the data better than the null model ($w = .58$). Similarly, the interactions were not statistically significant for the visual-spatial skills groups ($p = .8347$) and fluid reasoning groups ($p = .3263$), but the model probability for the model that grouped students by fluid reasoning performance indicated that the model comparing groups had an approximately 62% chance of being the better-fitting model, given the data.

Interactions among mathematics gains, condition, and each domain-general group variable on all other mathematics measures were also tested, and no statistically significant interactions were found. Notably, though, the association between high visual-spatial versus low visual-spatial group and gains on the ASPENS was larger for students who received Roots than for students in the comparison condition, but the interaction was not statistically significant ($p = .0698$), while the model probability indicated that the interaction model had an approximately 65% chance of being the better-fitting model. Model probabilities for all other models (i.e., models comparing gains on all mathematics measures across all domain-general skill groups) ranged from .26 to .52 (see Appendix Tables A5-A7).

Discussion

Establishing robust foundations in mathematics is a critical aspect of early elementary education. Students who exit kindergarten with low mathematics skills tend to continue to struggle with mathematics, and remain in the lowest percentiles of achievement in future years (Morgan et al., 2016). Thus, the development of targeted, effective interventions for students at risk for MD in the early grades has been a priority for the field (Gersten, Beckman et al., 2009). As the pool of generally efficacious interventions grows, identifying and capitalizing on factors that are positively associated with learning to improve interventions can help to lessen rates of non-response and sustain intervention gains in future years. Given the association between domain-general skills and mathematics achievement, specifically attending to cognitive skills may be especially critical to supporting early mathematics development for students at risk for mathematics difficulties. Modifying intervention protocols to leverage and support domain-general strengths and weaknesses may be a fruitful approach to limit non-response and reduce fadeout effects.

The current study sought to examine relations among domain-general skills and mathematics for students at risk for MD, and to explore how those relationships differ for students who receive intervention. Consistent with prior research findings, cognitive skills were associated with mathematics achievement, but associations between cognitive skills and mathematics achievement gains were not strong for kindergarten students at risk for MD in the current study. This finding suggests that mathematics gains for kindergarten students at risk for MD did not depend on their domain-general cognitive skills at pretest. Similarly, intervention response did not differ based on domain-general skills. That is, intervention participant gains in mathematics, as compared to the mathematics gains for control participants, did not differ as a function of domain-general skills.

Lastly, when holding mathematics skill constant, there were no statistically significant differences in intervention response for students with above-median versus below-median cognitive skills. This analysis was of particular interest given the variable, and in some cases, potentially limited exposure to formal education for kindergarten students. In other words, some students may have less developed early mathematics skills due to a lack of exposure to formal schooling and instruction (i.e., average or above-average domain-general skills, but lower mathematics skills), while other students may have had ample opportunities to learn early mathematics concepts, but may require more intensive instruction (i.e., below-average domain-general skills, and commensurate low mathematics skills). The intent of this analysis was to examine whether students with low cognitive skills and below-average mathematics skills responded differently to the Roots intervention than students with similar mathematics skills, but high cognitive skills.

Importantly, though, while not statistically significant, the finding of potentially noteworthy patterns of students with particular domain-general skills who received intervention demonstrating greater gains on mathematics outcome measures may provide useful areas for future research. The finding in the current study of trends suggesting that students with above-median phonological memory skills and students with above-median fluid reasoning skills demonstrated greater gains on a distal mathematics measure, while students with above-median visual-spatial skills demonstrated improved basic number skills as a result of intervention participation, provides some impetus to engage in targeted, deeper explorations of the role of domain-general skills in early mathematics intervention.

Limitations

The present study examined only a subset of individual domain-general skills for a large, but somewhat restricted sample of kindergarten students who were at risk for MD. Measures of domain-general constructs were chosen based on a range of factors including their hypothesized relation to features of the Roots intervention, ease of administration and scoring, sensitivity within the target population, and relevance to mathematics achievement and the field of early academic achievement, in general. Because a full cognitive battery was not administered, the extent to which the role of individual domain-general constructs can be distinguished from general intelligence (i.e., g; see Carroll, 1993) is unknown in the present study. In fact, it is likely that individual domain-general skills fall under the same umbrella of connected cognition referred to as g (Spearman, 1961). Yet while domain-general skills and g are closely related, research suggests that domain general skills and g are not identical (Conway et al., 2003) and the role of general intelligence in learning warrants continued research (Sternberg, 2019).

