Issues and affordances in studying children's drawings with a mathematical eye

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In this third consecutive MERGA symposium focused on young children's drawings, three separate groups of researchers discuss the benefits and issues of using drawings as a source of data in their studies. Although drawings are ubiquitous in early years classrooms and in studies of children's learning, there is no comprehensive framework for analysing children's drawings in mathematical contexts. The overarching purpose of these symposiums has been to explore the qualitative methods that researchers have developed in their distinct projects and advance our critical perspectives on interpreting drawings and understanding the role they can play in children's learning of mathematics.

Broadly, the researchers view drawings as an external representation of mathematical concepts, mathematical thinking, or perceptions of mathematical contexts. Typically, researchers trust that children's drawings express to some extent the developing internal systems of the child, including the affective domain. In studying the interplay between children's internal and external representations, researchers must grapple with the ambiguities of interpreting representational drawing, as explained in quotation below.

> "Internal systems, … include students' personal symbolization constructs and assignments of meaning to mathematical notations, as well as their natural language, their visual imagery and spatial representation, their problem-solving strategies and heuristics, and (very important) their affect in relation to mathematics. The *interaction* between internal and external representation is fundamental to effective teaching and learning. Whatever meanings and interpretations the teacher may bring to an external representation, it is the nature of the student's developing internal representation that must remain of primary interest." (Goldin & Shteingold, 2001, p.2).

In this symposium, as well as sharing results from recent research, the authors reflect on some of the issues and affordances in studying children's drawings with a mathematical eye.

Goldin, G. & Shteingold, N. (2001). Systems of representation and the development of mathematical concepts. In Cuoco, A. (Ed.), *The roles of representations in school mathematics,* NCTM 2001 Yearbook, (pp.1-23). Reston VA: NCTM.

Chair & Discussant: Jennifer Way

Paper 1: Jill Cheeseman, Ann Downton, Anne Roche & Sarah Ferguson *Drawings reveal young students' multiplicative visualisation*

Paper 2: Katherin Cartwright, Janette Bobis & Jennifer Way *Investigating students' drawings as communication and representation modes of mathematical fluency.*

Paper 3: Kate Quane, Mohan Chinnappan & Sven Trenholm *Children's drawings as a source of data to examine attitudes towards mathematics: Methodological affordances and issues*

2021. In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & S. L. Chin (Eds.*), Excellence in Mathematics Education: Foundations and Pathways (Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia)*, pp. 109. Singapore: MERGA.

Drawings reveal young students’ multiplicative visualisation

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In the context of a multiplicative problem, our study investigated young children’s ability to visualise and draw equal groups. This paper reports the results obtained from 18 Australian children in their first year of school (age 5-6 years). The task *12 Little Ducks*, taught by their classroom teacher, provoked children to visualise and to draw different solutions. Fifteen children (83%) could identify and create equal groups via drawings; eight of these children (44%) could also quantify the number of groups that were formed. These findings show that some young children can visualise multiplicative situations and can communicate their reasoning of equal group situations through drawing.

The accepted wisdom of earlier research was that the intuitive pathway for children to multiplication is through repeated addition (Anghileri, 1989). Research reported by Sullivan et al. (2001) showed a relatively large cognitive step for children to move from using models with counting to abstract multiplication. These authors recommended that the teaching of multiplication require children of 5-8 years of age to imagine objects as well as model with objects.

The theoretical framework of this research is a social constructivist theory of learning which holds that meaning is created between individuals through their interactions (Ernest, 1991). The mathematical content was framed by the research literature related to problem solving with children, early multiplication and division, and children’s drawings. The ability to solve problems is a fundamental life skill and develops naturally through experiences, conversations and imagination (Cheeseman, 2018). The perceived importance of problem solving stimulates educators to look for authentic problem-solving situations in which children behave as mathematicians (Baroody, 2000). The task reported in this paper is one such non-routine mathematical problem.

Multiplicative thinking involves making two kinds of relations: the many-to-one correspondence between the three units of one and the one unit of three (Clark & Kamii 1996). Doing so requires an ability to form visual images of composite unit structures and is fundamental to multiplicative thinking (Sullivan et al., 2001). Young children are only able to abstract this notion of a composite unit when they have constructed meaning in their own minds (Bobis, 2008). In order to determine children’s meaning of groups, this study used children’s drawings as a research tool, and to potentially be a “window into the mind of a child” (Woleck, 2001, p. 215). Children were asked to draw a picture of what they were visualising and to describe their thinking as they solved the problem. Materials and modelling were used only when a child was unable to solve the problem (Sullivan et al., 2001). We conjectured that many children make mental images and visualise quantities when situations provoke them to do so. Our challenge was to create a context that would elicit children’s thinking, and to interpret and understand what children imagine. The research question we set out to answer was: *How do children’s drawings, explanations and actions reveal the ways they visualise group structures?*

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Method

A teaching experiment methodology was used to explore and explain students' mathematical actions and thoughts about recognising and making equal groups. As researchers we wanted to experience, first-hand, students' mathematical learning and reasoning (Steffe & Thompson, 2000). The study included the four basic elements of teaching experiment methodology. The "teaching episode" in this case, a sequence of five consecutive days of mathematics lessons in one school with a class of 5-6 year-olds in their first year of school. Three researchers witnessed the teaching and video-recorded each lesson.

