MATH TEACHERS' SENSEMAKING AND ENACTMENT OF THE DISCOURSE OF "PERSEVERANCE"

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Students' opportunities to struggle with mathematical ideas have long been considered paramount to learning. However, there's little research on how teachers (1) draw on and make sense of the discourses of perseverance, (2) enact it in classroom, and (3) develop an expansive view of perseverance. To contribute to these lines of research, we build on a case study featuring a veteran mathematics middle-school teacher across two settings: his classroom where he facilitates students' engagement with a classical mathematical task, the Tower of Hanoi, and in a subsequent video-based debrief with his colleague and our research team. We propose a conceptualization of perseverance as upholding three dimensions of (a) persistence (b) sensemaking and (c) problem solving heuristics. We argue for its potential as a conceptual resource for operationalizing perseverance more comprehensively.

Keywords: Mathematics Teaching, Perseverance, Video-based coaching, Problem-solving

Objectives

Perseverance, grit, and productive struggle are a few of the descriptors signaling the importance of fostering students' sustained engagement in math classrooms to support mathematical sensemaking. Mathematics educators, researchers, and policymakers agree that cultivating students' dispositions towards seeing mathematics as a worthwhile and *effortful* pursuit supports students' success in engaging with conceptually-rich mathematics in flexible and fluent ways. Indeed, *make sense of problems and persevere in solving them* is the first of eight mathematical practices deemed essential for teachers to champion in their classrooms as outlined in the Common Core State Standards for Mathematics (National Governors Association, 2010). Similarly, in the influential document *Adding It Up* (Kilpatrick et al., 2001), mathematical proficiency is defined as the amalgamation of five strands, one of which is *productive disposition,* or the "habitual inclination to see mathematics as sensible and useful, coupled with a belief in one's own efficacy" (p. 5). In short, perseverance is one of the noncognitive factors that is highly consequential for academic success (e.g. SRI International, 2018, p. 1).

Despite a burgeoning discourse within education broadly and mathematics classrooms specifically, little research investigates how mathematics teachers interpret these discourses of perseverance. Yet these interpretations matter for how they take them up in their classrooms. Accordingly, inspired by Naraian (2011), we explore questions such as: What discourses do teachers draw on as they aspire to promote perseverance? How do teachers design for and enact perseverance in math classrooms? What discursive dilemmas emerge in this process? How can teacher educators support teachers' expansive view of perseverance?

In this analysis, we present an illustrative "best case" of a teacher, Ezio (all names of teachers and the school are pseudonyms), who identified perseverance as an explicit goal for his class. We analyze the discourses he drew on, his enactment of a lesson designed to support perseverance, and the discussion we conducted three days later as part of our research project. We identify the Opportunities To Learn (OTLs) that emerged in our debrief that supported the

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teacher's thinking about perseverance more expansively while also pinpointing where we fell short as teacher educators in our own limited understanding of perseverance, the same discourse to which we aim to contribute in this paper.

Theoretical Framework

Situating Teachers as Sensemakers

Despite strong efforts to shift teachers towards ambitious instruction, the field continues to struggle with effectively translating research-driven visions and practices so they might be taken up more frequently and more seamlessly by practitioners. We address this problem in our research, as well as in our practice as teacher educators, by taking a situated view on teaching (Greeno & MMAP, 1998; Horn & Kane, 2015). A situated view on teaching (and on teacher learning) highlights that teachers do more than simply implement pedagogical discourses, but rather they constantly make sense of their classroom context as it is negotiated by institutional norms and practices and wider socio-historical discourses (Horn & Little, 2010; Narian, 2011). In this vein, we take a situative approach in examining teachers' Opportunities To Learn (OTLs) by considering the various discourses teachers draw on such as the state standards, professional development organizations, and their own epistemic stances or beliefs about what can be known, how to know it, and why it matters (Horn & Kane, 2015). In this study, we are interested in the conceptual resources made available for teachers and teacher educators in their attempts to operationalize discourses around productive struggle. We view the two settings of classroom and video-based coaching session as informative in drawing out the discursive dilemmas in each other. In other words, classroom dilemmas inform teachers' OTLs, and pedagogical conversations with colleagues, at their best, inform classroom dilemmas. In light of that, we aim to conceptualize perseverance in a way that might support teachers and teacher educators in conceiving and enacting perseverance more expansively.

