Journal for Research in Mathematics Education

The Journal for Research in Mathematics Education is an official journal of the National Council of Teachers of Mathematics (NCTM). *JRME* is the premier research journal in mathematics education and is devoted to the interests of teachers and researchers at all levels—preschool through college.

ARTICLE TITLE:

What Early Algebra Knowledge Persists 1 Year After an Elementary Grades Intervention?

AUTHOR NAMES:

Ana Stephens, Rena Stroud, Susanne Strachota, Despina Stylianou, Maria Blanton, Eric Knuth, and Angela Gardiner

DIGITAL OBJECT IDENTIFIER:	VOLUME:	ISSUE NUMBER:
10.5951/jresematheduc-2020-0304	52	3

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CONTACT: jrme@nctm.org


JULY 2019 NUMBER 4 VOLUME 50

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Journal for Research in Mathematics Education
2021, Vol. 52, No. 3, 332–348

What Early Algebra Knowledge Persists 1 Year After an Elementary Grades Intervention?

Ana Stephens
University of Wisconsin–Madison

Rena Stroud
Merrimack College

Susanne Strachota
Ohio University

Despina Stylianou
City University of New York

Maria Blanton
TERC

Eric Knuth
University of Texas at Austin

Angela Gardiner
TERC

This research focuses on the retention of students' algebraic understandings 1 year following a 3-year early algebra intervention. Participants included 1,455 Grade 6 students who had taken part in a cluster randomized trial in Grades 3–5. The results show that, as was the case at the end of Grades 3, 4, and 5, treatment students significantly outperformed control students at the end of Grade 6 on a written assessment of algebraic understanding. However, treatment students experienced a significant decline and control students a significant increase in performance relative to their respective performance at the end of Grade 5. An item-by-item analysis performed within condition revealed the areas in which students in the two groups experienced a change in performance.

Keywords: Algebraic thinking; Early algebra; Elementary grades; Retention study

Follow-up studies of the impact of interventions in mathematics education are rare. In fact, they are almost nonexistent (for exceptions, see studies on preschoolers by Clements et al., 2013, and on undergraduates by Kwon et al., 2005). Typical pretest-intervention-posttest studies measure students' knowledge at the conclusion of an intervention but do not follow students beyond this time period

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The research reported here was supported in part by the U.S. Department of Education under IES Award R305A140092. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the U.S. Department of Education. This article was accepted under the editorship of Jinfa Cai.

to investigate long-term impact (Bailey et al., 2017). In this brief report, we share results from a retention study that took place 1 year after a Grades 3–5 early algebra intervention and use this to discuss the importance of retention studies in mathematics education more broadly. We also discuss what our results may suggest about the treatment of algebra in the elementary and middle grades.

Early Algebra and the Persistence of Knowledge Growth

Mathematics education scholars have advocated for some time that students be provided long-term experiences, beginning in the elementary grades, that can support the development of their algebraic thinking and build their algebra readiness for the middle grades and beyond. A broad aim of our work has been to develop and test an early algebra instructional intervention to produce research-vetted curricular progressions and instructional materials with which we can better understand early algebra's impact on students' algebraic thinking. This work has involved small-scale, cross-sectional, and longitudinal studies (Blanton, Isler-Baykal, et al., 2019; Blanton, Stephens, et al., 2015) and, more recently, a large-scale, longitudinal, cluster randomized trial (Blanton, Stroud, et al., 2019). In a process fully described in Fonger et al. (2018) and Blanton et al. (2018), this earlier work involved building a curricular framework and progression, instructional intervention, and associated assessments around the algebraic thinking practices of generalizing, representing, justifying, and reasoning with mathematical structure and relationships (see also Blanton et al., 2011). We also characterized three "Big Ideas" (Shin et al., 2009) in which these practices can occur: generalized arithmetic; equivalence, expressions, equations, and inequalities; and functional thinking.

We define generalized arithmetic as "generalizing arithmetic relationships, including fundamental properties of number and operation (e.g., the Commutative Property of Addition), and reasoning about the structure of arithmetic expressions rather than their computational value" (Blanton, Stephens, et al., 2015, p. 43). Students engaging in generalized arithmetic look across multiple computations; notice underlying structures; and represent, justify, and reason with these structures.

