TIME EXPRESSIONS AND ELEMENTARY STUDENTS' REASONING WITH THE ANALOG CLOCK

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*This paper investigates the interplay of words and expressions with students' efforts to indicate times on a clock. We consider how elementary students interpret precise times (e.g., 2:30, 4:30) as compared to relative times (e.g., half past 11) as they describe this intangible quantity using a clock. Interviews with students in grades 2 and 4 (*n*=42) revealed that precise times led to whole number descriptions whereas relative times led to descriptions of part-whole relations consistent with representations of measure. A subsequent analysis of assessment performance of students across elementary grades (*N *= 612) corroborates the differential performance based on expression used in the tasks. Findings suggest that elementary students need further support to treat this invisible quantity according to measurement properties. Implications for theory are discussed.*

Keywords: Time, Clock, Elementary math

Learning mathematics is inextricably linked with the words and expressions used to enact and communicate mathematical ideas. For this reason, theorists have offered sociocultural descriptions of learning that include explicit treatments of word meaning and use (e.g., Halliday, 1993; Sfard, 2008; Vygotsky, 1978, 1986). For the topic of time, modern practices include a variety of expressions both within and across languages around the world that reference time units in substantively different ways. The time 2:15 may be stated as "two fifteen," "fifteen past two," or "quarter past two" depending on the language and community (see Bock, Irwin, Davidson, & Levelt, 2002; Burny, Valcke, DeSoete, & Van Luit, 2013). Even within a language like English, speakers of the same language across countries may draw upon time expressions with different treatments of time unit (e.g., for 11:30, a common phrase in Ireland is "half 11," an expression unfamiliar to many in the United States). Just as a language's counting words are consequential to early understandings of numeracy (Miura, 2001; Ng & Rao, 2010), we investigate if analogous expressions for time with substantively different treatments of time unit lead to differentiated descriptions of time.

Our research from a sociocultural perspective has examined how children draw upon the analog clock as a tool to solve elapsed time problems (Earnest, 2017; Earnest, Gonzales, & Plant, 2018). We share two contentions with current approaches to time instruction. First, time is invisible and untouchable, and for this reason we contend that we must consider the consequential role of words and language as individuals describe time and interpret representations of time (Bock et al., 2002; Tillman, Marghetis, Barnet, & Srinivasan, 2016). Research has established the interplay between words and pathways of mathematical thinking, such as the role of counting words in some East Asian languages that highlight the structure of the base-ten number system (Miura, 2001; Ng & Rao, 2010). Building on Sfard's (2008) treatment of words and narratives (or, the story or description of the "relations between or activities with or by objects," p. 574) in mathematics discourse, we explore such interplay in order to generate a sociocultural description of students' thinking in relation to time words.

Second, as we further consider below in implications, we dispute the current treatment of time in elementary mathematics, which relegates the topic to early elementary grades with a focus on clock-reading procedures. Technology has advanced to the point where reading the analog clock in order to identify the time is no longer an appropriate endgoal—digital clocks easily enable this and are widely available. We do not suggest the analog clock is unimportant; rather, we propose that as a field we ought to re-think the role of time in K-12 education, in particular how representations of time such as the analog clock or the *x*-axis of a graph share structural similarities (Earnest, 2015) together with its relation to other elementary mathematics topics, such as fractions and partitioning (Earnest et al., 2018).

Researchers investigating the learning and teaching of time, including our team, have drawn attention to the minimal research conducted on learning this topic (Burny et al., 2013; Kamii & Russell, 2012). Moreover, even among those studies conducted over recent decades, experimental designs and identified data points suggest different visions for what is critical and necessary in order to investigate children's time-related ideas. Unlike the present analysis, which considers student thinking in tandem with the tool and language, much of the developmental research explored children's emerging intuitions related to duration independent from the standard tools or units we have for time (e.g., Long & Kamii; 2001; Piaget, 1969; Wilkening, Levin, & Druyan, 1987). Also from a constructivist perspective, Kamii and Russell (2012) explored children's elapsed time calculations in standard units, yet independent of an analog clock's spatial representation of time. In one of the few studies that considered how children themselves grappled with time-related ideas together with standard units of and tools for time, Williams (2012) conducted a cognitive ethnography in early elementary classrooms, identifying metaphors upon which children draw as they reason with an invisible and intangible entity (see also, Lakoff & Nuñez, 2000). We therefore see this topic within mathematics education as one in need of more research, in particular because of its underlying role in the mathematics of change that characterizes middle and high school mathematics. Because time is intangible, words and expressions for time take on a prominent role in the construal of meaning, leading us to our research questions focusing on the interplay of words and action.

