Singapore Enactment Project

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The *Enactment Project* is a Programmatic Research Project funded by the Ministry of Education, Singapore, and administered through the Office of Educational Research, National Institute of Education, Nanyang Technological University. The project began in 2016 and its aim is to study the enactment of the Singapore mathematics curriculum across the whole spectrum of secondary schools within the jurisdiction. There were two phases in the project: the first involved in-depth examination of 30 experienced and competent mathematics to draw out characteristics of their practices; in the second phase, we study the extent of these characteristics through a survey of 677 mathematics teachers. A symposium was organised in MERGA 42 in 2019 where the foundational elements of this project were presented; we would like to share more findings of this project in this year's conference.

Paper 1: Berinderjeet Kaur *Models of mathematics teaching practice in Singapore secondary schools*

This paper revisits the models of mathematics teaching practice that were proposed by earlier researchers of the Singapore mathematics classrooms: Traditional Instruction (TI), Direct Instruction (DI), and Teaching for Understanding (TfU). The data from the survey in this project point to hybridisation of these models.

Paper 2: Tin Lam Toh *An experienced and competent teacher's instructional practice for normal technical students: A case study*

This paper presents a case of how an experienced and competent teacher engaged mathematics "low-attainers" in the learning of mathematics in a way that was responsive to their learning needs while upholding the ambitious goal of helping them acquire relational understanding of mathematical concepts.

Paper 3: Joseph Boon Wooi Yeo *Imbuement of desired attitudes by experienced and competent Singapore secondary mathematics teachers*

One of the components of the Singapore Pentagonal curricular framework is "Attitude". This paper presents findings of a survey that point to specific strategies used by Singapore mathematics teacher to imbue positive attitude towards mathematics in their students.

2021. In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & S. L. Chin (Eds.), *Excellence in Mathematics Education: Foundations and Pathways (Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia)*, pp. 65-66. Singapore: MERGA.

Paper 4: Yew Hoong Leong & Lu Pien Cheng *Singapore mathematics teachers' design of instructional materials*

Case studies based on the data in Phase 1 of the project revealed that the teachers crafted their own instructional materials based on modifications of reference materials. This paper summarises some of the moves teachers adopted when designing instructional materials for their lessons.

Singapore mathematics teachers’ design of instructional materials

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This paper reports on one aspect of a bigger project: teachers’ design of instructional materials. We found a number of design moves used by the teachers in our study. In this paper, we report three of them: Making things explicit, making connections, and re-sequencing practice examples.

This paper focuses on one major component of the project which examined the enactment of the Singapore mathematics curriculum in the Secondary Schools: the design and use of instructional materials by the teachers. We define instructional materials to be classroom-ready materials that teachers incorporate into their lessons for students’ direct access for their learning. We make a distinction between *instructional materials (IM)* and *reference materials (RM)*. The latter are resources (including textbooks) which teachers refer to while planning for lessons; the former are the actual materials that are brought into their classrooms for use in their mathematics instruction. For most teachers which were the subjects of our study, their instructional materials differ substantially from their reference materials – it is this ‘transformational space’ that is an area of interest to us. For the rest of this paper, we will briefly describe a few such transformational moves as illustrated by some teachers in our study and their underlying intentions.

Transform Move 1: Making things explicit

The fuller version in the examination of this move is in Leong et al. (2019). We provide a brief description here. This move is illustrated by Teacher Teck Kim. Repeatedly, in the interviews with him, he mentioned “making explicit” as a major goal in the design of instructional materials. That is, in selecting and modifying from RM (mainly the textbook subscribed by the school), he considered some of the contents as displayed in the textbook not sufficiently clear to the students; in crafting the IM, he was thus consciously governed by the principle of making the mathematical content more explicit to the students.

Figure 1 shows an example of such an explication deliberated by Teacher Teck Kim. He made the following adaptations (among others): (i) ln the RM, the textual explanation of column vectors was located at a section that was separate from the vector diagram. in Teck Kim’s IM, he merged the textual mode into the visual representation of column vectors. Not only was the label of $\binom{-3}{4}$ placed beside the drawn vector, the explanation of translation of “-3” and “4” was also summarily fused into the diagram. This merging of representational modes was the way in which Teck Kim made explicit—in this case the links among the drawn vector, the column vector notation, and the translational significance. (ii) The two examples in the RM were $\binom{2}{3}$ and $\binom{-1}{-4}$. The two examples in Teck Kim’s notes were $\binom{-3}{4}$ and $\binom{-3}{-4}$[the latter is not shown in Figure 1 due to space constraints]. Apart from the fact that the magnitudes of these vectors yielded an integer value, not a surd, and thus potentially reduce computational complexity so that the focus was on the definition and method of obtaining the magnitude, the choice of $\binom{-3}{4}$ and $\binom{-3}{-4}$ shows a one-component variation only in the translation in the y-direction, allowing the teacher to focus students’ attention on the