According to Blanton, Stephens, et al. (2015),

> The big idea of *equivalence, expressions, equations, and inequalities* includes developing a relational understanding of the equal sign, representing and reasoning with expressions and equations in their symbolic form, and describing relationships between and among generalized quantities that may or may not be equivalent. (p. 43)

The fact that many elementary and middle school students hold the misconception that the equals sign indicates the need to produce an answer is well documented (e.g., Carpenter et al., 2003; Knuth et al., 2005; Molina & Ambrose, 2008). Student work around this big idea starts with developing an appropriate conception of this symbol that can then support more complex work with equations.

Finally, "*functional thinking* involves generalizing relationships between covarying quantities and representing and reasoning with those relationships through natural language, algebraic (symbolic) notation, tables, and graphs" (Blanton, Stephens, et al., 2015, p. 43). Research on functional thinking in the elementary and middle grades has explored issues such as how students understand linear relationships (e.g., Carraher, Martinez, & Schliemann, 2008), their covariational reasoning about relationships (e.g., Ellis, 2007), the role of figural patterns (e.g., Rivera & Becker, 2011), and progressions in how young children come to generalize and represent relationships between two quantities (e.g., Blanton, Brizuela, et al., 2015; Carraher, Martinez, & Schliemann, 2008). These choices of algebraic thinking practices and big ideas were informed by Kaput's (2008) content analysis of algebra and by the areas around which much of the early algebra literature had coalesced.

The retention study that is the subject of this article took place after our large-scale longitudinal study (Blanton, Stroud, et al., 2019). Given the rarity of retention studies in mathematics education, we borrow from the educational effectiveness literature in adopting the constructs of *persistence* and *fadeout* (Bailey et al., 2017) to help frame our work. In addition to comparing treatment and control student performance 1 year after the conclusion of the intervention, we were interested in examining what algebraic understandings around the big ideas and thinking practices we identified may persist and what knowledge may fade.

The impacts of educational interventions too often dissipate soon after the interventions are complete. Bailey et al. (2017) argue that interventions that sustain persistently beneficial impacts tend to target skills[1] that are malleable, fundamental, and would not eventually develop in the absence of the intervention. Malleable skills are ones that can be affected by an intervention, such as mathematics or literacy skills, SAT test preparation, and academic motivation. Bailey et al. define fundamental skills as "those upon which later skills are built, and that influence positive life outcomes" (p. 13) such as mathematics or literacy skills, social skills, or general intelligence.

Another process that Bailey et al. (2017) describe that is relevant for retention studies such as ours to consider is that of sustaining environments. They argue that high-quality postintervention environments are critical for sustaining earlier gains in skills. Indeed, Clements et al. (2013) found some support for this argument in a multiyear follow-up study of a preschool mathematics intervention. Students who participated in a program that connected the ideas of the intervention to the preceding mathematics content were significantly more successful than those who participated only in the intervention. In a review of retention studies across a variety of school subject areas, Semb and Ellis (1994) likewise found that continued practice, relearning, advanced training, or continued exposure to the content during the retention interval facilitated retention. We return to consider the nature of skills that persist and the construct of sustaining environments in the discussion of our results.

Previous Research: The Longitudinal Study

To provide background context, we briefly describe the longitudinal study of the Grades 3–5 intervention that preceded the retention study (see Blanton, Stroud,

[1] Bailey et al. (2017) define *skill* "broadly to encompass any skill, behavior, capacity or psychological resource that helps individuals attain successful outcomes" (p. 8).

et al., 2019, for a full description). The study used a cluster randomized trial design and included approximately 3,000 students from 46 elementary schools across three school districts, with half of the schools randomly assigned to each condition, treatment or control. Students in the treatment schools were taught our early algebra intervention, which consisted of eighteen 1-hr lessons per year, in Grades 3–5 during their regular mathematics instructional time by their classroom teachers. The lessons were designed to engage students in the aforementioned big algebraic ideas and thinking practices through problem-solving tasks involving work in small groups and whole-group discussion. These tasks were often adapted from those used successfully in previous research (e.g., Brizuela & Earnest, 2008; Carpenter et al., 2003; Carraher, Schliemann, & Schwartz, 2008) and generally allowed for multiple points of entry, so that students at varying levels of understanding could engage with some aspect of the tasks. Teachers took part in monthly professional development that helped prepare them to teach the intervention.