We address the following research questions: (a) How do students interpret precise and relative times on an analog clock? If students do so differently, (b) is there a difference in performance on such tasks across elementary grades?

Method

Participants for individual paper-and-pencil assessment included students across grades 2-5 ($N = 612$) from three school districts in urban, suburban, and rural areas in Massachusetts. Students were asked to, for example, "Show 2:30 on the clock" or "Show half past 11 on the clock" (Figure 1), which were two of 35 total items. For the focal tasks, we coded the position of the drawn hands as a point between 0.0 to 11.9, with accuracy coded as $\pm$ 0.2 from the accurate position (for example, an accurate hour hand position for 4:30 pointed between 4.3 and 4.7).

Using scores on the written assessment, a subset of students in grades 2 and 4 ($n = 42$) participated in interviews in which they solved problems involving an analog clock with independent hour and minute hands. For two focal tasks, the interviewer presented a card that stated the problem (i.e., "Show me 4:30" or "Show me half past 11") and asked the student to describe their solution approach. Our research group coded video data for accuracy of positions for hour and minute hands as described above. Further, we open coded (Corbin & Strauss, 2008) transcripts and video to identify emerging themes drawing upon utterances and activity in problem solving. Narrative categories emerged through multiple passes through the data. We present our findings beginning with interview analysis and, based on those results, turning back to the assessment data as a way to corroborate findings based on interviews. We note two decisions made due to space constraints in the present paper. First, we limit our interview analysis to narratives in data reflecting 10% or more of interviewee responses. Second, we limit our analysis to tasks to the half hour; our presentation will feature additional assessment and interview tasks featuring additional precise and relative times.

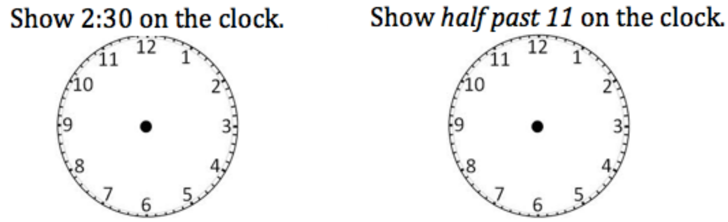

Figure 6: Focal Assessment Tasks

Results

We begin with a description of narratives (Sfard, 2008) that emerged in our open coding analysis of interviews before then detailing how such narratives emerged in students' interpretations of each time expression. Four dominant narratives emerged that reflected 10% or more of responses in interviews: Whole Number, Part-whole Relations, Conversion, and Hand Movement. Table 4 features narratives with frequencies across the 168 possible instances (2 subtasks for 2 problems for each of the 42 students)[1].

Table 1: Identified Narratives

Narrative	Number of Instances	% of total (*N*=168)
Whole Number	93	55.4%
Part-whole Relations	75	44.6%
Conversion	27	16.1%
Hand Movement	20	11.9%

Some solutions reflected ideas related to Whole Number, namely number matching and skip counting. For example, for 4:30, a second grader matched the hour hand directly (and inaccurately) on 4 and the minute hand directly (and accurately) on the 6 and stated: "It was easy, because I already knew *that* [6] was thirty, and I already knew *that* [4] was a four" (Figure 2a). Other students coded as Whole Number drew upon skip counting to count by 5s in order to position the minute hand (Figure 2b)

Otten, S., Candela, A. G., de Araujo, Z., Haines, C., & Munter, C. (2019). *Proceedings of the forty-first annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. St Louis, MO: University of Missouri.