Research on Perseverance

To conceptualize perseverance, we draw on several studies (Bass & Ball, 2015; Hiebert & Grouws, 2007; Sengupta-Irving & Agarwal, 2017; Stein et al. 2017), and propose a rich and productive conceptualization of perseverance as upholding the three conditions of (a) *persistence* (b) *sensemaking* and (c) problem solving *heuristics*.

Hiebert & Grouws (2007) documented the ways that certain forms of *persistence* or student struggle can be a key enabler for math learning. However, persistence alone is an insufficient condition for learning mathematics conceptually. For example, the results of a recent study by Stein et al. (2017) revealed that teachers disproportionately create conditions for students to struggle unproductively, with little explicit attention to and few opportunities for building conceptual understanding. For students to develop conceptual understanding, they need opportunities to *make sense* of mathematical ideas, construct new knowledge, and make connections. Consequently, teachers are tasked with finding challenging problems that require the right amount of struggle within students' conceptual reach or within their zone of proximal development (ZPD; Sengupta-Irving & Agarwal, 2017; Vygotsky, 1978). Less productive challenges were described by Sengupta-Irving and Agarwal (2017) as either "unnecessary struggle" or "no struggle" (see Figure 1).

Bass & Ball (2015) add a dimension to our understanding of perseverance related to problem solving *heuristics*. Analyzing a 5th grade class engaged in collective problem solving, they presented a macro-analysis of the lesson identifying (1) six stages of students' collaborative work; and (2) the teacher's instructional moves that oriented students engagement across the stages. Through their analysis of how teachers made perseverance "visible and learnable," they underscored how the teacher made explicit for students the different heuristics they exercised in making sense of the task, identifying adequate approaches and generating possible solutions. Indeed, research has shown heuristics to be an important tool for successful problem solvers (Schoenfeld, 1985).

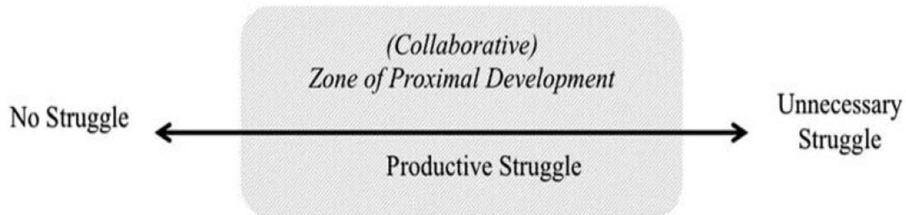

Figure 1: Productive Struggle and the ZPD (Sengupta-Irving & Agarwal, 2017)

Methods

Research Context

This analysis comes from a larger research-practice partnership aimed at understanding how video-based coaching, tied to teachers' instruction, can support urban mathematics teachers' development of ambitious instruction. To support and document teacher learning, our research team developed a Video-based Formative Feedback (VFF) cycle, a 4 step co-inquiry into practice. First, we prompt teachers about their goals and specific inquiries related to the class to be filmed, identifying a question we can use to inform our observation. Second, we film one classroom session using two cameras, a classroom wide-angle camera and teacher point-of-view camera. Additionally, we place four audio markers at different student groups to capture group-level conversations. These video and audio records serve as rich representations of teachers' practice from which to focus coaching conversations. Next, our team analyzes the captured footage to identify moments of interest that may address the teachers' inquiry questions. Finally, typically within two days of filming, we meet with the teachers to debrief the lesson, focusing on the identified video moments as a springboard for discussion. The debrief session is filmed for our research purposes.

