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Varieties of Numerical Estimation: A Unified Framework

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Abstract

There is an ongoing debate over the psychophysical functions that best fit human data from numerical estimation tasks. To test whether one psychophysical function could account for data across diverse tasks, we examined 40 kindergartners, 38 first graders, 40 second graders and 40 adults' estimates using two fully crossed 2 × 2 designs, crossing symbol (symbolic, non-symbolic) and boundedness (bounded, unbounded) on free number-line tasks (Experiment 1) and crossing the same factors on anchored number-line tasks (Experiment 2). This strategy yielded 4 novel tasks to assess the generalizability of the models. Across all 8 tasks, 90% of participants provided estimates better fit by a mixed log-linear model than other cognitive models, and the weight of the logarithmic component (λ) decreased with age. After controlling for age, the weight of the logarithmic component (λ) significantly predicted arithmetic skills, whereas parameters of other models failed to do so. Results suggest that the logarithmic-to-linear shift theory provides a unified account of numerical magnitude estimation and provides uniquely accurate predictions for mathematical proficiency.

Keywords: cognitive development; numerical cognition; number-line estimation; psychophysical function

Introduction

In this paper, we aimed to address an ongoing debate on the psychophysical functions that link numbers to their magnitude estimates and to provide a unified framework for understanding seemingly-conflicting data from a variety of studies (Barth & Paladino, 2011; Cohen & Sarnecka, 2014; Opfer, Thompson, & Kim, 2016; Siegler & Opfer, 2003; Slusser, Santiago, & Barth, 2013). Specifically, we sought to test whether models that fit data from old research methods could accurately predict data from new methods that differed in small increments that were thought to be psychologically meaningful. Finally, we aimed to test whether models that best accounted for numerical magnitude estimates also provided the best predictors of educational outcomes.

The classic theory about developmental change in numerical estimation is that the representation of numerical magnitudes follows a logarithmic-to-linear shift (Siegler & Opfer, 2003; Siegler, Thompson, & Opfer, 2009). This account was mostly based on a single version of the number-line task, which is the symbolic bounded free branch in the taxonomy in Figure 1.

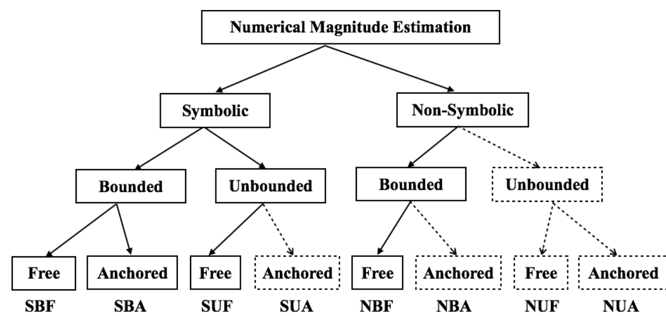

Figure 1: Taxonomy of number-line tasks. Branches connected by solid lines were examined in previous studies. Ones connected with dashed lines are new.

Two alternative accounts were recently proposed. One is the proportional-judgment account, claiming participants adopt proportion judgment strategies when estimating numerical magnitudes (Barth & Paladino, 2011; Slusser et al., 2013). The other is the measurement-skills account, claiming that data from number-line tasks arise from task-specific measurement skills (Cohen & Blanc-Goldhammer, 2011; Cohen & Sarnecka, 2014). Like the classic theory, these accounts also depended only on specific sets of number-line tasks (symbolic bounded anchored for proportional-judgment account; symbolic unbounded free for measurement-skills account (see Figure 1).

1. Symbolic vs. Non-symbolic

One potentially important variable is whether numerical magnitudes are presented symbolically or non-symbolically. Most studies have focused only on the symbolic magnitude estimates, though with different psychophysical functions being proposed (Barth & Paladino, 2011; Cohen & Sarnecka, 2014; Opfer et al., 2016; Siegler & Opfer, 2003; Slusser et al., 2013).

In contrast, when Dehaene et al. (2008) and Anobile et al. (2012) presented subjects with non-symbolic numeric magnitudes, they found that a mixed log-linear model (MLLM) provided a better fit to data than alternatives. Among these alternatives, however, the power models proposed by Slusser et al. (2013) and Cohen & Blanc-Goldhammer (2011) were not included.