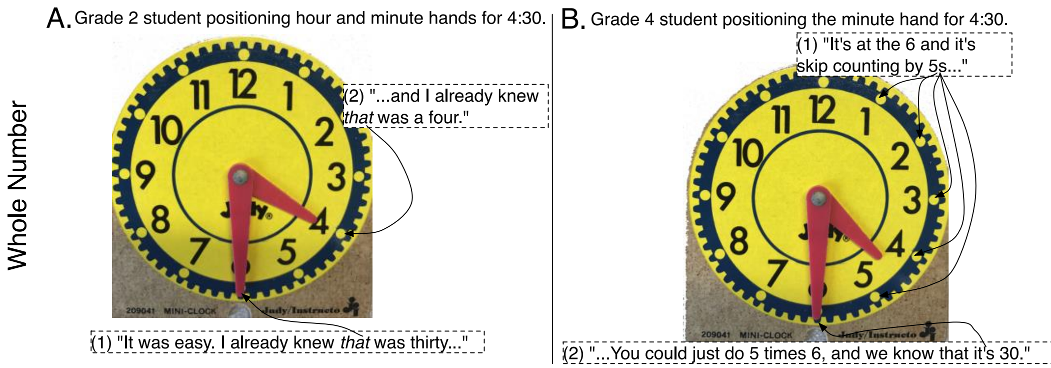

Figure 2: Whole Number Approaches to Hour and Minutes Hands

We coded approaches as Part-whole Relations (Figure 3a-c) when a student identified a unit interval or whole on the clock and described partitioning that unit or whole. For example, a second grader solving 4:30 positioned the hour hand at 4.5, stating, “It’s past the four <*sweeping his finger from the 4 to the 5*> by half, because it was 30 minutes into the hour” (Figure 3a).

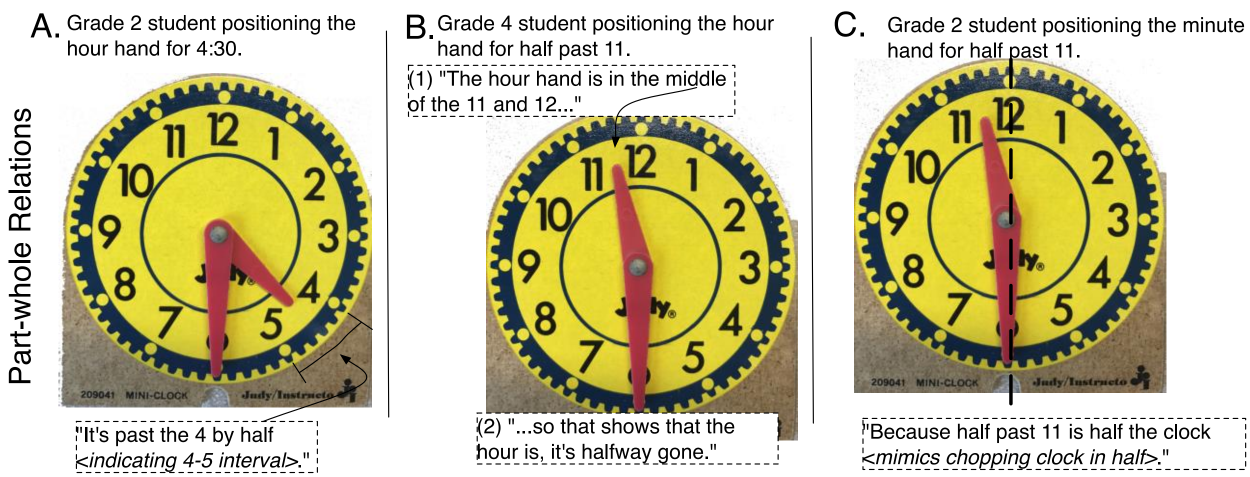

Figure 3: Part-whole Relations Approaches to Hour and Minutes Hands

We assigned the code Conversion when a student described a conversion between hours and minutes, which in our data was either one hour and 60 minutes or half an hour and 30 minutes (Figure 4a). We applied this code only 27 times out of a possible 168 solutions. Hand Movement was applied in only 20 of the 168 possible solutions. This particular code was applied when a student, first, referenced a particular numeral on the clock and, second, positioned the focal hand in tandem with a description of the moving hand; this description, however, did not include reference to a unit or whole (Figure 4b-c).