Keeping in mind the overlap among cognitive processes (Kovacs & Conway, 2016), relations between other domain-general skills and mathematics achievement may warrant future exploration, as well. For example, language skills have also been shown to adequately predict low mathematics performance in young children (Purpura et al., 2017), and processing speed, language skills, and rapid automatized naming are effective in discriminating between typical and at-risk learners in the early grades (Cirino et al., 2015). These findings suggest that language skills may be important to support the acquisition of basic numerical skills and engagement in more advanced, formalized mathematics. Future efforts are warranted to explore the extent to which other domain-general skills, and possibly even other measures of the constructs explored here, are associated with early mathematics intervention response.

Findings from this study are also limited in that they can only be generalized to the Roots intervention. Additional study is needed to determine the extent to which domain-general skills are associated with response to various interventions and instructional approaches. It should also be noted that implementation fidelity is a key construct when assessing differential response to an intervention, and that fidelity of implementation in authentic school contexts (i.e., the current study) is not completely uniform. Differences in intervention response can be attributed to a range of factors, especially when fidelity of implementation varies.

While there was no evidence of differential response to the Roots intervention based on domain-general skills, the findings of the present study contribute to the collaboratively derived research agenda of Alcock and colleagues (2016) that identified critical next steps in mathematical cognition research, saying that “the time is ripe for studies designed to disentangle the effects of different predictors and to map patterns of development” (p. 26). As discussed above, the effect of the Roots intervention did not differ based on domain-general skills, and there were no statistically significant differences in intervention effects based on domain-general skills; yet manipulating intervention components to provide targeted domain-general training and strategic domain-general supports may be a means to increase intervention effectiveness and facilitate sustained intervention gains to close achievement gaps nonetheless. Insights gained from studies that control and manipulate cognitive components may also help to distinguish young students with mathematics difficulties from students with more severe mathematics disabilities, and may make important contributions to efforts to define MD.

Future Directions

Given the findings of this study, further work is warranted as a response to calls to consider cognitive competences in the design of instructional materials and approaches

(Geary, Berch et al., 2017). Research in this area can inform intervention development and support the development of specific, individualized intervention provision protocols that can begin to close achievement gaps and have lasting effects. To date, the development of interventions specifically tailored to potential non-responders and students with disabilities has been the focus of limited study (Fuchs et al., 2014; Swanson et al., 2014), despite calls for such efforts (Koyama et al., 2016; Miller et al., 2014). This targeted intervention approach may offer a mechanism to better serve students with a mathematics disability, in contrast to students with low initial achievement who are more likely to respond to typical mathematics intervention protocols.

Extensive research has demonstrated associations between cognitive skills and academic achievement; it is also clear, however, that cognitive skills and academic tasks do not exist or operate in isolation (Gilmore et al., 2017). Instead, cognitive skills are part of a complex network of interconnected processes (Knops et al., 2017; LeFevre et al., 2010), and even the most basic academic tasks employ a range of requisite behaviors. In fact, research conducted with early elementary-aged students revealed that students who score similarly on a mathematics achievement test possess variable and often quite different cognitive profiles (Gilmore et al., 2017). Furthermore, relations between cognitive skills and mathematics achievement differ based on the type of mathematics skill assessed, and cognitive skills are often more closely related to complex mathematics tasks, as opposed to basic procedural tasks (Cirino et al., 2018; Fuchs et al., 2010). This finding suggests that research in this area requires both great specificity and thoughtful holistic examinations of the role of domain-general cognitive skills in mathematics achievement. This consideration is especially relevant for young learners, some of whom are experiencing formal schooling for the first time, and whose academic skills, cognitive skills, and learning behaviors are all developing in concert.

Given these complexities, it is also recommended that future efforts examine the concurrent development of domain-general skills and academic achievement. Knowledge of the relations between concurrent mathematics and cognitive development for early elementary students at risk for MD can inform future screening, intervention, and progress monitoring activities, and may help to identify critical target constructs related to MD risk. Integrated, evidence-based intervention systems that attend to the specific and complex learning needs of each individual student and are delivered via responsive instructional delivery platforms may be the key to improving intervention response and the maintenance of learning gains over time.

