Building the Staircase to Algebra through Grade 5 Mathematics

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Abstract

This program focused on improving mathematics content and pedagogy for fifth-grade teachers, including regular classroom teachers and exceptional education teachers, as they implemented the Common Core State Standards (CCSS) for Mathematics. Concentrating work with a single grade allowed for tightening breadth while increasing depth of content. The major focus of fifth-grade content includes operations with fractions, two-digit divisor and decimal division, and understanding of volume. All activities were correlated to fifth-grade CCSS. The eight CCSS Mathematical Practices were woven throughout the activities.

The Tennessee Educator Acceleration Model was reviewed with teachers, and activities were presented and analyzed with regard to the rubric. The 19 TEAM categories meshed with the program objectives.

Thirty-one teachers from partner school districts participated. The program timeline included one spring Saturday session, a 5-day summer academy, and two follow-up Saturday sessions in the fall (50 contact hours), and online discussion throughout the spring, summer, and fall. Hands-on activities that utilized a variety of mathematics manipulatives and constructivist strategies for teaching and learning were emphasized. Participants demonstrated a significant increase in mathematics content knowledge.

The program was funded through the Tennessee Higher Education Commission (THEC) Improving Teaching Quality Program.

Introduction

At an Ayers Institute (2014) summer training for teacher educators, Assistant Commission of Education Emily Barton stated that Tennessee students were weakest in their "staircase to algebra." Though educators believe that knowledge of patterns leads to success with algebra, Barton stated that data and studies reviewed suggest that knowledge of fractional operations is a much stronger predictor of success with algebra, and a much weaker area than patterns, for students. She stated that, from assessment questions reviewed, the depth of understanding required can be seen, and that this mastery of fractions will prepare students for more abstract thinking [in algebra] (pers. comm., 05/20/14). She presented examples of weaknesses on this staircase: third-grade student understanding of multiple representations of fractions; and fourth- and fifth-grade student understanding of how to represent fractional situations with pictorial models, especially when multiplying two fractions (an abstract skill). In middle school, cross-multiple forms (e.g., table and graph) in proportions and ratios were difficult to understand. Barton further stated that these were "actionable things" that could be addressed to build a stronger foundation.

This program focused on improving mathematics content and pedagogy for 31 fifth-grade teachers, including regular classroom and exceptional education teachers, as they continued to implement the Common Core State Standards for Mathematics (CCSS Initiative, 2014). Concentrating work with a single grade allowed for tightening breadth while increasing depth of content. The major focus of fifth-grade content is operations with fractions (5.NF.1-7), two-digit divisor and decimal division (5.NBT.1-7), and understanding of volume (5.MD.3-5), with supporting content from data (5.MD.1-2) and additional content from algebraic thinking (5.OA.1-3) and geometry (5.G.1-4) (Tennessee Department of Education, n.d.). Activities were

correlated to, and directly addressed, all 26 of the fifth-grade CCSS. The eight CCSS Mathematical Practices were woven throughout the activities.

The Tennessee Educator Acceleration Model (TEAM, n.d.) was reviewed with teachers, and activities were presented and analyzed, with regard to the rubric. The 19 TEAM categories meshed with the program objectives. Barton stated that the four competencies with which teachers struggle most, and that need the most focused attention in implementing higher standards, were questioning, thinking, problem solving, and academic feedback (pers. comm., 05/20/14). Attention was focused on those four areas of TEAM, in relation to program content and pedagogy. In tying together CCSS and TEAM, Barton stated that teachers who teach the CCSS well fare better on the teacher evaluation, and that students who write, at least once per month, in a subject other than English will fare better on assessments (pers. comm., 05/20/14). There was an emphasis on cross-multiple forms.

Review of Literature

Concrete and Formal Operations

Working with middle school students, Booth and Newton (2012) suggested that fraction knowledge was related to algebra readiness and supported the CCSS by stating that knowledge of fractions should be developed through the use of number lines. A number line was used in several of the selected activities in this program. Bezuk and Cramer (1989, cited in Brown & Quinn, 2007), provided guidance for the teaching of fraction concepts, which Brown and Quinn stated was in much of the published literature. Included were (a) the use of concrete manipulatives as fundamental in developing student understanding; (b) developing a conceptual base of fraction relationships, prior to sixth grade; (c) delaying fractional operations until students understood order and equivalence of fractions; and (d) computational work that limited