The exploratory teaching was undertaken by Sarah (fourth author). While not privy to the team's design of learning contexts, she contributed to the theoretical framing of the study, and was conversant with the purpose of the research. Sarah was familiar with the Launch, Explore, and Summarise lesson structure (Lappan, & Phillips, 2009), and she believed that children should not be shown possible solution strategies before they attempt a task. The research team noted that the lesson content was beyond the intended curriculum and would present conceptual challenges for 5-6-year-olds, as would the exploratory teaching. Analysis of the children's mathematical thinking was based on their drawings, mathematical language and actions, and on the researchers' theoretical interpretation of events in accordance with a teaching experiment methodology. We closely observed children's interactions to infer their thinking about multiplication as seeing "groups of groups".

Participants were 21 children (13 girls and 8 boys) from a primary school in a large rural city of Victoria, Australia. The mean age was 5 years and 6 months. Sarah's class provided a convenience sample for investigating our research question. The results are from the 18 children who were present on the day. We devised lessons as contexts in which 5-6 year-old children could be stimulated to recognise and create equal groups and to quantify those groups. One lesson, Twelve Little Ducks, is the setting for the results presented here. Sarah was given a lesson outline and encouraged to implement the ideas in any way that she felt suited her children. The problem was originally written as: *Can you make 12 little ducks into equal groups? Can you do it a different way? Draw or write what you did.* To introduce the task to her children, Sarah told a story:

> In order not to lose any of her ducklings the mother duck put them into some groups that were the same. She put them into equal groups, because it was easy for her to see that she still had her 12 baby ducks. Can you make a picture in your head of those 12 little ducklings? The mother duck put them into groups with the same number of ducks in each group. I wonder what groups she put them into … I would like you to draw a picture of what is in your head (video transcript).

Sarah chose not to show a picture of ducks or to model the problem with materials, she explained that it might interfere with children's thinking. She was keen to learn what her children could imagine without objects - in a context her children would understand. Sarah was conscious of the challenge of the task's mathematical vocabulary as her diary showed:

> These children have not heard the term "equal groups" from me at school at all until today. I did say "the same number in each group" but I didn't go into great detail about what I meant by equal groups.

These pedagogical decisions deliberately created a challenging for 5-6 year-olds. Blocks were not provided initially but a child was offered blocks when it was apparent that s/he could not begin to solve the problem.

Data were collected from two fixed video cameras, three tablet cameras operated by the observer-researchers recording children working or in conversation with an adult. Subsequently, photographs of work in progress, children's finished work samples, classroom observations, and the video and photographic data were closely examined and interrogated. Data analysis began with each university researcher describing in detail what they observed soon after the lesson. In this way, we built a shared understanding of the events in the classroom. Each child's work sample was examined. Tentative categories of responses were proposed and iteratively tested to refine category definitions.

Findings

Analysis of the work samples together with our observations, conversations with the children, and video evidence revealed that three distinct categories of thinking could be described in terms of demonstrated multiplicative thinking.

Evident - *could simultaneously quantify objects in groups and enumerate the groups as new units*

Eight children (44%) produced 12 ducks by drawing and simultaneously creating equal groups. The ducks in their drawing were located in identifiable groups, indicating that they had perceived or imagined such groups before drawing the ducks. Elise drew two groups of six, circled each group and labelled her drawing, "2 groop 6" (*sic*) (Figure 1). She could make equal groups and quantify the groups. It appears Elise had determined the group size prior to drawing her solution because the ducks are drawn in equal rows.

Figure 1. Elise's first solution

Partial - *having some awareness of the quantity of each group but not the number of groups shown*

Figure 2. Georgie's first solution.

Six children (33%) were categorised as having "partial" understanding because they made equal groups but were not able to quantify the number of groups. Georgie drew three groups of four ducks (Fig 2), and when asked about her groups she said:

Georgie:	There are four here, and four there and four there. (Pointing to each group.)
Teacher:	How many groups of four have you got?
Georgie:	Twelve.
Teacher:	Twelve altogether. How many groups of four?
Georgie:	Um, I'm not sure yet.

Emergent - *unable to find a solution – even with 12 cubes to model the problem*

The four children (22%) who we described as emergent thinkers had several observed misunderstandings. For example, Conrad was unable to make six groups of two, from his drawing. It appears that Conrad did not have a solution in mind when drawing the 12 ducks as they were not drawn in identifiable clusters or rows. The random arrangement may have contributed to the difficulty of circling groups of two. Other emergent thinkers were unable to make equal groups in their drawings or when provided blocks to do so.

Figure 3. Conrad's drawing

To Conclude

We investigated whether children could visualise and construct equal groups and recognise the composite units they formed. Our research question was answered. Some children can imagine and draw equal group structures and in doing so recognize composite units. Some children can also enumerate the composite units. More than 80% of the children in the present study exhibited early multiplicative thinking. Children seemed to have intuitive understandings of equal group structures based on their experiences because they came to the problem we posed without any prior formal instruction about equal groups. This finding is novel - we have found no studies that have reported similar results with 5-6 year-old children.

Children communicated their visualisation of equal group situations through their drawings and elaborated their meaning with verbal descriptions and gestures. Such drawings of visualisations represent abstract thinking and call into question the accepted view of the way early multiplication typically develops via direct modelling to partial modelling, then to thinking abstractly (e.g., Anghileri, 1989).

We argue that it is productive to require young children to abstract problems earlier. Requiring visualisation together with drawings is an alternative approach to direct modelling. We acknowledge this is a small study and the results are only indicative of the ability of young children to visualise multiplicative situations. Further research might investigate other provocations that elicit children's thinking about multiplication. Children's drawings of their mathematical reasoning are fascinating and the intuitive understandings that young children develop about aspects of multiplication are worthy of detailed examination.

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