Students completed written assessments prior to the start of the intervention in Grade 3 and at the end of Grades 3, 4, and 5. As described in Blanton, Stephens, et al. (2015), assessments were designed to measure understanding of the big algebraic ideas and algebraic thinking practices that formed the basis of the intervention. Assessment items were largely based on items that had performed well in previous research (e.g., Carpenter et al., 2003; Carraher, Schliemann, & Schwartz, 2008; Knuth et al., 2008). The majority of the assessment items were open-response questions that asked students to show their thinking. Responses to assessment items were scored for correctness as well as strategy use. We found no statistically significant differences with respect to correctness or strategy use between groups prior to the intervention, but treatment students demonstrated statistically significant gains on both variables relative to their control-group peers at each subsequent grade level.

Given the promising results of our Grades 3–5 study, we were interested in assessing these same students 1 year later to measure the intervention's longer term impact. Our research questions were the following:

1. How does the performance of students who took part in a Grades 3–5 early algebra intervention as part of their regular instruction compare with that of students who experienced only their regular Grades 3–5 mathematics curriculum 1 year after the conclusion of the intervention?
2. What was the nature of gains and losses for each group of students with respect to understanding of algebraic concepts and practices from Grade 5 to Grade 6?

Method

Participants

Sixth-grade retention data were collected from 1,455 students across 23 middle schools in the three school districts that took part in the Grades 3–5 longitudinal study. Of these students, 716 were from 23 control (elementary) schools and had received only their regular instruction in Grades 3–5, whereas 739 were from 23 treatment (elementary) schools and had been taught the intervention as part of their regular instruction in Grades 3–5. Table 1 shows the demographic data for

Table 1

Demographics of Participating Districts

District	Number of participating elementary schools	Number of participating middle schools	Free or reduced lunch	English language learners	Students of color
A	Treatment: 3; control: 3	2	20%	6%	36%
B	Treatment: 6; control: 7	10	54%	10%	40%
C	Treatment: 14; control: 13	11	62%	20%	82%

the three school districts in which the 23 middle schools are found. (The districts contain 28 middle schools in total. One school from District A, three schools from District B, and one school from District C declined to participate.)

Context

Though participating students were consistently in a control or treatment elementary school throughout Grades 3–5, all students moved to new (middle) schools in Grade 6 and were intermixed. The students who participated in the retention study experienced their schools' regular mathematics curriculum during Grade 6 and no instructional intervention from our research team. We did not observe Grade 6 classrooms and, thus, cannot characterize the instruction that students received with great confidence. The Grade 6 teachers who taught the students who participated in the retention study cited the use of more than 20 mathematics curricula or other instructional resources. In all cases, teachers were expected to align their instruction to their state's standard course of study, which is, in turn, aligned with the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO], 2010).

Data Collection

Students completed a written assessment at the end of Grade 6 consisting of 11 open-response items, most of which contained multiple parts. Nine of these 11 items (with a total of 24 individual item parts) also appeared on the Grade 5 assessment and will be the focus of the results that we share in this article (see Table 2; the numbering of items is consistent with the numbering as reported in Blanton, Stroud, et al., 2019).

Data Analysis

Coding schemes developed in previous work (see Blanton, Stephens, et al., 2015, for a full description of this process) were used to categorize student responses to assessment items with respect to both correctness and strategy use. In the results shared here, we focus only on correctness. (Strategy results paralleled correctness