the size of the denominator to 12 or less. Brown and Quinn noted that the students, 7 to 11 years of age, were in the concrete operational stage of Piaget's Theory of Cognitive Development. At this stage, adult-like logic appeared, but it was "limited to reasoning about concrete, real-life situations" (McDevitt & Ormrod, 2013, p. 199). Wu (2001, cited in Brown & Quinn), suggested teaching the process of abstraction as soon as possible, as the student, at 11 to 12 years of age, would be ready to move to the formal operational stage. Practice in abstraction, with fractions, may ease the passage to algebra. In the formal operations stage, "many capabilities essential for advanced reasoning in science and mathematics appear" (McDevitt & Ormrod, 2013, p. 199). This balance between the concrete and formal operational stages of cognitive development was addressed in this program when presenting and discussing activities.

Siegler et al. (2012) found that fifth-grade understanding of fractions and division predicted high school knowledge of algebra and overall math achievement. This demonstrated the immediate need to improve teaching and learning of fractions and division. Broad et al. (2006) found that the student gained a greater command of complex material, through inquiry-based teaching, as they answered some of the fundamental questions on their own. The instructor was a problem poser who guided the student to those answers without supplying answers to intermediate steps or biasing the direction that the student took in solving the problem. In this program, activities were standards-based, and promoted problem solving and inquiry.

Need for Professional Development

Gao and Lafortune (2019) detailed the adoption of the CCSS in California. Specific school districts were examined to see where they were in the process of implementing these standards. The major findings revealed that progress was uneven across the school districts and that the actual impact of the standards was difficult to gauge because of this uneven

implementation. The researchers recommended utilizing more comprehensive professional development sessions related to these standards to help resolve these issues.

Gwynne and Cowhy (2017) detailed the Chicago Public Schools' implementation of CCSS over several years. There were several major findings that were revealed during this process. One was that elementary teachers reported having participated in far more professional development sessions than high school teachers, and that the teachers who did participate in these professional development sessions reported feeling well prepared to teach according to these new standards.

Elementary schools with high amounts of professional development showed significant improvements in effective instruction following the introduction of new, Common Core Standards. These professional development experiences were focused specifically on explaining the standards in detail (Gwynne & Cashdollar, 2018).

Brown (2016) examined the level of preparedness teachers felt for implementing the recently-developed Common Core Standards after completing a professional development program focused on teaching the standards to them. Results demonstrated that the elementary school teachers surveyed felt well-prepared to teach these standards, but that further professional development was warranted.

Kober et al. (2013) sent a survey to many different state deputy superintendents to gather information on professional development and the implementation of Common Core Standards. In a majority of states surveyed, slightly more than half of the teachers had participated in this kind of professional development, though few states had provided training to a large number of teachers. The major challenges to providing this type of professional development were linked to a lack of funding and expertise.

Methods

In a July 2014 pilot study, a 5-day enrichment program for rising fifth-grade students served as a springboard to fifth-grade mathematics. This experience demanded that program faculty study the fifth-grade standards and plan hands-on activities to model fifth-grade mathematics. A small increase in student attitude toward mathematics was observed using Remmers' (1960) scale. Building upon work with students, this program provided professional development for fifth grade teachers at the interface of arithmetic and algebra.

Through an inquiry-based, concentrated study of mathematics concepts, this program focused on professional development as an agent to effect change in the fifth-grade classroom. The CCSS and Mathematical Practices were emphasized through activities drawn from several books published by the National Council of Teachers of Mathematics (NCTM), as well as other instructional materials. See Figure 1.

The selected texts provided needed work, especially through modeling with concrete objects and progressing through written notation and abstraction. Trends that emerged during the summer academy and through the NEON portal were revisited during the fall sessions. The National Governors Association (2008, abstract) found that teachers who attended all professional development sessions and implemented the project materials saw the greatest gain in student achievement. Program sessions and discussion allowed teachers the necessary time to study and implement new content and strategies.

The program included 50 contact hours for 31 teachers. Through teacher professional development, this program had the potential to increase student achievement in mathematics and decrease achievement gaps between subgroups in mathematics, as reported by the Tennessee Department of Education (2014). Further, *The Nation's Report Card* (National Center for

Education Statistics, n.d.) reported that Tennessee fourth-grade mathematics test results remained lower than the national average on the 2013 test. This program provided numerous pedagogical strategies to improve teaching for diverse groups of students through the hands-on activities with manipulatives and the introduction to long investigations. Another strategy, aimed at closing gaps, was the learning community (DuFour, DuFour, & Eaker, 2008). The NEON site was a learning community for the teacher to ask questions, comment on activities, and post a summary of the required presentation to colleagues. It was an important resource for all teachers.