The MLLM consists of a logarithmic and a linear component, and is defined by the following equation:

$$y = a\left((1-\lambda)x + \lambda \frac{U}{\ln(U)}\ln(x)\right),$$

where y represents an estimate of number x on a 0-U number line, a a scaling factor and λ a logarithmicity index that denotes the degree of logarithmic compression in estimates. The logarithmic component λ returns to a value between 0 and 1. If estimation is perfectly linear, λ converges to 0, whereas the value of the logarithmicity index gets close to 1 as estimation shows more logarithmic compression.

2. Bounded vs. Unbounded

Another potentially important feature of number-line estimation is whether an upper endpoint is provided (bounded) or not (unbounded). Cohen and his colleagues have suggested that extensions of cyclic models (CPMs) provide best fitting models for estimates in the bounded condition and that scallop power models (SPMs) provides best fitting for estimates in the unbounded condition (Cohen & Blanc-Goldhammer, 2011; Cohen & Sarnecka, 2014). Though they did not include the mixed log-linear model of Dehaene et al. (2008) among the alternatives tested, Kim and Opfer (in press) found the MLLM was a better predictor of estimates than CPMs and SPMs for symbolic bounded free and unbounded free number-line tasks.

In Kim and Opfer (in press)'s study, they modified a mixed cyclic power model (MCPM2) based on Cohen and Sarnecka (2014)'s extensions of cyclic models (CPMs) for the bounded task, which was given by the following equation:

$$y = w_1 \times SBCM + w_2 \times 1CPM + w_3 \times 2CPM,$$

where SBCM is a subtraction bias cyclic model, 1CPM and 2CPM are cycle power models with two and three reference points used. Each of w_1, w_2 and w_3 indicates a weight of each variant of the cyclic model respectively.

Also, based on Cohen and Blanc-Goldhammer (2011)'s study for unbounded tasks, they modified a mixed scallop model (MSPM), which was defined as:

$$y = w_1 \times 1SPM + w_2 \times 2SPM + w_3 \times MSPM,$$

where 1SPM denotes the single scallop model, 2SPM the dual scallop model, and MSPM the multiple scallop model.

3. Free vs. Anchored

A third potentially important variable is whether subjects are given the numeric magnitude of the half-way point on the number-line (anchored) or not (free). Slusser et al. (2013) showed 5- to 10-year-old children symbolic bounded anchored number lines and found children's estimates were better fit by one of three adapted cyclical power models (CPMs) (Hollands & Dyre, 2000) than a simple linear model. Subsequent studies, however, found the MLLM provided a better fit to both symbolic bounded free and symbolic founded anchored number-line estimates than mixtures of the CPMs (Opfer et al., 2016), which was called MCPM1 in Kim and Opfer (in press)'s study. The equation is as follows:

$$y = w_1 \times 0CPM + w_2 \times 1CPM + w_3 \times 2CPM,$$

where 0CPM, 1CPM and 2CPM indicate different variant of the cyclic power model, and w_1, w_2 and w_3 their weight.

The Current Study

In this study, we manipulated all three variables orthogonally to systematically test the mixed log-linear model against its competitors on 4 previously examined tasks and 4 novel tasks. Thus, we tested all the branches shown in Figure 1, with symbolic bounded free (SBF), symbolic unbounded free (SUF), non-symbolic bounded free (NBF), non-symbolic unbounded free (NUF) tasks in Experiment 1 and symbolic bounded anchored (SBA), symbolic unbounded anchored (SUA), non-symbolic bounded anchored (NBA), non-symbolic unbounded anchored (NUA) tasks in Experiment 2. At the end of Experiment 2, we also administrated a battery of math tests, including addition and subtraction, to each subject to determine which model parameters best predicted addition and subtraction proficiency. This issue has educational significance, but it also tests the key cognitive process claim of the measurement-skills account, viz. that unbounded number-line estimates are easier than the bounded ones because they require addition skills rather than subtraction skills.