Table 2

Items Common to the Grade 5 and Grade 6 Assessments

Item number	Big idea(s)	Item
1	Equivalence, expressions, equations	Fill in the blank with the value that makes the number sentence true. 7 + 3 = ____ + 4 Explain how you got your answer.
3	Generalized arithmetic	Marcy's teacher asks her to solve "23 + 15." She adds the two numbers and gets 38. The teacher then asks her to solve "15 + 23." Marcy already knows the answer is 38 because the numbers are just "turned around." a) Do you think Marcy's idea will work for any two numbers? Why or why not? b) Write an equation using variables (letters) to represent the idea that you can add two numbers in any order and get the same result.
4	Generalized arithmetic	Brian knows that if you add any three odd numbers, you will get an odd number. Explain why this is true.
5	Equivalence, expressions, equations	Tim and Angela each have a piggy bank. They know that their piggy banks each contain the same number of pennies, but they don't know how many. Angela also has 8 pennies in her hand. a) How would you represent the number of pennies Tim has? b) How would you represent the total number of pennies Angela has? c) Angela and Tim combine all of their pennies. How would you represent the number of pennies they have all together? Suppose Angela and Tim now count their pennies and find they have 16 all together. Write an equation with a variable (letter) that represents the relationship between this total and the expression you wrote above.

Table 2 (continued)

<table>
<tr><th>Item number</th><th>Big idea(s)</th><th>Item</th></tr>
<tr><td>9</td><td>Functional thinking</td><td>
Brady is celebrating his birthday at school.
He wants to make sure he has a seat for everyone.
He has square desks.
He can seat 2 people at one desk in the following way:

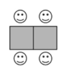
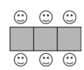
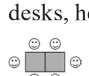
If he joins another desk to the first one, he can seat 4 people:

If he joins another desk to the second one, he can seat 6 people:

a) Fill in the table below to show how many people Brady can seat at different numbers of desks.
<table>
<tr><th>Number of desks</th><th>Number of people</th></tr>
<tr><td>1</td><td>2</td></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td></td></tr>
<tr><td>5</td><td></td></tr>
<tr><td>6</td><td></td></tr>
<tr><td>7</td><td></td></tr>
</table>
b) Do you see any patterns in the table from part a? If so, describe them.
c) Think about the relationship between the number of desks and the number of people.
Use words to write the rule that describes this relationship.
Use variables (letters) to write the rule that describes this relationship.
d) If Brady has 100 desks, how many people can he seat? Show how you got your answer.
e) Brady figured out he could seat more people if two people sat on the ends of the row of desks. For example, if Brady had 2 desks, he could seat 6 people.

How does this new information affect the rule you wrote in part c?
Use words to write your new rule. Use variables (letters) to write your new rule.
</td></tr>
</table>

Table 2 (continued)

<table>
<tr><th>Item number</th><th>Big idea(s)</th><th>Item</th></tr>
<tr><td>10</td><td>Functional thinking; equivalence, expressions, equations</td><td>The table below shows the relationship between two variables, k and p.

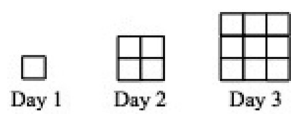
The rule $p = 2 \times k + 1$ describes their relationship.

a) Some numbers in the table are missing. Use this rule to fill in the missing numbers.

<table>
<tr><th>k</th><th>p</th></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>2</td><td></td></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td>9</td></tr>
</table>

b) What is the value of p when $k = 21$? Show how you got your answer.

c) What is the value of k when $p = 61$? Show how you got your answer.</td></tr>
<tr><td>14</td><td>Functional thinking</td><td>The following magic square is growing so that each day it is made up of more and more smaller squares.



The following table shows a given day and the number of small squares on that day:

<table>
<tr><th>Day</th><th>Number of small squares</th></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>3</td><td>9</td></tr>
<tr><td>4</td><td>16</td></tr>
<tr><td>5</td><td>25</td></tr>
<tr><td>6</td><td>36</td></tr>
</table>

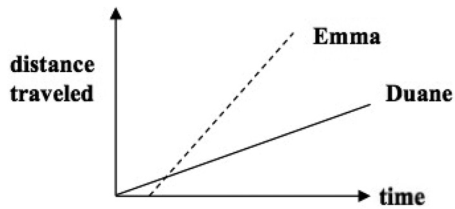
a) Think about the relationship between the number of days and the number of small squares.

Use words to write the rule that describes this relationship.