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Table 1

Descriptive Statistics for Student Characteristics by Condition

	Roots	Control	Total
	n (%)	*n* (%)	*n* (%)
Total sample (n)	441 (100%)	180 (100%)	621 (100%)
Sex			
Male	231 (52%)	103 (57%)	334 (54%)
Female	210 (48%)	77 (43%)	287 (46%)
Race			
American Indian or Alaskan Native	2 (1%)	3 (2%)	5 (1%)
Asian	10 (3%)	5 (3%)	15 (3%)
Black	20 (5%)	7 (5%)	27 (5%)
Hispanic	141 (36%)	62 (40%)	203 (37%)
Hawaiian/Pacific Islander	3 (1%)	0 (0%)	3 (1%)
White	198 (51%)	73 (47%)	271 (50%)
More than one	13 (3%)	5 (3%)	18 (3%)
Special education			
Eligible	246 (64%)	93 (60%)	339 (63%)
Not eligible	141 (36%)	62 (40%)	203 (37%)
	M (SD) n	*M (SD) n*	*M (SD) n*
Age	5.3 (0.4) 440	5.2 (0.4) 180	5.3 (0.4) 621

Note. Percentages represent the proportion of students for whom each variable was reported. The sample sizes (n) for each variable will not sum to the total sample (top row) due to missing responses. Race and ethnicity categories were exclusive, so we did not receive reports, for example, of students as White and Hispanic. M = mean, SD = standard deviation.

Table 2

Correlations (and % Overlapping Variance) between Cognitive Measures and Baseline Mathematics Measures and Gains in Mathematics from Fall to Spring

Measure	CTOPP PMC	WPPSI BD	WPPSI MR
Fall (Baseline)			
RAENS	.34 (12%)	.19 (4%)	.25 (6%)
Oral Counting	.34 (12%)	.07 (0%)	.13 (2%)
ASPENS	.33 (11%)	.19 (4%)	.26 (7%)
NSB	.36 (13%)	.15 (2%)	.27 (7%)
TEMA	.39 (15%)	.16 (3%)	.23 (5%)
Gains from Baseline to Post-Test			
RAENS	.01 (0%)	.00 (0%)	.05 (0%)
Oral Counting	.09 (1%)	.02 (0%)	.07 (0%)
ASPENS	.09 (1%)	.06 (0%)	.07 (0%)
NSB	.14 (2%)	.07 (0%)	.07 (0%)
TEMA	.06 (0%)	-.03 (0%)	.02 (0%)

Note. All correlations with baseline measures were statistically significant after Benjamini-Hochberg correction, except for the correlation between BD and baseline oral counting (pBH = .0658). No correlations with gains were statistically significant after Benjamini-Hochberg adjustment, except for the correlation between PMC and NSB gains (pBH = .0225).

Examining the Role of Domain-General Skills in Mathematics Learning
and Intervention Response in Kindergarten

Supplemental Appendix

Manuscript

Shanley, L. et al. (2021). *Examining the role of domain general skills in mathematics learning and intervention response in kindergarten*.

Detailed Results

This appendix provides detailed results from the tests of main effects of the Roots intervention and associations between domain-general cognitive skills and intervention response.

Main Effects

Table A1 reports the results of the tests for intervention condition differences in gains on mathematics skills. We assessed main effects of the intervention partially nested model described in Clarke, Doabler, Smolkowski, Kurtz Nelson et al. (2016) to account for individual students randomly assigned to condition and then nested within small groups only when assigned to the intervention condition. The models account for the potential heterogeneity among variances across conditions. Because the residual variances may have differed between the nested intervention condition and the unclustered control condition, we tested the assumption of homoscedasticity of residuals with a likelihood ratio test. Because we tested for equivalence, or for the noninferiority, of the simpler, homoscedastic model when compared to the more complex, heteroscedastic model, we reversed the null and alternative hypotheses (e.g., Dasgupta et al., 2010). For this reason, we compare models with likelihood ratio test using $\alpha = .20$ as our criterion Type I error rate, and report the more complex model unless we are relatively certain the two are equivalent.

The model tested pre-/post-data with Time, Condition, and the Time × Condition interaction. Condition represents the difference at baseline, Time the gains from pretest to post-test among control students, and Time × Condition represents condition differences in gains. We tabled additional test statistics for the Time × Condition interaction.

Interactions with Continuous Cognitive Variables

Tables A2 to A4 report results for tests of differential response to the intervention based on students' continuous domain-general skills. This is equivalent to differential associations between cognitive variables and mathematics gains based on participation in Roots. These models expanded on the main effects models by adding the cognitive predictor and its interactions with the Time, Condition, and Time × Condition terms. We tabled additional test statistics for the Predictor × Time × Condition interaction.