Experiment 1: Free Numerical Estimation

Methods

Participants Participants were 40 kindergartners (M=5.98 years; 47.5% female), 38 first-graders (M=7.13 years; 50% female), 40 second-graders (M=8.09 years; 57.5% female) and 40 adults (M=20.1 years; 50% female).

Materials and procedure Participants were administered four different number-line tasks using a 2 (symbolic/non-symbolic) by 2 (bounded/unbounded) fully-crossed design Order of tasks was determined by a balanced Latin square.

In symbolic conditions, participants were presented with 20 number-lines, with a number on each endpoint of the line. The to-be-estimated numerals were evenly sampled from 0 to 30. On each trial, numbers were shown 2s followed by random-noise mask. In non-symbolic conditions, procedure was similar, except that endpoints of lines and to-be-estimated numbers were dot arrays. Sizes of dots were controlled on 50% of trials, while areas covered by dots were controlled on the other 50%.

In bounded conditions, endpoints of the line were 0 and 30 (symbolic condition) or 0 and 30 dots (non-symbolic condition). In the unbounded condition, endpoints were 0 and 1 (symbolic condition) or 0 and 1 dots (non-symbolic condition). The instructions for the unbounded condition were taken from Cohen and Sarnecka (2014).

Results

1. Logarithmic-to-linear-shift theory accurately predicted median estimates and individual differences. We first fit median estimates for all four number-line tasks and age groups using MLLM. Across all tasks and age groups (Figure 2), fit of MLLM was very high (R^2 =.93 ~ 1). Analyses of the weight of logarithmic component (λ) revealed that with age, estimates changed from logarithmic patterns to linear ones, with λ decreasing from kindergartners to adults across all tasks (Figure 2). As expected, λ in non-symbolic conditions was higher than in symbolic ones. Also, λ in unbounded conditions were higher than in bounded ones regardless of symbolic format, which argues against the view that "the unbounded task requires less mathematical sophistication than the bounded task does" (Cohen and Sarnecka, 2014). To test whether individual performance revealed the same pattern, we computed λ for individual participants' data and conducted a mixed ANOVA, with symbolic format and boundedness as within-participant factors and age group as a between-participant factor. Results showed a main effect of symbolic format, $F(1,154)=74.19$, $p<.001$, boundedness, $F(1,154)=86.32$, $p<.001$, and age group, $F(3,154)=39.08$, $p<.001$. An interaction between symbolic format and boundedness, $F(1,154)=4.17$, $p<.05$, indicated that the effect of symbols was greater for the bounded tasks.

To test whether logarithmicity of estimates represented a stable pattern of individual differences, we correlated individual participants' λ among all tasks. Results showed that individual participant's λ among all the four number line tasks positively correlated (with correlation coefficient .70 (p<.001) between SBF and SUF tasks; .45 (p<.001) between SBF and NBF tasks; .35 (p<.001) between SBF and NUF tasks; .49 (p<.001) between SUF and NBF tasks; .39 (p<.001) between SUF and NUF tasks; and .54 (p<.001) between NBF and NUF tasks).

2. Model comparison. We next compared the fit of MLLM to that of its competitors: MLLM vs MCPM1 and MCPM2 for the bounded conditions and MLLM vs MSPM for the unbounded ones. The proportion of individual children who were best fit by the mixed log-linear model (MLLM) using AICc was calculated.

As illustrated in Table 1, estimates of 68% to 100% of participants were best fit by mixed log-linear model (MLLM) among all four free number line tasks. In the bounded condition, none of the MCPMs was the best fitting model for the majority of any age or task combination. In unbounded condition, we replicated the findings from Kim and Opfer (in press), with MLLM providing a better fit for 100% of participants' estimates compared to MSPM.