Data sources. Primary data sources for this analysis include: (1) Focal teacher's email describing the lesson activities and teacher's goals; (2) the 90-minute video-taped classroom session; (3) the 81-minute videotaped debrief of the lesson with three members of the research team—the two authors of this paper and the project Principal Investigator, Ilana Horn. Patricia Buenrostro facilitated the conversation with the focal teacher, Ezio, and a collaborating teacher, Veronica. The other researchers were primarily filming (Ehrenfeld) and taking field notes (Horn) but on occasion participated in the debrief conversation. Because our relationship with Ezio extends beyond this focal event, we draw on other conversations, observations, and interviews as secondary data in confirming or disconfirming evidence for tenuous claims.

Focal participant. The VFF cycle analyzed represents a debrief with Ezio Martin, a veteran, middle-school teacher with over 17 years of experience, and his colleague, Veronica Kennedy. Ezio demonstrated strong content knowledge during his teaching and in professional development workshops, underscored by his undergraduate and graduate math degrees. His participation in a 5-year professional development program showed his commitment to his professional growth. In addition, he had strikingly warm rapport with his students, who show respect and admiration for him. As a teacher with a strong math identity, the classroom was decorated with mathematical phrases, posters and books. The class episode under study featured animated talk, boisterous laughter, and wooden manipulatives in action. In short, it appeared a vibrant environment in which to learn and do math. Ezio's queries prior to filming centered around student dynamics (e.g., status issues). Notwithstanding, in the debrief, Ezio identified perseverance as an explicit goal for the featured task, the Tower of Hanoi.

Focal task, "Tower of Hanoi." The Tower of Hanoi activity consists of three poles and varying number of rings in different sizes (see Figure 2), arranged on one pole. The goal of the task is to move all the rings, one at a time, from the initial-pole to either of the two remaining poles, while avoiding the placement of a larger ring onto a smaller one *in the least number of moves*. For example, 3 rings can be transferred in only 7 moves. The task is to find the general rule for determining the minimal number of moves for any given number of rings [$f(n) = 2^n-1$]. Ezio wanted to expose students to a nonlinear function and to experience a challenging problem with which to persevere.

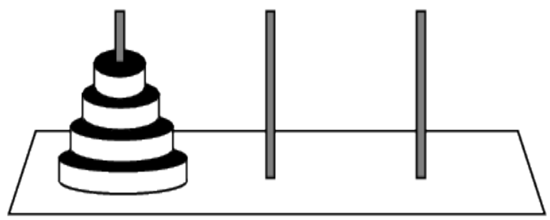

Figure 2: Illustration of Tower of Hanoi Kit with Three Poles and Four Rings

Methods of Analysis

To be clear, the centrality of Ezio's goal to have his students persevere emerged during the post-class debrief. Our overall methods represent an iterative process of analyzing both the debrief and classroom session to understand how Ezio made sense of, designed for, and enacted *perseverance*.

We looked across the data from both settings and used interaction analysis (Jordan & Henderson, 1995) with both sets of transcripts to examine the complementary and contradictory discourses that emerged. For the classroom, we looked at Ezio's design and enactment of perseverance with an eye towards the macro-analysis of lesson phases presented by Bass and Ball (2015) and expanded our definition of perseverance. For the debrief, we analyzed Ezio's conceptualization of perseverance as he discussed his teaching. Comparing the emergent views of *perseverance* arising from these analyses, we explored the discursive dilemmas (e.g., asking questions students are unable to answer) in both events—the classroom and debrief—which ultimately led to our expansive understanding and proposed characterization of perseverance described earlier.

Results

To understand the discourses that Ezio drew on in making sense of perseverance, we examined his epistemic stance on perseverance as corroborated from his email and the debrief conversation. Next, we report on the classroom events from the filmed lesson, followed by an analysis of the debrief conversation. Finally, we look across the two settings and reflect on how, in our role as coaches, we supported Ezio in re-imagining perseverance in his classroom, while also underscoring where we needed a richer conceptualization of perseverance.