Interactions with Dichotomous Cognitive Variables in Matched Sample

Tables A5 to A7 report results for tests of differential response to the intervention based on students' dichotomized domain-general skills in a sample that matched students on TEMA scores. The exact matching procedure matched all students who scored above the median on each cognitive measure with students who scored below the median. The analysis then proceeded as described for the continuous moderators, but with the dichotomized cognitive variables and a smaller, matched sample.

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Table A1

Main Effects of Condition Differences for the Roots Intervention

		RAENS	OC	ASPENS	NSB	TEMA
Heterogeneity of Variance Test	Likelihood Ratio χ^2	0.56	1.64	0.14	0.23	0.51
	p value	.7553	.4400	.9336	.8913	.7731
Fixed Effects	Intercept	10.97 (0.44)	18.21 (1.35)	18.36 (2.03)	11.86 (0.32)	16.03 (0.55)
	Time	6.54 (0.40)	21.48 (1.57)	41.95 (2.24)	5.91 (0.32)	7.15 (0.40)
	Condition	0.37 (0.57)	2.59 (1.72)	2.50 (2.61)	0.70 (0.42)	0.80 (0.73)
	Time × Condition	5.67 (0.48)	1.80 (1.93)	19.69 (2.70)	1.03 (0.38)	1.94 (0.48)
Variances	Group-Level Intercept	7.75 (1.75)	43.60 (13.09)	142.53 (32.59)	5.14 (0.96)	15.72 (2.91)
	Group-Level Gains	0.88 (0.59)	17.14 (10.55)	11.88 (21.25)	0.00 (0.36)	0.43 (0.66)
	Member-Level Intercept	13.49 (1.59)	76.52 (14.63)	168.15 (32.56)	4.90 (0.80)	24.84 (2.44)
	Residual (Error)	12.38 (0.90)	193.10 (14.49)	415.57 (31.54)	8.85 (0.63)	12.79 (0.99)
	ICC	.07	.08	.03	.00	.03
Test of Time × Condition Estimate	Hedges's *g*	0.92	0.08	0.56	0.20	0.25
	Model probability (*w*)	> .99	.36	> .99	.93	> .99
	p value	<.0001	.3506	<.0001	.0076	<.0001
	df	412	385	384	437	385

Note. Table entries show χ^2 values, *p* values, parameter estimates with standard errors in parentheses, intraclass correlation coefficients (ICCs), model probabilities (*w*), Hedges's *g* values, and degrees of freedom (*df*). The Likelihood ratio test in the first two rows compared homoscedastic residuals to heteroscedastic residuals with a criterion α of .20 and one degree of freedom; all models supported homoscedastic residuals. Tests of fixed effects in the next four rows accounted for small groups as the unit of analysis within the Roots intervention condition and unclustered individuals in the control condition. RAENS = Roots Assessment of Early Numeracy Skills; OC = oral counting; ASPENS = Assessing Student Proficiency in Early Number Sense; NSB = Number Sense Brief; TEMA = Test of Early Mathematics Ability-Third Edition.

Table A2

Interactions between Phonological Memory Skills and Condition Differences

		RAENS	OC	ASPENS	NSB	TEMA
Heterogeneity of Variance Test	Likelihood Ratio χ^2	1.25	1.90	0.15	0.62	0.98
	p value	.5349	.3865	.9259	.7323	.6113
Fixed Effects	Intercept	11.10 (0.42)	18.41 (1.31)	18.78 (2.01)	12.02 (0.31)	16.28 (0.51)
	Time	6.62 (0.40)	21.68 (1.57)	42.39 (2.25)	5.90 (0.32)	7.16 (0.40)
	Condition	0.27 (0.54)	2.42 (1.65)	2.21 (2.54)	0.57 (0.39)	0.56 (0.66)
	Time × Condition	5.56 (0.49)	1.50 (1.94)	19.34 (2.71)	0.99 (0.38)	1.91 (0.48)
	PMC	0.73 (0.20)	1.49 (0.61)	2.35 (0.95)	0.62 (0.14)	1.18 (0.24)
	PMC × Condition	0.12 (0.24)	0.65 (0.73)	-0.25 (1.13)	-0.16 (0.17)	-0.09 (0.29)
	PMC × Time	0.22 (0.19)	1.80 (0.74)	2.18 (1.05)	0.32 (0.15)	0.16 (0.19)
	PMC × Time × Condition	-0.32 (0.23)	-1.20 (0.89)	-1.30 (1.26)	-0.09 (0.18)	0.00 (0.23)
Standardized Estimates for Fixed Effects with PMC	PMC	.189	.146	.125	.238	.289
	PMC × Condition	.026	.053	-.011	-.052	-.018
	PMC × Time	.040	.123	.081	.086	.027
	PMC × Time × Condition	-.048	-.068	-.040	-.021	.000
Variances	Group-Level Intercept	4.85 (1.54)	26.43 (11.50)	104.81 (30.11)	3.29 (0.81)	9.78 (2.39)
	Group-Level Gains	0.86 (0.61)	17.01 (10.46)	6.37 (20.80)	0.04 (0.36)	0.57 (0.70)
	Member-Level Intercept	12.80 (1.59)	63.62 (13.96)	169.65 (33.37)	4.52 (0.77)	22.57 (2.30)
	Residual (Error)	12.50 (0.92)	190.36 (14.47)	416.30 (31.94)	8.57 (0.62)	12.60 (0.99)
Test of PMC × Time × Condition	Model probability (*w*)	.49	.47	.38	.29	.26
	p value	.1578	.1763	.3033	.6015	.9912
	df	564	571	561	564	555