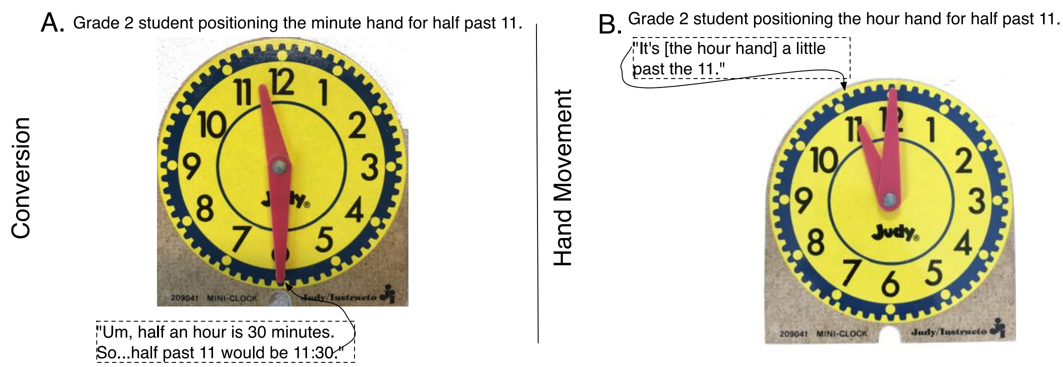

Figure 4: Conversion and Hand Movement Approaches to Hour and Minutes Hands

Expressions of Time and Corresponding Narratives

We now consider how the narratives manifested as a function of the expression in the prompt (4:30 or half past 11) and the hand (hour or minute). When presented as 4:30, students' solution approaches largely reflected ideas related to Whole Number (see left panels of Figure 5). In total, 20 of 42 students drew upon whole number ideas for the 4:30 hour hand and 36 of 42 did so for the 4:30 minute hand. For the hour hand specifically, Whole Number approaches—matching the number in the prompt with a number on the clock—were highly unsuccessful, despite being the most-represented approach for the 4:30 hour hand. Turning to half past 11 (right panels of Figure 5), students' solution approaches for both hour and minute hands largely reflected ideas related to Part-whole Relations, with 24 of 42 students drawing upon this narrative for both of the hands and mostly accurately.