Ezio's Epistemic Stance on Perseverance: "I want them to learn how to persevere"

During the debrief, Ezio interrupted the start of a video clip viewing to share the significance of having his students engage with a complex problem such as the Tower of Hanoi. He stated, "I think the Tower of Hanoi is complex enough, at least in middle school [that] I want them to learn how to persevere. It is one of the standards." He was not quite sure how to teach perseverance ("I don't know how to do it"), but he believed that posing a challenging task would create the conditions for his students to persevere. This notion is corroborated by the email Ezio sent us the day prior to filming his class in which he briefly laid out his goals and plans for the class. He noted that "most will not be able to determine the equation for Tower of Hanoi even with the hints I give them...I am willing to leave the problem 'unsolved' for those who are not able to figure it out." Ezio anticipated that "most" students would struggle, thereby, creating a situation in which they would persevere.

We learned in the debrief about Ezio's epistemic stance regarding the importance of perseverance in math learning. Ezio shared his "personal belief" that all students will experience a potentially insurmountable challenge in mathematics. He described this experience as "hit[ting] a wall," one that "everybody hits." Ezio described all math learners, including himself and the greatest of mathematicians, as eventually "hit[ting] a breaking point" but that the key is to "[learn] how to overcome [it]." Ezio distinguished those that "reach that breaking point and...give up" from those "who continue the struggle…[those] who will succeed in math." For Ezio, *hitting the wall* is a defining moment where one decides to either concede to not being capable enough or, contrastingly, persevere and "overcome that breaking point." While he said that he does not "do a good job of teaching [students] how to persevere over those obstacles," the Tower of Hanoi gives him "the opportunity to focus in on that."

Classroom Excerpt: "Maybe you haven't tried hard enough"

To understand how Ezio designed for and enacted perseverance in his classrooms, we describe what we termed as Ezio's "cycles of perseverance" in light of his epistemic stance. The cycle begins with a challenge beyond students' conceptual reach, in essence directing them to a "wall," and concludes with Ezio's scaffolding them over the wall of struggle.

The 90-minute lesson was replete with intensive mathematical work by 31 eighth graders. It was built around two mathematical tasks: a warm up with visual patterns representing a linear function (around 30 minutes), and the Tower of Hanoi (around 60 minutes). The episode discussed here is focused on the second task. It began with a playful introduction from Ezio, where he teased students about being "a bunch of babies" necessitating the use of toys to which students responded with smiles and baby voices.

Two cycles of perseverance. Students were seated in seven groups of four to six but working primarily in pairs. Ezio systematically circulated between the seven groups, conversing with students about their work, occasionally teasing them ("Maybe you haven't tried hard enough"). In our analysis of classroom events, we identified that Ezio designed for and enacted perseverance in his classroom in two cycles, each consisting of "a wall" and a scaffold (see Table 1). We highlight here two cases in which Ezio paused the groupwork to scaffold the whole class: first, to supply them with "a cheat", and second, to aid them in synthetically manipulating their empirical results to arrive at the function, $f(n) = 2^n-1$.

Table 1: Two Cycles of Perseverance, Each Consist of a "Wall" and a Scaffold

Cycle	The "wall"	The scaffold
Cycle 1	Ezio asking students repeatedly, how they know their result is the minimal possible number of moves?	"the cheat" - explains the recursive rule (i.e. how to find the answer for n rings according to the answer for n-1 rings).
Cycle 2	Ezio asking students, what would happen if they had 100 rings? (i.e. the recursive rule does not help)	the table - manipulate results to find the general function f(n)