Registration information was sent to each participating school district's curriculum coordinator for distribution to school principals. Partners were committed to both recruiting teachers who could effect positive change in student assessment outcomes and providing the opportunity for teachers to present an aspect of the program to colleagues as they returned for the academic year. Including teachers from regular and exceptional education addressed some of the mathematics proficiency gaps, preparing to boost mathematics proficiency on formal assessments for students from historically underperforming subgroups. Effort was made to recruit teachers from underrepresented groups. Effective recruitment was evidenced in six previous THEC programs in which 62 of 263 teachers (24%) were from underrepresented groups. The Chattanooga Chamber of Commerce (n.d.) reported area ethnicity data as 81% White and 19% from underrepresented groups. For this program, 4 of the 31 participants (13%) were teachers from underrepresented groups.

Mathematics received renewed attention through the CCSS. School districts focused attention on building the foundation of knowledge and skills required for students to advance in mathematics. Program activities emphasized multiple approaches to teaching and learning, were written to be used with students in the classroom, and provided implementation information. The

program timeline is presented in Figure 2.

Teachers focused on standards through manipulatives-based activity, collecting data, making connections across topics in mathematics, and communicating. Texts, Internet sites, and materials promoted important mathematics for all students and innovative practices. Constructivist strategies were emphasized, and included learning through posing problems, exploring possible answers, and focusing on global goals that specify general abilities such as problem solving, completing group work, and exploring open-ended questions (Roblyer, 2003). The summer academy daily schedule included activities in the morning, an introduction for the longer investigation to follow in the afternoon, and the formative assessment. Sample activities and resource references are presented in Figure 3. A list of classroom materials provided to participants is presented in Figure 4.

The program timeline included one spring Saturday session, a 5-day summer academy, two, follow-up, fall Saturday sessions (50 contact hours), and online discussion throughout the program. Hands-on activities, using a variety of mathematics manipulatives, and constructivist strategies for teaching and learning were emphasized. Emphasis was placed on presenting the abstractness of fractional operations with concrete materials. Relevant Internet resources were explored. An online group, through the NASA Educators Online Network (NEON, 2011), was established for communication. The goal was to provide high-quality, teacher professional development to Tennessee teachers to increase content knowledge and instructional skills aligned with the CCSS. Measurable objectives included the following:

1. There will be a statistically significant increase in teachers' scores on a 40-item mathematics quiz, between pre-test and post-test assessments.
2. There will be a statistically significant increase in teacher growth on observed

instances of teachers' problem-solving skills, as the program progresses (*Survey on Mathematics*, n.d.).

3. There will be a statistically significant improvement in teacher attitudes toward mathematics, as the program progresses (Remmers, 1960).
4. Teacher reporting of perception of student learning will reflect that 50% of students or more were proficient on all measures (University of Minnesota, Morris, 2000).

This partnership was designed to increase teacher content knowledge, as defined by the CCSS, and subsequent student mathematics achievement, as well as pedagogical change associated with the Mathematical Practices. According to 2014 TCAP data (Tennessee Department of Education, 2014), many school districts had made little or no progress, or had lost ground, as compared to the previous year's data for student growth in grades 3-8 mathematics. The percentage of students that were proficient or advanced in mathematics skills ranged from a little more than one-half down to one-third. Mathematics proficiency gap data for historically underperforming subgroups (Black/Hispanic/ Native American, Economically Disadvantaged, English Language Learners, Students with Disabilities), as compared to the comparison group, showed that each partnering school district included one to three subgroups for which the gap had increased since the previous year.

Results

Program evaluation was aligned with the measurable objectives presented in the above Methods section. In summary, data collection and analysis was concerned with change in mathematics content knowledge, problem-solving skills, attitude toward mathematics, and perception of student learning.

A 40-item, mathematics content pre-test/post-test was administered in April at the outset of the professional development sessions, in June at the close of the summer academy, and in September at the first session in the fall, allowing time for initial learning, classroom implementation, and reflection. Item sources included the Connecticut State Department of Education (2014), Engage NY (2014), the Louisiana Department of Education (2013), and the New York State Testing Program (n.d.). All items were aligned to grade 5 CCSS. Coefficient alpha was calculated as 0.795. Of the 31 teachers, 24 teachers completed the pre-test, the summer academy post-test, and the fall post-test. In a one-tailed *t*-test, a significant increase in learning was found between the summer academy and fall administrations of the post-test ($p<.05$).