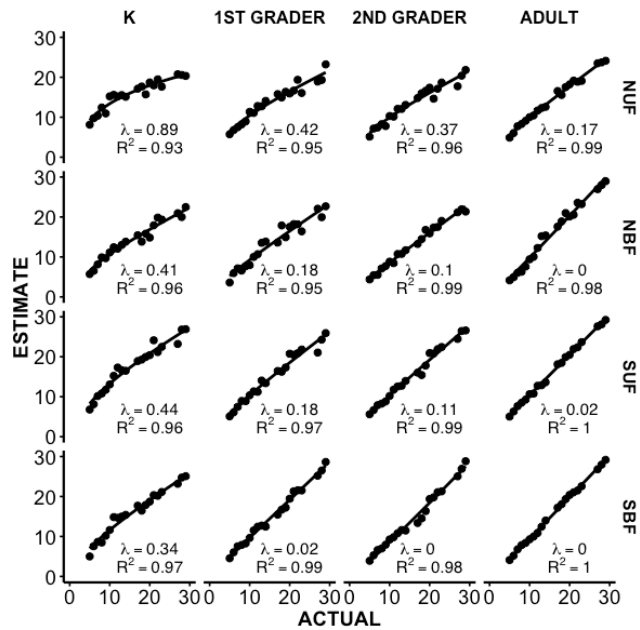

Figure 2: Median estimates on 0-30 free number lines for different age groups.

Table 1: Percent of participants best fit by MLLM for free number line tasks. K, kindergartners; 1, first graders; 2, second graders; A, adults.

	MLLM				
	K	1	2	A	All
SBF	95	89	83	68	84
NBF	95	95	93	90	93
SUF	100	100	100	100	100
NUF	100	100	100	100	100

Table 2: Partial correlation between λ in MLLM and math score after controlling for age across the free number line tasks.

			Partial correlation	
			Addition	Subtraction
MLLM				
	SBF	λ	-.40 ***	-.27 ***
	NBF	λ	-.27 ***	-.19 *
	SUF	λ	-.36 ***	-.24 **
	NUF	λ	-.17 *	-.17 *

Note. * p<.05, ** p<.01, *** p<.001

3. Predicting the mathematical performance. We next conducted partial correlation analysis between individual participant's addition and subtraction performance and the best-fitting parameter values from the models when controlling for age. The addition score was the sum score of simple and complex addition problems, and the subtraction score was the sum score of simple and complex subtraction problems.

As shown in Table 2, the logarithmicity parameter λ of the MLLM predicted both addition and subtraction performance across all tasks after controlling for age. In contrast, the correlations among the model parameters of the MLLM competitors were very small, inconsistent, and not expected by the theories that generated the models. Specifically, for bounded conditions, the negative correlation between the absolute value of βs-1 of the MCPMs and math performance was found in only a few of number line tasks, with the absolute value of β_{2CPM}-1 of MCPM1 negatively correlating with addition for the symbolic bounded free number line task (r=-.18, p<.05), the absolute value of β_{SBCM}-1 of MCPM2 negatively correlating with addition and subtraction for the non-symbolic bounded free task (r=-.17, p<.05 for addition; r=-.17, p<.05 for subtraction). Also, only the absolute value of s-1 in MCPM2 negatively correlating with subtraction was found in symbolic bounded free task (r=-.17, p<.05). For the unbounded condition, the negative correlation between the absolute value of β_{MSPM-1} in MSPM and addition was only found in symbolic unbounded free task (r=-.18, p<.05). These finding suggests that MLLM uniquely predicts math performance, regardless of tasks or age groups.

Experiment 2: Anchored Numerical Estimation

Methods

Participants Participants in Experiment 2 were the same as in Experiment 1.

Materials and procedure Participants received the same 2 (symbolic/non-symbolic) by 2 (bounded/unbounded) number line tasks as in Experiment 1, except that information was given about the location of 15 (or 15 dots) in each of the four tasks. Order of tasks followed a Latin square. After that, 200 arithmetic problems were presented for participants to solve as quickly as possible: simple addition, simple subtraction, complex addition and complex subtraction. For simple addition problems, each of the addends was a one-digit number and the sum was no more than 10 (e.g., 5+3, 2+1). For simple subtraction problems, the difference was less than 10 and both minuend and subtrahend were one-digit numbers (e.g., 9-3, 8-2). For complex addition problems, sums were bigger than 10 but less than 30, and addends were one- or two-digit numbers (e.g., 4+16, 14+15). For complex subtraction problems, differences were bigger than 10 but less than 30, with the minuend a two-digit number and the subtrahend one- or two-digit numbers (e.g., 16-5, 25-11).