Use variables (letters) to write the rule that describes this relationship.

b) Use your rule to predict how many small squares will be inside the big square on day 100. Show how you got your answer.</td></tr>
</table>

Table 2 (continued)

Item number	Big idea(s)	Item
21	Functional thinking	Duane and Emma each went for a bike ride. The graphs below represent the relationship between time spent riding and distance traveled for each rider.  a) Who started riding first? How can you tell? b) Who rode faster? How can you tell? c) Is Emma riding the same speed on her whole trip, or is she speeding up or slowing down? How can you tell?
23	Equivalence, expressions, equations	Do the following two equations have the same solution? Explain. $2 \times n + 15 = 31$ $2 \times n + 15 - 9 = 31 - 9$

results as they did in the Grades 3–5 data reported in Blanton, Stroud, et al., 2019.) Items were coded by trained coders unaware of students' treatment condition. A median-split technique was used to divide the elementary schools that students attended into high or low socioeconomic status (SES) categories on the basis of the percentage of students receiving free or reduced-price lunch. Changes in overall correctness from Grades 5 to 6 were assessed using a repeated measures analysis of variance (ANOVA), with treatment condition and SES as between-subjects factors. McNemar's test was then used to examine item-by-item changes in performance from Grade 5 to Grade 6 within each condition.

Results

The data reported here come from the 1,455 students who completed both the Grade 5 and Grade 6 assessments. The repeated measures ANOVA showed no significant effect of testing time across all students but did reveal a significant interaction between testing time and treatment condition, $F(1,\ 1{,}451) = 168.84$, $p < .001$. Overall, treatment students (M = 47.51% correct, SD = 21.54%) maintained their significant advantage over control students (M = 37.93% correct, SD = 19.74%) at the end of Grade 6, $F(1,\ 1{,}451) = 78.13$, $p < .001$, 1 year after the conclusion of the intervention. In practical terms, treatment students responded correctly to 2.3 more item parts (out of the total 24) than did control students. However, the gap between the two groups narrowed from Grade 5 to Grade 6, with control students experiencing an overall increase in performance from Grade 5 (M = 33.38%, SD = 19.43%) to Grade 6 (M = 37.93%, SD = 19.74%; an increase of 1.1 item parts correct) and treatment students experiencing an overall decrease in performance from Grade 5 (M = 53.40%, SD = 24.17%) to Grade 6 (M = 47.51%, SD = 21.54%; a decrease of 1.4 item parts correct; see Figure 1). We did not find a significant three-way interaction among Grade 5 to Grade 6 improvement, treatment condition, and SES, $F(1, 1{,}451) = 0.69$, $p = .41$.

McNemar's test was used to compare performance (percentage correct) across the two conditions on the nine individual assessment items (with a total of 24 individual item parts) common to Grades 5 and 6. The results for these items are given in Table 3.

As shown in Table 3, control students made gains in performance on individual assessment items much more so than did treatment students. Specifically, control students improved on 15 of 24 individual item parts and declined on three parts from Grade 5 to Grade 6. Treatment students, conversely, improved on just one part and declined on 11 parts from Grade 5 to Grade 6.

Of particular interest is the identification of the algebraic concepts and practices for which treatment and control students exhibited significant gains or losses. In which areas did control students make progress toward closing the gap between their performance and that of their treatment counterparts? In which areas did students in the treatment condition maintain or gain (i.e., demonstrate persistence) in their Grade 5 performance levels and in which areas did they decline (i.e., demonstrate fadeout) in the year following the intervention?

For control students, significant gains occurred across a range of big algebraic ideas and thinking practices. Students showed an increased understanding of equivalence and equations on some items (Items 1, 10b, and 10c). They were increasingly able to identify one-step function rules in words (Items 9c1 and 14a1) and, in one of these cases, make a far prediction (Item 14b). They also made gains identifying an arithmetic property (Item 3a) and representing unknowns,

Figure 1

Overall Percentage Correct on Common Assessment Items Across Conditions and Testing Times