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translational significance when “4” is replaced with “-4”, thus highlighting the need to attend carefully to signs. In other words, Teck Kim re-worked the examples to make explicit critical ideas (perhaps, even potential student mistakes) which may have otherwise been unnoticed by the students. (iii) [Not shown in Figure 1] The task implicit in the RM required students’ to “write” the given drawn vector in column vector notation; the task in Teck Kim’s IM [not shown in Figure 1] required students to do the reverse: to “draw” vector given its column vector notation. He made explicit by filling a gap in the textbook. In this case, the gap was the skill of drawing vectors.

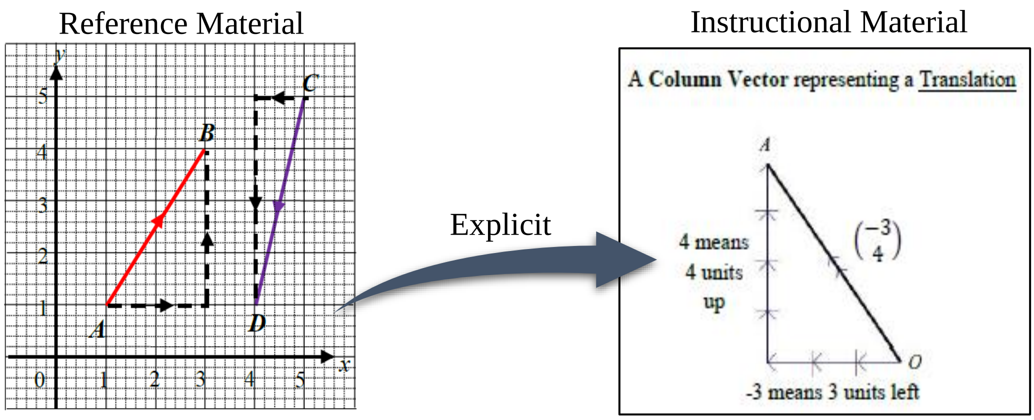

Figure 1. Making explicit from reference materials to instructional materials

Transform Move 2: Making connections

We illustrate this move by drawing upon the IM of Teacher Siew Ong. The phrase “making connections” – and similar phrases – occur frequently in her talk during our interview sessions with her. This move is particularly significant as connection-making in instructional work is highlighted as desirable in Singapore’s official documents: “connections refer to the ability to see and make linkages among mathematical ideas …” (Ministry of Education, 2012, p. 15, emphasis added).

The context was the method of “completing the square”. The RM presents an “investigative task” consisting of a table with four columns entitled (from left to right): “Quadratic Expression”, “Number that must be added to complete the square”, “Half the coefficient of x”, “Quadratic expression in the form $(x+a)^2 - b$”. An example as a row entry was then given for “$x^2 + 2x$” in the first column, “$1^2 = 1$” in the second column, “$\frac{2}{2} = 1$” in the third column, and the algebraic working to obtain $(x+1)^2 - 1$ in the last column. Other blank rows were given in the table below this first entry to provide working space for other samples of algebraic expressions of the form $x^2 + px$.

The IM designed by Teacher Siew Ong was an adaptation of the RM. She retained the four columns and kept largely to the titles of the first and the fourth columns (the ‘beginning form’ and the ‘targeted complete square form’). She renamed the middle two columns as “Geometric representation” (second column) and “Term to be added” (third column). Figure 2 shows how the entry in the second column looks like for the same example of $x^2 + 2x$.

Different from the RM, she intended to help students connect “square” in “completing the square” to a “geometric square”. There is thus a deliberate design decision to draw students’ attention to intermodal links – between the algebraic mode and the geometric mode of representation. The geometric square provided a more natural motivation and hint as to what value need “to be added” (language of Column 3) within the perforated small square to “complete the (geometric) square”. This shift of focus rendered the step in Column 3 of the RM (“half the coefficient …”) unnecessary as it would have become more intuitive from the geometric mode of representation within the context of forming a geometric square. [As an aside, the algebraic working in Column 4 now takes on a different function: it is not merely an algebraic procedure to complete; it is a static record (algebraically) of what happens dynamically over the entries in the last three columns. This further strengthens the algebraic-geometric connection].