For selected daily activities, observational data quantified teacher problem-solving skills through the *Survey on Mathematics* (n.d.). The survey served as a checklist of problem-solving behaviors, and instances of observed problem-solving behavior were tallied and compared over time. No significant difference in instances of observed problem solving was found from Monday to Friday of the summer academy.

The mathematics attitude survey was administered in April, June, and September to determine if an attitude change had occurred over the course of the program. Remmers' (1960) scale is interpreted through a score assigned to the item which is the median of the list to which the teacher agreed. No significant differences in attitude were found.

Prior to the October session, 24 teachers completed a survey regarding perception of student learning (University of Minnesota, Morris, 2000). Average teacher ratings of student learning ranged from 11% to 82%. Average question ratings ranged from 28.4% to 70.8%, with 70.8% being the only rating greater than 50%.

A daily evaluation was administered through a writing prompt and served both as a formative assessment of learning and a meeting of program objectives. The daily plan was modified as determined necessary through formative evaluation. Program effectiveness was assessed through a brief survey administered in October.

Discussion

With regard to the pre-test/post-test, the significant increase in mathematics content knowledge from the end of the summer academy to the fall administration could have been due, in part, to teachers working with students upon returning to school, with new teaching strategies, new content examples, and new materials to enhance content and pedagogy.

It was challenging to use a 25-item checklist to assess changes in problem-solving strategies among the participants. This was done through observation as activities occurred and did not provide much data. If used in the future, the items might be grouped so a smaller number of problem-solving behaviors would be tallied.

Attitude toward mathematics did not change. For those teachers who continued to teach mathematics, that could be viewed as a positive outcome. There was attrition during the two, fall Saturday sessions. Over the summer, several teaching assignments were changed. These were not planned, as of the spring enrollment, and were not known during the summer academy.

Teacher-reported perception of student learning was lower than anticipated. The only item in which average teacher response was above 50% was estimation of the percent of students who demonstrated basic, proficient, or advanced mathematics knowledge and skills. This was not the case for perception of modeling, problem formulation, problem-solving, interpretation, or communication skills. No data was collected on the level of the students (inclusion, English-language learners, etc.).

Overall, participants were highly satisfied with the professional development program. Participants expressed the need for continued sessions.

It was hoped that, upon program completion, teachers would have a sharpened focus toward identifying and implementing fifth-grade, standards-based activities and would have the demonstration materials and resources necessary for successful implementation. Teachers would understand the most critical areas of fifth-grade mathematics, be able to employ the targeted instructional practices of TEAM, and be able to address student needs through the integration of mathematics and other content areas.

The activity books and materials (manipulatives and children's books) were selected to have long-term use in the classroom. Activities were correlated to standards rather than to a particular publisher's curriculum. Activities and resources emphasized active learning, use of relevant materials, connections between mathematics and other content areas, deeper understanding of mathematical concepts, and improved problem solving. It was deemed important to develop meaningful learning environments for teachers as a model to use for creating such environments in their classrooms.

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Figure 1

A Summary of Program Texts and Internet Sites

- *Implementing the CCSSM through problem solving, grades 3-5* (NCTM, 2014). The text was used for discussion and activities at the April meeting. It contains rich problems that tie Common Core content to problem-solving skills.
- *Reasoning and sense-making problems and activities for grades 5-8* (NCTM, 2011) – The text contains problem-solving work with fractions and decimals.
- Books from the *Navigations* series (NCTM, 2001a, 2001b, 2007a, 2007b) - The *Navigations* activities fall under the general headings of number and operations (fractions, decimals, place value), problem solving and reasoning (extended investigations), algebra (equality), and geometry (volume).
- *The Super Source: Grades 5-6* (ETA hand2mind, 2007) contains activities for all topics, using seven manipulatives.
- Common Core materials are available on Internet sites such as Illustrative Mathematics (n.d.), and online manipulatives sites such as Illuminations (NCTM, n.d.), Interactivate (Shodor, 2014), and the National Library of Virtual Manipulatives (Utah State University, 2014).

Figure 2

Timeline to Outline the 50 contact Hours of the Program

Spring 2015

- January-March – Identify teachers, begin resource Web page. Order materials for April 11.
- April 11, Saturday session, 6.25 contact hours. Pre-test for content knowledge. Attitude survey. Formative evaluation. Begin standards-based activities, and online discussion through NEON.
- May – Order materials and prepare notebooks for the summer academy.