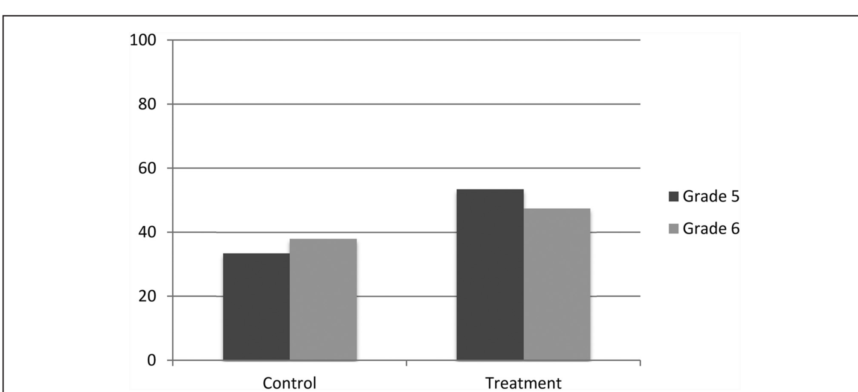


Table 3

Percentage Correct on Assessment Items Across Conditions and Testing Times

	Control		Treatment	
Item	Grade 5	Grade 6	Grade 5	Grade 6
1	67	71*	85	83
3a	54	63*	68	70
3b	22	35*	67	59**
4	10	8**	23	12**
5a	19	37*	60	56
5b	8	11*	30	20**
5c1	11	9	51	17**
5c2	9	5**	38	9**
9a	94	97*	95	96
9c1	22	16**	35	24**
9c2	27	32*	61	48**
9d	79	79	84	83
9e1	9	7	23	14**
9e2	16	17	42	32**
10a	56	59	74	72
10b	35	48*	58	61
10c	31	39*	45	48
14a1	8	16*	26	25
14a2	13	20*	50	45**
14b	36	43*	50	50
21a	62	75*	73	80*
21b	61	70*	73	74
21c	28	34*	46	39**
23	23	20	28	26
Total	33	38	53	48

*Items on which students showed significant gain from Grade 5 to Grade 6 ($p < .05$).
**Items on which students showed significant decline from Grade 5 to Grade 6 ($p < .05$).

arithmetic relationships, and one-step functional relationships with variables (Items 3b, 5a, 5b, 9c2, and 14a2). Although these gains are promising for control students and point to the role of early middle school experiences in developing all students' readiness for a more formal study of algebra, we emphasize that control students continued to lag behind treatment students on all these items. Control students did not experience gains—and occasionally even experienced a decline—on items involving advanced understanding of equivalence (Item 23), one-part (Item 9c1) and two-part (Items 9e1 and 9e2) function rules, using algebraic expressions of more than one term to represent problem situations (Items 5c1 and 5c2), and providing a general argument to justify a statement about the sum of odd numbers (Item 4).

Treatment students maintained their Grade 5 performance levels on simple and complex items explicitly addressing their understanding of the equals sign (Items 1 and 23) and work with equations (Items 10a, 10b, and 10c). They also maintained their ability to identify an arithmetic property (Item 3a), make far predications when working with functional relationships (Items 9d and 14b), and represent a simple unknown with a variable (Item 5a). The items on which treatment students showed a decline in performance from Grade 5 to Grade 6 included those on which they needed to identify one- or two-step functional relationships in words (Items 9c1 and 9e1), provide a general argument to justify a statement about the sum of odd numbers (Item 4), and provide a variable representation of more than one term to represent unknown quantities (Items 5b, 5c1, and 5c2), an arithmetic property (Item 3b), and functional relationships (Items 9c2 and 9e2).

Discussion

The results from our longitudinal Grades 3–5 study (Blanton, Stroud, et al., 2019) offered evidence that providing students with sustained early algebra experiences across a range of big algebraic ideas and thinking practices can, in fact, place them at an advantage with respect to algebraic understanding as they enter the middle grades relative to students who experience a more arithmetic-focused approach to elementary school mathematics. The study reported in this article assessed the algebraic understanding of these students 1 year after the conclusion of the Grades 3–5 intervention. We found that, 1 year postintervention, treatment students still significantly outperformed control students on an assessment measuring their understanding of big algebraic ideas and thinking practices.

How Did Performance Vary Across Big Ideas?