Note. Table entries show χ^2 values, *p* values, parameter estimates with standard errors in parentheses, model probabilities (*w*), and degrees of freedom (*df*). The Likelihood ratio test compared homoscedastic residuals to heteroscedastic residuals; all models supported homoscedastic residuals. Tests of fixed effects in accounted for the partially nested study design. PMC = Comprehensive Test of Phonological Processing Phonological Memory composite score; RAENS = Roots Assessment of Early Numeracy Skills; OC = oral counting; ASPENS = Assessing Student Proficiency in Early Number Sense; NSB = Number Sense Brief; TEMA = Test of Early Mathematics Ability.

Table A3

Interactions between Visual-Spatial Reasoning Skills and Condition Differences

		RAENS	OC	ASPENS	NSB	TEMA
Heterogeneity of Variance Test	Likelihood Ratio χ^2	0.57	1.68	0.34	0.44	0.44
	p value	.7502	.4318	.8426	.8039	.8012
Fixed Effects	Intercept	10.96 (0.43)	18.11 (1.36)	18.24 (2.03)	11.88 (0.32)	16.04 (0.54)
	Time	6.52 (0.40)	21.65 (1.58)	42.16 (2.23)	5.92 (0.32)	7.16 (0.40)
	Condition	0.41 (0.56)	2.84 (1.74)	2.77 (2.59)	0.72 (0.42)	0.81 (0.71)
	Time × Condition	5.71 (0.49)	1.72 (1.94)	19.77 (2.67)	1.00 (0.38)	1.95 (0.48)
	BD	0.54 (0.21)	0.50 (0.65)	1.81 (0.96)	0.29 (0.15)	0.67 (0.26)
	BD × Condition	-0.13 (0.24)	-0.14 (0.75)	-0.71 (1.11)	-0.12 (0.18)	-0.32 (0.30)
	BD × Time	0.21 (0.19)	0.57 (0.75)	-0.21 (1.07)	0.35 (0.15)	-0.05 (0.19)
	BD × Time × Condition	-0.21 (0.22)	-0.45 (0.88)	1.61 (1.25)	-0.28 (0.18)	-0.01 (0.22)
Standardized Estimates for Fixed Effects with BD	BD	.145	.051	.101	.115	.173
	BD × Condition	-.029	-.012	-.034	-.042	-.070
	BD × Time	.039	.040	-.008	.096	-.008
	BD × Time × Condition	-.033	-.027	.053	-.067	-.002
Variances	Group-Level Intercept	6.92 (1.67)	45.83 (13.37)	137.33 (32.30)	4.84 (0.92)	14.50 (2.80)
	Group-Level Gains	0.88 (0.60)	15.17 (10.64)	3.85 (20.45)	0.00 (0.36)	0.52 (0.68)
	Member-Level Intercept	12.97 (1.57)	71.21 (14.72)	162.73 (32.91)	4.61 (0.78)	24.30 (2.41)
	Residual (Error)	12.42 (0.91)	196.29 (14.88)	416.89 (31.75)	8.80 (0.63)	12.73 (0.99)
Test of BD × Time × Condition	Model probability (*w*)	.36	.29	.45	.55	.27
	p value	.3511	.6124	.1984	.1152	.9550
	df	575	590	581	585	569

Note. Table entries show χ^2 values, *p* values, parameter estimates with standard errors in parentheses, model probabilities (*w*), and degrees of freedom (*df*). The Likelihood ratio test compared homoscedastic residuals to heteroscedastic residuals; all models supported homoscedastic residuals. Tests of fixed effects in accounted for the partially nested study design. BD = Wechsler Preschool and Primary Scale of Intelligence Block Design scores; RAENS = Roots Assessment of Early Numeracy Skills; OC = oral counting; ASPENS = Assessing Student Proficiency in Early Number Sense; NSB = Number Sense Brief; TEMA = Test of Early Mathematics Ability.