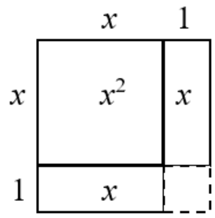

Figure 2. Geometric representation of $x^2 + 2x$ to set up for completing the square

In addition, to set up this way of thinking by students, that is, to view a quadratic expression as ‘almost a square’, she designed a prior page (not found in RM) where numbers (more accessible to students initially than algebraic expressions) were also represented geometrically as almost a square. As an example, 120 where written as $121 – 1 = 11^2 – 1$. This was also represented geometrically as a square of side 11 with a tiny square of 1^2 at the corner snipped off. This additional preamble that she designed revealed her deliberate effort at connection in at least these ways: (i) intermodal connections not only between algebraic and geometric representations, but also numerical to algebraic and geometric; (ii) conceptual connections – she recognised that students had prior familiarity with numerical perfect squares such as $121 = 11^2$. She drew from this prior conception to connect it to their other prior familiar imagery of geometric squares. These were then linked and further developed into ‘almost square’ in anticipation of connecting to the method of completing the square. In other words, she connected concepts by developing tightly from earlier concepts.

Transform Move 3: Re-sequencing practice examples

The details for this move can be found in Leong et al. (In press). As in the first move, we provide here a brief description. The teacher we studied for this move was Teacher Beng Choon. She designed the IM for the purpose of helping students gain proficiency with some ‘rules’ within the topic of differentiation. For the purpose of this paper, we restrict our consideration to the ‘formula’ of $\frac{d}{dx}(x^n) = nx^{n-1}$.

In her case, we were unsure as to the specific RM she relied upon most. Being an experienced teacher for many years, she could not specify a particular textbook she adapted from as her IM had evolved throughout the years over many rounds. For the purpose of this discussion, we referred to one common textbook to serve as a comparison to the examples she sequenced for this same section immediately after the introduction of the formula. The textbook provided three examples for application of this formula in this order: $\frac{1}{x^2}$, $\sqrt{x}$, and 1. The examples that appeared in Beng Choon’s IM were: x^3, 5, $\frac{1}{x}$, and $\sqrt{x}$. Figure 3 provides a summary of what she wrote on the board for each item and how she explained the procedure to obtain the final answer. Her main goal was to help students recognise the form x^n so that they can apply the formula correctly. As such, she needed to *vary the form* – so that they can 'see' how surface forms that do not initially look like x^n can be re-written in such a form for correct use of the formula. At the same time, she was cognizant that students did not get discouraged by difficulties and so she *proceeded gradually from simpler cases* of the form. A brief chronology: She started with x^3 as it is most recognisable as x^n. The switch to "5" was deliberate as she wanted to draw students away from fixation of formula-application; rather, they can think graphically and connect to differentiation as "finding gradient". The third and the fourth items show progressive complexity in recognising and rewriting into the form.

(a) $x^3 \xrightarrow[\text{formula}]{\text{apply}} 3x^2$

(b) $5 \xrightarrow{\text{gesture}} \text{hor. line} \xrightarrow[\text{gradient}]{\text{find}} 0$

(c) $\frac{1}{x} \xrightarrow{\text{rewrite}} x^{-1} \xrightarrow[\text{formula}]{\text{apply}} -x^{-2} \xrightarrow{\text{rewrite}} -\frac{1}{x^2}$

(d) $\sqrt{x} \xrightarrow{\text{rewrite}} x^{\frac{1}{2}} \xrightarrow[\text{formula}]{\text{apply}} \frac{1}{2}x^{-\frac{1}{2}} \xrightarrow{\text{rewrite}} \frac{1}{2\sqrt{x}}$

Figure 3. Summary of the procedures explained for each item by Teacher Beng Choon

Discussion

Clearly, these moves as described are not exhaustive nor are they unrelated. A cursory reflection would reveal that a teacher who wishes to adopt such moves may do so in an integrated way for the same activity – that is, making things explicit, making connections, and re-sequencing of practice examples can be applied concurrently. The purpose, however, of this article is to illustrate examples of each of these moves as they were adopted by the teachers in our study. This paper highlights that Singapore secondary mathematics teachers do not merely 'teach from the textbook'; rather, they make intentional moves to adapt the reference materials in ways that fit their instructional purposes which are largely 'sound' both from a theoretical perspective and in terms of concurrence to policy mandates. Often, these moves are elusive to a casual observer. The results of this study reminds us as researchers that we should avoid the simple route of pigeonholing pedagogical enactments based on cursory observations.

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