Summer 2015

- June 1-5 – 5-day summer academy, 31.25 contact hours. Activities and implementation strategies. Formative evaluation. Mid-point post-test for content knowledge. Attitude survey.
- June – December – Continue online discussion through NEON.

Fall 2015

- August – September – Each teacher will present a program aspect to colleagues.
- September 19 – Saturday session, 6.25 contact hours. Teachers report on information presented to colleagues. Continued work with standards-based activities and implementation strategies. Post-test for content knowledge. Attitude survey.
- October 24 – Saturday session, 6.25 contact hours. Continued work with standards-based activities. Focus on student assessment. Program evaluation. Collection of teacher perception survey.
- October to December – Classroom co-teaching and district professional development, as requested. Continue online discussion through NEON.

Figure 3

Sample Activities

- Operations and Algebraic Thinking - Going in Slow Motion. Time v. distance data is collected for two speeds of a toy car, then plotted on coordinate axes, and analyzed for rate changes and differences, using decimal division.
- Number and Operations in Base Ten – Modeling Multiplication. Base ten blocks are used to model whole number and decimal multiplication, and extended to division.
- Number and Operations – Fractions – Naming Rods. Cuisenaire rods are used to model equivalent fractions and addition of fractions.
- Number and Operations – Fractions – Dividing by One-Half. Paper half-circles are used to model division by a fraction.
- Measurement and Data – Exploring Packages. Volume and surface area are explored with nets and grid paper.
- Geometry – Roping in Quadrilaterals. Attributes of two-dimensional shapes are explored.
- Problem-solving investigation – Comparing Ourselves with Others. Scale and fractions are used to compare human height, speed, etc. with that of animals. The activity combines rich mathematics vocabulary with problem-solving and writing tasks.
- Activity sources include the following: Ellison (2014); ETA hand2mind (2007); Gleason (2003); Illustrative Mathematics (n.d.); National Air and Space Museum (1996); NCTM (2001a, 2001b, 2003, 2005a, 2005b, 2006, 2007a, 2007b, 2009, 2011); NYC Department of Education (2014); Rey (1976); Smarter Balanced Assessment Consortium (n.d.); The Math Forum (2014).

Figure 4

Classroom Materials Provided to Participants

- Books
  - National Council of Teachers of Mathematics. (2014). *Implementing the CCSSM through problem solving, grades 3-5*. NCTM
  - National Council of Teachers of Mathematics. (2007). *Navigating through problem solving and reasoning in grade 5.* NCTM.
  - National Council of Teachers of Mathematics. (2001). *Navigating through algebra in grades 3-5.* NCTM.
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  - National Council of Teachers of Mathematics. (2007). *Navigating through number and operations in grades 3-5.* NCTM.

- Children's literature
  - *A place for Zero*
  - *Biggest, strongest, fastest*
  - *What's faster than a speeding cheetah?*
  - *Jim and the beanstalk*
  - *The fly on the ceiling*
  - *Sir Cumference and the Viking's map*
  - *Sir Cumference and the roundabout battle*
  - *Sir Cumference and the off-the-charts dessert*
  - *Sir Cumference and all the king's tens*
  - *Multiplying menace: The revenge of Rumpelstiltskin*
  - *The multiplying menace divides*
  - *Pastry school in Paris: An adventure in capacity*
  - *Fractions, decimals, and percents*
  - *Working with fractions*
  - *Perimeter, area, and volume*
  - *The lion's share*
  - *Mummy math: An adventure in geometry*
  - *What's your angle, Pythagoras?*
  - *Pythagoras and the ratios*
  - *Equal shmequal*
  - *Sir Cumference and the first round table*
  - *Sir Cumference and the dragon of pi*
  - *Sir Cumference and the great knight of Angleland*
  - *Sir Cumference and the sword in the cone*
  - *Sir Cumference and the Isle of Immeter*

- Manipulatives for classroom demonstration
  - color tiles
  - base ten blocks
  - Cuisenaire rods
  - angle ruler
  - geoboard 5x5
  - geoboard 11x11
  - tangrams
  - snap cubes
  - centimeter cubes
  - pattern blocks
  - geosolids
  - battery-powered toy car
  - demonstration analog clock
  - dice
  - blank number cubes
  - stopwatch
  - measuring tape
  - spinner
  - fraction circles
  - measuring cups
  - graduated cylinders
  - Venn hoops
  - Polydron
  - hinged mirror