Table A4

Interactions between Fluid Reasoning Skills and Condition Differences

		RAENS	OC	ASPENS	NSB	TEMA
Heterogeneity of Variance Test	Likelihood Ratio χ^2	1.14	1.00	0.20	0.35	0.82
	p value	.5669	.6061	.9063	.8381	.6626
Fixed Effects	Intercept	11.01 (0.43)	18.20 (1.36)	18.47 (2.02)	11.93 (0.32)	16.14 (0.54)
	Time	6.57 (0.40)	21.77 (1.58)	41.91 (2.24)	5.96 (0.32)	7.16 (0.40)
	Condition	0.33 (0.55)	2.73 (1.72)	2.47 (2.57)	0.66 (0.41)	0.68 (0.71)
	Time × Condition	5.63 (0.49)	1.45 (1.93)	19.80 (2.69)	0.94 (0.39)	1.93 (0.48)
	MR	0.40 (0.17)	0.63 (0.53)	1.80 (0.80)	0.37 (0.12)	0.51 (0.21)
	MR × Condition	0.13 (0.20)	-0.08 (0.63)	-0.53 (0.94)	-0.07 (0.15)	0.03 (0.25)
	MR × Time	0.17 (0.16)	-0.27 (0.62)	0.48 (0.88)	0.25 (0.13)	-0.07 (0.16)
	MR × Time × Condition	-0.15 (0.19)	1.13 (0.74)	0.41 (1.05)	-0.20 (0.15)	0.15 (0.19)
Standardized Estimates for Fixed Effects with MR	MR	.124	.074	.115	.169	.150
	MR × Condition	.033	-.008	-.028	-.025	.009
	MR × Time	.037	-.022	.021	.079	-.015
	MR × Time × Condition	-.026	.076	.015	-.053	.025
Variances	Group-Level Intercept	6.59 (1.65)	43.04 (13.13)	124.01 (31.39)	4.45 (0.88)	14.17 (2.78)
	Group-Level Gains	0.86 (0.60)	14.38 (10.56)	8.08 (20.92)	0.02 (0.37)	0.47 (0.68)
	Member-Level Intercept	12.60 (1.56)	70.75 (14.67)	167.38 (32.94)	4.36 (0.76)	23.47 (2.37)
	Residual (Error)	12.44 (0.91)	195.62 (14.84)	412.46 (31.64)	8.79 (0.63)	12.78 (0.99)
Test of MR × Time × Condition	Model probability (*w*)	.33	.53	.28	.46	.33
	p value	.4285	.1279	.6974	.1873	.4325
	df	575	585	577	581	569

Note. Table entries show χ^2 values, *p* values, parameter estimates with standard errors in parentheses, model probabilities (*w*), and degrees of freedom (*df*). The Likelihood ratio test compared homoscedastic residuals to heteroscedastic residuals; all models supported homoscedastic residuals. Tests of fixed effects in accounted for the partially nested study design. MR = Wechsler Preschool and Primary Scale of Intelligence Matrix Reasoning scores; RAENS = Roots Assessment of Early Numeracy Skills; OC = oral counting; ASPENS = Assessing Student Proficiency in Early Number Sense; NSB = Number Sense Brief; TEMA = Test of Early Mathematics Ability.

Table A5

Matched-Sample Interactions between Phonological Memory Skills and Condition Differences