Cycle 1. Ezio introduced the Tower of Hanoi and ensured all student pairs understood the task. As students began to collect and annotate their initial empirical results, Ezio circulated to six of the seven groups asking them "how do you know that's the fewest [number of moves]?" He repeatedly asked this question without waiting for or following up with students for a response. Moreover, the students continued to explore it empirically (i.e. trying to move rings in less moves) without discussing the question posed. We account for this question as the first "wall". Ezio followed this up with a whole-class scaffold where he explained "the cheat". The cheat represents the recursive nature of the task which allows one to easily solve for any number of rings n, given the result for n-1. For example, one can determine the least number of moves for 4 rings by building on the least number of moves for 3 rings. (we do not bring the full explanation for lack of space). We want to be explicit regarding the assumptions under which we consider this teaching move to be an over-scaffold: explaining to students the recursive rule is over-scaffolding *if* the teacher aims for student discovery of mathematical ideas. In the case where the teacher's goal is to directly teach the idea of recursive functions (which is not the case in hand), we do not consider this move to be an over-scaffold but rather an instantiation of direct instruction. Upon explaining "the cheat," Ezio prompted several groups to replicate the "cheat" or recursive rule by asking questions such as "If you know [the number of moves for] 4 [rings] can you show me 5?".

Cycle 2. After having students demonstrate the recursive rule, Ezio introduced them to the next wall: how could they determine the minimal number of moves for 100 rings? Here the recursive rule only works if students have generated the minimum for 99, 98 etc. Ezio followed up with another scaffold in the form of a table (distributed on a hand-out) where they could determine the number of minimal moves for *n* rings, into the function $2^n - 1$, only by way of a synthetic manipulation. As one example, for n=3 rings, students were instructed to: (a) find the minimal number of moves (7); (b) add one (8); (c) do prime factorization (2^3); and (d) write the final power form [$f(n) = 2^3$]. Students were then asked to find a relationship between the final power form [$f(n) = 2^n$] and the function [$f(n) = 2^n - 1$]. Again, we view this as an over-scaffold in the sense that it does not center sense-making but rather guides students almost directly to generating the function.

Conclusion of classroom analysis. Our analysis shows that students moved back and forth between both edges of their collective ZPD as described by Sengupta-Irving and Agarwal (2017) as moments of “unnecessary struggle” and “no struggle” (see Figure 3). Ezio’s questions (the “wall”) placed students to the right edge of the ZPD students creating unnecessary struggle as students appeared to have few, if any, conceptual resources to answer the questions. Following their struggle, students were strongly scaffolded either directly by Ezio’s whole-class explanation of “the cheat” (rooted in his enactment of the task) , or by the table on the handout (rooted in the design of the task). To conclude, Ezio’s design and enactment of the Tower of Hanoi placed his *perseverance as persistence* at odds with sensemaking, all the while leaving out problem solving heuristics as a potential avenue for aiding students in discovery and sensemaking.

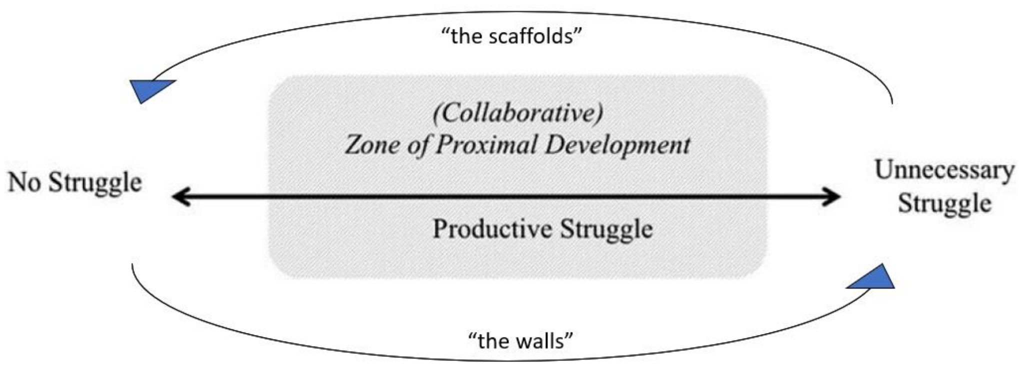

Figure 3: Cycles of “Walls” and Scaffolds Outside of Students’ ZPD (adapted from Sengupta-Irving and Agarwal, 2017)