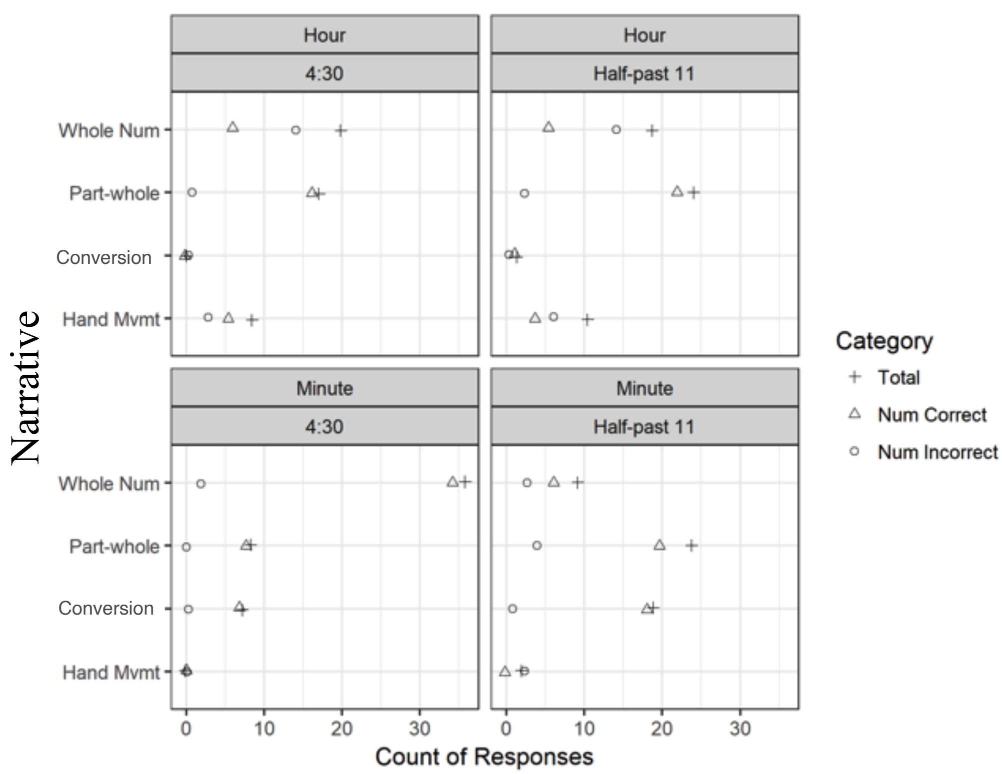

Figure 5: Narratives Emerging in Interviews

In sum, the Part-whole Relations narrative was deployed across hands and tasks with general success (73 instances out of 168; 90% accuracy). The Whole Number narrative was more common yet more variable with respect to accuracy (84 instances out of 168, 60% accuracy). Most students with Whole Number descriptions of the hour hand positioned it directly and inaccurately on the hour, whereas those with Whole Number descriptions of the minute hand successfully counted by 5s to position it. The units presented in the prompt were related to the narrative that emerged in children's descriptions, indicating the consequential interplay of time expression and children's descriptions of time, though as can be seen in Figure 6, those students drawing upon Part-whole Relations for the 4:30 hour hand (partitioning the hour interval) were largely successful, unlike those that drew upon Whole Number (matching the numeral 4 in the prompt with the 4 on the clock).

Given these differences in interview performances based on time units in the prompt, we turn back to assessment data for students across elementary grades to either corroborate or refute the trend in interview analysis. On the assessment, we found no statistical difference in performance on the two focal tasks when comparing the overall problems. However, when looking at hour and minute hands separately, a striking pattern emerged: accuracy for the placement of a particular hand in relation to time expression is statistically different. Across all grades, hour hand performance is better for half past 11 (Figure 7, left panel). The reverse is true for placing the minute hand; all grades perform this subtask better for the 2:30 task. Patterns in performance across subtasks are statistically significant. The minute-hand placement had lower accuracy rates relative to the 2:30 minute-hand subtask ($\widehat{\beta_s} = -0.29, \hat{\sigma} = 0.15, p < 0.05$). Hour hand position on the half past 11 subtask was more accurate than that for 2:30 ($\widehat{\beta_t} = -0.87, \hat{\sigma} = 0.12, p < 2 \cdot 10^{-13}$). Despite the fact that the two tasks featured analogous times to the half hour, accuracy in hand positioning appears to depend on which hand and on how time units are referenced in the prompt. Although the analysis involves a comparison of only two tasks and must therefore be interpreted humbly, findings corroborate qualitative trends that emerged in interview data: children are likely to interpret time meaning in relation to the time expression.