		RAENS	OC	ASPENS	NSB	TEMA
Heterogeneity of Variance Test	Likelihood Ratio χ^2	1.52	0.21	2.00	1.16	0.78
	p value	.4665	.8983	.3682	.5600	.6766
Fixed Effects	Intercept	10.97 (0.70)	16.97 (2.16)	17.27 (3.37)	11.41 (0.52)	15.86 (0.79)
	Time	7.07 (0.68)	20.22 (2.67)	44.57 (3.83)	6.04 (0.55)	7.07 (0.66)
	Condition	-0.38 (0.84)	1.67 (2.59)	1.53 (4.04)	0.68 (0.62)	0.33 (0.95)
	Time × Condition	5.40 (0.81)	3.32 (3.21)	17.42 (4.58)	0.80 (0.65)	1.31 (0.79)
	PMC	-0.23 (0.95)	1.26 (2.95)	1.25 (4.60)	1.26 (0.71)	0.12 (1.07)
	PMC × Condition	0.66 (1.13)	0.25 (3.50)	0.40 (5.48)	-0.92 (0.84)	-0.31 (1.27)
	PMC × Time	-0.33 (0.94)	3.87 (3.67)	-0.39 (5.26)	-0.20 (0.76)	-0.02 (0.92)
	PMC × Time × Condition	0.89 (1.13)	-3.22 (4.41)	0.31 (6.34)	0.68 (0.92)	1.82 (1.11)
Standardized Estimates for Fixed Effects with PMC	PMC	-.015	.031	.016	.121	.008
	PMC × Condition	.040	.006	.005	-.083	-.020
	PMC × Time	-.018	.081	-.004	-.016	-.001
	PMC × Time × Condition	.041	-.058	.003	.048	.088
Variances	Group-Level Intercept	2.97 (1.63)	21.39 (14.43)	68.65 (37.53)	3.29 (0.94)	6.28 (2.18)
	Group-Level Gains	1.14 (0.85)	24.05 (16.32)	27.88 (36.16)	-0.24 (0.58)	0.37 (1.05)
	Member-Level Intercept	11.87 (1.89)	43.57 (16.29)	169.94 (42.73)	3.56 (0.94)	16.97 (2.39)
	Residual (Error)	12.21 (1.17)	180.74 (19.10)	391.68 (43.52)	8.99 (0.88)	12.28 (1.35)
Test of PMC × Time × Condition	Model probability (*w*)	.33	.32	.26	.32	.58
	p value	.4348	.4660	.9605	.4552	.1005
	df	394	406	399	398	390

Note. Table entries show χ^2 values, *p* values, parameter estimates with standard errors in parentheses, model probabilities (*w*), and degrees of freedom (*df*). The Likelihood ratio test compared homoscedastic residuals to heteroscedastic residuals; all models supported homoscedastic residuals. Tests of fixed effects in accounted for the partially nested study design. PMC = Comprehensive Test of Phonological Processing Phonological Memory composite score; RAENS = Roots Assessment of Early Numeracy Skills; OC = oral counting; ASPENS = Assessing Student Proficiency in Early Number Sense; NSB = Number Sense Brief; TEMA = Test of Early Mathematics Ability.

Table A6

Matched-Sample Interactions between Visual-Spatial Reasoning Skills and Condition Differences

		RAENS	OC	ASPENS	NSB	TEMA
Heterogeneity of Variance Test	Likelihood Ratio χ^2	0.78	2.31	0.33	3.06	1.97
	p value	.6943	.3153	.8493	.2169	.3734
Fixed Effects	Intercept	10.27 (0.72)	18.38 (2.23)	17.30 (3.40)	11.84 (0.53)	15.48 (0.87)
	Time	6.08 (0.66)	19.02 (2.71)	42.07 (3.78)	5.35 (0.53)	7.07 (0.66)
	Condition	0.71 (0.86)	2.51 (2.67)	2.63 (4.07)	0.65 (0.63)	1.51 (1.04)
	Time × Condition	6.13 (0.78)	4.35 (3.20)	16.39 (4.46)	1.86 (0.62)	2.08 (0.77)
	BD	1.38 (0.98)	0.06 (3.06)	1.68 (4.65)	0.47 (0.72)	1.05 (1.19)
	BD × Condition	-0.67 (1.15)	-1.08 (3.62)	0.26 (5.47)	-0.17 (0.85)	-1.23 (1.39)
	BD × Time	0.62 (0.90)	2.93 (3.69)	-4.14 (5.16)	0.58 (0.72)	-0.02 (0.89)
	BD × Time × Condition	-0.41 (1.07)	-2.15 (4.40)	11.16 (6.14)	-1.02 (0.86)	0.22 (1.06)
Standardized Estimates for Fixed Effects with BD	BD	.085	.002	.021	.044	.063
	BD × Condition	-.039	-.024	.003	-.015	-.070
	BD × Time	.033	.059	-.045	.046	-.001
	BD × Time × Condition	-.019	-.037	.105	-.070	.010
Variances	Group-Level Intercept	6.42 (1.98)	33.88 (14.14)	125.93 (38.15)	3.88 (1.03)	12.31 (2.95)
	Group-Level Gains	0.58 (0.73)	11.57 (12.69)	9.95 (26.59)	-0.03 (0.45)	0.00 (.)
	Member-Level Intercept	13.25 (1.95)	63.35 (17.44)	176.81 (40.38)	5.53 (1.00)	22.99 (2.76)
	Residual (Error)	12.56 (1.10)	210.51 (18.43)	417.15 (38.03)	8.47 (0.73)	12.80 (0.87)
Test of BD × Time × Condition	Model probability (*w*)	.28	.29	.65	.42	.27
	p value	.7024	.6247	.0698	.2388	.8347
	df	443	454	450	448	441