Results

1. Logarithmic-to-linear-shift theory accurately predicted median estimates and individual differences. We first fit the median estimates for all four number line tasks and age groups using MLLM. As shown in Figure 3, across all tasks and age groups, the fit of MLLM was uniformly high (R^2 = .93 ~ 1). Analyses of λ revealed that with age, estimates changed from logarithmic patterns to linear ones, with λ decreasing from kindergartners to adults (Figure 3). As with Experiment 1, λ in non-symbolic conditions were higher than in symbolic ones, and λ in unbounded conditions were higher than in bounded conditions regardless of symbol. We also computed λ for individual participants' data. The mixed ANOVA results again showed a main effect of symbolic format, F(1, 154) = 83.17, p<.001, boundedness, F(1,154)=21.20, p<.001, and age group, F(3, 154) = 19.63, p<.001.

To test whether the logarithmic-to-linear-shift theory could also capture individual differences, we correlated individual participant's λ among tasks. The results showed that individual participant's λ among all the four number line tasks positively correlated (with correlation coefficient .81 (p<.001) between SBA and SUA tasks; .61 (p<.001) between SBA and NBA tasks; .48 (p<.001) between SBA and NUA tasks; .54 (p<.001) between SUA and NBA tasks; .43 (p<.001) between SUA and NUA tasks; and .61 (p<.001) between NBA and NUA tasks.

2. Model comparison. We next examined whether MLLM is the best model compared to other competitors. According to the previous studies (Cohen & Sarnecka, 2014; Opfer et al., 2016; Slusser et al., 2013), we compared the fit of MLLM, MCPM1, and MCPM2 on individual data for the bounded condition (which included SBA and NBA tasks). Since the unbounded anchored number-line tasks were new in this study, we compared the fit of all the four models for the unbounded condition (which included SUA and NUA tasks). The proportion of individual children who were best fit by the mixed log-linear model (MLLM) using AICc was calculated.

As illustrated in Table 3, the estimates of 63% to 100% of participants were best fit by mixed log-linear model (MLLM) among all four anchored tasks across all age groups. Specifically, in the bounded condition, no matter what types of symbol were given, against to the proportional account and subtraction or division-skill account, none of the MCPMs was the best fitting model for the majority. In the unbounded condition, our results showed that estimates of 78% to 98% of participants were best fitting by MLLM when compared the fitting of all the four models. All these results suggest the logarithmic-to-linear-shift account for all the anchored numerical magnitude representation, regardless of boundedness or symbolic format.

3. Predicting the mathematical performance. Similar with Experiment 1, we also conducted partial correlation analysis between individual participant's addition and subtraction performance and the best-fitting parameter values from the models when controlling for age. As shown

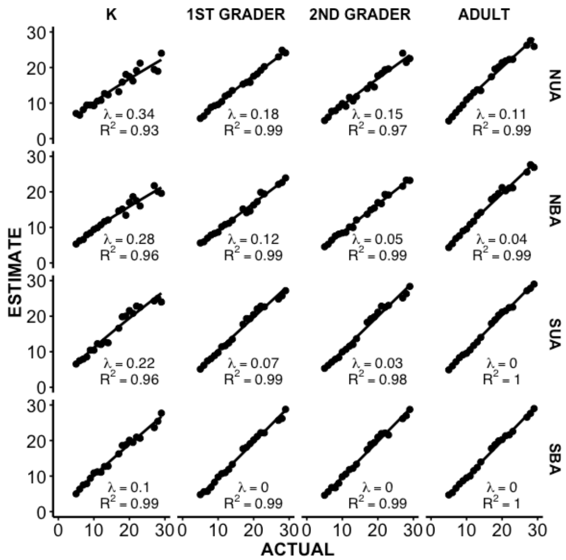

Figure 3: Median estimates on 0-30 anchored number lines for different age groups.

Table 3: Percent of participants best fit by MLLM for anchored number line tasks. K, kindergartners; 1, first graders; 2, second graders; A, adults.