Digging deeper into the item-by-item results revealed the areas in which treatment students were able to maintain their Grade 5 performance levels and the areas in which their performance dropped significantly. Likewise, we were able to identify areas in which control students made significant gains.

One important area in which treatment students maintained their performance was the concept of equivalence and the meaning of the equals sign. This was the core concept with which we started our Grade 3 lessons, and it was consistently revisited throughout the 3 years of the early algebra intervention. Although control students made some gains in this area as well, their gains were not consistent across items and did not include growth on our most advanced item involving the recognition of equivalent equations. This is a critical area given the role that equals sign understanding plays in algebra success (Knuth et al., 2006).

An important big idea in our Grades 3–5 intervention where treatment students' learning was not robustly maintained from Grade 5 to 6 is functional thinking. Although treatment students maintained their performance in identifying a simple exponential function in words, they experienced a decline on all other items requesting the identification of a functional relationship, whether in words or variable notation. Control students showed some gains on these items, but they still fell short of the performance of their treatment counterparts. Our work with students across Grades 3–5, as well as the work of Blanton, Brizuela, et al. (2015) with much younger students, has illustrated that elementary school students are capable of engaging in such thinking, beginning with very simple relationships that increase in complexity over time. This, coupled with the lack of success of traditional routes to algebra in which big ideas such as functional thinking appear for the first time in a traditional ninth-grade course with no previous development, leads us to argue for the sustainment of opportunities to engage in such thinking over time.

Only one item (Item 4) on our assessment explicitly asked students to build an argument to justify an arithmetic generalization. This was among the most difficult tasks for Grades 3–5 students in both conditions. By the end of Grade 5, 23% of treatment students in the Grade 5 to Grade 6 retention study were able to produce a general argument—very often a representation-based argument (Schifter, 2009)—to justify that the sum of three odd numbers is an odd number. By the end of Grade 6, only 12% of treatment students were able to do so. Control students likewise showed no improvement (in fact, they showed a slight decline) in their ability to produce such an argument. Like functional thinking, argumentation is an area in which we know elementary school students are capable of engaging. Our previous work (Blanton, Stephens, et al., 2015; Blanton, Stroud, et al., 2019), along with the work of others (e.g., Carpenter et al., 2003; Carpenter & Levi, 2000; Russell et al., 2011a, 2011b), illustrates that although building general arguments is challenging, students are capable of justifying mathematical claims in the context of arithmetic.

Finally, treatment students showed a decline from Grade 5 to Grade 6 in their abilities to represent an arithmetic property, functional relationships, and related unknown quantities using variable notation. Although control students showed some gains, the performance levels they reached were nowhere near that of their treatment counterparts. Evidence from our Grades 3–5 work as well as the work of others (e.g., Brizuela & Earnest, 2008), including with much younger students (e.g., Brizuela et al., 2015), shows that students are capable of representing varying quantities with algebraic notation and, in fact, are sometimes more successful with such representations than with verbal ones. Given the difficulties that secondary school students have with variable notation (MacGregor & Stacey, 1997), we believe that a sustained focus on the different roles that variables can take on (e.g., variable as varying quantity, variable as generalized number, or variables in relation to one another in a function rule) is appropriate.

What Happens Next? Fadeout or Long-Lasting Impacts?

As noted earlier, Bailey et al. (2017) argue that interventions with persistently beneficial and sustained impacts tend to target skills that are malleable, fundamental, and would not eventually develop in the absence of the intervention. We believe that the skills associated with early algebra content are indeed (a) malleable, as shown in our previous work as well as the work of other early algebra researchers, (b) fundamental, in that they address core concepts (e.g., variable and equivalence) that are crucial to success in future mathematics study, and (c) not currently treated in a comprehensive or sustained way in currently popular elementary school mathematics curricula and instruction.

However, as discussed earlier, treatment students showed persistence in their understanding of algebraic equivalence but demonstrated fadeout in several other areas. We believe that the argument of Bailey et al. (2017) for the importance of "sustaining environments" (p. 25) points to one possible and important explanation for the losses that we observed in treatment students' algebraic knowledge. Although we did not observe Grade 6 classrooms and, thus, cannot confidently characterize the instruction that took place during this year—a notable limitation of this study—we believe that looking to the Common Core (NGA Center & CCSSO, 2010) to gather information about the mathematical content to which Grade 6 students were likely exposed is appropriate.