Note. Table entries show χ^2 values, *p* values, parameter estimates with standard errors in parentheses, model probabilities (*w*), and degrees of freedom (*df*). The Likelihood ratio test compared homoscedastic residuals to heteroscedastic residuals; all models supported homoscedastic residuals. Tests of fixed effects in accounted for the partially nested study design. BD = Wechsler Preschool and Primary Scale of Intelligence Block Design scores; RAENS = Roots Assessment of Early Numeracy Skills; OC = oral counting; ASPENS = Assessing Student Proficiency in Early Number Sense; NSB = Number Sense Brief; TEMA = Test of Early Mathematics Ability.

Table A7

Matched-Sample Interactions between Fluid Reasoning Skills and Condition Differences

		RAENS	OC	ASPENS	NSB	TEMA
Heterogeneity of Variance Test	Likelihood Ratio χ^2	0.93	1.79	0.12	0.36	2.41
	p value	.6273	.4096	.9413	.8350	.3002
Fixed Effects	Intercept	10.08 (0.71)	15.77 (2.13)	14.64 (3.32)	11.03 (0.51)	14.79 (0.86)
	Time	6.23 (0.65)	23.70 (2.40)	42.36 (3.70)	5.28 (0.50)	7.43 (0.64)
	Condition	0.00 (0.89)	3.33 (2.63)	3.26 (4.12)	1.02 (0.63)	0.93 (1.07)
	Time × Condition	6.18 (0.81)	-2.86 (2.95)	15.59 (4.59)	1.41 (0.62)	1.40 (0.79)
	MR	0.75 (1.00)	2.73 (2.99)	3.55 (4.66)	0.91 (0.71)	0.58 (1.21)
	MR × Condition	-0.19 (1.22)	-3.12 (3.63)	-2.56 (5.65)	-0.76 (0.86)	-1.03 (1.46)
	MR × Time	-0.38 (0.92)	-4.76 (3.39)	-4.16 (5.22)	1.12 (0.71)	-0.58 (0.92)
	MR × Time × Condition	0.51 (1.13)	6.24 (4.16)	8.23 (6.38)	-1.05 (0.87)	1.11 (1.13)
Standardized Estimates for Fixed Effects with MR	MR	.046	.067	.046	.087	.035
	MR × Condition	-.011	-.072	-.031	-.068	-.059
	MR × Time	-.020	-.100	-.046	.092	-.030
	MR × Time × Condition	.023	.113	.079	-.074	.049
Variances	Group-Level Intercept	3.90 (2.25)	38.85 (18.10)	85.29 (43.10)	3.66 (1.08)	9.23 (3.33)
	Group-Level Gains	1.46 (0.95)	1.17 (13.43)	38.88 (32.08)	0.21 (0.48)	0.00 (.)
	Member-Level Intercept	16.20 (2.48)	74.93 (20.29)	201.22 (47.23)	5.26 (1.05)	26.23 (3.50)
	Residual (Error)	12.12 (1.22)	183.54 (18.82)	393.10 (40.69)	7.74 (0.73)	13.46 (0.98)
Test of MR × Time × Condition	Model probability (*w*)	.28	.52	.45	.43	.62
	p value	.6524	.1345	.1977	.2238	.3263
	df	387	394	390	391	384

Note. Table entries show χ^2 values, *p* values, parameter estimates with standard errors in parentheses, model probabilities (*w*), and degrees of freedom (*df*). The Likelihood ratio test compared homoscedastic residuals to heteroscedastic residuals; all models supported homoscedastic residuals. Tests of fixed effects in accounted for the partially nested study design. MR = Wechsler Preschool and Primary Scale of Intelligence Matrix Reasoning scores; RAENS = Roots Assessment of Early Numeracy Skills; OC = oral counting; ASPENS = Assessing Student Proficiency in Early Number Sense; NSB = Number Sense Brief; TEMA = Test of Early Mathematics Ability.