	MLLM				
	K	1	2	A	All
SBA	85	63	63	68	70
NBA	93	97	100	93	96
SUA	90	92	78	78	84
NUA	98	97	95	93	96

Table 4: Partial correlation between λ in MLLM and math score after controlling for age across the anchored number line tasks.

			Partial correlation	
			Addition	Subtraction
MLLM				
	SBA	λ	-.29 ***	-.20 *
	NBA	λ	-.22 **	-.15
	SUA	λ	-.30 ***	-.22 **
	NUA	λ	-.24 **	-.20 *

Note. * p<.05, ** p<.01, *** p<.001

in Table 4, λ in the MLLM predicted both addition and subtraction performance across almost all the anchored number line tasks after controlling for age. However, for bounded condition, the negative correlation between the absolute value of βs-1 of the MCPMs and math performance was only found for the symbolic bounded anchored task, with absolute value of β_{2CPM}-1 in MCPM2 negatively correlating with addition and subtraction (r=-.26, p<.001 for addition; r=-.26, p<.001 for subtraction). The finding suggests that MLLM uniquely predict math performance, regardless of tasks or age groups.

Discussion

Our experiments indicate that the logarithmic-to-linear shift account provides a unified framework that can account for data coming from a broad array of numerical estimation tasks. Specifically, we found a mixed log-linear model provided the best fitting model for the vast majority (90%) of children and adults. This finding held regardless of whether the symbolic format was symbolic or non-symbolic, whether the task was bounded or unbounded, and whether an additional reference was given or not. These results replicate those reported in Opfer et al. (2016) and Kim and Opfer (in press), as well as extending them to 4 novel number-line tasks.

Our results also showed that the logarithmic weight (λ) was not fixed, but depended on the developmental history and prior experiences of the subject, leading to lower λ values from kindergartners to adults. These findings met the overarching principle of the logarithmic-to-linear shift theory, which holds that the representation of numerical magnitude will change from the logarithmic pattern to linear one with age and experience (Opfer et al., 2011; Opfer & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003; Thompson & Opfer, 2008).

Finally, individual differences were stable across the eight tasks: children whose estimates were more logarithmic in one task were also more logarithmic in the other seven tasks, r(156)=.35 ~ .81, p<.001. This would not be expected if the eight tasks elicited radically different estimation strategies, and it suggests that the logarithmic-to-linear theory provides an accurate picture for mental representation of all kinds of numerical estimations.

Implications for alternative accounts

Broadly, our results undercut key claims of the proportion-judgment and measurement-skills accounts. A key claim of the proportion-judgment account is that developmental change involves a change in the degree of bias and use of implicit reference points. In this view, the degree of bias (β) was thought to gradually converge on 1, and more reference points would be utilized by the participants, "from an unbounded power to a one-cycle proportional to a two-cycle proportional version of the model" (Slusser et al., 2013, p.5). If these views were correct, the weights for 0-cyclic power model (w_1) and 1-cyclic power model (w_2) in MCPM1 would be expected to decrease with age and the weight for 2-cyclic power model (w_3) would be expected to increase –at the very least among the bounded tasks in Experiment 1 and 2. However, we found no support for this developmental pattern among any of our eight tasks. Additionally, there was no stable pattern of individual differences in the degree of bias and use of reference points. Given the relatively poor fits of these models, this lack of predictive power might not be surprising, but it does warrant caution about the psychological meaning of the parameter values.

Our results also provide robust evidence against the measurements-skills account. First, according to Cohen and Sarnecka (2014), "the implicit addition needed for the unbounded task is less mathematically sophisticated than the implicit subtraction needed for the bounded task, [therefore] children should perform better on the unbounded task at a younger age" (p. 1643). Against this contention, we found greater accuracy for bounded than unbounded tasks regardless of age, symbolic format, or provision of anchors. Far from being easier, the unbounded tasks were more difficult and actually yielded the highest logarithmicity scores. Even more critically, the parameter values of the models associated with this account (subtraction and scallop bias) were thought to track general subtraction and addition skill. If so, one would expect them to predict subtraction and addition skill when subjects actually performed subtraction and addition. However, we found no evidence that this was the case. Again, given the relatively poor fits of these models, its lack of predictive power should not be surprising.

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