VFF Debrief Conversation

While we posit that Ezio’s enactment of perseverance fell short of engaging students’ sense-making in the task, we also acknowledge our own shortcomings, as teacher educators, in unpacking perseverance. During the debrief we did mention all three dimensions of perseverance, albeit in isolation from one another. For example, the coach (author one) confirmed that, in her view students were persevering (read: persisting), an observation supported by the partner teacher and the other researchers present. Secondly, she recognized that students were persisting through the activity but not verbalizing their thinking of how to reproduce the same number of (minimal) moves from one trial to the next. Consequently, the coach offered Ezio the heuristic of “solve a simpler problem first” (Polya, 1945/2004) strategy as a possible scaffold to center students’ sensemaking in the lesson. The coach suggested this heuristic to Ezio as a scaffold to have students think about how and why they can consistently reproduce the transfer of 3 rings in 7 moves. By understanding the logic of the simpler problem, students could then discuss with their peers how to reproduce the transfer in the fewest moves and apply it to 4 rings, 5 rings, and so on. Essentially, they would be discovering and making sense of the “cheat.”

As Ezio reflected on his “fear [that] they would waste so much time on the counting…[and not get] in-depth with anything else,” he acknowledged the suggested heuristic as an important scaffold that would have accomplished students going “in-depth” with the problem. Veronica

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also acknowledged the value of giving challenging problems, as a first step, and the importance of teaching students problem solving heuristics. Ezio returned to the suggested heuristic several times as his big takeaway from the debrief in thinking about future ways to aid his students discovery of the "cheat" on their own.

Discussion

In the debrief, we discussed all the units of persistence, sense-making, and heuristics but did not explicitly connect the dots between them as constituting what we now propose as a conceptualization of perseverance. As researchers, we have the privilege to grapple in-depth with the in-vivo problems of practice we encounter as teacher educators. The discursive resources our analysis has generated were not available to us at the time of the debrief. Without an elucidated conceptualization of perseverance, we were also conflating perseverance as persistence as we too noted students not giving up in the absence of sense-making. Although the coach suggested a heuristic scaffold, it fell short of addressing the full mathematical practice of *make sense of problems and persevere to solve them*, by not connecting persistence to sensemaking.

Interestingly, as Ezio described the "wall" and hitting that breaking point, he reiterated that those that make it (i.e., persevere) in mathematics "learn *how* to overcome it" as opposed to those who "mentally give up on math." It bears noting here that Ezio is wondering how to help students overcome that breaking point. Learning *how* to overcome is distinct from pushing through (persistence). Overcoming a breaking point implies being able to access some internal or external resources that could shed some insight into the given hurdle, a resource such as a problem solving heuristic. Moreover, his comment about "mentally giv[ing] up on math" implicates the cognitive demand of mathematics which is more than the will and heart to continue--more than persistence. "Mentally giv[ing] up on math," seems to suggest a person not making sense of the mathematics or hitting a wall in which a path forward seems insurmountable. Although Ezio is unable to tease this out, and we are unable to support him in doing so, he does have the initial instinct and foresight to know that he wants to offer students some type of tool or heuristic that they can use in a general sense to overcome mathematical challenges.

In sum, a deeper analysis of Ezio's stance on perseverance merited more attention. As coaches, we were limited by an insufficient conceptualization of perseverance beyond persistence much like Ezio in his design and enactment of it in the classroom. While the coach was able to suggest a scaffold that served as a heuristic for the sake of making sense of the task, we were limited in explicating a formidable relationship between persistence, sense-making, and problem solving heuristics as constituting perseverance, his long term goal for his students.

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