The areas in which treatment students demonstrated fadeout included functional thinking, argumentation in the context of generalized arithmetic, and the production of variable representations. These areas were emphasized in the Grades 3–5 intervention but are not prioritized in the Common Core's (NGA Center & CCSSO, 2010) Grade 6 standards. Clements et al. (2013) found that the alignment of preschool and early elementary school instruction mitigated fadeout following a preschool mathematics intervention. We suggest that a similar elementary-to-middle-school alignment has the potential to do the same.

What will happen as these students progress through secondary school mathematics and begin to encounter functional thinking, argumentation and proof, and variable representations in their regular mathematics curricula and instruction? Although studies that follow students for as long as we have are uncommon, we recognize the importance of even longer term studies to investigate the impacts of interventions on students' learning years after the completion of the interventions. Many open questions persist around these issues. Though the control students in this study could potentially fully close the gap with their treatment counterparts in the continued absence of any intervention, the knowledge that treatment students gained throughout the course of the intervention could also possibly be "activated" if they are in environments that call on this knowledge. What kind of environments will best activate this knowledge? Are traditional algebra courses such environments, or are reform-minded programs that purposefully build on the intuitive-to-more-formalized conceptions developed in our intervention necessary for such activation to occur? Moreover, although our focus at the beginning of the longitudinal study was on the development of algebraic knowledge, we now wonder whether our intervention may have developed malleable and fundamental skills such as self-concept, confidence, and academic motivation that were not measured. If so, what will be the long-term impact of these skills? Challenges with research funding cycles and researcher and participant sustainment make extended research very difficult, but this is exactly the kind of work that is needed to answer questions about the long-term impact of educational innovations.

Conclusion

More than 20 years ago, Kaput (1998) called for an end to "the most pernicious curricular element of today's school mathematics—late, abrupt, isolated, and superficial high-school algebra courses" (p. 25). He (and others) argued that we could do so by viewing algebra as a K–12 experience, integrating algebraic thinking and reasoning throughout the mathematics curriculum. Our work developing and testing a Grades 3–5 early algebra intervention coupled with our Grade 5 to Grade 6 retention study lends support to Kaput's argument. Despite unanswered questions about the even longer term impact and potential unmeasured effects of our intervention, we found that, 1 year after our intervention's conclusion, treatment students retained a significant advantage over their control peers in their understandings of important big algebraic ideas and thinking practices. Importantly, we believe that the fact that treatment students' performance declined from Grade 5 to Grade 6 lends support to the argument that algebra must be treated as a continuous K–12 strand of thinking and not as a subject that can be infused into students' elementary school curricula for a few years only to be cast aside until its more formal treatment.

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Authors

Ana Stephens, Wisconsin Center for Education Research, University of Wisconsin–Madison, 683 Educational Sciences Building, 1025 W. Johnson Street, Madison, WI 53706; acstephens@wisc.edu

Rena Stroud, School of Education and Social Policy, Merrimack College, 315 Turnpike Street, North Andover, MA 01887; stroudr@merrimack.edu

Susanne Strachota, The Patton College of Education, Ohio University, 100 East Union Street, McCracken Hall 309J, Athens, OH 45701; strachot@ohio.edu

Despina Stylianou, School of Education, City University of New York, Covent Avenue at 138th Street, New York, NY 10031; dstylianou@ccny.cuny.edu

Maria Blanton, TERC, 2067 Massachusetts Avenue, Cambridge, MA 02140; Maria_Blanton@terc.edu

Eric Knuth, College of Education, University of Texas at Austin, 1912 Speedway STOP D5700, Austin, TX 78712; eric.knuth@austin.utexas.edu

Angela Gardiner, TERC, 2067 Massachusetts Avenue, Cambridge, MA 02140; Angela_Gardiner@terc.edu

Submitted June 17, 2019

Accepted December 23, 2019

doi:10.5951/jresematheduc-2020-0304