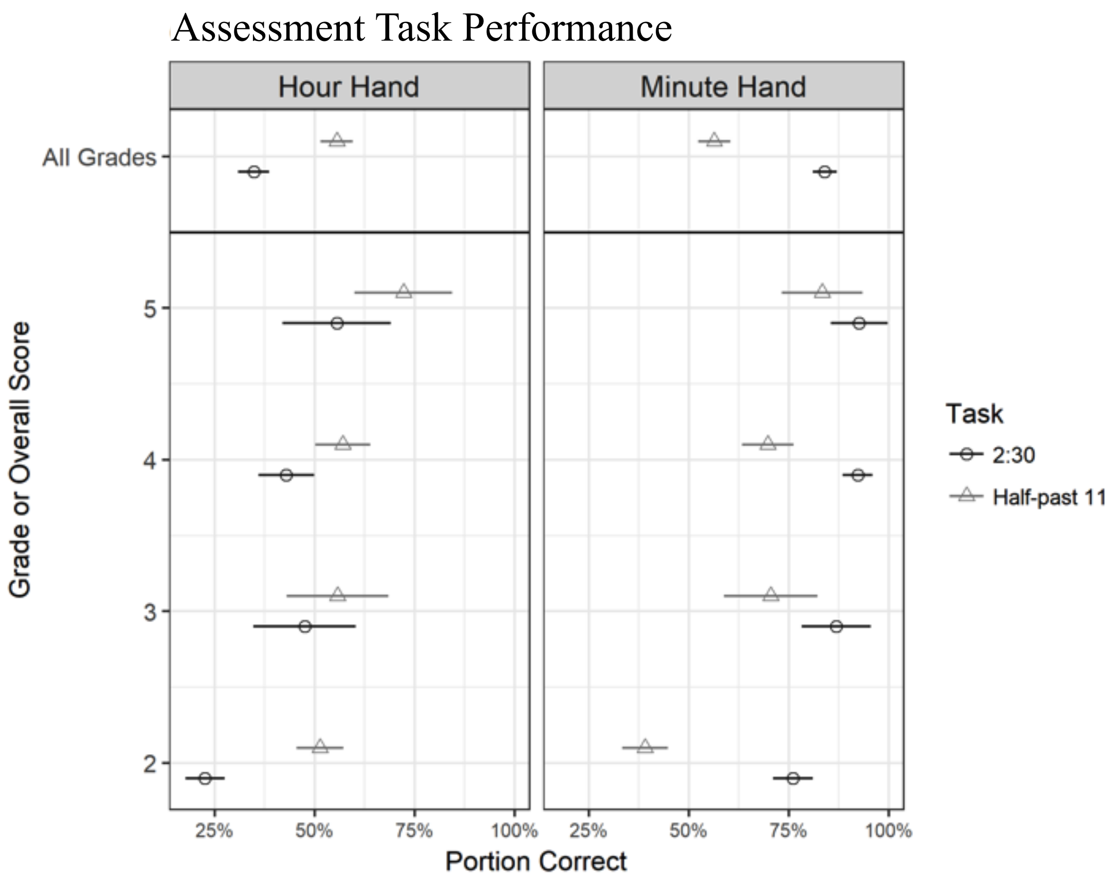

Figure 7: Performance for Hour (left) and Minute Hand (right) Placement for Each Task

Discussion

Time is a challenging topic in mathematics education due in part to its invisible character. Our mixed methods analysis indicates that expressions with substantively different treatments of time units lead to different interpretations of time. Each expression, both of which are appropriate ways to reference the time of day, encodes information about agreed-upon conventions related to unit. The times 4:30 or 2:30 are typically stated with whole number words that highlight whole number features of the clock and, in our data, were less likely to make salient clock features related to fractions and partitioning. Instead, the words are the same as those children have long used to reference whole numbers, a topic that almost assuredly has constituted the bulk of their elementary mathematics experiences thus far. Meanwhile, when presented with the expression half past *h*, many students drew upon part-whole relations, a narrative related to partitioning and measurement. This was evidenced even among the second graders on the assessment, among whom more than 50% positioned the hour hand for half past *h* accurately as compared to less than 25% for *h*:30 (see bottom left of Figure 7); of note is that both tasks are currently identified as grade 1 benchmarks (CCSSO, 2010), even though grade 2 students in our sample clearly struggled with these same ideas.

Why are these results relevant for mathematics education? Time pervades the mathematics of change in later grades as well as basic scientific investigations of the world yet, surprisingly, current time instruction highlights little more than clock-reading procedures in the early elementary grades. We attribute Whole Number narratives above as related to the over-emphasis on clock-reading procedures. Our findings contribute to emerging research with implications more broadly for a need to re-think K-12 instruction related to time. Part-whole Relations narratives—those that typically accompanied the "half past *h*" expression—well reflect measurement principles. Extrapolating these findings to later grades when students encounter other contexts involving representations of time, such as the *x*-axis of a function graph, we conjecture that a Part-whole Relations narrative for working with a clock will be generative and continue to support mathematical insights when working with such representations with structural similarities (see also, Earnest, 2015). Although further research is needed, we are less confident that Whole Number narratives well provide a foundation to support later mathematical ideas involving interval properties of time, potentially leaving students unprepared for complex mathematical ideas involving time.

A narrative is a story of relations or activities among objects (Sfard, 2008). As we found, the words for time expressions are interwoven with particular mathematical narratives that may or may not be consistent with the endorsed ideas about time and time representations. Our current instructional emphasis on clock-reading procedures may emphasize whole number properties of the clock at the expense of interval properties related to fractions and partitioning.

What should children learn about time, and when should they learn it? We do not interpret the findings above to indicate that one phrase ought to be used in curriculum and instruction absent of the other; in fact, such a claim would poorly reflect reality. Rather, our findings highlight the interplay of words and time ideas and underscore some of the complexity we as a field have been overlooking related to emerging conceptions of time. Findings suggest that we ought to be strategic and thoughtful about when and how we draw upon particular expressions and representational systems (i.e., digital and analog clock), how they may support children's age-appropriate learning of specific time-related ideas, and what the target content goals of doing so are. Further research is necessary to determine how such expressions could be leveraged to support interval interpretations of an analog clock.

Endnotes

Note that occasionally a solution approach was given more than one code, the result of which is that the cumulative percentage in Table 4 is greater than 100%.

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